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MODELING TOTAL FERTILITY RATE IN INDONESIA: A COMPARISON OF FOURIER SERIES REGRESSION AND ELASTIC NET REGRESSION

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ABSTRACT

The Total Fertility Rate (TFR) describes population growth and socioeconomic development of a country. This statistic plays an important role in predicting future social and economic conditions. Indonesia has experienced a steady decline in TFR over the past few decades, which can be a serious problem if this trend continues. Therefore, the factor influencing the decline must be found. The independent variables include the percentage of women graduating high school, percentage of the poor population, poverty gap index, poverty severity index, prevalence of inadequate food consumption, proportion of people living below 50 percent of median income, unemployment rate, infant mortality rate, child mortality rate, and percentage of ever-married women aged 15–49 years using contraception methods. The aim of this study is to compare both Fourier Series Regression and Elastic Net Regression models to see which approximation can capture the TRF phenomenon that occurs in Indonesia and identify the causes of its decline. Fourier Regression is chosen because there is a repetition of patterns in several variables. Moreover, this data is experiencing multicollinearity; hence, Elastic-net Regression is the best way because this method overcomes the limitations of each Ridge and Lasso approach. These models are compared to see which is more suitable to capture the relationships between these factors and TFR. The best model obtained will provide a clearer understanding of Indonesia's underlying drivers of fertility decline. The result is that the Fourier Series Regression can model all variables better than the Elastic-net Regression, and the independent variables can explain the proportion of variance in the dependent variables by 97.91%, with all the independent variables significantly affecting the Total Fertility Rate.

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1. INTRODUCTION

The Total Fertility Rate (TFR) is a demographic indicator that measures the average number of children a woman would have if she experienced current age-specific fertility rates throughout her reproductive years [1]. TFR illustrates the population dynamics and demographic changes that help in determining whether a population is growing, shrinking, or remaining stable. A TFR of about 2.1 children per woman is considered the replacement rate in many countries [2][3]. A TFR below this threshold can lead to population decline. Based on the population census by the Indonesian Central Bureau of Statistics (BPS), the TFR in Indonesia tends to decrease over the years. It can be seen from the graph below.

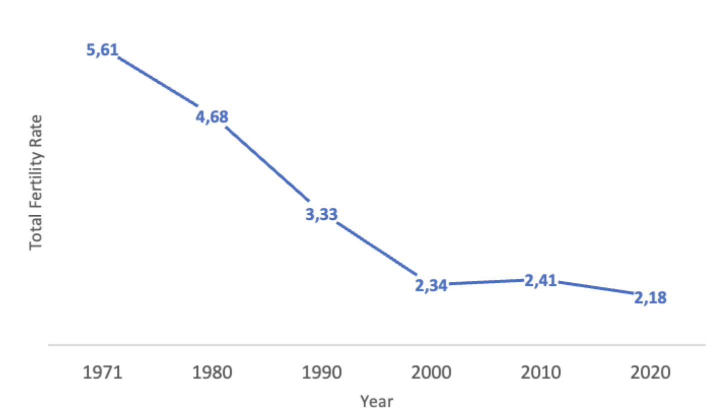


Figure 1. Total Fertility Rate in Indonesia

Based on Figure 1, it can be seen that the TFR in Indonesia declined from 5.61 in 1971 to 2.18 in 2020. Indonesia's TFR is currently only 0.08 points above the established threshold. If this pattern continues, it will become a problem. The same condition applied both in developed and developing countries [4]. The decline in TFR presents both opportunities and challenges, such as changes in labor force dynamics, aging populations, and pressures on social support systems. Therefore, finding the factors causing the decline in the TFR needs to be done.

The TFR is influenced by many aspects such as social, technological, environmental, economic, political, and health [5]. This study will focus on socio-economic factors. One of the most interesting factors is the educational attainment of women. Research shows that higher education among women is associated with lower fertility rates. Women with higher education tend to have fewer children and delay childbearing [6], [7], [8]. Besides education, economy, religion, and contraception have a correlation with TFR [9]. Moreover, several studies show a negative association between infant mortality rate and fertility [10], [11]. The same pattern is found in child mortality rate and fertility [12]. Hence, the factors examined in this paper are the percentage of women graduating high school, percentage of the poor population, poverty gap index, poverty severity index, prevalence of inadequate food consumption, proportion of people living below 50 percent of median income, unemployment rate, infant mortality rate, child mortality rate, and percentage of ever-married women aged 15–49 years using contraception methods.

