BAREKENG: Journal of Mathematics and Its Applications
December 2025 Volume 19 Issue 4 Page 2467–2480
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol19iss4pp2467-2480

BIRESPONSE SPLINE TRUNCATED NONPARAMETRIC REGRESSION MODELING FOR LONGITUDINAL DATA ON MONTHLY STOCK PRICES OF THREE PRIVATE BANKS IN INDONESIA

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Article History:

Received: 1st January 2025
Revised: 6th April 2025
Accepted: 15th June 2025
Available online: 1st September 2025

Keywords:

Biresponse Modeling;
Longitudinal Data;
Nonparametric Regression;
Stock Prices in the Banking Sector;
Truncated Spline;
Weighted Least Squares.

ABSTRACT

This study investigates the application of a truncated spline nonparametric regression model for biresponse analysis of longitudinal data, focusing on modeling monthly stock prices specifically opening and closing prices of three private banks in Indonesia: Bank Mayapada, Bank Mega, and Bank Sinar Mas. The data used in this research are secondary data sourced from the website Id.Investing.com and monthly financial statement publications of three private banks in Indonesia. Longitudinal data, combining cross-sectional and time-series dimensions, are utilized to capture trends and patterns not detectable in traditional cross-sectional analysis. The truncated spline method is selected for its adaptability to nonlinear relationships and abrupt data behavior changes. The model incorporates three predictor variables traded stock volume, total assets, and total liabilities and evaluates their influence on stock prices. Assumptions of longitudinal data are validated using the Ljung-Box autocorrelation test, Bartlett's sphericity test, and Pearson correlation. Results confirm significant within-subject correlations, independence between subjects, and strong interdependence between response variables. The optimal configuration is determined using Generalized Cross Validation (GCV), with up to three knots considered for segmentation. Weighted Least Squares (WLS) is employed for parameter estimation, accounting for within-subject correlations. Model evaluation based on Mean Absolute Percentage Error (MAPE) indicates high accuracy, with all MAPE values below 5%. The highest MAPE value is 4.41% for the closing price of Bank Mayapada, while the lowest is 2.65% for the opening price of the same bank. The segmentation analysis reveals that traded stock volume and total assets positively influence stock prices, while total liabilities exhibit a predominantly negative impact. The model is limited to internal financial indicators and does not include external macroeconomic factors such as interest rates or inflation. This study is the first to apply a biresponse truncated spline nonparametric regression approach to analyze stock prices of private banks in Indonesia by simultaneously modeling both opening and closing prices, providing a flexible and effective method for capturing complex patterns in longitudinal financial data.

How to cite this article:

R. Pahlepi, I. Sriliana, W. Agwil and C. R. Oktarina., "BIRESPONSE SPLINE TRUNCATED NONPARAMETRIC REGRESSION MODELING FOR LONGITUDINAL DATA ON MONTHLY STOCK PRICES OF THREE PRIVATE BANKS IN INDONESIA," *BAREKENG: J. Math. & App.*, vol. 19, no. 4, pp. 2467-2480, December, 2025.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*
Research Article · Open Access

1. INTRODUCTION

Regression analysis is a statistical technique used to investigate and model the relationship between variables [1]. Regression analysis typically utilizes cross-sectional data but can also be applied to longitudinal data. Longitudinal data combines time series and cross-sectional data, encompassing observations from multiple subjects at various time points over a specific period [2]. Through longitudinal data analysis, trends or patterns that are not evident in cross-sectional data can be effectively captured [3].

Spline is one of the approaches used in nonparametric regression modeling. Other approaches in nonparametric regression modeling include Fourier Series, Kernel, and Wavelet. The primary advantage of spline is its ability to adapt to complex data distributions without assuming a specific functional form for the relationship between predictor and response variables [4]. By utilizing connecting points called knots, spline provides flexibility in capturing intricate or nonlinear data patterns [4][5]. This feature allows for more adaptive and data-driven modeling without being restricted to predetermined structures or patterns, as in parametric approaches [4]. Consequently, spline is often a preferred choice for nonparametric regression modeling as it can handle complex and unexpected variations in data [6].

Studies utilizing truncated splines in nonparametric regression have been widely applied across various fields. [7] examined the estimation of nonparametric regression models using truncated splines. [8] and [9] applied nonparametric regression with the spline approach in the health sector. [3] employed truncated splines in modeling poverty in districts/cities in Papua Province. Based on these studies, no research has yet applied truncated spline nonparametric regression to the case of biresponse modeling. To gain a comprehensive understanding of a problem, involving two response variables in regression modeling known as biresponse regression is necessary. Biresponse or multi-response modeling requires a strong and significant correlation between the response variables [10].

The study uses stock prices in the banking sector, including opening and closing prices, modeled with truncated spline nonparametric regression due to its adaptability to extreme price changes. Stock prices reflect investor views on a company's intrinsic value and are influenced by market dynamics and trading activities. Factors such as market capitalization, trading records, financial history, asset volume, and liabilities also affect stock price formation. Previous studies using truncated spline regression, such as [11], focused on single-response models and external factors like inflation and exchange rates. This research is the first to apply a biresponse truncated spline nonparametric regression approach to longitudinal data for modeling stock prices, by simultaneously considering two response variables opening and closing prices of three private banks in Indonesia. It expands the number of response variables and predictors with the aim of developing a more accurate regression model for predicting stock prices in the banking sector. To address the limitations of previous studies, which generally used only a single response variable and predictors derived from external factors, this study introduces a new approach by incorporating multiple response variables along with predictor variables that represent internal financial factors. This approach is expected to improve the model's precision in capturing the complex dynamics of stock prices. By leveraging the flexibility of truncated spline nonparametric regression, the model is able to accommodate nonlinear relationships and extreme variations often found in longitudinal financial data. The focus on three private banks in Indonesia provides a more comprehensive and contextual framework for understanding stock price dynamics under the influence of internal factors.

