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PRIME LABELING OF AMALGAMATION OF FLOWER GRAPHS

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ABSTRACT

Graph labeling is the assigning of labels represented by integers or symbols to graph elements, edges and/or vertices (or both) of a graph. Consider a simple graph G with a vertex-set $V(G)$ and an edge-set $E(G)$. The order of graph G, denoted by $|V(G)|$, is the number of vertices on G. The prime labeling is a bijective function $f: V(G) \to \{1,2,\dots,|V(G)|\}$, such that the labels of any two adjacent vertices in G are relatively prime or $gcd(f(u), f(v)) = 1$, for every two adjacent vertices u and v in G. If a graph can be labeled with prime labeling, then the graph can be said to be a prime graph. A flower graph is a graph formed by helm graph H_n by connecting its pendant vertices (the vertices have degree one) to the central vertex of H_n, such a flower graph is denoted as $Fl(n)$. In this research, we employ constructive and analytical methods to investigate prime labelings on specific graph classes. Definitions, lemmas, and theorems are developed as the main results in this research. The amalgamation $Amal(G_i, v_{0i})$ is a graph formed by taking all by taking all the G_i's and identifying their fixed vertices v_{0i}. If $G_i = G_j = G$, then we write $Amal(G_i, v_{0i})$ with $Amal(G)_m$. In previous research, it has been shown that the flower graphs $Fl(n)$, for $n \geq 3$ are prime graphs. Continuing the research, we prove that two classes of amalgamation of flower graphs $Amal(Fl(n)_m)$ are prime graphs.

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1. INTRODUCTION

According to Ralph Faudree and Spencer Joel (1988), graph theory is a branch of discrete mathematics that examines mathematical structures representing interactions between entities called vertices and edges. Graph theory is also an ever-evolving theory, as evidenced by discoveries regarding various graphs, labeling techniques, and labeling methods. Graph labeling is a strategy to assign labels to the vertices or edges of a graph with the aim of satisfying certain rules or explaining specific properties. Graph labeling is a significant area of study in graph theory. According to a survey conducted by Gallian [1], the concept of graph labeling emerged in the mid-1960s. Since then, more than 200 labeling techniques have been explored, published in over 3000 research papers. Graph labeling [2] involves assigning numbers, typically positive or non-negative, to specific components of a graph. These components are usually either vertices or edges. When the assigned numbers correspond to vertices, the process is referred to as vertex labeling. There are many types of vertices labeling that have been developed in previous research, such as prime labeling, vertex-magic total labeling, total vertex irregularity strength, radio labeling, and L-labeling. Several articles have studied vertex labeling, including radio labeling of banana graphs [3], the total vertex irregularity strength on a series parallel graph $sp(m,r,4)$ [4], anti-fuzzy graph magic labeling [5], and bipolar anti-fuzzy graph magic labeling [6] on path, star, and cycle graphs.

One type of vertex labeling that continues to develop is prime labeling. The concept of prime labeling stems from Entriger's conjecture in 1980, which proposed that every tree graph could possess a prime labeling. The conjecture's progress was most recently advanced in 2011 by Haxell, et al. [7], who demonstrated that all tree graphs with a sufficiently large number of vertices qualify as prime graphs. Building on this conjecture, Tout et al. [8] formally introduced the notion of prime labeling in a paper published in 1982. Before defining prime labeling, the definition of greatest common divisor (gcd) is first given. According to [9], the gcd of two integers a and b denoted as $\gcd(a,b)$ is the largest positive integer that can exactly divide both a and b. Two integers a and b are called relatively prime if $\gcd(a,b)=1$. The Euclidean algorithm is an efficient algorithm for finding the gcd of two integers. If a and b are two integers with $a>b$, then the $\gcd(a,b)=\gcd(a \bmod b, b)$. This process is repeated until one of the numbers becomes 0 [10]. This concept and algorithm will be used in prime labeling.

Prime labeling is a bijective function, let G be a simple graph with the vertex set $V(G)$ and the edge set $E(G)$. A prime labeling on G is a labeling of the vertices on G with different integers in $V(G)$ such that the labels of any two adjacent vertices in G are relatively prime. This means that prime labeling is a bijective function $f: V(G) \to \{1,2,\dots,|V(G)|\}$ with $|V(G)|$ being the number of vertices in G, such that for any two distinct vertices $u,v \in V(G)$ with vertex u adjacent to vertex v in G, $\gcd(f(u),f(v))=1$, where $f(v), f(u) \in \{1,2,\dots,|V(G)|\}$. If a graph can be labeled with prime labeling, then the graph can be said to be prime graph [11].

Let $I \subseteq V(G)$ be a subset of vertices in graph G. An independent set $I \subseteq V(G)$ is defined as a set of vertices such that no two vertices in I are adjacent, meaning $uv \notin E(G)$ for any $u,v \in I$. The maximum size of an independent set in G is known as the independence number of G and is denoted by $\beta_0(G)$. In [12], they gave a necessary condition for any graph to be prime, it must satisfy the condition $\beta_0(G) \geq \left\lfloor \frac{|V(G)|}{2} \right\rfloor$.

