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NONLINEAR TRACKING CONTROL FOR PREY STABILIZATION IN PREDATOR-PREY MODEL USING BACKSTEPPING

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ABSTRACT

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The common method used in population dynamics is optimal control, which employs Pontryagin's minimum principle. This method minimizes costs, with the constraint function being the model dynamics. Unfortunately, if the main objective of the control function is to modify the population's behavior to follow a specific pattern, this method is challenging to apply. This article introduces a control function to the predator-prey model for the tracking problem using the backstepping method. The control function drives the population from the initial value towards the given trajectory. The goal is to maintain the balance between predator and prey populations in the habitat, with the chosen trajectory being the equilibrium point. The application of backstepping to the predator-prey model is combined with input-output feedback linearization to obtain a normal form, enabling the implementation of backstepping. Simulation results show that the controller successfully drives the predator-prey populations toward the equilibrium point with a relatively small control function and excellent performance.

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1. INTRODUCTION

The interaction between predators and prey is one of ecosystems' most fundamental ecological dynamics, reflecting the complex reciprocal relationships between species. Predator-prey mathematical models such as the Lotka-Volterra model, developed by Alfred J. Lotka and Vito Volterra in the early 20th century, have long been used as an effective analytical tool to understand the population fluctuation patterns of these two types of organisms [1], [2]. This model illustrates typical population cycles and provides insights into ecosystem stability, the impact of environmental interventions, and the potential for chaos in population dynamics. By integrating biological parameters such as prey growth rates, predation rates, and predator energy efficiency, this model is an essential foundation for developing conservation strategies and sustainable natural resource management.

The predator-prey model has been widely studied and developed. Developments include the use of interaction schemes between populations, the application of fractional models, or even the use of partial differential equations to accommodate population movement. In [3], it discusses a predator-prey model in the form of difference equations. The research develops a Ricker-type predator-prey model by changing the step size to an arbitrary constant step size. In [4], the predator-prey model uses partial differential equations. The predation process, which involves hunting prey, is accommodated in the form of an equation that represents the movement of the prey, which simultaneously causes the movement of the predators. The movement of the predators is also assumed to be uneven due to competition among predators within the same population, leading to the spread of hunting areas depending on the competition abilities. [5] discusses the predator-prey model using a system of partial differential equations. The use of partial differential equations is intended to accommodate the movement of the predator population. This occurs because the prey population is not evenly distributed, requiring predators to gather in areas where sufficient prey is available. [6] uses fractional differential equations to discuss a predator-prey model with one predator and two prey species.

One of the prey species has food that also serves as additional food for the predator, allowing for competition to occur. In [7], [8] discuss predator-prey models with Holling Type II response and the presence of protection. In [7], protection is provided for the prey population, while in [8], protection is provided for the predator population, and cannibalism is added as a factor within the predator population. Cannibalism in the predator population can reduce the predator population and decrease the predation rate, as the predators' needs are partly met through cannibalistic behavior. Still using the Holling Type II response function, [9] modified the predator-prey model into a food chain model involving three populations. The model formed was also developed by involving competition between predators and predator harvesting as a form of control. [10] and [11] discuss predator-prey models involving food sharing. In [10], it is assumed that there is fear between predator populations due to their ability to secrete toxins. The prey in this model is food sharing because both predator populations live in the same environment. [11] discusses a time-dependent model for the predator-prey problem with food sharing and time budgeting. The developed model divides the predator population into two categories with different capabilities: adult predators, which are capable of hunting and reproducing, and young predators, which are not yet able to hunt or reproduce. This food sharing occurs between the adult and young predator populations, as the young population still depends on the adult population. In [12], a predator-prey model was developed with the presence of an infected population. The existence of an infected population makes the prey more susceptible to predation, necessitating a treatment scheme, either care or vaccination, to enhance immunity.

The predator-prey model is a system of nonlinear differential equations. Control design for nonlinear systems is still challenging and heavily depends on the specific model [13]. One method that is quite popular for nonlinear systems is the backstepping method. Backstepping is widespread in various problems, such as in [14] for electrical induction in photovoltaic generators. In [15], [16], [17], backstepping is used to design control for transportation systems, implemented in the problem of quadrotor unmanned aerial vehicles (UAVs). Backstepping can also be applied in the healthcare field, such as in brain tumor treatment, as done in [18]. In [19], backstepping was used to determine the control function for treating the spread of COVID-19, which occurred in early 2020, and was modeled using the SIRD model. In [20], backstepping was applied to control the spread of COVID-19 with a vaccination scheme to keep the population at a certain threshold level. Backstepping is also very effective in addressing systems that involve uncertainty. In [21], [22], [23], [24], the use of backstepping is specifically discussed for systems that involve disturbance functions or uncertain parameters, combined with the input-output feedback linearization method to transform the nonlinear system into normal form.

In this article, we discuss the predator-prey model controlled using nonlinear control. The nonlinear control used is the backstepping method. Applying the backstepping method to the predator-prey model is combined with input-output feedback linearization to form a strict feedback form. The goal of the control design in this article is to bring the system output from the initial value to a chosen trajectory. It is assumed that the control strategy is a conservation measure for the predator population, which typically has a smaller population size. It is also known that prey is the primary source of life for the predator population, so its availability needs to be monitored. Therefore, the control function is applied to the predator population, while the sensor for detecting the output is placed on the prey population.

The contribution of this article lies in the application of nonlinear control to the predator-prey model. The control typically used in population models is optimal, as seen in [12], [25], [26], [27], [28], [29], and [30]. Our article applies nonlinear control using the backstepping method. The optimal control used in population or epidemiological models, such as in [31], [32], [33], and [34], aims at the stabilization problem. This is less relevant to the predator-prey problem, as stabilizing a population, which essentially means making the population extinct, would lead to ecosystem damage. In this article, we apply control to conserve the population by bringing it to a specific point, representing a tracking problem.

This article is organized as follows. The second section presents the research methods, which include the predator-prey model, the input-output feedback linearization method, and the backstepping method. The discussion section presents the control design for both the stabilization problem and the tracking problem. Numerical simulations complement the discussion to compare the controlled system and examine the effect of parameter variations on the controller. The final section of the article provides a summary.