2. RESEARCH METHODS

2.1 Nonparametric Regression

Nonparametric regression is a statistical method to model the relationship between predictor variables and response variables assuming that the pattern of the relationship between predictor variables and response variables is unknown [4]. Nonparametric regression has high flexibility in modeling data patterns [6]. If the regression function or past information of the regression function is unknown, a nonparametric regression approach can be used. According to [5], the nonparametric regression approach can be applied to all forms of data distribution because it does not have to follow a certain pattern. [6] gives the general nonparametric regression equation as follows:

$$y_i = f(x_i) + \varepsilon_i, \qquad i = 1,2,\dots,n \tag{1}$$
$$\text{with } \varepsilon_i \sim IID\ N(0,\sigma^2)$$

The function $f(x_i)$ is a regression function whose relationship pattern is unknown and is assumed to be smooth and contained in a certain function space [4]. There are several functions used in nonparametric regression modeling. Some of the most popular are splines, kernels, Fourier series, wavelets, and MARS.

2.2 Longitudinal Data

The type of data in regression analysis is crucial in determining the direction of research. Typically, regression analysis uses cross-sectional data, which involves observations of multiple individuals at a single point in time [12]. Regression can also use longitudinal data, collected from subjects observed at multiple time points over a specific period [2]. In longitudinal data, observations within the same subject are dependent or correlated [13]. There are two correlation assumptions to address in biresponse regression modeling for longitudinal data: correlations between observations within the same subject and correlations between response variables [14]. The Ljung-Box autocorrelation test can identify correlations between observations within the same subject by checking for autocorrelation across lags. Meanwhile, correlations between response variables can be assessed using Pearson's correlation test. Additionally, according to [13], longitudinal data analysis is generally simpler because subjects are often assumed to be independent. The Bartlett's sphericity test can be applied to verify the independence assumption between subjects in longitudinal data.

1. Ljung-Box Autocorrelation Test: The Ljung-Box autocorrelation test is a statistical test used to evaluate whether autocorrelation exists in a time series dataset. Autocorrelation refers to the correlation between the same values in a time series at different time points [15].

   H_0: $\rho_1 = \rho_2 = \cdots = \rho_{(n-1)} = 0$
   H_1: There is at least one $\rho_c \neq 0$, $c = 1, 2, \cdots, (n-1)$

   Ljung-Box autocorrelation test statistics:

$$D = n(n+2)\sum_{c=1}^{n-1}\frac{r_c^2}{n-c} \tag{2}$$

   With:

$$r_C = \frac{\sum_{i=1}^{n-c}(y_{i+c}-\bar{y})(y_i-\bar{y})}{\sum_{i=1}^{n}(y_i-\bar{y})^2} \tag{3}$$

   The null hypothesis H_0 is rejected if $D > \chi^2_{\alpha;c}$, indicating that There is at least one $\rho_c \neq 0$.

2. Bartlett Sphericity Correlation Test: Bartlett's sphericity test is a statistical method to evaluate relationships among variables in multivariate cases [16]. For longitudinal data, it assesses whether subjects are independent by comparing the observed correlation matrix to an identity matrix, where deviations indicate interdependence among subjects.

   H_0: $\mathbf{R} = \mathbf{I}$ (between subjects are not correlated or independent)
   H_1: $\mathbf{R} \neq \mathbf{I}$ (between subjects are correlated or dependent)

   The test statistics used in the Barlett sphericity correlation test are:

$$\chi^2 = -\left(n - \frac{(2m+5)}{6}\right)\log_e|\mathbf{R}| \tag{4}$$

   With:

$$\mathbf{R} = \begin{pmatrix} \hat{\rho}_{11} & \hat{\rho}_{12} & \cdots & \hat{\rho}_{1m} \\ \hat{\rho}_{21} & \hat{\rho}_{22} & \cdots & \hat{\rho}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\rho}_{m1} & \hat{\rho}_{m2} & \cdots & \hat{\rho}_{mm} \end{pmatrix}, \quad \mathbf{I} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix}$$

   The null hypothesis is rejected if $\chi^2 > \chi^2_{\alpha;\frac{m(m-1)}{2}}$, indicating that between subjects are correlated.

Each of these tests serves a specific purpose: Ljung-Box evaluates within-subject autocorrelation, Bartlett tests inter-subject independence, and Pearson assesses correlation between multiple responses, all of which are critical in ensuring the validity of longitudinal modeling.

2.3 Spline Truncated

Splines are segmented polynomial pieces that have separate and continuous properties [2]. Splines have the advantage of overcoming changes in data patterns with the help of knot points, and the resulting curve is relatively smooth. A knot point is a point where there is a change in the pattern of data behavior of a function at a certain sub-interval [5]. In this study, the polynomial function and additional spline basis functions are limited to a linear order so that the linear order truncated spline function for one predictor can be expressed as follows:

$$\boldsymbol{f(x_i) = \theta_0 + \theta\, x_i + \sum_{g=1}^{G} \delta_g \left(x_i - K_g\right)_+ , g = 1, 2, \cdots, G} \tag{5}$$

Function $(x_i - K_g)_+$ is a truncated basis with

$$\left(x_i - K_g\right)_+ = \begin{cases} \left(x_i - K_g\right) & , x_i \geq K_g \\ 0 & , x_i < K_g \end{cases} \tag{6}$$

