BAREKENG: Journal of Mathematics and Its Applications
September 2025 Volume 19 Issue 3 Page 2191-2206
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol19iss3pp2191-2206

MULTIOBJECTIVE MODEL PREDICTIVE CONTROL IN STOCK PORTFOLIO OPTIMIZATION

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Article History:

Received: 11th January 2025
Revised: 3rd March 2025
Accepted: 18th April 2025
Published: 1st July 2025

Keywords:

Model Predictive Control;
Multi-Objective Optimization;
Particle Swarm Optimization;
Stock Optimization.

ABSTRACT

This paper proposes a Multi-Objective Model Predictive Control (MO-MPC) framework for stock portfolio optimization, designed to achieve an optimal balance between return maximization and risk minimization in volatile financial markets. This approach integrates Stochastic Model Predictive Control (SMPC) to predict asset returns and dynamically adjust portfolio allocation based on a discrete-time state-space model. The optimization problem is formulated as a multi-objective optimization and is solved using Multi-Objective Particle Swarm Optimization (MOPSO). Simulation results show that the MO-MPC approach significantly outperforms conventional methods regarding wealth maximization and risk minimization. Moreover, SMPC performs better than MOPSO in maximizing portfolio value and reducing risk. These findings confirm the potential of SMPC as an adaptive and reliable strategy for financial decision-making under uncertainty.

How to cite this article:

Marlina. Y and Solikhatun., "MULTIOBJECTIVE MODEL PREDICTIVE CONTROL IN STOCK PORTFOLIO OPTIMIZATION," *BAREKENG: J. Math. & App.*, vol. 19, no. 3, pp. 2191-2206, September, 2025.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

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1. INTRODUCTION

Stock portfolio management has numerous challenges that can significantly impact investment performance. One of the primary challenges in stock portfolio management is the difficulty in accurately predicting future stock prices. As noted by Sen [1], the problem becomes increasingly complex when attempting to optimize future returns and risk values, as predicting stock prices is inherently uncertain. This uncertainty is compounded by market volatility and external factors such as economic indicators, geopolitical events, and market sentiment. These factors can influence the stock performance unpredictably [2]. Furthermore, the empirical study by Becker et al. [3] highlights the challenge of developing models that can effectively capture the multifaceted nature of stock performance across various investment criteria. The case indicates that single-objective models may not be sufficient for comprehensive portfolio management. Another significant challenge is allocating capital among a diverse set of stocks. Individual investors often struggle with managing a portfolio that includes too many different stocks, as research suggests that an optimal portfolio typically holds an average of seven stocks [4]. These challenges are increasingly complex due to market uncertainty, changes in global economic conditions, and operational constraints such as transaction costs and investment regulations. Therefore, a more adaptive and efficient method is needed to deal with changing market dynamics.

The Multi-Objective Model Predictive Control (MO-MPC) approach presents a promising solution to portfolio optimization challenges. At its core, Model Predictive Control (MPC) is a widely adopted strategy that determines optimal control actions by minimizing a cost function over a finite prediction horizon while adhering to system constraints. MPC facilitates real-time feedback and effective disturbance rejection by continuously updating the optimization at each time step using the most recent state information. Building on this foundation, MO-MPC extends the standard MPC framework by incorporating multiple objectives—such as maximizing returns and minimizing risk—thereby enabling dynamic and adaptive management of stock portfolios in uncertain market conditions [5]. Multi-objective Model Predictive Control (MO-MPC) is a sophisticated prediction-based method that enables dynamic management of stock portfolios. One of the key advantages of MO-MPC is its ability to utilize predictive models to forecast future asset prices and market trends. This predictive capability is essential in a volatile market environment, where timely decisions can significantly impact investment performance [6], [7]. The dynamic nature of MO-MPC allows for continuous updates to the portfolio based on the latest market information, enabling managers to respond proactively to changes in asset values and market conditions [8]. Moreover, MO-MPC facilitates the consideration of multiple objectives in portfolio management. Traditional portfolio optimization methods often focus on a single objective, such as maximizing returns or minimizing risk.

In contrast, MO-MPC allows for the simultaneous optimization of various objectives, which is crucial for achieving a balanced investment strategy. For instance, it can incorporate constraints related to risk tolerance, investment horizon, and liquidity requirements, ensuring that the portfolio aligns with the investor's overall financial goals [9], [10]. This multi-objective framework enhances the robustness of portfolio management strategies, as it accounts for the trade-offs between competing objectives [11].

