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MULTIPLE STATE MODEL FOR PREMIUM CALCULATION OF THE ELDERLY LONG-TERM CARE INSURANCE

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ABSTRACT

As Indonesia enters an ageing population phase—with 12% of its population categorized as elderly—the need for financial protection in later life is increasingly urgent. The elderly population faces declining economic productivity, increased health risks, and a growing need for long-term care support. To address this, the present study develops an actuarial framework for calculating net annual premiums for elderly long-term care (LTC) insurance using the equivalence principle and a five-state multiple state model. Unlike previous research that focused on critical illness insurance, the proposed model reflects elderly care needs more realistically by incorporates transitions into and out of dependency where the elderly need assistance to perform daily activities. The results show that premiums rise significantly with later enrollment ages due to higher dependency and mortality risks along with shorter contribution periods. Furthermore, cash flow simulations based on 500 life table iterations demonstrate that LTC insurance can remain financially sustainable when accurately priced and supported by stable investment returns. This study offers a novel actuarial approach to developing sustainable LTC insurance products for Indonesia's ageing population.

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1. INTRODUCTION

Indonesia has entered an ageing population phase, with 12.00% of its population consisting of elderly individuals aged 60 years and above. In 2024, the average life expectancy in Indonesia is estimated to be 72.39 years [1]. Sustainable development in health, education, employment, and other socio-economic sectors have contributed to declining mortality rates and increasing life expectancy, thereby leading to a continuous rise in both the number and proportion of the elderly population. However, the elderly are considered a vulnerable demographic due to three main factors: reduced economic productivity, increased health risks, and the need for assistance to provide long-term care [2]. Moreover, only a small proportion of the elderly are covered by social health insurance or employment-based social security (pension plan) to support their financial needs in old age [3]. According to the Central Bureau of Statistics Indonesia, the elderly dependency ratio stands at 17.76, implying that each elderly person relies on support from approximately six working-age individuals (15–59 years) [1]. One potential solution that could be implemented early to ensure financial protection and mitigate risks during old age is long-term care insurance.

Long-term care (LTC) insurance is a type of life insurance that provides benefits in the form of allowances or long-term care services to elderly individuals or those with critical illnesses [4]. A common approach to modeling LTC insurance product is the multiple state model. In countries such as the United States and Canada, which have developed LTC insurance for the elderly, the states in the multiple state model for LTC insurance are related to the insured individual's ability to independently perform Activities of Daily Living (ADLs) [5]. ADLs refer to basic self-care tasks performed daily such as feeding, bathing, grooming, dressing, bowel control, bladder control, toilet use, transfers, mobility on level surfaces, and stairs [6]. As a result of physical changes associated with aging and various illnesses, elderly individuals may experience a decline in their ability to perform ADLs over time [7].

Certain countries with rapidly aging populations have implemented LTC insurance systems to address rising healthcare demands among the elderly. In South Korea, individuals aged 65 and older accounted for 10.3% of the population in 2008 but incurred approximately 30% of the nation’s healthcare costs. To mitigate this burden, the government introduced LTC insurance that year, providing in-kind benefits such as institutional, home-based, and community-based services [8]. Similarly, Japan launched its LTC insurance system in 2000, mandating all residents aged 40 and above to enroll and contribute premiums. Individuals aged 65 and older who are certified as needing LTC are eligible to receive reimbursement for care services, up to a ceiling based on their assessed level of need. The Japanese system is financed through a combination of premiums and public funds, with regional variation in premiums based on expenditure levels [9].

Several studies have examined the calculation of LTC insurance premiums; however, most of the studies focus on critical illness insurance rather than elderly LTC. For example, Putri et al. [10] developed a formula for calculating the premium of critical illness LTC insurance products using an eight-state model. Syamdena et al. [4] calculated the single premium for LTC insurance based on a four-state diabetes progression model with constant transition rates between states. Other researchers have utilized critical illnesses prevalence data to estimate transition probabilities between states in the model, as demonstrated by Gumauti et al. [11] with a three-state model, Perdana et al. [12] with an eight-state model, and Satyahadewi et al. [13] with a ten-state model.

Despite the growing body of research, there is currently no LTC insurance product available for the elderly in Indonesia, nor are there actuarial studies specifically tailored to local demographic and healthcare characteristics. Moreover, LTC services are not covered under Indonesia’s National Health Insurance (JKN) due to cost limitations. This study aims to fill this gap by developing a multiple state model for calculating the net annual premium of LTC insurance for the Indonesian elderly population. The proposed model comprises five states: active, retired, dependent, previously dependent, and dead. It incorporates the possibility of recovery from the dependent to the previously dependent state, allowing for a more realistic representation of transitions in dependency status. Additionally, a cash flow simulation is performed to assess the financial sustainability of the proposed insurance product from the insurer’s perspective.