2. RESEARCH METHODS

2.1 Problem Formulation

The predator-prey model is given by:

$$\begin{cases} \dot{m}(t) &= -\alpha_1 m(t)p(t) + \beta m(t) \\ \dot{p}(t) &= \alpha_2 m(t)p(t) - \gamma p(t) \end{cases} \tag{1}$$

where $m(t)$[individual] is the prey population in the habitat at time t, $p(t)$[individual] is the predator population in the same habitat at time t, $\alpha_1[\text{individual}^{-1}\text{time}^{-1}]$, $\alpha_2[\text{individual}^{-1}\text{time}^{-1}]$ is the predation rate constant resulting from the interaction between predators and prey, $\beta[\text{time}^{-1}]$ is the prey growth rate constant in the absence of predators, and $\gamma[\text{time}^{-1}]$ is the predator mortality rate constant in the absence of prey. All variables and parameters are assumed to be positive.

The equilibrium points of the predator-prey model are $(m^* = 0, p^* = 0)$ and $\left(m^* = \frac{\gamma}{\alpha_2}, p^* = \frac{\beta}{\alpha_1}\right)$. The stability of the model can be analyzed locally using the Jacobian matrix. The Jacobian matrix of Equation (1) is:

$$\mathbf{A} = \begin{bmatrix} -\alpha_1 p(t) + \beta & -\alpha_1 m(t) \\ \alpha_2 p(t) & \alpha_2 m(t) - \gamma \end{bmatrix} \tag{2}$$

Substitute equilibrium $(m^* = 0, p^* = 0)$ into (2) to obtain the Jacobian matrix for the first equilibrium point, $\mathbf{A}_1 = \begin{bmatrix} \beta & 0 \\ 0 & -\gamma \end{bmatrix}$, with eigenvalues $\lambda = \{\beta, -\gamma\}$. This means that the equilibrium point $(m^* = 0, p^* = 0)$ is an unstable equilibrium point. The Jacobian matrix for the second equilibrium point is $\mathbf{A}_2 = \begin{bmatrix} 0 & -\gamma \\ \beta & 0 \end{bmatrix}$, with eigenvalues $\lambda = \pm\sqrt{\gamma\beta}I$. The second equilibrium point is stable equilibrium.

Control action refers to the intervention applied to the population. In this article, the control function is applied to the predator population, as it is generally the predator population that is more vulnerable and scarcer compared to the prey population. Thus, the control function here serves as a conservative measure for the predator population. Since the only resource relied upon for the growth of the predator population is the adequate supply of the prey population, the sensor for this problem is placed on the prey population. Therefore, Equation (1) is subsequently written as

$$\begin{cases} \dot{m}(t) &= -\alpha_1 m(t)p(t) + \beta m(t) \\ \dot{p}(t) &= \alpha_2 m(t)p(t) - \gamma p(t) + u(t) \\ y(t) &= m(t) \end{cases} \tag{3}$$

where $y(t) \in \mathbb{R}$ is the measurable output of the model, and $u(t) \in \mathbb{R}$ is the control function. In this article, the goal of the control $u(t)$ is to maintain the predator-prey populations stable. The target stability condition for the control is population conservation, so the desired control trajectory is the second equilibrium point $\left(m^* = \frac{\gamma}{\alpha_2}, p^* = \frac{\beta}{\alpha_1}\right)$.

2.2 Input-Output Feedback Linearization

Given a single-input, single-output (SISO) nonlinear control system:

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t)) + \mathbf{g}(\mathbf{x}(t))u(t) \\ y(t) &= h(\mathbf{x}(t)) \end{cases} \tag{4}$$

Nonlinear control systems remain a challenging problem to solve. One proposed solution is to transform the system into a linear system. Input-output feedback linearization (IOFL) can convert a nonlinear control system into a linear one. Unlike linearization using the Jacobian, IOFL is a coordinate transformation, meaning it does not have the limitations of Jacobian linearization, which depends on the linearization point. The coordinate transformation is performed by modifying the system while considering the relative degrees of the system.

Definition 1. *The relative degree, denoted by* $1 \le \rho \le n$, *is a natural number that represents the number of derivatives required on the system output* Equation (4) *until the control function* $u(t)$ *appears explicitly.* This is expressed as

$$\begin{cases} L_{\mathbf{g}} L_{\mathbf{f}}^{k-1} h(\mathbf{x}) = 0 \quad , \quad k = 1,2,\dots,\rho - 1 \\ L_{\mathbf{g}} L_{\mathbf{f}}^{\rho-1} h(\mathbf{x}) \neq 0 \quad , \end{cases} \tag{5}$$

$L_{\mathbf{f}} h(\mathbf{x})$ in Equation (5) is Lie derivative defined as $L_{\mathbf{f}} h(\mathbf{x}) = \nabla h \, \mathbf{f}(\mathbf{x})$, and $\nabla h = \left[\frac{\partial h}{\partial x_1} \quad \frac{\partial h}{\partial x_2} \quad \cdots \quad \frac{\partial h}{\partial x_n}\right]$.

This article is limited to the case $\rho = n$. The coordinate transformation for the nonlinear control system $\mathbf{z} = T(\mathbf{x})$ is expressed as

$$z_i = L_{\mathbf{f}}^{i-1} h(\mathbf{x}), i = 1,2,\dots,n \tag{6}$$

The derivative of $z_i(t)$ in Equation (6) with respect to t results in

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \boldsymbol{B}v(t) \tag{7}$$

where $\mathbf{z} = \langle z_1, z_2, \dots, z_n \rangle$ and $v(t) = L_{\mathbf{f}}^n h(\mathbf{x}) + L_{\mathbf{g}} L_{\mathbf{f}}^{n-1} h(\mathbf{x}) u(t)$. The control function $v(t)$ in Equation (7) is the control function applicable to the transformed system $\mathbf{z}(t)$, which can be designed using linear control theory. The relation between $u(t)$ and $v(t)$ is given by

$$u(t) = \frac{v(t) - L_{\mathbf{f}}^n h(\mathbf{x})}{L_{\mathbf{g}} L_{\mathbf{f}}^{n-1} h(\mathbf{x})} \tag{8}$$

The control function $u(t)$ in Equation (8) will then be used in nonlinear control Equation (4) and the predator-prey model in Equation (3).

2.2 Backstepping

Backstepping is a control design method based on Lyapunov stability, achieved through an integrator process.