Applying stochastic elements into MO-MPC enhances the control strategy's adaptability to changing market conditions. By continuously updating the probabilistic models based on new market data, the MO-MPC framework can adjust the portfolio allocation in real-time, responding to shifts in asset performance or market volatility [12]. In addition, stochastic factors allow for the inclusion of probabilistic constraints, which specify acceptable levels of risk or the likelihood of achieving specific performance metrics [13].

Incorporating stochastic factors and including multiple variables—such as portfolio cardinality and transaction constraints—significantly increase the complexity of the optimization problem in Multi-Objective Model Predictive Control (MO-MPC). Consequently, an effective and robust optimization method is essential to navigate the high-dimensional solution space and identify optimal asset allocations. The Particle Swarm Optimization (PSO) algorithm is employed to address this challenge. PSO, an evolutionary algorithm introduced by Eberhart and Kennedy, has proven particularly effective for multi-objective optimization problems due to its ability to explore significant and complex solution spaces [14] efficiently. As a gradient-free global optimization technique within Swarm Intelligence, PSO offers several advantages: it is simple to implement, robust in performance, and converges quickly [15]. Its population-based approach maintains solution diversity, critical in dynamic financial markets where conditions can shift rapidly [16], [17]. The algorithm's population-based nature allows it to converge rapidly towards optimal solutions while maintaining diversity among potential solutions, which is crucial in financial markets where conditions can change. In predictive portfolio control, PSO facilitates the construction of diversified and resilient portfolios by efficiently traversing non-convex and discontinuous solution landscapes [18].

In portfolio optimization, PSO has been employed to tackle various challenges, including cardinality constraints and transaction costs. Moreover, the performance of PSO in portfolio optimization has been validated through comparative studies. Research by Chen et al. indicates that PSO can yield superior results in constructing portfolios compared to traditional optimization methods, particularly in high-dimensional constrained optimization scenarios [14]. Integrating MO-MPC with heuristic optimization techniques, such as Particle Swarm Optimization (PSO), further enhances its effectiveness. PSO is known for its efficiency in exploring large solution spaces and can be employed to optimize the parameters of the MO-MPC model. By combining MO-MPC with PSO, portfolio managers can achieve more effective and efficient optimization results, particularly in complex and high-dimensional portfolio problems [19], [20]. This hybrid approach allows for the dynamic adjustment of portfolio weights based on predictive insights, leading to improved returns and risk management [21], [22].

The synergy between MO-MPC and PSO enhances the decision-making process in stock portfolio management. By utilizing MO-MPC's predictive capabilities alongside PSO's optimization strengths, investors can dynamically adjust their portfolios in response to market fluctuations while simultaneously optimizing multiple objectives. This integrated approach improves the robustness of investment strategies and aligns with modern portfolio theory principles, emphasizing the importance of diversification and risk management. Therefore, this paper is organized into six sections: Introduction, Research Methods, Results and Discussion, Conclusions, Acknowledgment, and References.

2. RESEARCH METHODS

This research seeks to develop a predictive control model for multi-objective optimization in stock portfolios, utilizing the Particle Swarm Optimization (PSO) algorithm. Through a systematic mathematical framework, this study aims to provide an efficient solution to achieving an optimal balance between profit maximization and risk minimization. The research methodology adopted in this study comprises two primary stages: a literature review and numerical simulation, as illustrated in Figure 1. In the first stage, the author explored relevant theoretical frameworks applied to address optimization problems during the initial stage.

The second phase begins with collecting historical stock data from the Yahoo Finance platform, emphasizing specific sectors or targeted stock market indices. The subsequent step involves formulating a stock portfolio optimization model to maximize wealth and minimize risk, incorporating stochastic factors into the objective function. Following this, the constructed model is solved to generate optimal portfolio configurations. Finally, a comprehensive analysis of the results is conducted to assess the efficacy of employing Stochastic Model Predictive Control (SMPC) in improving the performance of the optimized stock portfolio. SMPC is an advanced control strategy to optimize decision-making in financial systems by explicitly incorporating uncertainty and stochastic elements into future predictions. It forecasts the system's evolution over a predefined prediction horizon and solves an optimization problem that balances expected returns and associated risks. This process is executed in a receding horizon manner, meaning that at each time step, the model is updated with newly available information to refine subsequent decisions.