Previous studies have explored these relationships using correlation or traditional regression models. However, traditional regression models are often limited in their ability to capture non-linear relationships between these factors and fertility rates. Whereas, capturing the complex, potentially non-linear relationships among these variables requires more flexible approaches. Nonparametric regression methods can be the solution because of their flexibility. One of the approaches in nonparametric regression is the Fourier Series Regression. This method provides greater flexibility in modeling complex, cyclical patterns, which can be particularly useful when dealing with fertility data that may exhibit seasonal or periodic trends [13].

Fourier Series Regression is a nonparametric approach that utilizes trigonometric functions to model periodic data or data with a repeatable pattern [14], [15]. As a nonparametric regression approach, Fourier Series Regression will produce a better estimation curve than parametric and is free of assumptions about its form [16], [17]. This method does not assume a specific functional form for the relationship between variables, allowing it to adapt to the underlying characteristics of the data. In contrast, Parametric Linear Regression relies on a predefined linear relationship between independent and dependent variables, which may not adequately represent complex data structures. Parametric regression is also strict with the assumption

[18]. Its reliance on the assumption of linearity can lead to biased results if the true relationship is nonlinear. Furthermore, Linear Regression can be sensitive to outliers and may not perform well when the data does not meet the assumptions [19], [20]. In this case, the data have multicollinearity. One of the best ways to overcome this situation is by using elastic-net.

Elastic net is one of the penalization techniques that have been proposed in order to improve Ordinary Least Squares (OLS). This method is a regularization method for linear regression models that combines the properties of Ridge Regression and Lasso Regression. This method was developed to overcome the limitations of each Ridge and Lasso approach, especially in data with a large number of variables, high multicollinearity, or highly correlated features. When variables are correlated, there is a tendency for them to enter or exit the model simultaneously, so this condition must be overcome [21], [22]. Based on previous research, it was proven that Elastic Net is a powerful technique to address multicollinearity and perform variable selection in high-dimensional data [23],[24]. Elastic net has been applied in various fields such as medical [25], social science [26], and psychology [22]. Therefore, the study aims to apply a nonparametric regression approach using the Fourier series and Elastic net Regression to model the relationship between TFR and explanatory variables. By comparing both approaches of regression, the research results are expected to provide insights into the factors driving Indonesia's fertility decline.

2. RESEARCH METHODS

This study applied Fourier Series Regression and Elastic Net Regression in modeling the Total Fertility Rate in Indonesia with several factors.

2.1 Data and Variables

The data used in this study is from 34 provinces in Indonesia in 2020. Data selection is adjusted to the availability of Total Fertility Rate data obtained through a population census conducted every 10 years. The data are retrieved from BPS-Statistics (Central Bureau of Statistics) Indonesia. The variables used are:

Table 1. Variables and Definitions

Variable	Definition
Y	Total Fertility Rate
X_1	The Percentage of Women Graduating from High School
X_2	The Percentage of The Poor Population
X_3	Poverty Gap Index
X_4	Poverty Severity Index
X_5	Prevalence of Inadequate Food Consumption
X_6	Proportion of People Living Below 50% of Median Income
X_7	Unemployment Rate
X_8	Infant Mortality Rate
X_9	Child Mortality Rate
X_{10}	Percentage of Ever-Married Women Aged 15-49 Years Using Contraception Method

2.2 Regression Analysis

A regression curve describes the relationship between dependent and independent variables. Regression analysis can be divided into 3 based on the regression function: parametric, nonparametric, and semiparametric. The regression relationship can be modeled as:

$$Y_i = m(X_i) + \varepsilon_i \tag{1}$$

where m is regression function.