2.4 Truncated Spline Biresponse Nonparametric Regression on Longitudinal Data

In longitudinal data if there are j subjects and i observations in each subject with s predictor variables then the spline function can be defined as a function f and with g knot points and j subjects. If the truncated spline nonparametric regression modeling on longitudinal data is done biresponse, then according to [17] the truncated spline nonparametric regression model can be formulated as follows:

$$y_{kji} = \sum_{s=1}^{p} f_{kji}\left(x_{jsi}\right) + \varepsilon_{kji} \tag{7}$$

with

$$f_{kji}\left(x_{kji}\right) = \theta_0 + \sum_{s=1}^{p}\left(\theta_{kjs}x_{jsi} + \sum_{g=1}^{G}\delta_{kjsg}\left(x_{jsi} - K_{jsg}\right)_+\right) \tag{8}$$

with

$$\left(x_{jsi} - K_{gjs}\right)_+ = \begin{cases} \left(x_{jsi} - K_{gjs}\right) & , x_{jsi} \geq K_{gjs} \\ 0 & , x_{jsi} < K_{gjs} \end{cases} \tag{9}$$

This study applies the Weighted Least Squares (WLS) estimation method, which incorporates weights (V) into parameter estimation [18]. WLS accounts for correlations between observations within the same subject, a key characteristic of longitudinal data [13]. In biresponse regression, the correlation between multiple response variables is essential because it reflects underlying dependencies that, if ignored, may reduce the efficiency and accuracy of parameter estimates. By accounting for these correlations, the model better captures the joint variation in the responses, leading to more reliable inference and improved predictive performance. Parameters θ and δ in vector B are estimated by minimizing as outlined by [19].

$$\begin{aligned} \boldsymbol{Z} &= (\mathbf{y} - \hat{\mathbf{y}})^{\mathbf{T}}\boldsymbol{V}(\mathbf{y} - \hat{\mathbf{y}}) \\ &= (\mathbf{y} - \mathbf{X}(\mathbf{K})\mathbf{B})^{\mathbf{T}}\boldsymbol{V}(\mathbf{y} - \mathbf{X}(\mathbf{K})\mathbf{B}) \\ &= \left(\mathbf{y}^{\mathbf{T}} - \mathbf{B}^{\mathbf{T}}\mathbf{X}(\mathbf{K})^{\mathbf{T}}\right)(\boldsymbol{V}\mathbf{y} - \boldsymbol{V}\mathbf{X}(\mathbf{K})\mathbf{B}) \\ &= \mathbf{y}^{\mathbf{T}}\boldsymbol{V}\mathbf{y} - \mathbf{y}^{\mathbf{T}}\boldsymbol{V}\mathbf{X}(\mathbf{K})\,\mathbf{B} - \mathbf{B}^{\mathbf{T}}\mathbf{X}(\mathbf{K})^{\mathbf{T}}\boldsymbol{V}\mathbf{y} + \mathbf{B}^{\mathbf{T}}\mathbf{X}(\mathbf{K})^{\mathbf{T}}\boldsymbol{V}\mathbf{X}(\mathbf{K})\mathbf{B} \\ &= \mathbf{y}^{\mathbf{T}}\boldsymbol{V}\mathbf{y} - 2\mathbf{B}^{\mathbf{T}}\mathbf{X}(\mathbf{K})^{\mathbf{T}}\boldsymbol{V}\mathbf{y} + \mathbf{B}^{\mathbf{T}}\mathbf{X}(\mathbf{K})^{\mathbf{T}}\boldsymbol{V}\mathbf{X}(\mathbf{K})\mathbf{B} \end{aligned} \tag{10}$$

According to [20] the estimation of B is obtained from the partial derivative of the Z matrix equation in **Equation (10)**. The derivative of **Equation (10)** to B is as follows:

$$-2\mathbf{X}(\mathbf{K})^{\mathbf{T}}\boldsymbol{V}\mathbf{y} + 2\mathbf{X}(\mathbf{K})^{\mathbf{T}}\boldsymbol{V}\mathbf{X}(\mathbf{K})\hat{\mathbf{B}}$$

After obtaining the derivative of **Equation (10)** against B, then the derivative is equated to zero

$$-\mathbf{2X(K)^T}\boldsymbol{V}\mathbf{y}+\mathbf{2X(K)^T}\boldsymbol{V}\mathbf{X(K)}\widehat{\mathbf{B}}=\mathbf{0}$$

so that $\widehat{\mathrm{B}}$ will be obtained as follows:

$$\widehat{\mathbf{B}}=\left(\mathbf{X(K)^T}\boldsymbol{V}\,\mathbf{X(K)}\right)^{-\mathbf{1}}\mathbf{X(K)^T}\boldsymbol{V}\mathbf{y} \tag{11}$$

If truncated spline nonparametric regression modeling is performed using longitudinal data, then according to [2] the weight matrix is defined as follows:

$$\mathbf{V}=\mathrm{diag}(\boldsymbol{V}_{\mathbf{1}},\boldsymbol{V}_{\mathbf{2}},\ldots.,\boldsymbol{V}_{\mathbf{m}}) \tag{12}$$

There are three methods to determine the specification of the weight matrix (**V**), namely:

1. $\mathbf{V}=N^{-1}\mathbf{I}_n$ where $N=n\times m$ each observation is treated the same.
2. $\mathbf{V}=(n)^{-1}\mathbf{I}_n$, where each observation within the same subject is treated the same, according to the number of observations within the subject.
3. $\mathbf{V}=\mathbf{W}^{-\mathbf{1}}$ where $\mathbf{W}=\mathrm{Cov}(\boldsymbol{y})$ provided that the correlation of observations within the same subject is taken into account. **W** is a covariance matrix of size $(n\times m\times 2)\times(n\times m\times 2)$

In the truncated spline nonparametric regression model, the optimal location and number of knots must be selected. [16] stated that the Generalized Cross Validation (GCV) method is one approach developed to determine optimal knots. GCV is an extension of the Cross Validation (CV) method, differing in the factors used to divide residuals. The GCV function for biresponse truncated spline regression in longitudinal data, as described by [21], is as follows:

$$GCV(\mathrm{K})=\frac{MSE(\mathbf{K})}{\left[\frac{1}{2mn}\,trace\left(\mathbf{I}-\mathbf{A}(\mathbf{K})\right)\right]^{\mathbf{2}}} \tag{13}$$

With $MSE(\mathrm{K})=\frac{1}{2mn}\sum_{k=1}^{2}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(y_{kji}-\hat{y}_{kji}\right)^2$, and $\mathbf{A}(\mathbf{K})$ is a hat matrix or projection matrix obtained from the relationship $\hat{\mathrm{y}}=\mathbf{A}(\mathbf{K})\mathrm{y}$. The selection of the most optimal knot point with the GCV method is done by selecting the knot point that has the smallest GCV value [22].