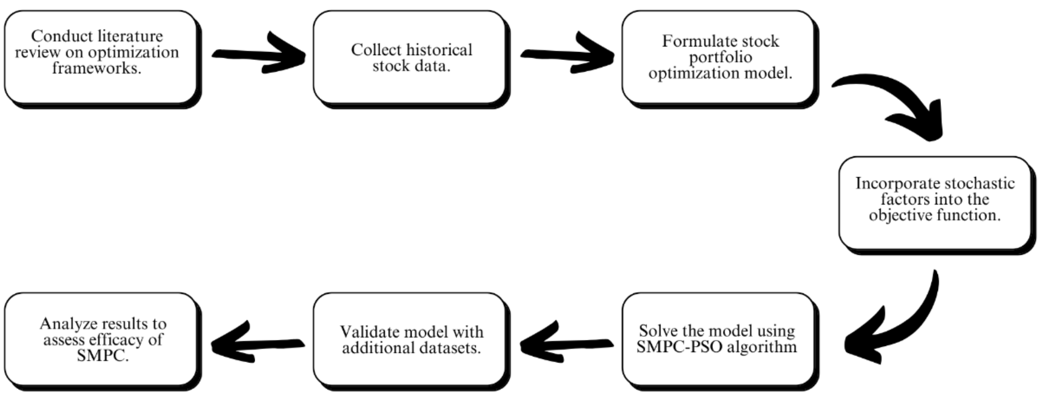

Figure 1. Reserch Flow Diagram

3. RESULTS AND DISCUSSION

This section presents the portfolio optimization model formulated by model predictive control.

3.1 Model Wealth Value in Portfolio

Consider an investment portfolio comprising n risky assets alongside a single risk-free asset. Let $x_i(k) \geq 0$, where $i = 1, 2, \ldots, n$ represent the capital allocated to the i-th risky asset at time k, and suppose $x_{n+1}(k) \geq 0$ denote the capital allocated to the risk-free asset at time k which the return of risk-free asset assumed fixed. The total wealth at time k can thus be expressed as follows:

$$W(k) = \sum_{i=1}^{n+1} x_i(k)$$

for each risky asset, the return at time k is denote by $\vartheta_i(k)$, where , $i = 1, 2, \ldots, n$. At time $k+1$, the return on the i-th risky asset over the period $[k, k+1]$, represented as $\vartheta_i(k+1)$, is defined as follows:

$$\vartheta_i(k+1) = \frac{b_i(k+1) - b_i(k)}{b_i(k)}$$

for each $i = 1, 2, \ldots, n$, let $b_i(k)$ represent the closing price of asset i at time k. Consequently, the total wealth associated with asset i at time k can be defined as follows:

$$\begin{aligned} W(k+1) &= W(k) + \sum_{i=1}^{n+1} \vartheta_i(k+1) x_i(k) \\ &= \sum_{i=1}^{n+1} x_i(k) + \sum_{i=1}^{n+1} \vartheta_i(k+1) x_i(k) \\ &= W(k) + \sum_{i=1}^{n+1} \vartheta_i(k+1) x_i(k) \end{aligned} \quad (1)$$

for each $i = 1, 2, \cdots, n$.

Let $R(k+1) = [\vartheta_1(k+1), \ldots, \vartheta_n(k+1), r_0]^T$, r_0 is prosentage of the ruturn of risk-free, and $X(k) = [x_1(k), \ldots, x_n(k), x_{n+1}(k)]^T$, **Equation (1)** can be reformulated as follows:

$$W(k+1) = W(k) + R^T(k+1)X(k). \quad (2)$$

Based on **Equation (2)**, the expected value of total wealth at time $k+1$ is given by:

$$E[W(k+1)] = W(k) + E[R^T(k+1)]X(k). \quad (3)$$

Because the returns and the capitals are independent, then if **Equation (3)** is carried out along the prediction horizon m, the following equation will be obtained,

$$E[W(k+1)] = W(k) + \sum_{j=0}^{m-1} E[R^T(k+1+j)]X(i+j) \quad (4)$$

where $E[R(k+j+1)] = [\ E[\vartheta_1(k+j+1)], E[\vartheta_2(k+j+1)], \ldots, E[\vartheta_n(k+j+1)], r_0], j = 1,2, \ldots,$ $m-1$. Suppose $E\big[\vartheta(k+1) = \hat{R}(k+1)\big]$.