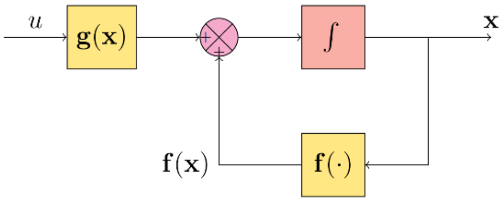


Figure 1. The Block Diagram of the Control System that Stabilizes the Nonlinear System Equation (4)

Assume there exists a control $\varphi(\mathbf{x})$ that stabilizes Equation (4) and a Lyapunov function that satisfies:

$$\dot{V}(\mathbf{x}) = \frac{\partial V}{\partial \mathbf{x}}[\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\varphi(\mathbf{x})] \leq -W(\mathbf{x}) < 0 \tag{9}$$

where $W(\mathbf{x})$ is a continuous and positive function.

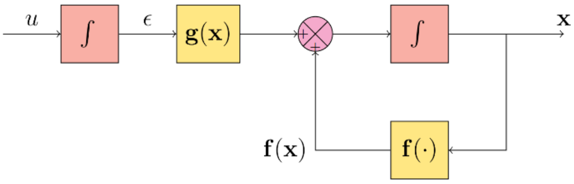


Figure 2. The Block Diagram of the Control System with an Integrator in the Backstepping Method for the Nonlinear System Equation (4)

The nonlinear control system Equation (4) can be extended with an integrator, as shown in Figure 2, resulting in:

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}\big(\mathbf{x}(t)\big) + \mathbf{g}\big(\mathbf{x}(t)\big)\,\varepsilon(t) \\ \dot{\varepsilon}(t) & = & u(t) \end{cases} \tag{10}$$

$\varepsilon(t)$ in the first equation in Equation (10) represents the control function, while it is a state variable in the second equation. We have assumed that Equation (10) is stabilized by $\varphi(\mathbf{x})$, so we express $\varepsilon(t)$ as a virtual control function.

Define the difference between the virtual control and the control $\varphi(\mathbf{x})$ as:

$$w(t) = \varepsilon(t) - \varphi(\mathbf{x}) \tag{11}$$

Differentiate Equation (11) and modify the system Equation (10) in terms of the variable $w(t)$ to obtain

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\,\varphi(\mathbf{x}) + \mathbf{g}(\mathbf{x})[\varepsilon(t) - \varphi(\mathbf{x})] \\ \dot{w}(t) & = & u(t) - \dot{\varphi}(\mathbf{x}) \end{cases} \tag{12}$$

The term $\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\,\varphi(\mathbf{x})$ in Equation (12) has been stabilized by the control $\varphi(\mathbf{x})$. Define the Lyapunov function as:

$$V(\mathbf{x}, w) = V(\mathbf{x}) + \frac{1}{2}w(t)^2 \tag{13}$$

The derivative of Equation (13) with respect to t and using Equation (9) results in:

$$\dot{V}(\mathbf{x}, w) = \dot{V}(\mathbf{x}) + w\dot{w} \leq -W(\mathbf{x}) + w\left[\frac{\partial V}{\partial \mathbf{x}}\mathbf{g}(\mathbf{x}) + u(t) - \frac{\partial \varphi}{\partial \mathbf{x}}(\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\varphi(\mathbf{x}) + \mathbf{g}(\mathbf{x})w)\right] \tag{14}$$

The control $u(t)$ that stabilizes the system is the one that ensures $\dot{V}(\mathbf{x}, w) < 0$, which can be chosen as:

$$u(t) = -r_w w(t) - \frac{\partial V}{\partial \mathbf{x}}\mathbf{g}(\mathbf{x}) + \frac{\partial \varphi}{\partial \mathbf{x}}[\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\varphi(\mathbf{x}) + \mathbf{g}(\mathbf{x})w(t)] \tag{15}$$

with $r_w \in \mathbb{R}^+$ and the Lyapunov function **Equation (14)** then becomes:

$$\dot{V} \leq -W(\mathbf{x}) - r_w w^2(t) < 0 \tag{16}$$

Equation (16) indicates that control $u(t)$ in **Equation (15)** stabilizes the system in **Equation (10)** asymptotically.

3. RESULTS AND DISCUSSION

The discussion begins by transforming the predator-prey model into its normal form, specifically the strict feedback form. This form is ideal for applying the backstepping method. Subsequently, control designs are presented to solve the stabilization and tracking problems. Illustrations are then provided to demonstrate the control's performance.

3.1 The Normal Form of a Predator-Prey Model with Control in Prey and Output in Prey

Before determining the control function, we must transform the predator-prey model into a normal form because backstepping requires a strict feedback form. The process flow in the control design is shown in **Figure 3**. The predator-prey model is transformed into a normal form, a linear control system that forms a strict feedback form. The transformation to the normal form is a linearization transformation. This linearization process cannot use Jacobian but must use IOFL because we want to control the system output to follow a given trajectory in the system we are studying. Using the normal form, the control function is designed using backstepping. The obtained control function is then applied to the predator-prey model utilizing the inverse of the IOFL transformation.

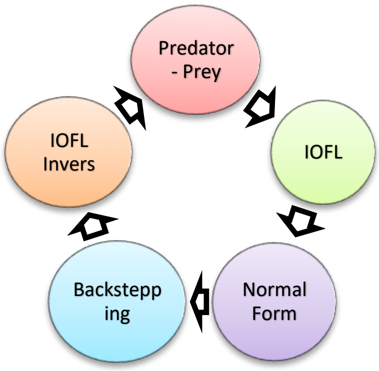


Figure 3. Diagram of Control Design using Backstepping on Predator-Prey

Theorem 1. *Consider the system in* **Equation (3)**. *Using the output* $y(t) = m(t)$, **Equation (3)** *can be exactly linearized.*

Proof. Consider **Equation (3)**. Define $\mathbf{x} = \langle m, p \rangle$. Since $y(t) = m(t)$, the output can be rewritten as $y(t) = \mathbf{hx}(t)$, where $\mathbf{h}^T = \langle 1, 0 \rangle$. **Equation (3)** can be expressed in the form of a nonlinear control system as $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u(t)$ with

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} -\alpha_1\, m(t)p(t) + \beta\, m(t) \\ \alpha_2\, m(t)p(t) - \gamma\, p(t) \end{pmatrix}, \mathbf{g}(\mathbf{x}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{17}$$