2. RESEARCH METHODS

The data used in this study are hypothetical and constructed based on secondary sources, including the BPJS Ketenagakerjaan Mortality Table 2022 (TMJ-22) [14], published in May 2023, and the "Dependency Proportions among Individuals Aged ≥ 60 Years" from the Indonesia Basic Health Research (Riskesdas) 2018 report [15], published by the Health Research and Development Agency of the Ministry of Health of the Republic of Indonesia in December 2018.

The mortality data are obtained from TMJ-22, which provides one-year mortality probabilities for males aged x. The dataset assumes a limiting age of 115 years, meaning that no individual is expected to survive beyond age 116. The dependency data are sourced from Riskesdas 2018, specifically the table titled "Proportion of Dependency Levels in the Population Aged ≥ 60 Years by Age Group Characteristics", which reflects the level of independence in performing Activities of Daily Living (ADLs) among individuals aged 60 and above, as shown in Table 1. These data are used to estimate both the one-year probability of an individual remaining independent and the transition probability from the retired state to the dependent state.

Table 1. Proportion of Dependency Levels in the Population Aged ≥ 60 Years by Age Group Characteristics

Age group	Dependency Levels (%)				
	Independent	Mildly Dependent	Moderately Dependent	Severely Dependent	Totally Dependent
60 – 69	80.30	17.50	0.60	0.53	1.07
70 – 79	68.09	27.22	1.50	1.33	1.86
80+	50.05	38.03	3.97	3.56	4.39

***Data source:** Indonesia Basic Health Research (Riskesdas) 2018*

In **Table 1**, the elderly dependency levels were assessed using an instrument based on the Barthel Index of Activities of Daily Living. The Barthel Index is used to evaluate an individual's level of independence in performing ten ADLs (feeding, bathing, grooming, dressing, bowel control, bladder control, toilet use, transfers, mobility on level surfaces, and stairs), with different levels of independence and weighted scores assigned to each ADL.

2.1 Multiple State Model

A multiple state model is a model for a stochastic process used to represent the probabilities of an individual's movement between a finite set of states [16]. These movements are referred to as transitions or events. The structure of a multiple state model depends on the number of states and the transition probabilities between them [10]. In this study, the net annual premium for LTC insurance is calculated based on multiple state model presented in **Figure 1**:

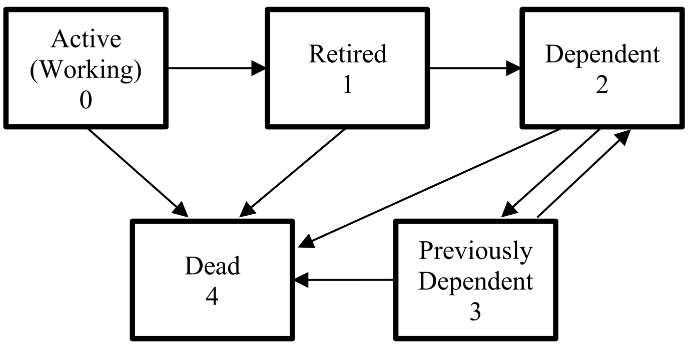

Figure 1. LTC Insurance Multiple State Model

Figure 1 illustrates that the LTC insurance model consists of five states, with the dependent state occurring when the insured requires assistance or becomes dependent on performing ADLs, necessitating LTC services. The dependent state is assumed to occur only after the insured transitions to the retired state, beginning at age 60. In this model, the insured may recover from the dependent state to the previously dependent state, where they regain independence in performing ADLs. The previously dependent state can only be accessed from the dependent state, allowing for two-way transitions between these two states.