Furthermore, as stated in [13], if $n \geq 4$ the complete graph K_n is not a prime graph. One of the graphs that has a prime labeling is the cycle graph C_n. According to [14], every cycle graph C_n is a prime graph for $n \geq 3$. The cycle graph C_n can be labeled with prime labeling because in that graph, every pair of adjacent vertices u and v in C_n satisfies $\gcd(f(u),f(v))=1$. Therefore, the cycle graph C_n is a prime graph. Several experts have proven the concept of prime labeling among others, prime labeling on the semi-total vertices graph of the broom graph $R^k\, B(n)$ [15], prime labeling for some helm-related graph [16], prime and coprime labeling on the graphs $P_m K_n$ and $P_m P_n$ [17], prime labeling on the crown graph C_n^* [18], prime labeling on trees [19], prime labeling on web graphs without center [12].

The flower graph denoted as $Fl(n)$ is a graph formed by a helm graph H_n by connecting each pendant vertex to the center vertex of H_n. The flower graph $Fl(n)$ has the vertex set $V(Fl(n)) = \{v, v_i, u_i \mid i = 1,2,3,\dots,n\}$ so that $|V(Fl(n))| = 2n+1$ and the edge set $E(Fl(n)) = \{vu_i, vv_i, u_iv_i, v_iv_{i+1} \mid i = 1,2,3,\dots,n\}$ where $v_{n+1} = v_1$. The flower graph $Fl(n)$ consists of three types of vertices, that is v as the central vertex, $v_1, v_2, \dots, v_n$ as vertices of degree four, and $u_1, u_2, \dots, u_n$ as vertices of degree two [20]. Let $\{G_i \mid i \in \{1,2,3,\dots,m\}\}$ for $m \in \mathbb{N}$ and $m \geq 2$ be a collection of finite graphs and each G_i has a fixed vertex, say v_{0i}. The following definition of an amalgamation of graph is referred from [21]. The amalgamation

$Amal(G_i, v_{0i})$ is a graph formed by taking all by taking all the G_i's and identifying their fixed vertices v_{0i}. If $G_i = G_j = G$, then we write $Amal(G_i, v_{0i})$ with $Amal(G)_m$. In this paper, we consider the amalgamation of flower graphs $Fl(n)$. In particular, let $G_i = Fl(n_i) = Fl(n)$, where v_{0i} be a central vertex of $Fl(n)$. We denote the amalgamation of m flower graphs $Fl(n)$ by $Amal(Fl(n)_m)$.

In [20], Ashokkumar and Maragathavalli gave prime labeling on the flower graph $Fl(n)$. Based on the results of the research, it is known that the flower graph $Fl(n)$ with $n \geq 3$ is a prime graph. From this research, we are interested to determine prime labeling on amalgamation of flower graphs $Amal(Fl(n)_m)$, for any $n \geq 3$ and $m \geq 2$.

2. RESEARCH METHODS

This research uses a literature study aimed at reviewing and analyzing graph theory books and research journals that study prime labeling. This research also employs constructive and analytical methods to investigate prime labeling on some classes of graph amalgamations. Definitions, lemmas, and theorems are developed to support the labeling construction and validity proofs. Here are the steps used:

1. Conducting a literature review on prime labeling in graphs.
2. Define amalgamation of flower graphs $Amal(Fl(n)_m)$, for positive integers $m \geq 2$ and $n \geq 3$.
3. Formulate theorems and prove mathematically that amalgamation of flower graphs $Amal(Fl(n)_m)$ are prime graphs. To achieve this, several steps will be followed:
    a. Based on the known properties and structural characteristics, formulate two new lemmas and two new theorems.
    b. Provide proofs for the formulated theorems and lemmas. The following properties from earlier studies support the proof of theorems.

       Theorem 1. [22] *Let A and B be two finite sets and let $f: A \to B$. If $|A| = |B|$, then function f is injective equivalent f is surjective equivalent f is bijective.*

       The function f is said to be bijective if satisfy injective and surjective. In this research to prove function f is bijective by proving that f is injective, then prove that the cardinality of the domain of f is equal to the cardinality of the codomain of f.

       Corollary 1. [23] *Two integers a and b are relatively prime integers if and only if there exist integers c and d such that $ac + bd = 1$. Since a and b are relatively prime, it follows that $gcd(a, b) = 1$.*

       Theorem 2. [24] *If a and b are two consecutive integers such that $a = n$ and $b = n + 1$, where n is an integer, then a and b are relatively prime.*

       Theorem 3. [24] *Let a, b, and c be integers. Then $(b + ca, a) = (a, b)$.*

       Then, for $c = -1$, we have $\gcd(b - a, a) = \gcd(a, b)$.
4. Validation of mathematical proof analytically.
5. Concluding the research findings and providing recommendations for open problems related to the studied research topic.

3. RESULTS AND DISCUSSION

In this section, we will prove that the amalgamation of flower graphs Amal $(Fl(n)_m)$ is a prime graph. This section presents definitions, lemma, main theorem, and corollaries.