2.5 Goodness of Model

Mean Absolute Percentage Error (MAPE) is one measure of model goodness that provides a clue as to how much the average absolute error of the estimated results is compared to the true value. The formula for calculating MAPE according to [23] is as follows:

$$\boldsymbol{MAPE}=\frac{\mathbf{1}}{\boldsymbol{n}}\sum_{\boldsymbol{i}=\mathbf{0}}^{\boldsymbol{n}}\left|\frac{\boldsymbol{y}_{\boldsymbol{i}}-\widehat{\boldsymbol{y}_{\boldsymbol{i}}}}{\boldsymbol{y}_{\boldsymbol{i}}}\right|\times\mathbf{100\%} \tag{14}$$

There is an analysis of the MAPE value which is interpreted with 4 categories, namely:

1. If the MAPE value is <10% then the model is categorized as very well used in estimating the response variable.
2. If the MAPE value is between 10% and 20% then the model is categorized as good to use in estimating the response variable.
3. If the MAPE value is between 20% and 50%, the model is categorized as good to use in estimating the response variable.
4. If the MAPE value is > 50% then the model is categorized as not good to use in estimating the response variable.

3. RESULTS AND DISCUSSION

3.1 Data and Analysis

This study uses secondary data sourced from *id.investing.com* and monthly financial reports of three private banks in Indonesia Bank Mayapada (MAYA), Bank Mega (MEGA), and Bank Sinar Mas (BSIM) with stock price observations from October 2021 to August 2023. The response variables are closing stock price (Y_1) and opening stock price (Y_2), while the predictor variables include traded stock volume (X_1), total assets (X_2), and total liabilities (X_3). Closing stock price refers to the last trading price at market close, opening stock price is the first price recorded at market opening, traded stock volume represents the total shares traded during a period, total assets denote the bank's resources, and total liabilities represent financial obligations.

1. Testing Longitudinal Data Assumptions
   a. Test correlation within the same subject using Ljung-Box autocorrelation.
   b. Test independence between subjects using Bartlett's sphericity.
   c. Test correlation between response variables using Pearson correlation.
2. Data Exploration and Visualization
   a. Perform descriptive statistics.
   b. Use scatter plots to examine relationships between response and predictor variables.
3. Constructing a General Biresponse Nonparametric Regression Model
4. Model Estimation Using WLS
   a. Define weights for the model.
   b. Estimate the model parameters.
   c. Optimal Knot Selection by minimizing GCV values.
5. Final Model Construction
6. Model Visualization & Model Segmentation
7. Interpret segmented results for each subject.
8. Evaluate model accuracy using MAPE.

3.2 Assumption Test of Longitudinal Biresponse Data

The assumption tests for longitudinal biresponse data ensure the validity of the analysis. These tests include evaluating within-subject correlation using the Ljung-Box autocorrelation test, assessing between-subject independence with Bartlett's sphericity test, and examining the correlation between response variables using Pearson correlation. These steps are crucial to confirm that the data meet the requirements for accurate modeling and reliable interpretation.

Table 1. Longitudinal Data Assumptions

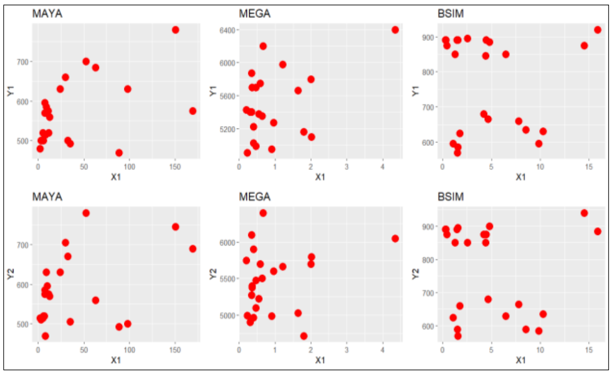
Test	Subject/Response	Statistic	p-value	Conclusion
Autocorrelation (y_1)	MAYA	5.711	0.017	Significant
Autocorrelation (y_1)	MEGA	8.256	0.004	Significant
Autocorrelation (y_1)	BSIM	16.193	5.72×10^{-5}	Significant
Autocorrelation (y_2)	MAYA	8.708	0.003	Significant
Autocorrelation (y_2)	MEGA	8.755	0.003	Significant
Autocorrelation (y_2)	BSIM	16.129	5.92×10^{-5}	Significant
Independence (y_1)	y_1	8.355	0.039	Independent
Independence (y_2)	y_2	10.659	0.014	Independent
Correlation (ρ)	MAYA	0.699	0.0002	Significant
Correlation (ρ)	MEGA	0.952	8.19×10^{-13}	Significant

Test	Subject/Response	Statistic	p-value	Conclusion
Correlation (ρ)	BSIM	0.997	2.20×10^{-16}	Significant
Correlation (ρ)	Overall	0.996	2.20×10^{-16}	Significant

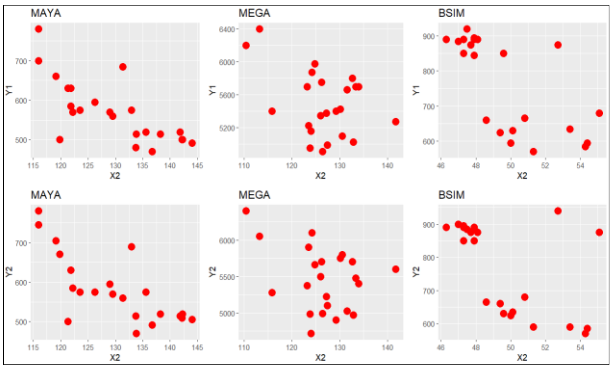
The assumption test results confirm that the dataset is suitable for longitudinal biresponse modeling. Significant autocorrelation across all subjects and responses indicates temporal dependence within subjects, justifying a longitudinal approach. The non-significant independence tests suggest that subjects can be considered independent, simplifying the model structure. Additionally, the strong and significant correlations between the two responses support the use of a biresponse model, as modeling them jointly can better capture their interdependence and improve estimation accuracy.