2.2 State-dependent Annuity and Insurance Benefit

Let the annual effective compound interest rate be constant at r. The formula for the discount factor (v), used to estimate the present value [17], is given in Equation (1) as follows:

$$v = \frac{1}{1+r}. \tag{1}$$

Suppose an individual aged x, denoted as (x), currently be in state i. The actuarial present value (APV) of a state-dependent annuity, paid annually at the beginning of each year for 1 unit over a period of n years, or while (x) remains in state j within the n-year period, is defined in Equation (2) as follows [5]:

$$\ddot{a}^{ij}_{x:\overline{n}|} = \sum_{t=0}^{n-1} v^t \ {}_tp^{ij}_x. \tag{2}$$

The APV of a state-dependent annuity-immediate for an individual (x), currently in state i, paid annually at the end of each year in the amount of 1 unit while (x) remains in state j, is expressed by Equation (3) as follows:

$$a^{ij}_x = \sum_{t=1}^{\infty} v^t \ {}_tp^{ij}_x. \tag{3}$$

The actuarial present value of a lump sum payment of 1 unit for an individual (x), currently in state i, paid at the end of the year upon the (x)'s transition from state i to state j in a multiple state model with $m + 1$ states, is formulated by Equation (4) as follows:

$$A^{ij}_x = \sum_{t=0}^{\infty} \sum_{k=0,k\neq j}^{m} v^{t+1} \ {}_tp^{ik}_x \ p^{kj}_{x+t}. \tag{4}$$

2.3 Policy Design and Assumption

The long-term care insurance policy design and assumptions used in this paper are as follows:

1. The annual effective compound interest rate used is constant at $r = 6.00\%$.
2. The insured individual is male.
3. The insured individual enrolls in the LTC insurance program in a healthy and actively working state (state 0). The insurance contract begins at various ages, ranging from 35 to 50 years old.
4. The insured individual retires at the age of 60.
5. According to the mortality probabilities for males in the BPJS Ketenagakerjaan 2022 Mortality Table (TMJ-22), if the insured individual exceeds the age of 116 years, they are assumed to have passed away, and benefits corresponding to the deceased state will be paid.
6. The net annual premium for LTC insurance is calculated using the equivalence principle.
7. The multiple state model of LTC insurance is presented in
8. Figure 1 with premium payment scheme and benefit structure for each state are outlined in Table 2 and Table 3.

Table 2. Payment Scheme of Premium and Benefit

State	Payment Scheme
0 (active)	The insured pays annual term premiums at the beginning of each year while they remain healthy and actively employed, continuing until retirement at the age of 60.
1 (retired)	At the age of 60, the insured transitions to the retired state and receives an annual annuity benefit, payable at the end of each year while remaining in this state.
2 (dependent)	If the insured becomes dependent and is unable to perform ADLs, they receive an annual annuity benefit, payable at the end of each year while remaining in this state. It is assumed that the dependency occurs only after the insured reaches retirement age.
3 (previously dependent)	If the insured recovers from the dependent state, they transition to the previously dependent state, during which no benefits are received, nor premiums paid.
4 (dead)	In the event of the insured's death, a lump sum benefit is paid to their beneficiaries at the end of the year in which death occurs.

Table 3. The Benefit Amount for Each State

State	Notation	Benefit Amount
1 (retired)	$B^{(1)}$	An annual annuity of IDR 30,000,000 is paid at the end of each year while the insured individual remains at state 1.
2 (dependent)	$B^{(2)}$	An annual annuity of IDR 50,000,000 is paid at the end of each year while the insured individual remains at state 2.
4 (dead)	$B^{(4)}$	A lump sum payment of IDR 60,000,000 is paid at the end of the year in which the insured individual passes away.

3. RESULTS AND DISCUSSION

3.1 One-year Transition Probability Table

A transition probability represents the likelihood of an individual aged x transitioning from state i to state j within time t. It is expressed as the conditional probability $P(Y_{x+t} = j|Y_x = i)$ where $x \geq 0$, $t \geq 0$, and $i, j \in S$, denoted as **Equation (5) [5]**:

$$ {}_tp_x^{ij} = P(Y_{x+t} = j|Y_x = i). \tag{5}$$

For $t = 1$, the notation can be expressed as follows:

$$ {}_1p_x^{ij} = p_x^{ij}. $$

Based on the multiple state model in

Figure 1 and the established policy assumptions, the transition probability matrix for one upcoming year for an individual aged x is as follows:

$$\begin{pmatrix} p_x^{00} & p_x^{01} & p_x^{02} & p_x^{03} & p_x^{04} \\ p_x^{10} & p_x^{11} & p_x^{12} & p_x^{13} & p_x^{14} \\ p_x^{20} & p_x^{21} & p_x^{22} & p_x^{23} & p_x^{24} \\ p_x^{30} & p_x^{31} & p_x^{32} & p_x^{33} & p_x^{34} \\ p_x^{40} & p_x^{41} & p_x^{42} & p_x^{43} & p_x^{44} \end{pmatrix} = \begin{pmatrix} p_x^{00} & p_x^{01} & 0 & 0 & p_x^{04} \\ 0 & p_x^{11} & p_x^{12} & 0 & p_x^{14} \\ 0 & 0 & p_x^{22} & p_x^{23} & p_x^{24} \\ 0 & 0 & p_x^{32} & p_x^{33} & p_x^{34} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

Based on the matrix in **Equation (6)**, it can be observed that the insured individual may transition to state 2 (dependent) only after experiencing state 1 (retired) at age 60. Furthermore, the insured is permitted to transition to state 3 (previously dependent) only from state 2. The model also allows for two-way transitions between states 2 and 3, with state 4 acting as an absorbing state.