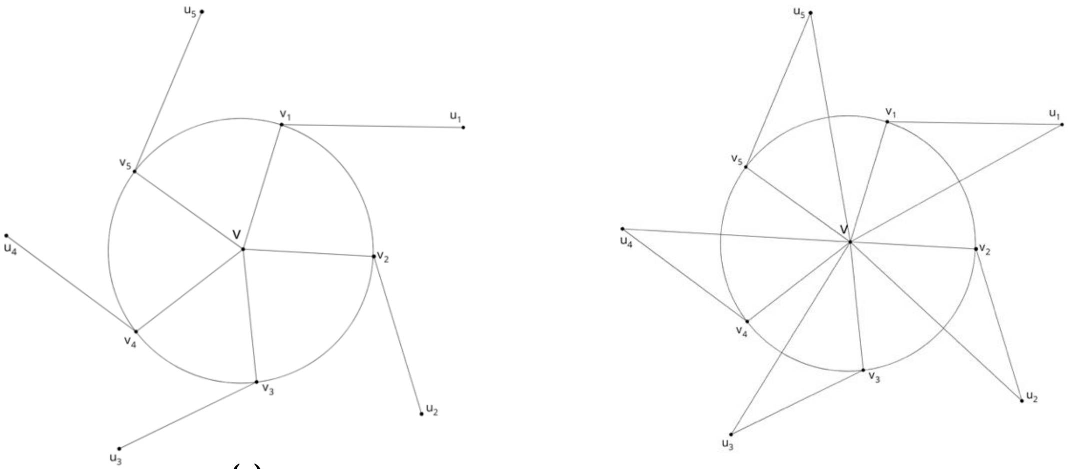
Lemma 1. [19] *Let $\beta_0(G)$ denote the independence number of a graph G. If G is a prime graph, then*

$$\beta_0(G) \geq \left\lfloor \frac{|V(G)|}{2} \right\rfloor.$$

Definition 1. **[20]** For any $n \geq 3$, the flower graph, denoted as $Fl(n)$, is a graph formed by adding n new edges to the helm graph H_n connecting the independent (pendant) vertex to the center vertex H_n.

As an illustration, a helm graph H_6 and a flower graph $Fl(6)$ is given in **Figure 1**.

In **[20]**, it was found that the flower graph $Fl(n)$ with $n \geq 3$ is a prime graph.



(a) (b)

Figure 1. (a). The Helm Graph H_6, (b). The flower graph $Fl(6)$

Now, we give the definition on amalgamation of m flower graphs, denoted by $Amal(Fl(n)_m)$. We depict $Amal(Fl(n)_m)$ as in **Figure 2**.

Definition 2. The amalgamation of flower graphs Amal $(Fl(n)_m)$ is a graph that constructed from the graph $Fl(n)$ by duplicating it m times and then uniting their central vertices. $Amal(Fl(n)_m)$ has a vertex-set

$$V(Amal(Fl(n)_m)) = \{v, u_i^j, v_i^j \mid 1 \leq i \leq n,\ \ 1 \leq j \leq m\}, \text{ and}$$

and an edge-set.

$$E(Amal(Fl(n)_m) = \{vu_i^j, vv_i^j, u_i^j v_i^j, v_i^j v_{i+1}^j \mid 1 \leq i \leq n,\ \ 1 \leq j \leq m,\ \ v_{n+1}^j = v_1^j\}.$$

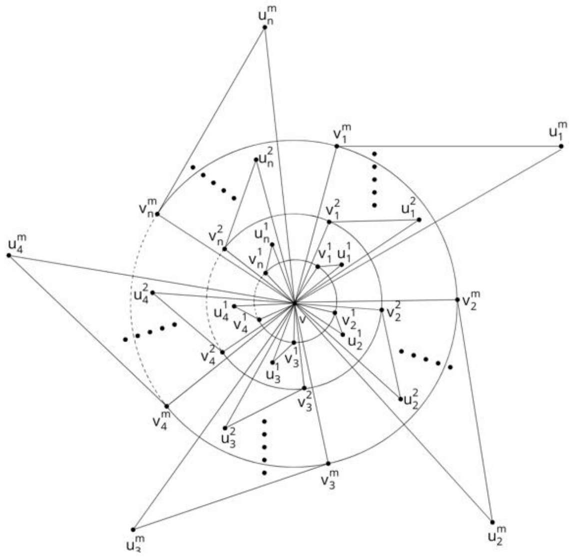


Figure 2. The Graph $Amal(Fl(n)_m)$

Then, $|V(Amal(Fl(n)_m)| = 2mn + 1$ and $|E(Amal(Fl(n)_m)| = 4mn$.

Lemma 2. *If $G = Amal(Fl(n)_m)$ then $\beta_0(G) = mn$, for any $n \geq 3$ and $m \geq 2$.*

Proof. Let $I(Amal(Fl(n)_m)$ be an arbitrary independent set of the graph $Amal(Fl(n)_m$. Consider the triangles with common vertex v that are $vu_i^j v_i^j$, for every $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$. Since each vertex

v is adjacent to u_i^j and v_i^j, it is clear that the set $I(Amal(Fl(n)_m)$ contains at most n vertices from each set $\{v, u_i^1, v_i^1 | i = 1, 2, \ldots, n\}$, $\{v, u_i^2, v_i^2 | i = 1, 2, \ldots, n\}$, …, $\{v, u_i^m, v_i^m | i = 1, 2, \ldots, n\}$. Thus, $\beta_0(Amal(Fl(n)_m)) \leq nm$.