The first derivative of $y(t)$ with respect to t and using **Equation (17)** result in:

$$\dot{y}(t) = \frac{\partial y}{\partial \mathbf{x}}\dot{\mathbf{x}} = \mathbf{h}\dot{\mathbf{x}} = -\alpha_1 m(t)p(t) + \beta m(t) \tag{18}$$

The second derivative of $y(t)$ using Equation (18) produce

$$\frac{d\dot{y}}{dt} = \frac{\partial \dot{y}}{\partial \mathbf{x}}\dot{\mathbf{x}} = -\alpha_1 m(t)u(t) + m(t)[-\alpha_1\alpha_2 m(t)p(t) + \alpha_1^2 p^2(t) - 2\alpha_1\beta p(t) + \alpha_1\gamma p(t) + \beta^2] \tag{19}$$

Since the control $u(t)$ appears in the second derivative of the output $y(t)$ in Equation (19), the relative degree of Equation (3) with the output $y(t) = m(t)$ is $\rho = 2$. Based on [35], the system can be exactly linearized. ∎

Corollary 1. *Consider Equation (3) with the output $y(t) = m(t)$ having a relative degree of $\rho = 2$. A change of variables is defined as $\mathbf{z} = T(\boldsymbol{x})$, where:*

$$\begin{cases} z_1(t) &= h(\mathbf{x}) \\ z_2(t) &= L_{\mathbf{f}}h(\mathbf{x}) \end{cases} \tag{20}$$

The derivative of Equation (20) with respect to t yields

$$\begin{cases} \dot{z}_1(t) &= z_2(t) \\ \dot{z}_2(t) &= v(t) \end{cases} \tag{21}$$

where $v(t)$ is the control function in the transformed system, expressed as:

$$v(t) = -\alpha_1 m(t)u(t) + m(t)[-\alpha_1\alpha_2 m(t)p(t) + \alpha_1^2 p^2(t) - 2\alpha_1\beta p(t) + \alpha_1\gamma p(t) + \beta^2] \tag{22}$$

Control $u(t)$ using the predator-prey model in Equation (3) is obtained from Equation (22) using the relation between $u(t)$ and $v(t)$ in (8).

Theorem 2. *Consider Equation (3) with the output $y(t) = m(t)$, which is transformed into Equation (21). Equation (21) is a system that can be controlled using $v(t)$.*

Proof. Equation (21) is expressed in the form $\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{b}v(t)$, with $\mathbf{z} = \langle z_1, z_2\rangle$, and

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{23}$$

The controllability matrix $\mathbf{M}$ for a second-order linear system is $\mathbf{M} = [\mathbf{b} \quad \mathbf{Ab}]$. Using the values of $\mathbf{A}$ and $\mathbf{B}$ from Equation (23), it is obtained that $\mathbf{M} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, which has $\text{rank}(\mathbf{M}) = 2$. Therefore, the system Equation (21) is controllable. ∎

3.2 Control Design for Stabilization Problem

Consider Equation (21) The block diagram illustrating the application of the integrator backstepping method for the stabilization problem is shown in Figure 4.

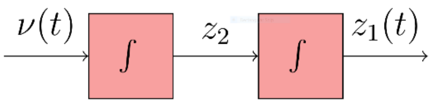


Figure 4. The Block Diagram of the Backstepping Method for Stabilizing Equation (21)

Assumption 1. There exists a control $\varphi(z_1)$ that stabilizes $z_1(t) = \varphi(z_1)$, and there exists a Lyapunov function $V(z_1)$ that satisfies

$$\dot{V}(z_1) = \frac{dV(z_1)}{dz_1}\varphi(z_1) \leq -W(z_1) < 0 \tag{24}$$

with $W(z_1)$ being a positive, continuous, and bounded function.

Theorem 3. *Given* **Equation (21)** *that satisfies Assumption* 1, *the control* $v(t)$ *that stabilizes* **Equation (21)** *using backstepping is*

$$v(t) = -z_1(t)(1 + r_1 r_2) - z_2(t)(r_1 + r_2) \tag{25}$$

where $r_1, r_2 \in \mathbb{R}^+$ are the backstepping control parameters.

Proof. Let $\varphi(z_1)$ be a feedback control that stabilizes $z_1(t) = \varphi(z_1)$. Here, we can choose $\varphi(z_1) = -r_1 z_1(t)$ with $r_1 \in \mathbb{R}^+$. The candidate Lyapunov function for $z_1(t)$ is defined as

$$V(z_1) = \frac{1}{2} z_1(t)^2 \tag{26}$$

The derivative of **Equation (26)** with respect to t, using the control $\varphi(z_1)$, results in

$$\dot{V}(z_1) = z_1(t)\dot{z}_1(t) = -r_1 z_1^2 < 0 \tag{27}$$

Based on **Equation (27)**, we can conclude that $W(z_1) = r_1 z_1^2$, which is a positive function. The diagram in **Figure 4** shows that $z_2(t)$ is the control function for $\dot{z}_1(t)$, which is the result of the integrator. Therefore, $z_2(t)$ is a virtual control that approximates $\varphi(z_1)$, so the difference between $\varphi(z_1)$ and the virtual control $z_2(t)$ can be defined as

$$w(t) = z_2(t) - \varphi(z_1) = z_2(t) + r_1 z_1(t) \tag{28}$$

Using the value of $w(t)$ in the dynamics of z_1 to obtain

$$\dot{z}_1(t) = w(t) - r_1 z_1(t) \tag{29}$$

The derivative of $w(t)$ in **Equation (28)** with respect to t, using **Equation (21)** and **Equation (29)**, will result in

$$\dot{w}(t) = v(t) + r_1 w(t) - r_1^2 z_1(t) \tag{30}$$

The second step is stabilizing $w(t)$ using the control $v(t)$. Choose a candidate Lyapunov function to determine the stability of $w(t)$, which combines with the Lyapunov function for $z_1(t)$, given by

$$V(w, z_1) = V(z_1) + \frac{1}{2} w(t)^2 \tag{31}$$

The derivative of **Equation (31)** with respect to t results in

$$\dot{V}(w, z_1) = \frac{\partial V(z_1)}{\partial z_1} \varphi(z_1) + w(t) \left[\frac{\partial V(z_1)}{\partial z_1} + \dot{w}(t) \right] \tag{32}$$