Assume a transaction cost proportion $c > 0$, representing the expense associated with each transaction, whether it involves purchasing or selling asset i. Consequently, **Equation (4)** can be reformulated as follows:

$$\begin{aligned} E[W(k+m)] = W(k) + \sum_{i=1}^{m-1} E[R^T(k+j+1)]X(k+j) \\ - c\sum_{i=1}^{m-1} 1\left|X(k+j) - \left(I_{n+1} + diag(E[R^T(k+j)])\right)X(k+j-1)\right| \end{aligned} \tag{5}$$

where 1 = [1 … 1].

3.2 Vector Auto Regressive (VAR)

Assume that the predicted returns of risky assets are generated by a first-order Vector Autoregressive (VAR) model, specified as:

$$\vartheta(k+1) = v + A_1\vartheta(k) + e(k+1) \tag{6}$$

where A_1 is an $n\ x\ n$ coefficient matrix, $v = (I_n - A_1)\mu$ is an $n\ x\ 1$ vector of intercept terms, $\mu = E[\vartheta(k)]$ representing the mean return. Here $e(k+1)$ denotes the white noise or random disturbance at time $k+1$ an n-dimensional vector with

$$E[e(k+1)] = 0,\ E[e(k+1)]e^T(k+1) = \sigma, \text{ and } E[e(k+i)e^T(k+j) = 0,\ i \neq j,$$

where σ is a $n \times n$ non-singular covariance matrix. For a prediction horizon of m periods, the expected return prediction, derived from **Equation (6)**, is given by:

$$\begin{aligned} \vartheta(k+1) &= v + A_1\vartheta(k) + e(k+1) \\ \vartheta(k+2) &= v + A_1\vartheta(k+1) + e(k+2) \\ &= v + A_1\left(v + A_1\vartheta(k) + e(k+1)\right) + e(k+2) \\ &= v + A_1v + {A_1}^2\vartheta(k) + A_1e(k+1) + e(k+2) \\ \vartheta(k+3) &= v + A_1\vartheta(k+2) + e(k+3) \\ &= v + A_1v + {A_1}^2v + {A_1}^3\vartheta(k) + {A_1}^2(k+1) + A_1e(k+2) + e(k+3) \\ &\vdots \\ \vartheta(k+m) &= v + A_1\vartheta(k+m-1) + e(k+m). \end{aligned}$$

As a result, the following equation is obtained

$$\vartheta(k+m) = A_1\vartheta(k) + \sum_{i=0}^{m-1}[A_i^j v + A_i^{m-j-1}e(k+j+1)]$$

where $e(k+j+1)$ capturing the stochastic component of the predicted returns.

3.3 Expected Portfolio Risk (Variance)

The risk of a portfolio is calculated using the investment weights and the covariance matrix as follow as:

$$\sigma_p^2 = \sum_{i=1}^{n}\ \sum_{j=1}^{n} W_iW_j\mathrm{Cov}(\hat{\vartheta}_i\hat{\vartheta}_j) \tag{7}$$

where $0 \le W_i, W_j \le 1$, for all $i, j = 1, 2, \dots, n$ are the allocation weights for each assets.

3.4 Expected Portfolio Risk (CvaR)

In the portfolio optimization problem, the loss function is critical for evaluating potential losses associated with investment decisions under changing market conditions. Conditional Value at Risk (CVaR) is frequently employed as an optimization criterion, as it measures the expected loss beyond a specific threshold, known as Value at Risk (VaR).

Definition 1. *Consider $\check{X} \in R^{mn}$ represents the decision vector $\check{X} = [\check{x}_1, \dots, \check{x}_n, \check{x}_{mn}]^T$ and $V \in R^{mn}$ is a random variable. The loss function $g(\check{X}, V)$: $R^{mn\,x\,mn} \rightarrow R$, where $\check{X}, V \in R^{mn}$.*