On other hand, the set $\{u_i^j, 1 \leq i \leq n, \; 1 \leq j \leq m\}$ where $|\{u_i^j, 1 \leq i \leq n, \; 1 \leq j \leq m\}| = mn$ is clearly an independent set $I(Amal(Fl(n)_m)$. Therefore, the lemma is proven. ■

We present the following **Lemma 3**, which will be used in the proof of **Theorem 3**.

Lemma 3. *For all positive integers $m, i, j \in \mathbb{Z}^+$, the following holds:*

$$\gcd(2(m(i-1)+j)+1, 2(mi+j)+1) = 1.$$

Proof. Let

$$a = 2(m(i-1)+j)+1 = 2mi - 2m + 2j + 1$$

$$b = 2(mi+j)+1 = 2mi + 2j + 1$$

Compute the difference:

$$b - a = (2mi + 2j + 1) - (2mi - 2m + 2j + 1) = 2m$$

By **Theorem 3**

$$\gcd(a, b) = \gcd(a, b-a) = \gcd(a, 2m)$$

Now, observe that

$$a = 2(m(i-1)+j)+1 = 2m(i-1) + 2j + 1$$

Let $d = \gcd(a, 2m)$. Since $d | 2m$, d must be an even divisor. In addition, $d | a = 2m(i-1) + (2j+1)$. However, $2j+1$ is odd number and $2m$ is even number. Therefore, the only positive integer that divides both an even and an odd number is 1. Thus,

$$\gcd(a, 2m) = \gcd(a, b) = \gcd(2(m(i-1)+j)+1, 2(mi+j)+1) = 1. \blacksquare$$

Next, we prove that amalgamation of flower graphs $Amal(Fl(n)_m)$, for any $n \geq 3$ and $m \geq 2$ are prime graphs. This proof will be divided into two cases, for odd m and even m as in the following **Theorem 4**.

Theorem 4. *The graphs $Amal(Fl(n)_m)$ are prime graphs, for any $n \geq 3$ and even $m \geq 2$.*

Proof. Let $G = Amal(Fl(n)_m)$, for any $n \geq 3$ and even $m \geq 2$. First, we will prove that the vertex labeling $f: V(G) \to \{1,2,3, \ldots, 2mn+1\}$ is a bijective function.

Define $f: V(G) \to \{1,2,3, \ldots, 2mn+1\}$ as
$f(v) = 1$
$f(v_1^j) = 2j$, $\quad for\ 1 \leq j \leq m$
$f(u_1^j) = 2j+1$, $\quad for\ 1 \leq j \leq m$
$f(v_i^j) = 2(m(i-1)+j)+1$, $\quad for\ \ 2 \leq i \leq n, 1 \leq j \leq m,$
$f(u_i^j) = 2(m(i-1)+j)$, $\quad for\ \ 2 \leq i \leq n, 1 \leq j \leq m,$
For $1 \leq i \leq n$, we obtain
$f(v) = 1$
$\{f(v_1^j), 1 \leq j \leq m\} = \{2, 4, 6, \ldots, 2m\}$
$\{f(u_1^j), 1 \leq j \leq m\} = \{3, 5, 7, \ldots, 2m+1\}$
$\{f(v_i^j), 2 \leq i \leq n, 1 \leq j \leq m\} = \{2m+3, 2m+5, 2m+7, \ldots, 2mn+1\}$
$\{f(u_i^j), 2 \leq i \leq n, 1 \leq j \leq m\} = \{2m+2, 2m+4, 2m+6, \ldots, 2mn\}$

It can be seen that $1 \leq f(v), f(u_i^j), f(v_i^j) \leq 2mn+1$ for every $v, v_i^j, u_i^j \in V(G)$ and the labels of all vertices are different. Thus, it is clear that $f: G \to \{1,2,3, \ldots, 2mn+1\}$ is an injective function. Since $|V(G)| = 2mn+1 = |\{1,2,3, \ldots, 2mn+1\}|$ and it is shown that f is injective, then using the **Theorem 1** we have f is bijective function. Now, we will prove that for any two adjacent vertices in G are relatively prime.

Let x and y be adjacent vertices in G. Then

1. If $x = v$ and y is any vertex adjacent to x (By **Definition 2**, $x = u_i^j$ or $x = v_i^j$)

   Since $f(v) = 1$, it is clear that $\gcd(f(x), f(y)) = 1$

2. If $x = u_i^j$ and $y = v_i^j$.

   Since $f(u_i^j)$ and $f(v_i^j)$ are two consecutive integers then by **Theorem 2**, $\gcd(f(x), f(y)) = 1$.

3. If $x = v_i^j$ and $y = v_{i+1}^j$.

   Since $f(v_i^j) = 2(m(i-1)+j)+1$ and $f(v_{i+1}^j) = 2(mi+j)+1$, by **Lemma 3**, $\gcd(f(x), f(y)) = 1$.