3.3 Visualization of The Relationship Between Predictor Variables and Response Variables

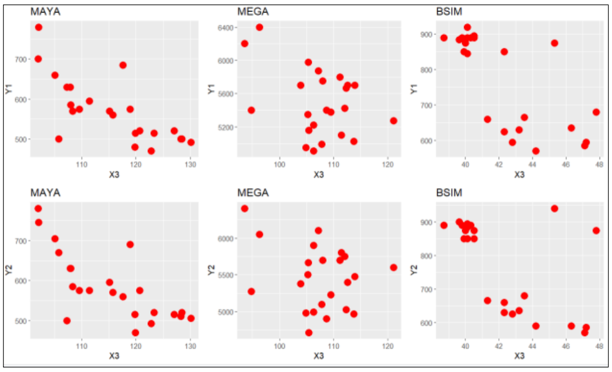
Furthermore, it is necessary to conduct a preliminary study and explore how the data spreads between predictor variables and response variables. The following is a visualization of the relationship between predictor variables and response variables for each subject.



(a)



(b)



(c)

Figure 1. Relationship between Predictor Variables and Response Variable for Each Subject (a) Trading Volume, (b) Total Assets, (c) Total Liabilities

Based on the visualizations of the relationship between predictor variables (X_1, X_2, X_3) and response variables (Y_1, Y_2) across subjects (MAYA, MEGA, BSIM), it is evident that the patterns of relationships are not easily identifiable. The scatter plots do not clearly indicate linear, quadratic, or other specific parametric trends. This supports the use of nonparametric regression methods, such as truncated spline regression, which are more suitable for capturing complex and undefined relationships in the data.

3.4 General Model of Truncated Spline Biresponse Nonparametric Regression

Based on **Equation (8)**, the general model of biresponse spline truncated nonparametric regression for longitudinal data with k responses, j subjects, and 3 predictor variables using 1, 2, and 3 knot points with $k = 1,2$, $j = 1,2,3$, and $i = 1,2,\cdots,n$ can be written as follows:

Model for 1 knot point

$$y_{kji} = \sum_{s=1}^{3} f_{kji}(x_{jsi}) + \varepsilon_{11i}$$
$$= \theta_0 + \theta_{kj1}x_{j1i} + \delta_{kj11}(x_{j1i} - K_{j11})_+ + \theta_{kj2}x_{j2i} + \delta_{kj21}(x_{j2i} - K_{j21})_+ + \theta_{kj3}x_{j3i} + \delta_{kj31}(x_{j3i} - K_{j31})_+ + \varepsilon_{kji} \quad (15)$$

Model for 2 knot points

$$y_{kji} = \sum_{s=1}^{3} f_{kji}(x_{jsi}) + \varepsilon_{kji}$$
$$= \theta_0 + \theta_{kj1}x_{j1i} + \delta_{kj11}(x_{j1i} - K_{j11})_+ + \delta_{kj12}(x_{j1i} - K_{j12})_+ + \theta_{kj2}x_{j2i} + \delta_{kj21}(x_{j2i} - K_{j21})_+ + \delta_{kj22}(x_{j2i} - K_{j22})_+ + \theta_{kj3}x_{j3i} + \delta_{kj31}(x_{j3i} - K_{j31})_+ + \delta_{kj32}(x_{j3i} - K_{j32})_+ + \varepsilon_{kji} \quad (16)$$

Model for 3 knot points

$$y_{kji} = \sum_{s=1}^{3} f_{kji}(x_{jsi}) + \varepsilon_{kji}$$
$$= \theta_0 + \theta_{kj1}x_{j1i} + \delta_{kj11}(x_{j1i} - K_{j11})_+ + \delta_{kj12}(x_{j1i} - K_{j12})_+ + \delta_{kj13}(x_{j1i} - K_{j13})_+ + \theta_{kj2}x_{j2i} + \delta_{kj21}(x_{j2i} - K_{j21})_+ + \delta_{kj22}(x_{j2i} - K_{j22})_+ + \delta_{kj23}(x_{j2i} - K_{j23})_+ + \theta_{kj3}x_{j3i} + \delta_{kj31}(x_{j3i} - K_{j31})_+ + \delta_{kj32}(x_{j3i} - K_{j32})_+ + \delta_{kj33}(x_{j3i} - K_{j33})_+ + \varepsilon_{kji} \quad (17)$$

3.5 Estimation of Biresponse Spline Truncated Nonparametric Regression Model on Longitudinal Data

Parameter estimation of the biresponse spline truncated nonparametric regression model in this study was carried out using the Weighted Least Square (WLS) method, which involves a weighting matrix in estimating the model parameters. In this study, the weighting matrix used is $\mathbf{V} = \mathbf{W}^{-1}$ where $\mathbf{W}$ is the covariance variance matrix of the response variable. or the first response variable (Y_1) of the first subject (MAYA), the weighting matrix is specifically derived based on the covariance structure of Y_1 for subject MAYA, ensuring accurate parameter estimation for this specific subset of the data.

$$\mathbf{W}_{111} = \begin{bmatrix} 6106.36 & 0 & \cdots & 0 \\ 0 & 6106.36 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 6106.36 \end{bmatrix} \quad (18)$$

Truncated spline nonparametric regression utilizes knots, points where changes in data patterns occur. These knots divide the plot between response and predictor variables into segments to better estimate the response variable according to its data distribution. Optimal knot location and number are determined by minimizing the GCV value, ensuring the estimation closely aligns with the original data.