Parametric Regression has a known function and has assumptions to be met, such as linear, quadratic, cubic, and so on. In the parametric approach, $m(.)$ is a function involving parameters whose model is known. In Parametric Linear Regression, the form of $m(.)$ is

$$\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k \tag{2}$$

Parametric linear regression relies on several assumptions, including linearity, homoscedasticity, and absence of multicollinearity and autocorrelation [27]. Violations of these assumptions can significantly impact model performance and interpretation. In this study, parametric linear regression will be used as one of the methods. However, elastic-net will be added as the variable selection method. Meanwhile, Nonparametric regression is a flexible form of regression analysis where the regression function, representing the underlying relationship between variables, is derived directly from the data rather than assuming a predetermined form. Unlike parametric methods, it does not impose strict assumptions (e.g., linearity or polynomial structure) on the regression curve, allowing it to adapt more closely to patterns observed in the scatterplot. This approach is often described as data-driven, as it lets the data speak for itself by determining the shape of the function based on the observed pairs. Nonparametric Regression merely assumes that $m(.)$ is a smooth function. The Nonparametric Regression function is unknown, hence we cannot perform pre-specification to determine the form of the function [28].

2.3 Elastic Net Regression

Elastic Net was developed to overcome the weaknesses of Ridge Regression and Lasso Regression. Combining the strengths of ridge and lasso regression, Elastic Net addresses multicollinearity while simultaneously selecting relevant variables in datasets with numerous predictors. During feature selection, elastic net utilizes the L_1 penalty so that some coefficients can be zero. The L_2 penalty makes Elastic Net able to handle highly correlated features better. The α parameter in elastic net is in the range of 0 and 1, so it is more flexible than the Ridge penalty (α = 0) and Lasso penalty (α = 1) [21], [29]. Parameter estimation in Elastic Net is as follows [30]:

$$\min_{(\beta_0,\beta)} \left[\frac{1}{2N} \sum_{i=1}^{N} \left(y_i - \beta_0 - x_i^T \beta \right)^2 + \lambda P_\alpha(\beta) \right] \tag{3}$$

where

$$\begin{aligned} P_\alpha(\beta) &= (1-\alpha)\frac{1}{2}||\beta||_{\ell_2}^2 + \alpha||\beta||_{\ell_1} \\ &= \sum_{j=1}^{p} \left(\frac{1}{2}(1-\alpha)\beta_j^2 + \alpha|\beta_j| \right) \end{aligned} \tag{4}$$

The Elastic Net penalty is P_α.

2.4 Fourier Series Nonparametric Regression

Fourier Series Regression is a nonparametric approach that uses trigonometric polynomials to model complex data patterns. In Fourier Series Regression, the classical assumptions of regression, such as linearity, normality, and homoscedasticity, are not strictly required. This is primarily due to the nonparametric nature of the Fourier series approach, which allows for greater flexibility in modeling complex relationships without adhering to traditional regression assumptions. The number of parameters (K) in the Fourier series can be optimized to improve model fit [14], [31]. The function of the Fourier Series [32]:

$$f(x) = bx + \frac{1}{2}a_0 + \sum_{k=1}^{K} a_k \cos kx \tag{5}$$

$f(x)$ estimated using:

$$\hat{f}_\lambda(x) = b(\lambda)x + \frac{1}{2}a_0(\lambda) + \sum_{k=1}^{K} a_k(\lambda)\cos kx \tag{6}$$

λ is the oscillation parameter or smoothing parameter that controls the balance of the function. The value of λ need to be optimized in order to avoid over-smoothing or under-smoothing. One method that can be used is Generalized Cross Validation (GCV) [33].

$$GCV = \frac{MSE(\lambda)}{(n^{-1}trace(\boldsymbol{I} - \boldsymbol{H}(x))^2} \tag{7}$$

2.5 The Steps of Analysis

The steps taken in this research are:

1. Data exploration and data distribution patterns recognition using scatter plots
2. Data modeling using Fourier Series Regression, the steps are:
   a. Modeling data using 1, 2, and 3 oscillation parameters (K)
   b. GCV calculation to determine the best model that has the minimum GCV
   c. Parameter estimation
   d. Model evaluation based on R^2
   e. Model visualization using scatterplot
3. Data modeling using Linear Regression, the steps are:
   a. Modeling data
   b. Classic assumption testing
   c. Parameter estimation
   d. Model evaluation based on R^2
4. Data modeling using Elastic Net Regression, the steps are:
   a. Model estimation
   b. Model evaluation based on R^2
5. Comparing between Fourier Series Regression model and the Elastic Net Regression model
6. Conclusion

3. RESULTS AND DISCUSSION

3.1 Data Exploration

First step before modeling the data used is to determine the characteristics of each variable by presenting descriptive statistics of the data. Descriptive statistics of each variable are presented in Table 2.