Use Assumption 1 and the values in **Equation (24)** and **Equation (30)** so that **Equation (32)** becomes

$$\dot{V}(w, z_1) \leq -W(z_1) + w(t)[v(t) + z_1(t) + r_1 w(t) - r_1^2 z_1(t)] \tag{33}$$

Using the control function $v(t)$ in **Equation (25)**, then **Equation (33)** becomes

$$\dot{V}(w, z_1) \leq -W(z_1) - r_2 w(t)^2 \tag{34}$$

Since $W(z_1)$ is a positive function and $r_2 w(t)^2 > 0$, we have **Equation (34)** satisfy $\dot{V}(w, z_1) < 0$, which implies that the system is asymptotically stable. ■

3.3 Control Design for Tracking Output to The Equilibrium

Theorem 4. *Given* **Equation (21)** *that satisfies Assumption* 1, *the control* $v(t)$ *that drives the output* $y(t) = m(t)$ *to follow the smooth trajectory* $y_d(t)$ *using backstepping is*

$$v(t) = \ddot{y}_d(t) + (r_1^2 - 1)e_1(t) - (r_1 + r_2)e_2(t) \tag{35}$$

with $e_1(t) = z_1(t) - y_d(t), e_2(t) = z_2(t) - \dot{y}_d(t) + r_1 e_1(t)$, and $r_1, r_2 \in \mathbb{R}^+$ are the control parameters.

Proof. The difference between the system output and the trajectory $y_d(t)$ is

$$e_1(t) = y(t) - y_d(t) \tag{36}$$

The derivative of **Equation (36)** with respect to t is

$$\dot{e}_1(t) = z_2(t) - \dot{y}_d(t) \tag{37}$$

Define the candidate Lyapunov function

$$V(e_1) = \frac{1}{2} e_1(t)^2 \tag{38}$$

The derivative of **Equation (38)** with respect to t dan using **Equation (37)** will obtain

$$\dot{V}(e_1) = e_1(t)[z_2(t) - \dot{y}_d(t)] \tag{39}$$

For $e_1(t)$ to be asymptotically stable, **Equation (39)** must satisfy $\dot{V}(e_1) < 0$, which is fulfilled by assuming $\dot{V}(e_1) = -r_1 e_1(t)^2$, leading to

$$z_2(t) - \dot{y}_d(t) = -r_1 e_1(t) \tag{40}$$

A new state variable is defined from **Equation (40)** as

$$e_2(t) = z_2(t) - \dot{y}_d(t) + r_1 e_1(t) \tag{41}$$

The derivative of **Equation (41)** with respect to t, using **Equation (21)** and **Equation (37)**, results in

$$\dot{e}_2(t) = v(t) - \ddot{y}_d(t) + r_1 e_2(t) - r_1^2 e_1(t) \tag{42}$$

The next step is stabilizing $e_2(t)$ using the control $v(t)$. Choose a candidate Lyapunov function that involves the Lyapunov function from the previous process in the form of

$$V(e_2, e_1) = V(e_1) + \frac{1}{2} e_2(t)^2 \tag{43}$$

Take the derivative of **Equation (43)** with respect to t, using **Equation (39)** and **Equation (42)**, then apply it together with **Equation (41)** to **Equation (39)**, resulting in

$$\dot{V}(e_2, e_1) = -r_1 e_1(t)^2 + e_2(t)[e_1(t) + v(t) - y_d(t) + r_1 e_2(t) - r_1^2 e_1(t)] \tag{44}$$

Using the control $v(t)$ in **Equation (35)** and applying it to **Equation (44)**, we obtain

$$\dot{V}(e_2, e_1) = -r_1 e_1(t)^2 - r_2 e_2(t)^2 < 0 \tag{45}$$

which results in the system being asymptotically stable. ■

Corollary 2. *It has been obtained that using the control $v(t)$, the system can be asymptotically stabilized, and a Lyapunov function is obtained that satisfies*

$$\dot{V}(e_2, e_1) = -r_1 e_1(t)^2 - r_2 e_2(t)^2 \tag{46}$$

From **Equation (45)**, since $V(e_2, e_1) \leq -W(\mathbf{e})$, where $W(\mathbf{e}) = r_1 e_1(t)^2 + r_2 e_2(t)^2$ is a positive and continuous function, based on the LaSalle-Yoshizawa theorem **[22]**, it can be concluded that

$$\lim_{t\to\infty} W(\mathbf{e}) = 0 \tag{47}$$

$W(\mathbf{e}) = e_1(t)^2 + e_2(t)^2 = 0$ will be satisfied only by $e_1(t) = e_2(t) = 0$. Since $e_1(t) = y(t) - y_d(t)$, it can be concluded that the tracking error goes to zero, resulting in perfect tracking. As a consequence of the LaSalle-Yoshizawa theorem, it can be concluded that the predator-prey model **Equation (3)** controlled using backstepping **Equation (35)** is uniform globally asymptotically stable **[36]**.

3.4 Numerical Simulation

Table 1 shows the data required for the simulation. The simulation is performed using a fourth-order Runge-Kutta scheme. The numerical simulation is carried out with several scenarios, including comparing the predator-prey dynamics without control and with controlled dynamics, comparing the predator-prey dynamics and control performance for various control parameter variations, and testing the relationship between control parameters and control performance. We also compare our proposed method with the optimum control method to bring the system to its equilibrium point.

Table 1. Data Parameter for 5

No	Parameter	Value	Description
1	α_1	0.2	The rate at which prey decrease in population due to predation of predator
2	α_2	0.1	The rate at which predators increase in population due to the consumption of prey
3	β	0.6	The natural growth rate of the prey population in the absence of predators
4	γ	0.3	The natural death rate of the predator population in the absence of prey
5	r_1	0.5,5,7	Parameter controls for backstepping
6	r_2	0.1,3,15	Parameter controls for backstepping
7	$m(0)$	3	The initial value for prey
8	$p(0)$	5	The initial value for predator
9	t	[0,20]	Time domain for simulation
10	$y_d(t)$	x_e	Trajectory for tracking
11	h	$1e-3$	Time step size

The first simulation compares the predator-prey model's dynamics without control and with control using backstepping. The values of the backstepping control parameters used are $\{r_1 = 0.5, r_2 = 0.1\}$. The trajectory to be followed by the system output is the equilibrium point, which is