Hence, it has been shown that for any two distinct vertices $u, v \in V(G)$ with vertex u adjacent to vertex v in G, $\gcd(f(u), f(v)) = 1$. Therefore, $Amal(Fl(n)_m)$ are prime, for any $n \geq 3$ and even $m \geq 2$. ■

To clarify **Theorem 4**, prime labeling of graph $Amal(Fl(5)_2)$ is given in **Figure 3**.

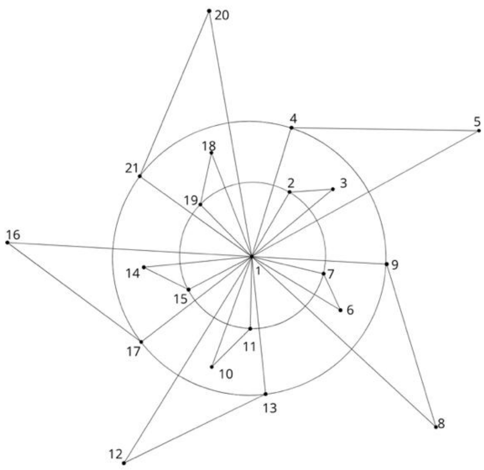


Figure 3. Prime Labeling of $Amal(Fl(5)_2)$

We conjecture that the graphs $Amal(Fl(n)_m)$ are prime, for any $n \geq 3$ and odd $m \geq 3$ also have a prime labeling. However, proving that the graphs have prime labeling requires more complex proof steps. Therefore, for the case when m is odd, we focus on proving that the graphs $Amal(Fl(n)_3)$ have prime labeling.

We depict the graph $Amal(Fl(n)_3)$ as in **Figure 4**.

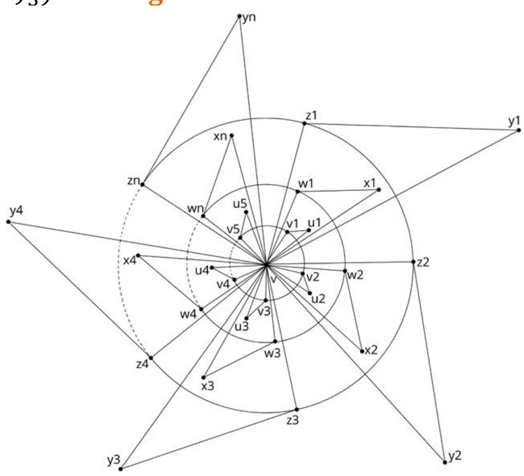


Figure 4. The Graph $Amal(Fl(n)_3)$

Theorem 5. *Graphs $Amal(Fl(n)_3)$, for any $n \geq 3$ are prime graphs.*

Proof. To simplify the labelling process, specific notations for the vertices and edges of the graphs $Amal(Fl(n)_3$ are introduced. Let $G = Amal(Fl(n)_3)$ with a set of vertices

$$V(Amal(Fl(n)_3)) = \{u_i, v_i, w_i, x_i, y_i, z_i | i = 1,2,3, \dots, n\},$$

and a set of edges

$$E(Amal(Fl(n)_3) = \{vu_i, vv_i, vw_i, vx_i, vy_i, vz_i, u_iv_i, w_ix_i, y_iz_i, v_iv_{i+1}, w_iw_{i+1}, z_iz_{i+1} \mid i = 1, 2, \dots, n$$

where $v_{i+1} = v_1,\ \ w_{i+1} = w_1,\ \ z_{i+1} = z_1\}$.

Then, $|V(Amal(Fl(n)_3)| = 6n + 1$ and $|E(\,Fl_m(n))| = 12n$.

First, we will prove that the vertex labeling $f: V(G) \to \{1,2,3, \dots, 6n + 1\}$ is a bijective function.
Define $f: V(Amal(Fl(n)_3) \to \{1,2,3, \dots, 6n + 1\}$ as

$$f(v) = 1$$

$$f(u_i) = \begin{cases} 7, & \text{for } i = 1 \\ 6i, & \text{for } 2 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} 6, & \text{for } i = 1 \\ 6i + 1, & \text{for } 2 \le i \le n \end{cases}$$

$$f(w_i) = \begin{cases} 4, & \text{for } i = 1 \\ 6i - 3, & \text{for odd } i \ge 3 \\ 6i - 1, & \text{for even } i \end{cases}$$

$$f(x_i) = \begin{cases} 5, & \text{for } i = 1 \\ 6i - 2, & \text{for } 2 \le i \le n \end{cases}$$

$$f(y_i) = \begin{cases} 3, & \text{for } i = 1 \\ 6i - 4, & \text{for } 2 \le i \le n \end{cases}$$

$$f(z_i) = \begin{cases} 2, & \text{for } i = 1 \\ 6i - 1, & \text{for } odd\ i \ge 3 \\ 6i - 3, & \text{for even } i \end{cases}$$