The transition probability table for one upcoming year for an individual aged x in this study is constructed based on the data of the elderly dependency levels from the Riskesdas 2018 in Table 1, as well as data from the BPJS Ketenagakerjaan Mortality Table 2022 (TMJ-22), as detailed in the following assumptions:

p_x^{04} : The one-year mortality probability for a male (x) aged 35–59 years from the TMJ-22.

p_x^{00} : $1 - p_x^{04}$, for $35 \le x \le 58$; 0, for $59 \le x \le 115$.

p_x^{01} : $1 - p_x^{04}$, for $x = 59$; 0, for $35 \le x \le 58$; and $60 \le x \le 115$.

p_x^{14} : The one-year mortality probability for a male (x) aged 60–115 years from the TMJ-22.

p_x^{11} : 0, for $35 \le x \le 59$; for $60 \le x \le 115$; $(1 - p_x^{14})$ is multiplied by the proportion for the independent category, corresponding to the age group in **Table 1**.

p_x^{12} : 0, for $35 \le x \le 59$; for $60 \le x \le 115$; $(1 - p_x^{14})$ is multiplied by the sum of the proportions for the mild, moderate, severe, and total dependency categories, corresponding to the age group in **Table 1**.

p_x^{24} : 0, for $35 \le x \le 59$; $1.2\, p_x^{14}$, for $60 \le x \le 115$ where $0 < p_x^{14} < \frac{5}{6}$; 1, for $60 \le x \le 115$ where $\frac{5}{6} \le p_x^{14} < 1$, assumed $p_x^{24} > p_x^{14}$ and $0 < p_x^{24} \le 1$.

p_x^{23} : 0, for $35 \le x \le 59$; $0.5\, p_x^{12}$, for $60 \le x \le 115$ where $0 < p_x^{14} < \frac{5}{6}$; 0, for $60 \le x \le 115$ where $\frac{5}{6} \le p_x^{14} < 1$; assumed $p_x^{23} < p_x^{12}$ and $0 < p_x^{23} \le 1$.

p_x^{22} : 0, for $35 \le x \le 59$; $1 - p_x^{23} - p_x^{24}$, for $60 \le x \le 115$.

p_x^{34} : 0, for $35 \le x \le 59$; $1.1\, p_x^{14}$, for $60 \le x \le 115$ where $0 < p_x^{14} < \frac{10}{11}$; 1, for $60 \le x \le 115$ where $\frac{10}{11} \le p_x^{14} < 1$; assumed $p_x^{24} > p_x^{34} > p_x^{14}$ and $0 < p_x^{34} \le 1$.

p_x^{32} : 0, for $35 \le x \le 59$; $1.2\, p_x^{12}$, for $60 \le x \le 115$ where $0 < p_x^{14} < \frac{10}{11}$; 0, for $60 \le x \le 115$ where $\frac{10}{11} \le p_x^{14} < 1$; assumed $p_x^{32} > p_x^{12} > p_x^{23}$ and $0 < p_x^{32} \le 1$.

p_x^{33} : 0, for $35 \le x \le 59$; $1 - p_x^{32} - p_x^{34}$, for $60 \le x \le 115$.

Based on the assumptions, the one-year transition probability table is constructed for male individuals aged 35–115 years. Some of the data is presented in **Table 4** as follows:

Table 4. One-Year Transition Probability Table

Age (x)	p_x^{00}	p_x^{01}	p_x^{04}	p_x^{11}	p_x^{12}	p_x^{14}	p_x^{22}	p_x^{23}	p_x^{24}	p_x^{32}	p_x^{33}	p_x^{34}
35	0.9988	0	0.0012	0	0	0	0	0	0	0	0	0
36	0.9987	0	0.0013	0	0	0	0	0	0	0	0	0
37	0.9985	0	0.0014	0	0	0	0	0	0	0	0	0
⋮												
58	0.9864	0	0.0135	0	0	0	0	0	0	0	0	0
59	0	0.9857	0.0143	0	0	0	0	0	0	0	0	0
60	0	0	0	0.7908	0.1940	0.0152	0.8848	0.0970	0.0182	0.2328	0.7505	0.0167
⋮												
111	0	0	0	0.1097	0.1094	0.7809	0.0082	0.0547	0.9370	0.1313	0.0097	0.8589
112	0	0	0	0.0764	0.0763	0.8473	0	0	1	0.0680	0	0.9320
113	0	0	0	0.0401	0.0400	0.9199	0	0	1	0	0	1
114	0	0	0	0.0001	0.0001	0.9997	0	0	1	0	0	1
115	0	0	0	0	0	1	0	0	1	0	0	1

According to **Table 4**, it is shown that the transition from state 0 (active) to state 1 (retired) occurs exclusively from age 59 to 60, meaning that after age 60, no individuals remain in state 0. Additionally, no individuals transition into states 2 (dependent) or 3 (previously dependent) between ages 35 and 59, as these states are assumed to only occur after transitioning through state 1.

3.2 t-year Transition Probability

To calculate the net annual premium, it is necessary to determine the transition probabilities from state 0 to other states over t years to evaluate the actuarial present value of premium payments and insurance benefits. The t-year transition probabilities can be computed recursively using the Chapman–Kolmogorov equation [5], as described in Equation (7), based on the one-year transition probability table developed for the LTC insurance model.

$$ {}_{t+s}p_x^{ij} = \sum_{k=0}^{m} {}_{t}p_x^{ik} \, {}_{s}p_{x+t}^{kj}. \tag{7}$$

Using Equation (7) and the one-year transition probability data provided in Table 4, the probability of remaining in state 0 for t years, as well as the transition probabilities from state 0 to states 1, 2, and 3 over t years for an individual aged x, can be calculated respectively with Equation (8), Equation (9), Equation (10), and Equation (11).

$$ {}_{t}p_x^{00} = {}_{t-1}p_x^{00} \, p_{x+t-1}^{00}, \tag{8}$$

$$ {}_{t}p_x^{01} = {}_{t-1}p_x^{00} \, p_{x+t-1}^{01} + {}_{t-1}p_x^{01} \, p_{x+t-1}^{11}, \tag{9}$$

$$ {}_{t}p_x^{02} = {}_{t-1}p_x^{01} \, p_{x+t-1}^{12} + {}_{t-1}p_x^{02} \, p_{x+t-1}^{22} + {}_{t-1}p_x^{03} \, p_{x+t-1}^{32}, \tag{10}$$

$$ {}_{t}p_x^{03} = {}_{t-1}p_x^{02} \, p_{x+t-1}^{23} + {}_{t-1}p_x^{03} \, p_{x+t-1}^{33}. \tag{11}$$

3.3 Net Annual Premium Calculation

The net annual premium for LTC insurance under the established assumptions, for a male aged x who is currently in state 0 upon joining the LTC insurance program, can be calculated using the equivalence principle [5], as expressed in Equation (12).

$$ \text{APV of premium income} = \text{APV of benefit outgo}. \tag{12}$$

The APV of the net annual premium (P), paid annually at the beginning of each year until retirement at age 60 for an individual aged x, is formulated in Equation (13).

$$ \text{APV of premium income} = P \, \ddot{a}^{00}_{x:\overline{60-x}|} \tag{13}$$

The APV of the LTC insurance benefits is calculated as the sum of the APVs of benefits provided in states 1, 2, and 4, which formulated in Equation (14).

$$ \text{APV of benefit outgo} = B^{(1)} a_x^{01} + B^{(2)} a_x^{02} + B^{(4)} A_x^{04} \tag{14}$$

where

$$ a_x^{01} = \sum_{t=1}^{\infty} v^t \, {}_{t}p_x^{01} = \sum_{t=1}^{\infty} v^t \left({}_{t-1}p_x^{00} \, p_{x+t-1}^{01} + {}_{t-1}p_x^{01} \, p_{x+t-1}^{11} \right)$$

$$ a_x^{02} = \sum_{t=1}^{\infty} v^t \, {}_{t}p_x^{02} = \sum_{t=1}^{\infty} v^t \left({}_{t-1}p_x^{01} \, p_{x+t-1}^{12} + {}_{t-1}p_x^{02} \, p_{x+t-1}^{22} + {}_{t-1}p_x^{03} \, p_{x+t-1}^{32} \right)$$