Consider $f_\beta(V)$ denote the probability density function of V, where β representing the confidence level where $0 \leq \beta \leq 1$. The β-VaR is defined as the smallest value ξ such that, with probability β, the portfolio loss does not exceed ξ. Formally, this is expressed as:

$$\xi_\beta(\check{X}) = min_x\{\xi \in R \mid \psi(\check{X}, \xi) \geq \beta\}$$

where the cumulative distribution function of the loss g is represented as:

$$\psi(\check{X}, \xi) = P(V \mid g(U, V) \leq \xi) = \int_{g(U,V) \leq \xi} f_\beta(V)\ dV.$$

While VaR provides an estimate of the maximum loss at a specified confidence level, it does not quantify the extent of losses that exceed this threshold. Therefore, Conditional Value at Risk (CVaR) or β-CVaR is introduced as the expected loss given that it exceeds β-VaR. Mathematically, β-CVaR is defined as:

$$\begin{aligned} \varphi_\beta(\check{X}) &= E[g(\check{X}, V) \mid g(\check{X}, V) \geq \xi_\beta(\check{X})] \\ &= \frac{1}{1-\beta} \int_{g(\check{X},V) \geq \xi_\beta(\check{X})} g(\check{X}, V)\, f_\beta(V)\ dV. \end{aligned}$$

An alternative approach to calculating CVaR involves minimizing the cumulative distribution function $F_\beta(\check{X}, \xi)$:

$$\varphi_\beta(\check{X}) = \min_{\xi \in R} F_\beta(\check{X}, \xi)$$

where

$$F_\beta(\check{X}, \xi) = \xi + \frac{1}{1-\beta} \int_{V \in R^m} [g(\check{X}, V) - \xi]^+\ f_p(V) dV.$$

If the distribution of V is approximated with samples $V^1, \dots, V^m$, an empirical approximation of $F_\beta(\check{X}, \xi)$ can be represented as:

$$F_\beta(\check{X}, \xi) = \xi + \frac{1}{1-\beta} \sum_{t=1}^{M} [g(\check{X}, V\text{^}T) - \xi]^+.$$

In multi-period settings, CVaR is calculated by incorporating stochastic scenarios that represent potential future trajectories of risky assets. The expected wealth at time $k + m$ is given by:

$$E[W(k + m)] = \frac{1}{S} \sum_{s=1}^{S} E[W_s(k + m)]$$

where S denotes the set of s scenarios and $E[W_s(k + m)]$ is expected wealth at $k + m$ for s scenario. Hence, the multi-period CVaR, with a loss function $-E[W_s(k + m)]$, is formally defined as:

$$E[F_\beta(X,\xi,k,m)] = \xi + \frac{1}{(1-\beta)S}\sum_{s=1}^{S}[-E[W_s(k+m)]-\xi]^+. \tag{8}$$

3.5 Multi-objective Model Predictive Control

3.5.1 State-Space

The state-space model for portfolio management includes three main variables:

a. State variable $\hat{x}(k)$: represents the proportion of investment distribution at time k, where x_i indicates the amount of capital in risky asset i, and x_{n+1} the amount of capital in a risk-free asset.

b. Control variable $\hat{u}(k)$: consists proportion of transfer amounts capital p (to risky assets) and q (back to risk-free assets).

c. Output variable $\hat{y}(k)$: represents total wealth $W(k)$ at time k, or $\hat{y}(k) = W(k)$.

The discrete state-space model is expressed as:

$$\hat{x}(k+1) = A\hat{x}(k) + B\hat{u}(k)$$
$$\hat{y}(k+1) = C\hat{x}(k),$$

where, $\hat{x}(k+1) = \begin{bmatrix} x_1(k+1) \\ \vdots \\ x_n(k+1) \\ x_{n+1}(k+1) \end{bmatrix}$, $\hat{u}(k) = \begin{bmatrix} p_1(k) \\ \vdots \\ p_n(k) \\ q_1(k) \\ \vdots \\ q_n(k) \end{bmatrix}$

and matrices A, B, and C are constructed based on asset returns and allocation rules, follows:

$$A_{(n+1)\times(n+1)} = \begin{bmatrix} 1+\hat{R}_1(k) & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1+\hat{R}_n(k) & 0 \\ 0 & \cdots & 0 & 1+r_0 \end{bmatrix},$$

$$B_{(n+1)\times 2n} = \begin{bmatrix} 1+\hat{R}_1(k) & \cdots & 0 & -(1+\hat{R}_1(k)) & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1+\hat{R}_n(k) & 0 & \cdots & -(1+\hat{R}_n(k)) \\ (1+r_0)(-1-c) & \cdots & (1+r_0)(-1-c) & (1+r_0)(1+c) & \cdots & (1+r_0)(1+c) \end{bmatrix},$$

$$C_{1\times(n+1)} = [1 \quad \cdots \quad 1 \quad 1],$$

r_0 represents the return of risk-free asset, and c denotes the transaction fee, where r_0 and c are constants.