Table 2. Descriptive Statistics of Data

Variable	N	Maximum	Minimum	Mean	Standard Deviation
Y	34	2.79	1.75	2.301	0.232
X_1	34	84.54	27.44	59.069	10.898

Variable	N	Maximum	Minimum	Mean	Standard Deviation
X_2	34	26.64	3.78	10.426	5.437
X_3	34	6.16	0.52	1.864	1.341
X_4	34	2.08	0.1	0.491	0.465
X_5	34	37.54	2.08	11.389	9.017
X_6	34	31.45	0.07	10.577	8.029
X_7	34	10.95	3.32	6.033	2.013
X_8	34	38.17	10.38	19.737	7.118
X_9	34	10.88	1.64	3.848	2.204
X_{10}	34	69.37	21.23	53.455	10.068

Based on **Table 2**, it can be inferred that some variables have diversity and others tend to be homogeneous. The variable with the highest dispersion is the Percentage of Women Graduating from High School; meanwhile, the most homogeneous is the Total Fertility Rate. This can be seen from the standard deviation value of each variable. Then, the correlation between variables will be examined as follows

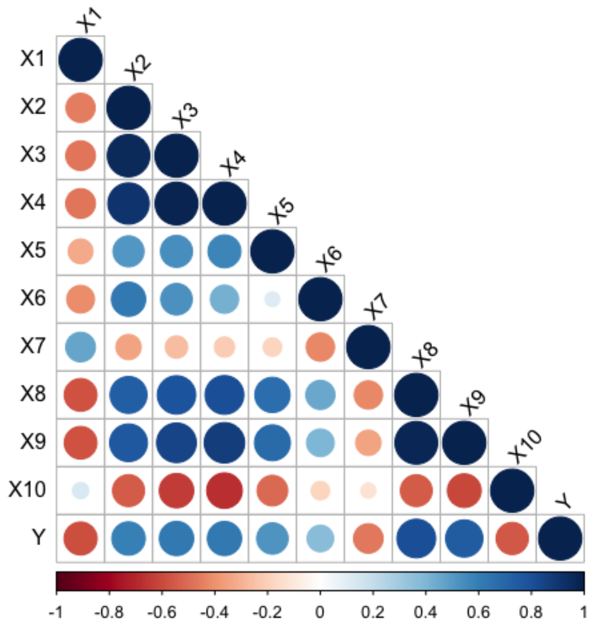
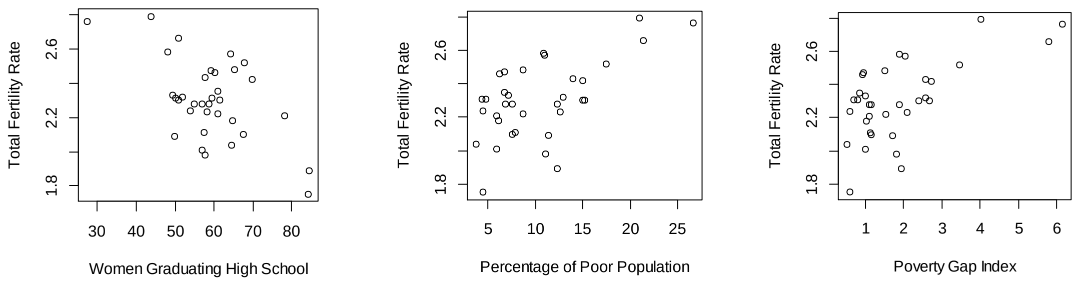


Figure 2. Correlation of The Variables

In **Figure 2**, we can see dots that provide an overview of the correlation between variables. The larger the size of the dot and the more intense the color, the higher the correlation between the variables. Based on **Figure 2**, it can be inferred that numerous variables have a strong correlation with each other. This situation may cause problems in modeling.