and

$$A_x^{04} = \sum_{t=0}^{\infty} \sum_{k=0,k\neq j}^{4} v^{t+1} \ {}_tp_x^{0k} \ p_{x+t}^{k4} = \sum_{t=0}^{\infty} \sum_{k=0}^{3} v^{t+1} \ {}_tp_x^{0k} \ p_{x+t}^{k4}$$

$$= \sum_{t=0}^{\infty} v^{t+1} ({}_tp_x^{00} \ p_{x+t}^{04} + {}_tp_x^{01} \ p_{x+t}^{14} + {}_tp_x^{02} \ p_{x+t}^{24} + {}_tp_x^{03} \ p_{x+t}^{34})$$

It is important to note that, according to TMJ-22 data, males are not expected to reach the age of 116 years. Therefore, a limiting age of $x = 115$ years is applied in the calculation of the APV for lifetime annuities and lump sum benefits. Using the equivalence principle and the established assumptions, the net annual premium for LTC insurance expressed as **Equation (15)**.

$$P \ \ddot{a}_{x:\overline{60-x}|}^{00} = B^{(1)} a_x^{01} + B^{(2)} a_x^{02} + B^{(4)} A_x^{04}$$

$$P = \frac{B^{(1)} a_x^{01} + B^{(2)} a_x^{02} + B^{(4)} A_x^{04}}{\ddot{a}_{x:\overline{60-x}|}^{00}} \tag{15}$$

where $B^{(1)} =$ Rp30,000,000, $B^{(2)} =$ Rp50,000,000, dan $B^{(4)} =$ Rp60,000,000.

The calculated net annual premiums for LTC insurance, with entry ages (x) ranging from 35 to 50 years, are presented in **Table 5** and illustrated in **Figure 2**. The insured individual pays the annual premium at the beginning of each year while remaining in the healthy and actively employed state (state 0), continuing until reaching retirement age at 60.

Table 5. The Net Annual Premium for LTC Insurance

Entry Age	Net Annual Premium (IDR)	Entry Age	Net Annual Premium (IDR)
35	6,790,388	43	13,398,069
36	7,342,038	44	14,758,809
37	7,951,031	45	16,319,461
38	8,625,465	46	18,123,600
39	9,375,003	47	20,228,405
40	10,211,298	48	22,709,924
41	11,148,380	49	25,671,960
42	12,203,491	50	29,259,796

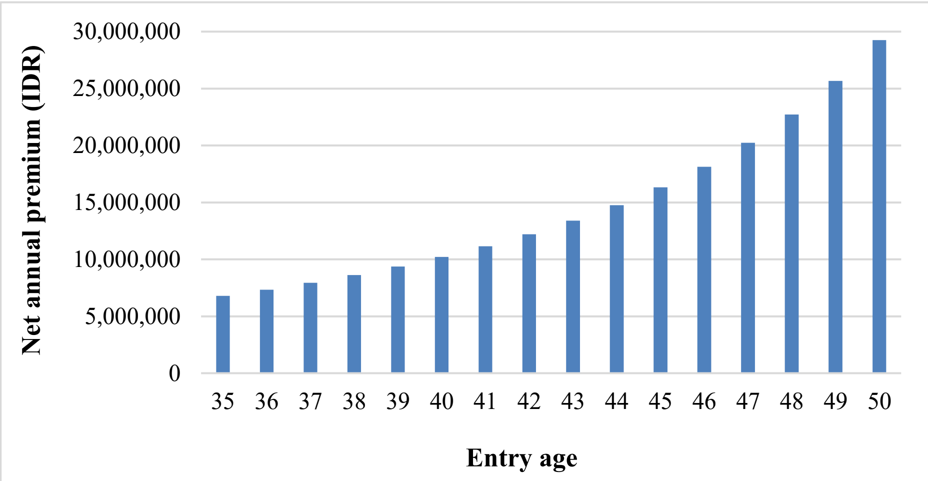

Figure 2. The Net Annual Premium for LTC Insurance

As observed in **Table 5** and **Figure 2**, the net annual premium increases significantly with the insured's age at the time of enrollment. Between ages 35 and 40, premiums increase by approximately 50.36%, followed by a 59.80% increase from age 40 to 45. The sharpest growth occurs between ages 45 and 50, with an increase of 79.26%. In addition, the year-over-year premium growth rate rises from 8.13% (age 35–36) to 13.97% (age 49–50), revealing a progressively steeper cost burden for late enrollees. This pattern results from the rising probabilities of developing ADLs dependency and mortality as individuals age. Consequently, the APVs of insurance benefits increase, while the premium payment period shortens, leading to higher annual premiums. These findings highlight the financial advantage of enrolling in LTC insurance at a younger age to secure lower premiums and manage long-term care costs more effectively.