Clearly for every $1 \le i \le n$, we have

$f(v) = 1$

$f(u_1) = 7$

$f(v_1) = 6$

$f(w_1) = 4$

$f(x_1) = 5$

$f(y_1) = 3$

$f(z_1) = 2$

$\{f(u_i) |\ 2 \le i \le n\} = \{12,18,24, \dots, 6n\}$

$\{f(v_i) |\ 2 \le i \le n\} = \{13,19,25, \dots, 6n + 1\}$

$\{f(w_i) |\ 2 \le i \le n, for\ odd\ i\} = \{15,27,39, \dots, 6n - 3\}$

$\{f(w_i) |\ 2 \le i \le n, for\ even\ i\} = \{11,23,35, \dots, 6n - 1\}$

$\{f(x_i) |\ 2 \le i \le n\} = \{10,16,22, \dots, 6n - 2\}$

$\{f(y_i) |\ 2 \le i \le n\} = \{14,20,26, \dots, 6n + 2\}$

$\{f(z_i) |\ 2 \le i \le n, for\ odd\ i\} = \{17,29,41, \dots, 6n - 1\}$

$\{f(z_i) |\ 2 \le i \le n, for\ even\ i\} = \{9,21,33, \dots, 6n - 3\}$

It can be seen that $1 \le f(v), f(u_i), f(v_i), f(w_i), f(x_i), f(y_i), f(z_i) \le 6n + 1$ for every $u, v, w, x, y, z \in V(\,G)$ and the labels of all vertices are different. Since $|V(G)| = 6n + 1 = |\{1,2,3, \dots, 6n + 1\}|$ and it is shown that f is injective, then using the **Theorem 1** we have f is bijective function. Now, we will prove that for any two adjacent vertices in G are relatively prime. Let x and y be adjacent vertices in G. Using the **Corollary 1**, we determine integers c and d such that $f(x)c + f(y)d = 1$. Without loss of generality, let $f(x) = a$ and $f(y) = b$. The integers c and d are presented in **Table 1**.

Table 1. The Integers c and d such that $f(x)c + f(y)d = 1$

Description	Condition	a	b	c	d
$f(v)$ is adjacent to $f(u_i)$	$i = 1$	$f(v) = 1$	$f(u_i) = 7$	-6	1
	$2 \leq i \leq n$		$f(u_i) = 6i$	$6i + 1$	-1
$f(v)$ is adjacent to $f(v_i)$	i=1	$f(v) = 1$	$f(v_i) = 6$	-5	1
	2≤i≤n		$f(v_i) = 6i + 1$	$-6i$	1
$f(v)$ is adjacent to $f(w_i)$	i=1	$f(v) = 1$	$f(w_i) = 4$	-3	1
	For odd $i \geq 3$		$f(w_i) = 6i - 3$	$6i - 2$	-1
	For even i		$f(w_i) = 6i - 1$	$6i$	-1
$f(v)$ is adjacent to $f(x_i)$	i=1	$f(v) = 1$	$f(x_i) = 5$	-4	1
	2≤i≤n		$f(x_i) = 6i - 2$	$6i - 1$	-1
$f(v)$ is adjacent to $f(y_i)$	$i = 1$	$f(v) = 1$	$f(y_i) = 3$	-2	1
	$2 \leq i \leq n$		$f(y_i) = 6i - 4$	$6i - 3$	-1
$f(v)$ is adjacent to $f(z_i)$	$i = 1$	$f(v) = 1$	$f(z_i) = 2$	-1	1
	For odd $i \geq 3$		$f(z_i) = 6i - 1$	$6i$	-1
	For even i		$f(z_i) = 6i - 3$	$6i - 2$	-1
$f(v_i)$ is adjacent to $f(u_i)$	$i = 1$	$f(v_i) = 6$	$f(u_i) = 7$	-1	1
	$2 \leq i \leq n$	$f(v_i) = 6i + 1$	$f(u_i) = 6i$	1	-1
$f(v_i)$ is adjacent to $f(v_{i+1})$	$i = 1$	$f(v_i) = 6$	$f(v_{i+1}) = 13$	-2	1
	$2 \leq i \leq n$	$f(v_i) = 6i + 1$	$f(v_{i+1}) = 6i + 7$	$i + 1$	$-i$
$f(v_i)$ is adjacent to $f(v_n)$	$i = 1$	$f(v_i) = 6$	$f(v_n) = 6n + 1$	$-n$	1
$f(w_i)$ is adjacent to $f(x_i)$	$i = 1$	$f(w_i) = 4$	$f(x_i) = 5$	-1	1
	For odd $i \geq 3$	$f(w_i) = 6i - 3$	$f(x_i) = 6i - 2$	-1	1
	For even i	$f(w_i) = 6i - 1$		1	-1
$f(w_i)$ is adjacent to $f(w_{i+1})$	$i = 1$	$f(w_i) = 4$	$f(w_{i+1}) = 11$	3	-1
	For odd i	$f(w_i) = 6i - 3$	$f(w_{i+1}) = 6(i + 1) - 1 = 6i + 5$	There exists	There exists
				Since $b - a = 6i + 5 - (6i - 3) =$ 8 and its know that $\gcd(a, b) =$	