3.5.2 Objective Function

The optimization problem to be formulated is designed to maximize expected wealth and minimize risk. Based on **Equation (5)**, the equation for maximizing future returns over the prediction horizon m is defined as follows:

Maximizing Expected Wealth:

$$\max E[W(k+m)]. \tag{9}$$

Next, based on **Equation (8)**, the equation for minimizing Conditional Value-at-Risk (CVaR), a risk measure that estimates potential losses in worst-case scenarios, is defined as follows:

Minimizing Risk (CVaR):

$$\min E[\hat{F}_\beta(X, \xi, k, m)]. \tag{10}$$

In the Multi-Objective Model Predictive Control (MO-MPC) framework, both objectives in **Equation (9)** and **Equation (10)** can be optimized simultaneously by assigning weights to each objective. The combined objective function in MO-MPC integrates both maximizing returns and minimizing risk using CVaR, as follows:

Combined Objective Function in MO-MPC:

$$J = \alpha_1(-E[W(k+m)]) + \alpha_2\big(E[F_\beta(\check{X}, \xi, k, m)]\,\big) \tag{11}$$

where α_1 and α_2 are weights indicating the investor's priority in maximizing returns and minimizing risk, respectively, $\alpha_1 \in [0,1]$, $\alpha_2 = 1 - \alpha_1$. Furthermore, we will minimizing the objective function J.

3.5.3 Constraints

The first constraint applied to the portfolio is the lower and upper bounds, derived from a percentage of wealth:

$$x_i^{min}(k) = \delta W(k)\ and\, x_i^{max}(k) = \delta W(k)\ for\ i\ =\ 1, \dots, n+1.$$

It is known that the portfolio is self-financing, meaning that initially, there is an injection of capital, but subsequently, no further capital inflows or outflows occur; all investment gains are reinvested in the portfolio. This constraint is expressed as follows:

$$\bar{\bar{X}}_{min}(k) = \begin{bmatrix} x_1^{min}(k) \\ x_2^{min}(k) \\ \vdots \\ x_{n+1}^{min}(k) \\ W(k) \end{bmatrix} \text{and } \bar{\bar{X}}_{max}(k) = \begin{bmatrix} x_1^{max}(k) \\ x_2^{max}(k) \\ \vdots \\ x_{n+1}^{max}(k) \\ W(k) \end{bmatrix}.$$

Suppose $M = \begin{bmatrix} I_{n+1} \\ E \end{bmatrix}$, with $E = [1 \ \cdots 1]$, then each control action must be bounded by:

$$x_{min}(k) \leq Mx(k) \leq x_{max}(k).$$

Since control will be applied over the time horizon m, the following constraint is defined:

$$\bar{\bar{X}}_{min}(k) \leq \mathcal{M}(k)\bar{\bar{X}}(k) \leq \bar{\bar{X}}_{max}(k)$$

where $\bar{\bar{X}}(k) = [x^T(k|k), \dots, x^T(k+m-1|k)\,]_{m(n+2)x1}$,

$\bar{\bar{X}}_{min}(k) = [x_{min}^T(k|k), \dots, x_{min}^T(k+m-1|k)\,]_{m(n+2)x1}$,

$\bar{\bar{X}}_{max}(k) = [x_{max}^T(k|k), \dots, x_{max}^T(k+m-1|k)\,]_{m(n+2)x1}$, and $\mathcal{M} = diag\ (M, \dots, M)_{m(n+2)x1}$.

The number of selected assets must fall within the range $[K_{inf}, K_{sup}]$:

$$K_{inf} \leq \sum_{j=1}^{n+1} z_j \leq K_{sup},$$

where z_j is an auxiliary variable that counts the number of assets in the portfolio:

$$z_j = \begin{cases} 1, \text{jika } u_j > 0, j = 1, \dots, n+1, \\ \qquad 0, \text{otherwise.} \end{cases}$$

To specify the portfolio cardinality, set $K_{inf} = K_{sup} = K$. Based on **Equation (12)**, this yields:

$$\sum_{j=1}^{n+1} z_j = K.$$

3.6 Multi-objective PSO

The optimization model using MOPSO is developed based on the principles outlined in **Equation (4)** and **Equation (7)** and is formulated as follows.