Table 2. Knot Point with the Smallest GCV

	Subject	Knot			GCV
		x_1	x_2	x_3	
1 Knot	MAYA	144.725	134.857	120.924	2.96×10^{-6}
	MEGA	3.002	131.547	112.120	
	BSIM	10.822	52.227	44.829	
2 Knot	MAYA	70.376	127.429	113.469	1.101×10^{-6}
		107.902	133.714	129.778	
	MEGA	1.898	123.216	104.824	
		2.832	130.265	110.998	

	Subject	Knot			GCV
		x_1	x_2	x_3	
	BSIM	6.679	49.892	42.414	
		10.184	51.867	44.457	
3 Knot	MAYA	73.787	128	114.043	4.56×10^{-7}
		80.61	129.143	115.189	
		90.845	130.857	116.91	
	MEGA	1.983	123.857	105.386	
		2.152	125.139	106.508	
		2.407	127.061	108.192	
	BSIM	6.998	50.071	42.6	
		7.635	50.431	42.971	
		8.591	50.969	43.528	

Based on **Table 2**, it is known that the smallest GCV value for the most optimal knot point combination in modeling with 3 knots is equal to 4.56×10^{-7}. GCV results for all knot point combinations used in modeling with 3 knots. Thus, parameter estimation results for the three private banks using 3 knots are obtained as follows:

Table 3. Parameter Estimation

MAYA		MEGA		BSIM	
y_1	y_2	y_1	y_2	y_1	y_2
957.57	1998.19	13630.34	16550.42	2594.61	4167.79
1.61	1.10	3.40	-317.91	13.06	4.47
51.83	29.74	-149.77	-272.96	52.21	8.61
-62.22	-46.52	96.95	215.81	-105.78	-92.99
-70.44	54.32	-5513.66	31905.67	-380.82	-201.43
31.44	67.71	738.88	875.49	-1918.63	-2631.16
37.99	34.32	-247.24	23.05	-486.75	7.94
110.42	-115.13	3425.61	-19020.99	621.64	296.66
-11.85	-2.03	-1396.83	-1521.63	3355.98	4694.90
-72.72	-134.39	788.82	64.77	1048.95	519.15
-41.16	63.64	3030.35	-16826.26	-255.93	-97.61
-63.13	-60.87	918.85	925.72	-670.27	-787.68
84.91	110.58	-762.93	-261.11	-1308.18	-1736.97

3.6 Spline Truncated Nonparametric Regression Model with Optimal Knot Points

After obtaining the optimal knot points to be used in modeling, the next step is to estimate the parameters and form a model with the most optimal knot point combination. Parameter estimation results for each model (response) on each subject:

The model formed for the MAYA subject is:

$$\begin{aligned}\hat{y}_{11i} = {} & 957.57 + 1.61x_{11i} - 70.44(x_{11i} - 73.787)_+ + 31.44(x_{11i} - 80.61)_+ \\ & + 37.99(x_{11i} - 90.845)_+ + 51.83x_{12i} + 110.42(x_{12i} - 128)_+ \\ & - 11.85(x_{12i} - 129.143)_+ - 72.72(x_{12i} - 130.857)_+ - 62.22x_{13i} \\ & - 41.16(x_{13i} - 114.043)_+ - 63.13(x_{13i} - 115.189)_+ + 84.91(x_{13i} - 116.91)_+\end{aligned}$$

$$\begin{aligned}\hat{y}_{21i} = \ & 1998.19 + 1.10x_{11i} + 54.32(x_{11i} - 73.787)_+ + 67.71(x_{11i} - 80.61)_+ \\ & + 34.32(x_{11i} - 90.845)_+ + 29.74x_{12i} - 115.13(x_{12i} - 128)_+ \\ & - 2.03(x_{12i} - 129.143)_+ - 134.39(x_{12i} - 130.857)_+ - 46.52x_{13i} \\ & + 63.64(x_{13i} - 114.043)_+ - 60.87(x_{13i} - 115.189)_+ \\ & + 110.58(x_{13i} - 116.91)_+\end{aligned}$$

The model formed for MEGA subject is:

$$\begin{aligned}\hat{y}_{12i} = \ & 13630.34 + 3.40x_{21i} - 5513.66(x_{21i} - 1.983)_+ + 738.88(x_{21i} - 2.152)_+ \\ & - 247.24(x_{21i} - 2.407)_+ - 149.77x_{22i} + 3425.61(x_{22i} - 123.857)_+ \\ & - 1396.83(x_{22i} - 125.139)_+ + 788.82(x_{22i} - 127.061)_+ + 96.95x_{23i} \\ & + 3030.35(x_{23i} - 105.386)_+ + 918.85(x_{23i} - 106.508)_+ \\ & - 762.93(x_{23i} - 108.192)_+\end{aligned}$$

$$\begin{aligned}\hat{y}_{22i} = \ & 16550.42 - 317.91x_{21i} + 31905.67(x_{21i} - 1.983)_+ + 875.49(x_{21i} - 2.152)_+ + 23.05 \\ & - 272.96x_{22i} - 19020.99(x_{22i} - 123.857)_+ - 1521.63(x_{22i} - 125.139)_+ \\ & + 64.77(x_{22i} - 127.061)_+ + 215.81x_{23i} - 16826.26(x_{23i} - 105.386)_+ \\ & + 925.72(x_{23i} - 106.508)_+ - 261.11(x_{23i} - 108.192)_+\end{aligned}$$