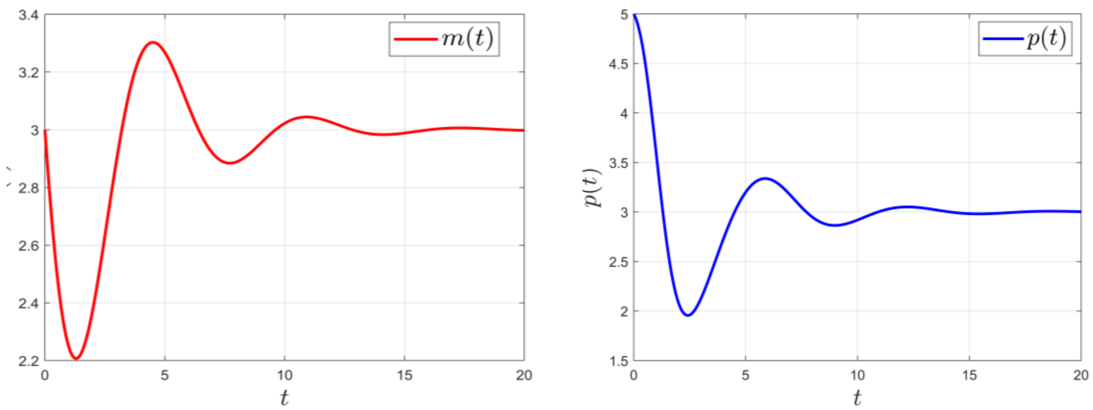
$$\mathbf{x}_e = (m^* = 3, p^* = 3) \tag{48}$$

Using $\{r_1 = 0.5, r_2 = 0.1\}$ from **Table 1**, the control function that drives the output to the equilibrium point in **Equation (48)** is

$$v(t) = (r_1^2 - 1)e_1(t) - (r_1 + r_2)e_2(t) \tag{49}$$

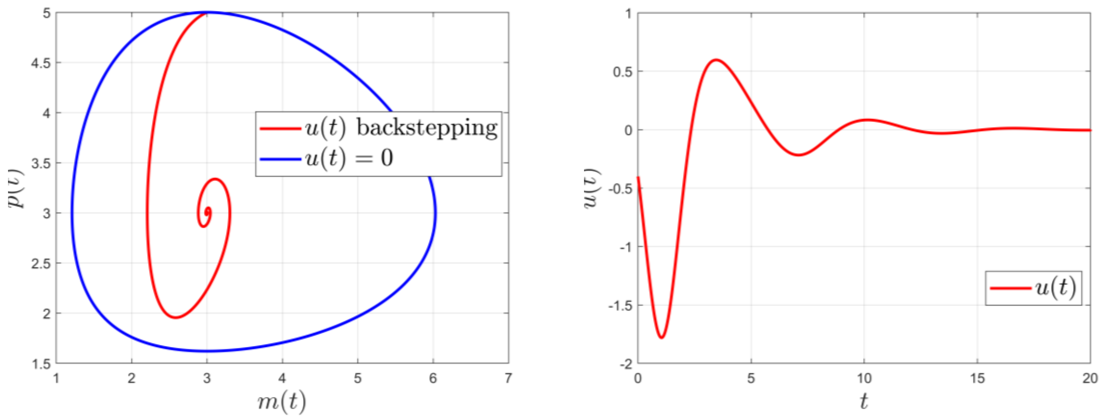
Figure 5 illustrates the dynamics of the predator and prey populations. It moves the controlled predator and prey populations from the initial values toward the equilibrium point. Oscillations occur between the initial point and the target point, which is possible because the control parameters used are relatively small. **Figure 6** shows a comparison between the controlled and uncontrolled populations and the control profile required to drive the population to the equilibrium point. The controlled population moves in a centered spiral pattern, consistent with the simulation results shown in Figure 5. Figure 6 indicates that the control profile needed to stabilize the population at the equilibrium point is relatively small. Negative values in the control profile represent actions to reduce the predator population. This can be achieved through harvesting, hunting, or relocating individuals from the observed habitat. Positive values in the control profile indicate the need to increase the predator population in the observed habitat. This can be accomplished by releasing additional individuals into the wild or implementing artificial breeding programs.

Controlling predator populations can cause problems if it is related to the status of the predator population, especially in animals that are vulnerable to extinction. However, suppose the predator population is not controlled. In that case, the large number that is not comparable to the availability of food sources has the potential to disrupt the growth of the population itself. The growth rate of the predator population is influenced by the value of α_2, which indicates the success of predation and is converted into new population growth. The number of predator populations that are too abundant will reduce the value of α_2 due to high competition. It needs to be added, as in [8] and [30], is the presence of a cannibalistic predator population. An abundant population, few food sources, and a cannibalistic population are a good combination for damaging the population as a whole. Therefore, direct control of the predator population is one strategy that can be taken to maintain the population while still paying attention to the methods used. In [37], it is explained that one of the methods in population management is to relocate the population to another area. Relocating to an area that provides better life support for several types of animals has positive results in improving the condition of the population that are vulnerable to extinction.



(a) (b)

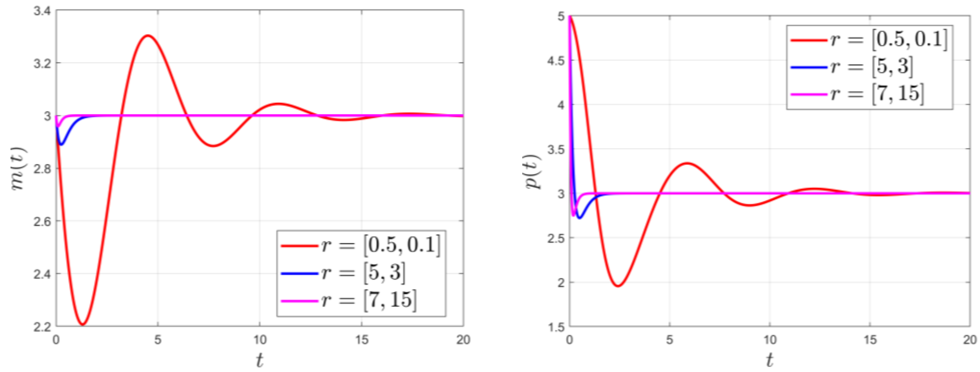
Figure 5. The Controlled Population Dynamics: (a) Prey Population, (b) Predator Population.