Description	Condition	a	b	c	d
				$\gcd(b-a,a)=\gcd(8,a)$. Since a is odd integer then by **Theorem 3**, $\gcd(8,a)=1$. Therefore, there exist c and d such that $ac+bd=1$	
	For even i	$f(w_i)=6i-1$	$f(w_{i+1})=6i+3$	$\frac{-3i-2}{2}$	$\frac{3i}{2}$
$f(w_i)$ is adjacent to $f(w_n)$	$i=1$ and For odd n	$f(w_i)=4$	$f(w_n)=6n-3$	$\frac{3n-1}{2}$	-1
	$i=1$ and For even n		$f(w_n)=6n-1$	$\frac{3n}{2}$	-1
$f(z_i)$ is adjacent to $f(y_i)$	$i=1$	$f(z_i)=2$	$f(y_i)=3$	-1	1
	For odd $i\geq 3$	$f(z_i)=6i-1$	$f(y_i)=6i-4$	$2i-1$	$-2i$
	For even i	$f(z_i)=6i-3$		1	-1

From **Table 1**, we find that there exist c and d such that $f(x)c+f(y)d=1$, for every two adjacent vertices x and y in G . Therefore, the graphs $Amal(Fl(n)_3)$ are prime graphs. ∎

To facilitate a better understanding of **Theorem 5**, prime labeling of the graph $Amal(Fl(5)_3)$ is given in **Figure 5**.

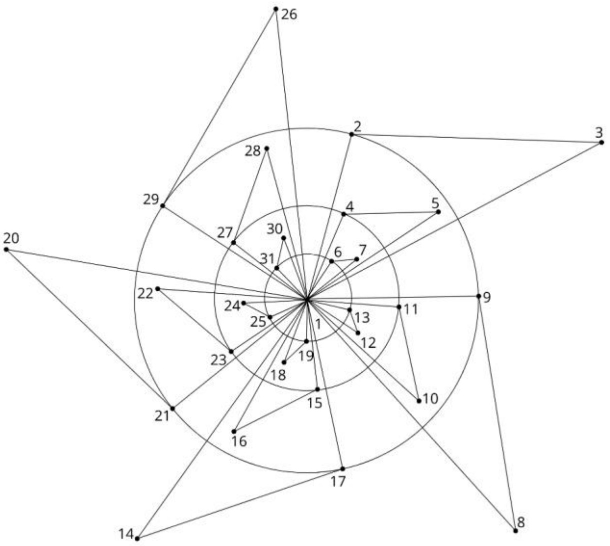


Figure 5. Prime Labeling of $Amal(Fl(5))_3$

4. CONCLUSION

Previous research [20] has explored the prime labeling of flower graphs $Fl(n)$. Based on the results of the research, it is known that the flower graph $Fl(n)$ with $n\geq 3$ is a prime graph. In this research, we are interested to determine prime labeling on amalgamation of flower graphs. We define amalgamation of m flower graphs, denoted by $Amal(Fl(n)_m)$ as a graph that constructed from the flower graph $Fl(n)$ by duplicating it m times and then uniting their central vertices. We proved that the graphs $Amal(Fl(n)_m))$ are prime graphs for any $n\geq 3$ and even $m\geq 3$. In Particular, for the case when m is odd, we focus on proving that the graphs $Amal(Fl(n)_3)$ have prime labeling. We conjecture that the graphs $Amal(Fl(n)_m)$ are prime, for any odd $m\geq 3$ also have a prime labeling. However, proving that the graphs have prime labeling requires more complex proof steps. Consequently, we recommend further research to study that the graphs $Amal(Fl(n)_m)$ are prime, for any odd $m\geq 3$. This opens the possibility of developing new methods or approaches to prove the conjecture for higher values of m. Exploring the structural properties of $Amal(Fl(n)_m)$ may also shed light on its prime labelling properties.

AUTHOR CONTRIBUTIONS

Desi Rahmadani: Conceptualization, Funding Acquisition, Writing-Original Draft. Ardi Aldianyah: Investigation, Visualization. Dina Pratiwi: Visualization, Writing-Original Draft. Mahmuddin Yunus: Data Curation, Writing-Review and Editing. Vita Kusumasari: Formal Analysis, Review and Editing. All authors discussed the results and contributed to the final manuscript.

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CONFLICT OF INTEREST

The authors declare no conflicts of interest to report study

REFERENCES

[1] J. A. Gallian, "A DYNAMIC SURVEY OF GRAPH LABELING," *Electron. J. Combin.*, vol. 1000, p. DS6: Dec 2, Dec. 2022, doi: https://doi.org/10.37236/11668.

[2] W.D. Wallis, *Magic Graphs*. Birkhäuser, Boston, 2001.