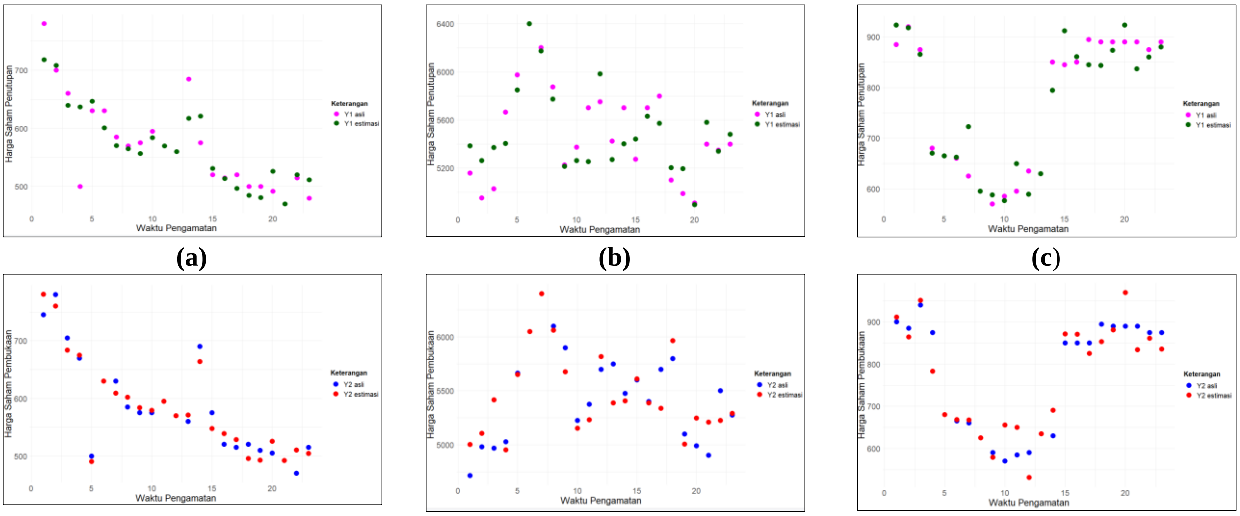
The model formed for the BSIM subject is:

$$\begin{aligned}\hat{y}_{13i} = \ & 2594.61 + 13.06x_{31i} \\ & - 380.82(x_{31i} - 6.998)_+ - 1918.63(x_{31i} - 7.635)_+ - 486.75(x_{31i} - 8.591)_+ \\ & + 52.21x_{32i} + 621.64(x_{32i} - 50.071)_+ + 3355.98(x_{32i} - 50.431)_+ \\ & + 1048.95(x_{32i} - 50.969)_+ - 105.78x_{33i} - 255.93(x_{33i} - 42.6)_+ \\ & - 670.27(x_{33i} - 42.971)_+ - 1308.18(x_{33i} - 43.528)_+\end{aligned}$$

$$\begin{aligned}\hat{y}_{23i} = \ & 4167.79 + 4.47x_{31i} \\ & - 201.43(x_{31i} - 6.998)_+ - 2631.16(x_{31i} - 7.635)_+ + 7.94(x_{31i} - 8.591)_+ \\ & + 8.61x_{32i} + 296.66(x_{32i} - 50.071)_+ + 4694.90(x_{32i} - 50.431)_+ \\ & + 519.15(x_{32i} - 50.969)_+ - 92.99x_{33i} - 97.61(x_{33i} - 42.6)_+ \\ & - 787.68(x_{33i} - 42.971)_+ - 1736.97(x_{33i} - 43.528)_+\end{aligned}$$

3.7 Model Visualization and Model Segmentation

After obtaining the truncated spline nonparametric regression model for each response and each subject, the value of the response variable can then be estimated. The following is a comparison of the estimated response variable value and the real value of the response variable:



(a) (b) (c)

(d) (e) (f)

Figure 2. Comparison Between Estimated Y Function Outputs and Actual Data
(a) Y_1 MAYA, (b) Y_1 MEGA, (c) Y_1 BSIM, (d) Y_2 MAYA, (e) Y_2 MEGA, (f) Y_2 BSIM

After modeling, the resulting model is used to predict how much each predictor variable affects the formation of closing stock prices and opening stock prices of three private banks in Indonesia at certain intervals through segmentation of the model that has been formed for each predictor variable. Model formed for Bank Mayapada:

$$\begin{aligned}\hat{y}_{11i} = {} & 957.57 + 1.61x_{11i} - 70.44(x_{11i} - 73.787)_+ + 31.44(x_{11i} - 80.61)_+ + 37.99(x_{11i} - 90.845)_+ \\ & + 51.83x_{12i} + 110.42(x_{12i} - 128)_+ - 11.85(x_{12i} - 129.143)_+ \\ & - 72.72(x_{12i} - 130.857)_+ - 62.22x_{13i} - 41.16(x_{13i} - 114.043)_+ \\ & - 63.13(x_{13i} - 115.189)_+ + 84.91(x_{13i} - 116.91)_+ \\ \hat{y}_{21i} = {} & 1998.19 + 1.10x_{11i} + 54.32(x_{11i} - 73.787)_+ + 67.71(x_{11i} - 80.61)_+ + 34.32(x_{11i} - 90.845)_+ \\ & + 29.74x_{12i} - 115.13(x_{12i} - 128)_+ - 2.03(x_{12i} - 129.143)_+ \\ & - 134.39(x_{12i} - 130.857)_+ - 46.52x_{13i} + 63.64(x_{13i} - 114.043)_+ \\ & - 60.87(x_{13i} - 115.189)_+ + 110.58(x_{13i} - 116.91)_+\end{aligned}$$