3.4 Simulation of The Insurance Company's Cash Flow

A simulation of the insurance company's cash flow was conducted to evaluate the profits or losses generated from the sale of LTC insurance products. This study simulated the cash flow calculation under the assumption that 10,000 male policyholders, aged 35 years, enrolled in the LTC insurance program at the beginning of the insurance period. Before calculating the cash flow, a life table was simulated, showing the number of insured individuals in each state at every age, based on the one-year transition probability table. The cash flow calculation was subsequently performed using the life table. In the cash flow analysis, the company's revenue comprises premium income and investment returns at an annual effective interest rate, while expenses are limited to benefit payments for insurance claims.

The life table was constructed by generating random numbers that follow specific distributions corresponding to the transition probability table. For ages 35–60, there are only two possible transitions: remaining in the active state or transitioning to the dead state. Thus, the life table for this age range was developed by generating random numbers with a Bernoulli distribution, where the probability of success corresponds to the probability of remaining in the active state (p_x^{00}). While specifically, for the transition from age 59 to 60, the probability of transitioning from the active state to the retired state $\left(p_{59}^{01}\right)$ was applied. For ages 61–115, the life table was constructed using random numbers following a multinomial distribution, where the probabilities of each outcome align with the transition probabilities for each state in the model.

A sample output of the simulated life table, detailing the number of insured individuals in each state at every age, is presented in Table 6. The simulation assumes that 10,000 insured individuals aged 35 enrolled at the beginning of the insurance period $\left(l_{35}^{(0)}\right)$.

Table 6. The Generated LTC Insurance Life Table

Age (x)	$l_x^{(0)}$	$l_x^{(1)}$	$l_x^{(2)}$	$l_x^{(3)}$	$l_x^{(4)}$
35	10000	0	0	0	0
36	9990	0	0	0	10
37	9979	0	0	0	11
⋮					
59	8690	0	0	0	120
60	0	8564	0	0	126
61	0	6678	1746	0	140
62	0	5192	2910	173	149
⋮					
79	0	3	3412	1429	236
80	0	0	3235	1357	252
⋮					
106	0	0	9	1	4
107	0	0	7	0	3
⋮					
111	0	0	1	0	1
112	0	0	0	0	1
113	0	0	0	0	0
114	0	0	0	0	0
115	0	0	0	0	0

Table 6 indicates that no individuals are in states 1 (retired), 2 (dependent), or 3 (previously dependent) between the ages of 35 and 59, as the life table was constructed based on the transition probability table presented in Table 4. Upon reaching the retirement age of 60, those who remain in state 0 (active) transition to state 1 (retired), leaving no individuals in state 0 beyond age 60. After age 60, individuals are eligible to transition to state 2 (dependent). In this model, transitions to state 3 (previously dependent) are only possible from state 2. In this simulation results show that the last surviving individual reaches age 111 in state 2 and passes away at age 112.

Based on the life table in Table 6, the cash flow calculation over a period of 80 years from the start of the insurance term is presented in Table 7.

Table 7. LTC Insurance Cash Flow

t	Balance (IDR billion)	Income (IDR billion)	Expenditure (IDR billion)
0	0.00	67.90	0.00
1	71.38	67.84	0.60
2	146.91	67.76	0.66
⋮			
24	3,433.60	59.01	7.20
25	3,437.69	0.00	264.48
26	3,347.91	0.00	296.04
⋮			
76	1,446.87	0.00	0.11
77	1,533.62	0.00	0.06
78	1,625.64	0.00	0.00
79	1,723.18	0.00	0.00
80	1,826.57	0.00	0.00

In the LTC insurance cash flow calculation in Table 7, the net annual premium used is IDR 6,790,400, which corresponds to the net annual premium for an insured individual aged 35 years at the time of enrollment. The company's balance of cash is invested at an annual effective interest rate of 6.00%. The company's expenditures consist of insurance benefits for states 1, 2, and 4, as per the assumptions. Premium payments are made at the beginning of each year from the start of the insurance period until the insured reaches retirement age at $t = 24$. According to the life table in Table 6, the final expenditure occurs at $t = 77$, which is the death benefit payment for an insured individual who passes away at age 112, made at the end of the year of death. Table 7 shows that the company's balance continues to be invested at the effective annual interest rate until $t = 80$, although no further expenditures occur in the last three years. The company's final balance at $t = 80$ for the LTC insurance is IDR 1,826,570,949,140.