[3] Sarbaini, Nazaruddin, M. Rizki, M.I.H. Umam, M. L. Hamzah, and T. A. Prasetyo, "RADIO LABELING OF BANANA GRAPHS" *BAREKENG : Journal of Mathematics and Its Applications*, vol. 17, issue 1, pp. 0165-3017, 2023.doi: https://doi.org/10.30598/barekengvol17iss1pp0165-0170

[4] C. C. Marzuki, A. Utami, M. Elviyenti, and Y. Mudao, "ON THE TOTAL VERTEX IRREGULARITY STRENGTH OF SERIES PARALLEL GRAPH $SP(m, r, 4)$, "*BAREKENG : Journal of Mathematics and Its Applications*, vol. 18, issue 1, pp. 0213-0222, 2024.doi: https://doi.org/10.30598/barekengvol18iss1pp0213-0222

[5] A. S. Brata, T. Nusantara, D. Rahmadani, A. Gani, and V. Kusumasari, "ON THE PROPERTIES OF ANTI-FUZZY GRAPH MAGIC LABELING, "*AIP Conference Proceedings*, 2194, 2194, 020012, 2019.doi ; https://doi.org/10.1063/1.5139744

[6] I. Firmansa, T. Nusantara, D. Rahmadani, A. Gani, H. Susanto, and Purwanto, "ON THE PROPERTIES OF BIPOLAR ANTI FUZZY GRAPH MAGIC LABELING," *AIP Conference Proceedings*, 2194, 020028, 2019.doi: https://doi.org/10.1063/1.5139760

[7] P. Haxell, O. Pikhurko, and Taraz, "PRIMALITY OF TREES," *J. Combinatorics*, vol. 2, pp. 481-500, 2011. doi: https://doi.org/10.4310/JOC.2011.v2.n4.a1

[8] A. Tout, A. Dabboucy, K. Howalla, "PRIME LABELING OF GRAPHS," *Nat. Acad. Sci. Lett.*, vol.11, pp. 365–368, 1982

[9] K. H. Rosen, *DISCRETE MATHEMATICS AND ITS APPLICATIONS.* M.cGraw-Hill., vol. 6, 1999. https://www.academia.edu/download/28147631/syllabus.pdf

[10] Cormen and H. Thomas, *INTRODUCTION TO ALGORITHMS.* MIT Press, 2022

[11] S. K. Vaidya and U. M. Prajapati, "SOME NEW RESULTS ON PRIME GRAPHS," *OJDM*, vol. 02, no. 03, pp. 99–104, 2012, doi: https://doi.org/10.4236/ojdm.2012.23019.

[12] A. Kansagara and S. Patel, "PRIME LABELING IN THE CONTEXT OF WEB GRAPHS WITHOUT CENTER," *AKCE International Journal of Graphs and Combinatorics*, vol. 18, no.3, pp. 132-142, 2021. doi: https://doi.org/10.1080/09728600.2021.1974286
[13] Lee, S., Wui, I., and Yeh, J. 1988. ON THE AMALGAMATION OF PRIME GRAPHS. BULL. MALAYSIAN MATH. SOC. (SECOND SERIES), 11 59-67.
[14] G. Chartrand, C. Egan, and P. Zhang, *HOW TO LABEL A GRAPH*. in SpringerBriefs in Mathematics. Cham: Springer International Publishing, 2019. doi: https://doi.org/10.1007/978-3-030-16863-6.
[15] D. R. Putra, M. Kiftiah, and F. Fran, "PELABELAN PRIMA PADA GRAF SIMPUL SEMI TOTAL DARI GRAF SIKAT," *Var*, vol. 5, no. 2, p. 117, Oct. 2022, doi: https://doi.org/10.26737/var.v5i2.2699.
[16] D. S. Meena and K. Vaithilingam, "PRIME LABELING FOR SOME HELM RELATED GRAPHS," *International Journal of Innovative Research in Science, Engineering and Technology*, vol. 2, no. 4, 2013.
[17] H. Komarullah, "PELABELAN PRIMA DAN KOPRIMA PADA GRAF $P_M K_N$ DAN GRAF $P_M P_N$," *Prosiding Seminar Pendidikan Matematika dan Matematika UNY*, vol. 2, 2023. doi: https://doi.org/10.21831/pspmm.v8i2.312
[18] K. Vaithilingam and D. S. Meena, "PRIME LABELING FOR SOME CROWN RELATED GRAPHS," *International Journal of Scientific & Technology Research*, vol. 2, no. 3, 2013.
[19] H.-L. Fu and K.-C. Huang, "ON PRIME LABELLINGS," *Discrete Math*., vol. 127, no. 1–3, pp. 181–186, 1994.doi: https://doi.org/10.1016/0012-365X(92)00477-9
[20] S. Ashokkumar and S. Maragathavalli, "PRIME LABELLING OF SOME SPECIAL GRAPHS," *IOSR Journal of Mathematics (IOSR-JM)*, vol. 11, no. 1, pp. 01-05, 2015.
[21] K. Carlson, "GENERALIZED BOOKS AND C_m-Snakes Are Prime Graphs," *Ars Combin*., vol. 80, pp. 215-221, 2006.
[22] T. Koshy, *DISCRETE MATHEMATICS WITH APPLICATIONS*. Elsevier Academic Press, 2004.
[23] K. H. Rosen, *ELEMENTARY NUMBER THEORY AND ITS APPLICATIONS.* AddisonWesley, 2011.
[24] N. Mwamba, "THE RELATIVELY PRIME NATURE OF CONSECUTIVE INTEGERS," *International Journal of Mathematics And its Applications*, vol. 10, no. 1, 97–100, 2022.