Then, in order to find out the pattern of data, data visualization is carried out using a scatter plot. Scatterplot between Independent and Dependent Variables is as follows:



(a) **(b)** **(c)**

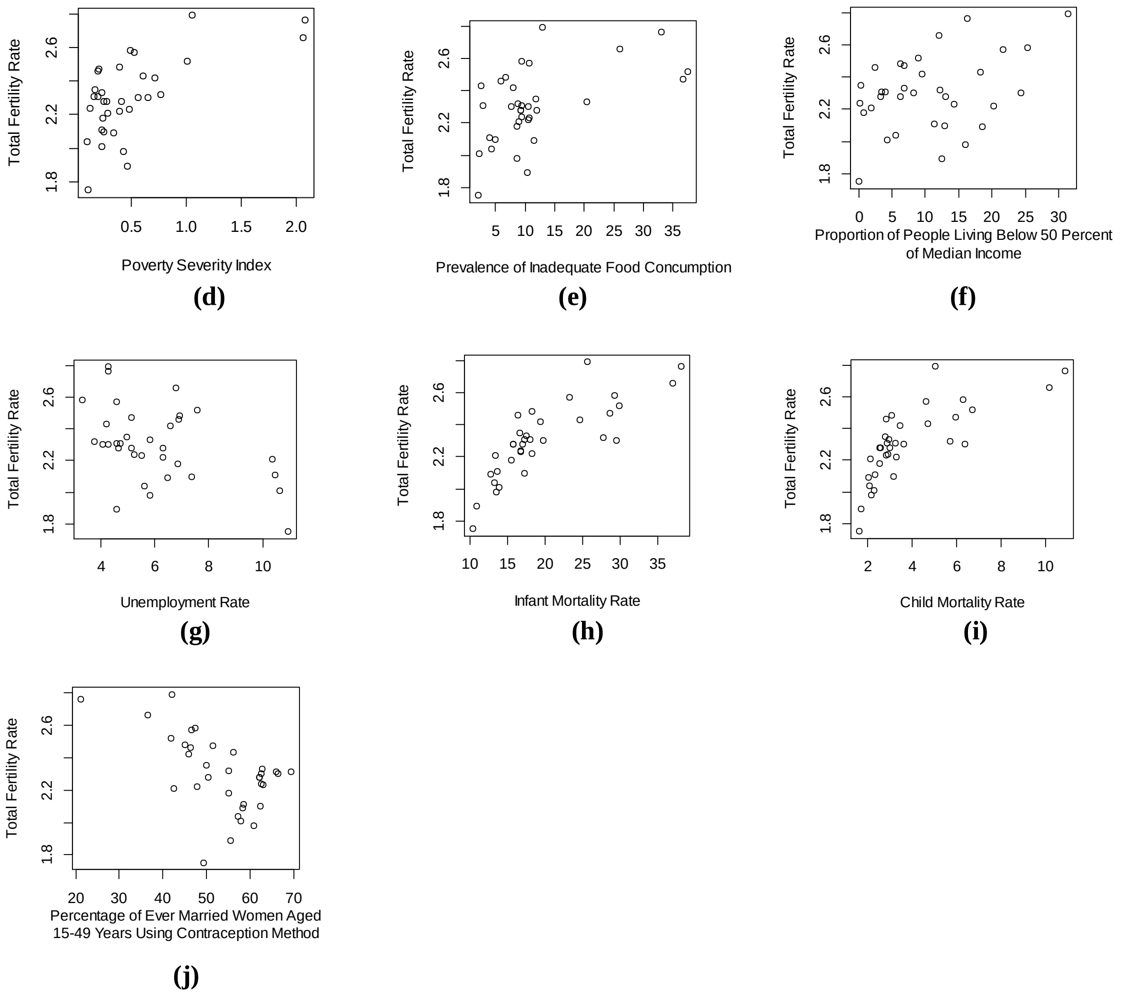


Figure 3. Scatterplot of Dependent Variable with Independent Variable, (a) Scatterplot between TFR and The Percentage of Women Graduating High School, (b) Scatterplot between TFR and The Percentage of The Poor Population, (c) Scatterplot between TFR and Poverty Gap Index, (d) Scatterplot between TFR and Poverty Severity Index, (e) Scatterplot between TFR and Prevalence of Inadequate Food Consumption, (f) Scatterplot between TFR and Proportion of People Living Below 50% of Median Income, (g) Scatterplot between TFR and Unemployment Rate, (h) Scatterplot between TFR and Infant Mortality Rate, (i) Scatterplot between TFR and Child Mortality Rate, (j) Scatterplot between TFR and Percentage of Ever Married Women Aged 15-49 Years Using Contraception Method

Figure 3 shows the plot between TFR and each independent variable. This step is done to see whether the relationship pattern that occurs follows a certain form of function, for example, linear or quadratic. If it follows a certain pattern, parametric regression analysis can be done, but if not, nonparametric regression analysis should be used because model pre-specification cannot be done. Based on Figure 3, it can be seen that the data does not follow a particular pattern; hence, pre-specification cannot be carried out. Therefore, it is suitable for modeling with nonparametric regression. From the plot, it is also shown that there is a repetition of patterns in several variables, so that we can use the Fourier Series Regression to model this case.