A simulation must be performed repeatedly to account for various possible scenarios, ensuring that the results are representative of real-world conditions [18]. In this study, 500 iterations of the cash flow calculations for the LTC insurance product were conducted. The average company balance over these 500 iterations was then calculated. The average company balance over an 80-year period from the start of the LTC insurance program, based on the 500 iterations, is illustrated in Figure 3.

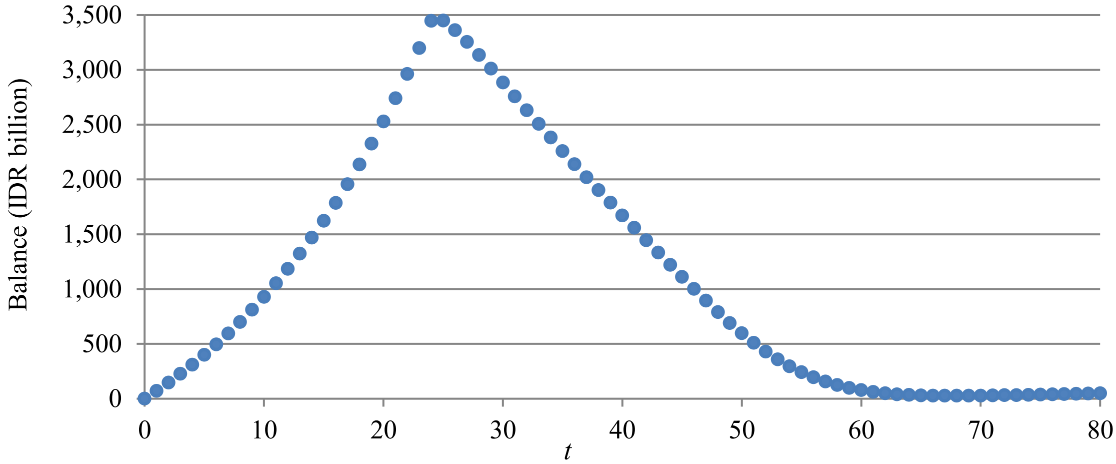

Figure 3. Average Company Balance for LTC Insurance Product

In **Figure 3**, the average company balance increases from $t = 0$ to $t = 25$, reaching its peak at IDR 3,449,762,281,129. After this point, the average balance decreases until $t = 67$, hitting its lowest value of IDR 26,238,461,900. This decline occurs because, during this period, the company relies solely on investment returns from its cash reserves at the annual effective interest rate, as premium payments ceased after $t = 24$. Subsequently, the average balance experiences a slight increase towards the end of the insurance period, with the final average balance recorded at IDR 48,955,391,056. This late-period increase is influenced by the reduced number of insured individuals receiving LTC benefits compared to the mid-period. Out of 500 simulations, not all cash flow calculations resulted in a positive final balance. A total of 244 simulations produced a negative final balance, indicating a 48.8% probability of the insurance company incurring a loss by the end of the insurance period.

4. CONCLUSION

This research establishes a practical actuarial framework for calculating net premiums and simulating cash flows for elderly long-term care (LTC) insurance in Indonesia using a five-state multiple state model. Unlike previous studies that focus on critical illness insurance, this model specifically addresses elderly care needs by capturing realistic health transitions, including recovery from dependency. The findings confirm that premium levels are sensitive to entry age and transition probabilities. Early enrollment leads to more affordable premiums, while later enrollment significantly increases costs due to higher risks of dependency and mortality, as well as shorter contribution periods. The cash flow simulation demonstrates that the LTC product can be financially sustainable when priced accurately and supported by prudent investment returns. The proposed model offers actionable insights for insurers developing LTC products and for policymakers aiming to protect Indonesia's aging population amid limited public health coverage.

AUTHOR CONTRIBUTIONS

Nadhira Fajri Aini: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Visualization, Writing - Original Draft, Writing - Review and Editing. Ruhiyat: Conceptualization, Data curation, Formal analysis, Funding acquisition, Methodology, Project administration, Resources, Supervision, Writing - Review and Editing. Windiani Erliana: Formal analysis, Supervision, Validation, Writing - Review and Editing. All authors discussed the results and contributed to the final manuscript.

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CONFLICT OF INTEREST

The authors declare no conflicts of interest regarding the publication of this paper.

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