(a) (b)

Figure 6. The Dynamics of the Population Comparison Between Uncontrolled and Controlled Scenarios, along with the Control Profile: (a) Comparison of Dynamics, (b) Control Profile (49)

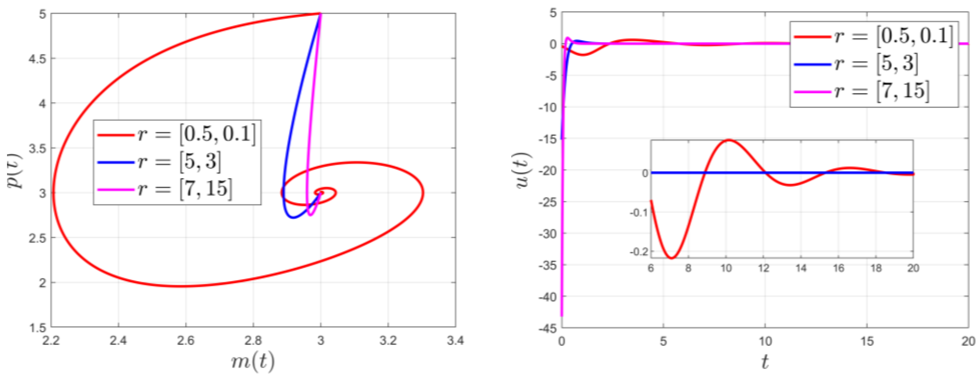
The following simulation examines the effect of control parameters on the stability rate in the controlled system. For this purpose, several variations are used: $\{(r_1 = 0.5, r_2 = 0.1), (r_1 = 5, r_2 = 3), (r_1 = 7, r_2 = 15)\}$. Figure 7 shows the dynamics of prey and predator populations for variations in the backstepping control parameters. It can be observed that the backstepping control parameters affect the stability rate and the stability's dynamic nature. Small control parameters result in oscillations in the population during its journey toward the equilibrium point. Meanwhile, increasing the values of the backstepping control parameters transforms the stability into exponential stability. It is also evident from Figure 8 that more extensive control parameters lead to faster stabilization of the population but has a significant spike in the predator population as a consequence.



(a) (b)

Figure 7. The Effect of Variations in Backstepping Control Parameters on the Controlled System: (a) Prey Population, (b) Predator Population

Figure 8 compares control profiles and population dynamics on a parametric curve due to backstepping control parameter value variations. Smaller parameters allow the population to oscillate, while more extensive control parameters force the population to move toward the desired equilibrium point. Regarding the control profile, the third variation of the backstepping control parameter exhibits a very high spike at the beginning of the simulation. Compared to the second variation, the control profile of the third variation converges earlier. This indicates that the population under the third variation achieves stability earlier than under the first and second variations.



(a) (b)

Figure 8. The Impact of Variations in the Backstepping Control Parameter on: (a) Population Dynamics, and (b) Control Profiles.

It is important to note that the second and third variations differ in their values and the relationship between the parameters. In the second variation, the control parameters satisfy $r_1 > r_2$, while in the third variation, $r_1 < r_2$. The next step is to examine the relationship between the backstepping control parameters $r_1, r_2,$ and control performance. Based on Theorem 4, the entire system can be asymptotically stabilized to equilibrium. Thus, we can randomly select values for r_1 and r_2 then compare them using a criterion that evaluates control performance. In this study, we use the integral absolute error, expressed as

$$\text{IAE} = \int_{t_0}^{t_f} |y_d(t) - y(t)|\, dt$$

with t_0 and t_f representing the initial and final observation times, respectively; these times do not necessarily coincide with the simulation duration because the system output may require additional time to reach the specified trajectory. Referring to Figure 7 (a), we can choose the starting point for the output to follow the trajectory at $t_0 = 1$. The final observation time is set equal to the simulation duration, $t_f = 20$. In this simulation, we use $\alpha_1 = \alpha_2 = 0.2$ and the rest use **Table 1**. The selected parameter controller variations are $r_1 \in [1, 10]$ and $r_2 \in [1, 10]$, partitioned into 37 points. Consequently, there are 1369 combinations of (r_1, r_2) and their corresponding impacts on the IAE (Integral Absolute Error) values. Control performance improves as the IAE value decreases, indicating that the output closely follows the trajectory. The simulation results, showing the IAE values for each combination of r_1 and r_2, are presented in **Figure 9**.

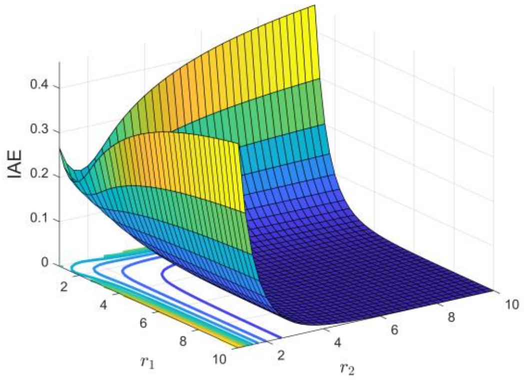


Figure 9. The 3D Curve of IAE Values Against Variations in $r_1 = [1, 10]$ and $r_2 = [1, 10]$, Partitioned into 1369 Points

Figure 9 shows that the IAE values fluctuate when the combinations of r_1 or r_2 are pretty extreme. Meanwhile, when both values are relatively small, the IAE values are pretty high, whereas the IAE values are very low when r_1 and r_2 are combined at more significant values. To examine the relationship between r_1, r_2, and IAE values, a Spearman correlation test was conducted, with the results displayed in **Table 2**. Furthermore, the influence weights of r_1 and r_2 on IAE variations are presented as a linear function, calculated using multiple linear regression, with the results also shown in **Table 2**. The correlation test results indicate a strong relationship between the increase in the control parameters r_1 and r_2 and control performance. An increase in r_1 and r_2 leads to decreased IAE values, signifying improved control performance. Unfortunately, the influence weight of the changes in control parameters is relatively small, at only 0.01507.