3.2 Fourier Series Regression

Modeling begins by determining the optimal oscillation parameter values obtained based on the smallest GCV value. In addition to the GCV value, Mean Squared Error (MSE) and Coefficient of Determination (R^2) will also be calculated. The following output is obtained

Table 3. GCV for Each Variable

K	GCV	MSE	R^2
K=1	7.125726e-03	0.002	95.31%
K=2	1.407166e-03	0.001	97.91%

K	GCV	MSE	R^2
K=3	8.993639e-25	8.472382e-25	100%

A scatterplot is presented with the regression curves of each K value to ensure the best model is selected.

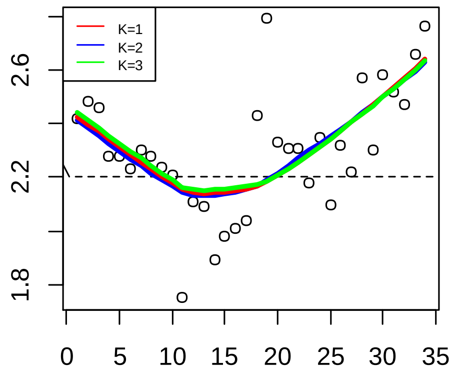


Figure 4. Comparison Plot between Each K

From Figure 4, it can be seen that the three regression curves have almost similar and overlapping patterns. A thing to consider in selecting a model is to avoid overfitting or underfitting. Therefore, 2 oscillation parameters were selected. The model estimation results are as follows

$$\hat{y} = 3.999 - 0.002x_{1i} + 0.005\cos x_{1i} + 0.016\cos 2x_{1i} - 0.049\,x_{2i} - 0.027\cos\,x_{2i} + 0.032\cos 2x_{2i} + 0.672x_{3i} - 0.130\cos x_{3i} - 0.003\cos 2x_{3i} - 2.885x_{4i} - 1.731\cos x_{4i} + 0.015\cos 2x_{4i} + 0.009x_{5i} + 0.038\cos x_{5i} + 0.051\cos 2x_{5i} + 0.003x_{6i} - 0.028\cos x_{6i} + 0.039\cos 2x_{6i} - 0.015x_{7i} - 0.140\cos x_{7i} + 0.067\cos 2x_{7i} + 0.092x_{8i} - 0.037\cos x_{8i} - 0.063\cos 2x_{8i} - 0.210x_{9i} - 0.284\cos x_{9i} - 0.051\cos 2x_{9i} - 0.010x_{10i} - 0.005\cos x_{10i} - 0.030\cos 2x_{10i}$$

R^2 = 97.91% means that 97.91% of the variation in the dependent variable (Y) can be explained by the independent variable (X) in the model. The rest is explained by other variables outside the model or random error.

3.3 Elastic Net Regression

Before being modeled with Elastic Net, modeling was carried out using linear regression. The estimated model of linear regression is as follows

$$Y = 2.8027 - 0.0072X_1 - 0.0327X_2 + 0.2796X_3 - 0.2758X_4 + 0.0015X_5 - 0.0053X_6 - 0.0193X_7 + 0.0847X_8 - 0.2734X_9 - 0.0109X_{10}$$

With Adjusted R^2 79.84%. Adjusted R^2 means that 79.84% of the variation in the dependent variable (Y) can be explained by the independent variables (X) in the model, after adjusting for the number of predictors. The rest is explained by variables outside the model or random error. A classical assumption test was carried out on the model, and the results showed that the model met the assumptions of normality, heteroscedasticity, and autocorrelation. Problems arise when checking for multicollinearity. The test results are as follows

Table 4. Multicollinearity Test Result

Variable	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
VIF	1.90	224.08	1676.31	879.92	2.62	3.73	2.49	103.95	159.15	2.57

From Table 4, it can be seen that several variables have a high VIF result. It means that multicollinearity occurs in this model. Multicollinearity that occurs must be overcome to obtain a better model. If we continue to use linear regression, in the best model, there are several variables that must be removed from the model. Therefore, Elastic Net Regression will be used. The result of modeling using Elastic Net Regression is:

Table 5. Model Estimation of Elastic Net Regression

Variable	Estimated Coefficient
Intercept	2.7842
X_1	-0.0070
X_2	-0.0217
X_3	0.1725
X_4	-0.0969
X_5	0.0011
X_6	-0.0049
X_7	-0.0187
X_8	0.0828
X_9	-0.2648
X_{10}	-0.0108

The MSE for this model is -0.0108 with R^2 85.88%. It means that 85.88% of the variation in the dependent variable (Y) can be explained by the independent variable (X) in the model. The rest is explained by other variables outside the model or random error. Based on these results, it can be seen that the elastic net can model all independent variables with dependent variables and has a higher coefficient of determination value than the linear regression model.

3.4 The Comparison

The Fourier series regression method, which is part of nonparametric regression, will be compared with elastic net regression as a parametric regression. In parametric regression, the model is easy to interpret, but it has assumptions that must be met for accurate estimation. In nonparametric regression, the model avoids assumptions that limit the form of the regression function, making it more flexible. However, this model is difficult to interpret; therefore, the model is visually presented in a scatterplot. The comparison was made based on the number of variables in the model, the Coefficient of Determination, and the Mean Squared Error as a measure of model validation.

Table 6. Comparison Between Two Models

Method	Number of Variables	R^2	MSE
Fourier Series Regression	10	97.91%	0.001
Elastic Net Regression	10	85.88%	-0.0108

The Fourier Series Regression Model is able to model all variables well, and the independent variables are able to explain the proportion of variance in the dependent variables by 97.91%, with MSE as a measure of model accuracy of 0.001. However, in the Elastic Net Regression model, the proportion of variance in TFR that can be explained in the model is 85.88% with all variables included in the model, and MSE equals -0.0108. Therefore, the variables that significantly affect Total Fertility Rate are The Percentage of Women Graduating High School, The Percentage of The Poor Population, Prevalence of Inadequate Food Consumption, Proportion of People Living Below 50% of Median Income, Unemployment Rate, Infant Mortality Rate, and Percentage of Ever Married Women Aged 15-49 Years Using Contraception Method.

4. CONCLUSIONS

It can be concluded that Fourier Series Regression can model the Total Fertility Rate phenomenon in Indonesia better than Elastic Net Regression. Fourier Series Regression model includes all independent variables and 97.91% of the variation in Total Fertility Rate is accounted for by the percentage of women graduating high school, percentage of the poor population, poverty gap index, poverty severity index, prevalence of inadequate food consumption, proportion of people living below 50 percent of median income, unemployment rate, infant mortality rate, child mortality rate, and percentage of ever-married women aged 15–49 years using contraception methods. The model is:

$$\begin{aligned}\hat{y} = {} & 3.999 - 0.002x_{1i} + 0.005\cos x_{1i} + 0.016\cos 2x_{1i} - 0.049\,x_{2i} - 0.027\cos\, x_{2i} + 0.032\cos 2x_{2i} \\ & + 0.672x_{3i} - 0.130\cos x_{3i} - 0.003\cos 2x_{3i} - 2.885x_{4i} - 1.731\cos x_{4i} + 0.015\cos 2x_{4i} + \\ & 0.009x_{5i} + 0.038\cos x_{5i} + 0.051\cos 2x_{5i} + 0.003x_{6i} - 0.028\cos x_{6i} + 0.039\cos 2x_{6i} - \\ & 0.015x_{7i} - 0.140\cos x_{7i} + 0.067\cos 2x_{7i} + 0.092x_{8i} - 0.037\cos x_{8i} - 0.063\cos 2x_{8i} - \\ & 0.210x_{9i} - 0.284\cos x_{9i} - 0.051\cos 2x_{9i} - 0.010x_{10i} - 0.005\cos x_{10i} - 0.030\cos 2x_{10i}\end{aligned}$$

In summary, while Elastic Net Regression can handle multicollinearity well, Fourier Series Regression provides a more flexible and robust framework for modeling data with an unknown pattern. The choice between these two methodologies should be guided by the nature of the data and the specific research questions being addressed. For datasets characterized by cyclical patterns or non-linear relationships, Fourier Series Regression is likely to yield more accurate and meaningful insights.

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