The following are the results of model segmentation for the predictor variable x_1 (Volume of shares traded) with the assumption that other variables are constant:

$$\hat{y}_{11i} = \begin{cases} 957.57 + 1.61x_{11i} & ; \quad x_{11i} \le 73.787 \\ 5197.56 - 68.83x_{11i} & ; \quad 73.787 < x_{11i} \le 80.61 \\ -2534.37 - 37.39x_{11i} & ; \quad 80.61 < x_{11i} \le 90.845 \\ -3451.21 + 0.6x_{11i} & ; \quad x_{11i} > 90.845 \end{cases}$$

$$\hat{y}_{21i} = \begin{cases} 1998.19 + 1.10x_{11i} & ; \quad x_{11i} \le 73.787 \\ -4008.11 + 55.42x_{11i} & ; \quad 73.787 < x_{11i} \le 80.61 \\ -5216.27 + 123.13x_{11i} & ; \quad 80.61 < x_{11i} \le 90.845 \\ -3117.8 + 157.45x_{11i} & ; \quad x_{11i} > 90.845 \end{cases}$$

3.8 Interpretation Segmented Results for Each Subject

Segmentation in this analysis plays a crucial role in identifying differential effects of trading volume on stock prices across specific intervals. By applying a truncated spline regression approach, the model captures nonlinear relationships between the explanatory variable (trading volume) and response variables (closing and opening prices). As an illustration, the following segmented interpretation is presented specifically for Bank Mayapada to demonstrate how segment-wise relationships are interpreted. For the remaining banks (Bank Mega and Bank Sinarmas) and other variables, the interpretation approach remains consistent guided by the same knot placement and segment-specific regression coefficients:

1. Volume ≤ 73.787 million shares: At relatively low trading volumes (less than or equal to 73.787 million shares), an increase of 1 million shares tends to raise the closing price by 1.61 Rupiah and also leads to an increase in the opening price in the following month by 1.10 Rupiah. This indicates that at lower volumes, the market responds positively to increased trading activity, which is reflected in both immediate and delayed price increases.
2. Volume between 73.787 and 80.61 million shares: In this range, a 1 million share increase tends to decrease the closing price by 68.83 Rupiah, while at the same time raises the opening price in the next month by 55.42 Rupiah. This pattern shows a shift in market reaction: although there may be short-term selling pressure (reflected in the falling closing price), optimism remains, which is evident in the strong rise in the opening price.
3. Volume between 80.61 and 90.845 million shares: For volumes within this range, a 1 million share increase lowers the closing price by 37.39 Rupiah but significantly raises the opening price in the following month by 123.13 Rupiah. This suggests that despite considerable closing-time selling pressure, market sentiment heading into the next period is very strong and positive.
4. Volume > 90.845 million shares: At very high trading volumes (greater than 90.845 million shares), a 1 million share increase causes a modest increase in the closing price by 0.6 Rupiah, while the opening price jumps significantly by 157.45 Rupiah in the following month. This shows that when trading volume is very high, the market tends to stabilize at closing, but there is strong optimism during the next opening possibly due to high expectations or positive market news.

These segmented insights offer practical value for investors and portfolio managers. By understanding how trading volume affects stock prices differently across intervals, investors can better time their buy and sell decisions. For instance, identifying volume thresholds that signal strong opening price surges may guide short-term investment strategies, while recognizing patterns of market correction during high volume can help in anticipating stabilization. Thus, the segmented model not only enhances prediction but also provides actionable information for market participants.

3.9 Evaluate Model Accuracy using MAPE

Based on **Equation (14)**, it is known that there are six spline truncated nonparametric regression models formed from modeling by utilizing longitudinal biresponse data, where the subjects used consist of three private banks in Indonesia. The MAPE value for each model can be seen in **Table 4**.

Table 4. Evaluate Model

Subject	Response	MAPE
MAYA	y_1	4.41%
	y_2	2.65%
MEGA	y_1	2.96%
	y_2	2.85%
BSIM	y_1	3.68%
	y_2	4.31%

The results indicate that six truncated spline nonparametric regression models were successfully constructed using longitudinal biresponse data from three private banks in Indonesia. The MAPE values for all models are below 5%, with the highest being 4,41% for y_1 (closing price) in the MAYA subject and the lowest being 2,65% for y_2 (opening price) in the same subject. These low MAPE values demonstrate that the models have a high level of accuracy in estimating both opening and closing stock prices for all three banks. This confirms the effectiveness of the truncated spline nonparametric regression approach in modeling longitudinal stock price data.

4. CONCLUSION

The best truncated spline nonparametric biresponse regression model for the longitudinal data of monthly stock prices from three private banks in Indonesia uses three optimal knots with a GCV value of 4.56×10^{-7}. The MAPE for all models across response variables and subjects is less than 10%, indicating high accuracy in estimating both closing and opening stock prices. The segmentation results show that the traded stock volume (x_1) generally has a positive influence on stock prices across most segments. The total assets (x_2) positively affect stock price formation, particularly for opening prices, while total liabilities (x_3) mostly have a negative impact on stock prices. However, this study is limited to internal financial indicators, and does not yet incorporate external macroeconomic factors such as interest rates or inflation, which may also influence stock price dynamics.

AUTHOR CONTRIBUTIONS

Reza Pahlepi: Data Curation, Formal Analysis, Visualization. Idhia Srilana: Conceptualization, Methodology, Supervision, Writing – Original Draft. Winalia Agwil: Investigation, Validation, Project Administration. Cinta Rizki Oktarina: Software, Resources, Writing – Review and Editing. All authors discussed the results and contributed to the final manuscript.

FUNDING STATEMENT

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

ACKNOWLEDGMENT

I would like to express my deepest gratitude to my fellow "spline" companions, who have shared this challenging yet rewarding journey with me. Your unwavering support, collaboration, and encouragement have been an immense source of motivation. Thank you all for making this journey memorable and enriching.

CONFLICT OF INTEREST

The authors declare no conflicts of interest to report study.

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