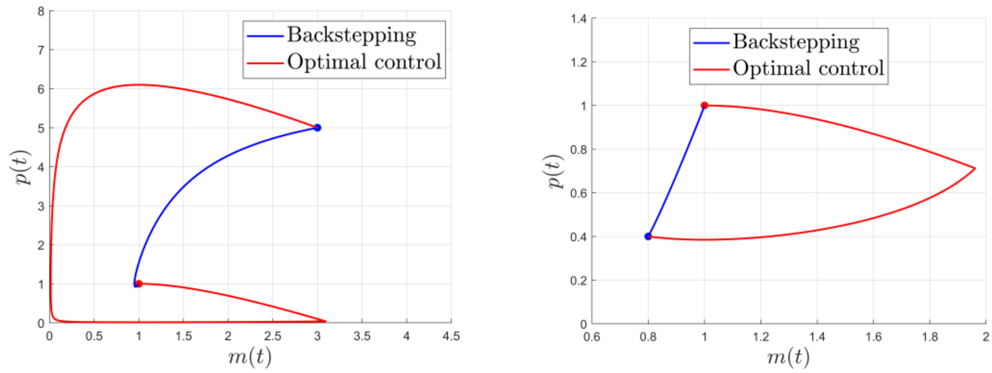
Table 2. Correlation and Linear Regression Results

Parameter	Correlation	Regression
r_1	-0.61058	-0.01507
r_2	-0.61057	-0.01507
Intercept		0.23215

The last simulation compares our proposed method with the optimum control given in **[27]**. Although the control context given is different from the model we studied, both models control the population to its equilibrium point. In optimum control, a strategy is used in the form of using an exchange point that separates the uncontrolled system and the fully controlled system. For the simulation parameters, we follow the parameter values given in **[27]**, namely $\{\alpha_1 = 1, \alpha_2 = 1, \beta = 1, \gamma = 1, b_1 = 0.5\}$. For the backstepping control, the parameters used are $\{r_1 = 3, r_2 = 5\}$. The simulation was carried out with two different initial values : $\{m_0 = 3, p_0 = 5\}$ and $\{m_0 = 0.8, p_0 = 4\}$. **Figure 10** shows the simulation results in the form of a comparison of predator-prey behavior with two different initial values and **Figure 11** shows the control profile used to bring the predator-prey to move towards its equilibrium point.

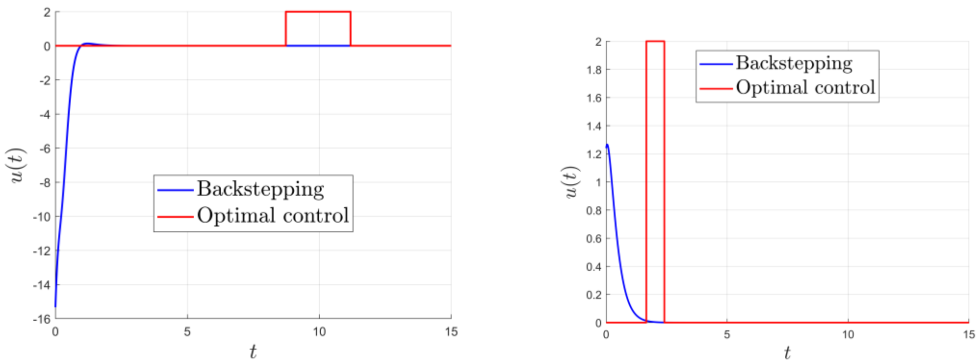
The simulation results show that both control methods successfully bring the predator-prey to its equilibrium point. However, the system's behavior controlled using backstepping tends to be more direct towards the trajectory, while the system controlled using optimal control moves along the trajectory until the exchange point. The control profile used to bring the system to the equilibrium point also has very different behavior. In the optimal control method, the system is not controlled at the beginning of the simulation because the system has not reached the exchange point; then, the system is fully controlled. Meanwhile, the system is controlled using the backstepping method; the control function works from the beginning so that the result of the system moves directly to the intended point, and the control stops when the goal has been achieved.

Figure 11shows that backstepping control can work directly at the beginning of the simulation so that the population can be controlled directly from the start. However, Figure 11 (a) shows that backstepping control requires a more significant control value than optimum control. From a real-life perspective, this is often irrelevant. The control function as an external action to modify system behavior requires resources and costs that are generally limited. Large control in backstepping results in a fast stability rate, but in real-world applications, this is difficult to do because of resource limitations. Therefore, in the context of real-world applications, it is necessary to adjust the control parameters to suit the existing resources, or it is necessary to develop a control design in the form of a combination with an optimum control method to provide control value limits to suit real-life conditions.



(a) (b)

Figure 10. Comparison of Controlled System using Optimal Control and Backstepping: (a) $\{m_0 = 3, p_0 = 5\}$, and (b) $\{m_0 = 0.8, p_0 = 0.4\}$. Red Circle is Equilibrium Point and Blue Circle is Initial Point



(a) (b)

Figure 11. Comparison of Optimal Control and Backstepping in Order to Drive State into the Equilibrium Point: (a) $\{m_0 = 3, p_0 = 5\}$, and (b) $\{m_0 = 0.8, p_0 = 0.4\}$.

4. CONCLUSIONS

This article discusses the application of nonlinear control using the backstepping method in a predator-prey model. The controlled population consists of predators, generally more vulnerable to extinction. Observation sensors are focused on the prey population to ensure their numbers remain sufficient to meet the predators' needs. The technical design of the backstepping control is combined with input-output feedback linearization to form a strict feedback form. Input-output feedback linearization is used because an output system needs to be controlled to follow a particular trajectory. Using input-output feedback linearization will transform the system with output into a regular dynamical, linear control system. It has also been demonstrated that the control design can drive the prey population from its initial value to the equilibrium point. Computational results show that both populations can be successfully brought from their initial values to the equilibrium point with a constrained control profile. Correlation and regression tests to examine the relationship between control parameters and performance, evaluated using the Integral Absolute Error (IAE), indicate that increasing the control parameter values leads to improved control performance.

The research in this article still focuses on the use of synthetic data. We have not validated the simulation results produced using real data. In addition, the model used in this article is the most basic predator-prey model, so it is still possible to do a lot of development. A model closer to natural phenomena can be developed in further research, such as using a spatial model to accommodate growth and interaction based on position. Development can also be done by considering the uncertainty factor in uncertain parameters and external disturbances, so adaptive control design is needed to adapt to uncertain state conditions. In control design, nonlinear control still assumes that the control quantity is not limited. This is less relevant than real life, so a combination of nonlinear control design and optimum control needs to be developed.

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