The prediction wealth for a portfolio is given by:

$$\max E[W(k + 1)].$$

Portfolio risk, defined as the variance, is given by:

$$\min \sigma_p^2$$

Where constrain as follows:

a. The allocation of capital to each asset x_i must be non-negative:

$$0 \leq x_i \leq 1, \text{for all } i = 1, 2, \ldots, n.$$

b. The allocation weights W_i for each asset are restricted to lie within the interval [0, 1]:

$$0 \leq W_i \leq 1, \text{for all } i = 1, 2, \ldots, n.$$

c. The total weight of the portfolio allocation must sum to 1:

$$\sum_{i=1}^{n} W_i = 1.$$

3.7 Implementation of PSO Algorithm

The multi-objective MPC controller is integrated with the PSO algorithm and operates as a control function block executed over each sample time interval T, where T indicates the number of days in the sample dataset. For each interval $k = 1, \ldots, T$, the solution procedure is outlined as follows:

a. Particle Initialization

   Particles are initialized with a random position vector $X_i(k)$, and velocity vector $V_i(k)$, for each particle i. Lower and upper bounds are set on portfolio allocations to ensure valid values.

b. Cost Function Definition

   The cost function for the MO-MPC algorithm is defined using **Equation (11)**.

c. Fitness Calculation

   Each particle's fitness is evaluated by computing the return fitness to estimate expected wealth, $E[W(k + m)]$, based on its position $X_i(k)$, and the risk fitness (CVaR) to measure expected risk, $E[F_\beta(X, \xi, k, m)]$, at position $X_i(k)$.

d. Updating Velocity and Position Using PSO

   The PSO algorithm updates particle velocity and position using the following equations:

   Velocity update:

$$V_i(k + 1) = \omega V_i(k) + c_1 r_1 \left(P_{best}(k) - X_i(k)\right) + c_2 r_2 \left(P_{best}(k) - X_i(k)\right)$$

Position update:

$$X_i(k + 1) = X_i(k) + V_i (k + 1)$$

where

$V_i(k)$ is the particle i velocity at time k,

$X_i(k)$ is the position of particle i, representing the portfolio allocation vector,

ω is the inertia factor, influencing the particle's momentum,

c_1 and c_2 are learning coefficients guiding the particle toward its personal best position,

P_{best} is the best solution of an individual particle,

G_{best} is the best P_{best} position of an individual,

r_1 and r_2 are random variables that maintain the algorithm's stochastic properties.

e. Optimal Solution Identification Using the Pareto Front

Once each particle has stored its P_{best} and G_{best}, the optimal solution is identified through the Pareto front—a set of solutions where no objective can be improved without detriment to another. Solutions on this front represent the optimal trade-offs between maximizing returns and minimizing risks.

f. Iteration and Convergence

The iteration process proceeds until convergence is achieved, either through minimal changes in particle position or upon reaching the maximum iteration threshold.

g. Results of Pareto Front Solutions

Following the completion of iterations, the Pareto front results provide optimal solutions that capture the best trade-offs between return maximization and risk minimization.

3.8 Simulation and Evaluation

3.8.1 Parameter Initialization

Portfolio optimization begins with determining the relevant parameters. These parameters encompass fundamental variables that significantly affect the portfolio's performance. We assume the relevant parameters which are summarized in **Table 1** below.

Table 1. Parameter Values

Variable	Value
β	5%
c	0.03%
r_0	6%
γ	1%
δ	90%
W(0)	100,000,000 IDR

3.8.2 Data

This study analyzed historical closing prices of selected companies within the LQ45 index, specifically PT Astra International Tbk (ASII.JK), Bank Central Asia (BBCA.JK), Bank Indonesia (BBSI.JK), PT Bumi Serpong Damai Tbk (BSDE.JK), PT Indofood Sukses Makmur Tbk (INDF.JK), and PT XL Axiata Tbk (EXCL.JK). The data from Yahoo Finance cover the period from January 1 to December 31, 2023. Fluctuations in returns are illustrated in **Figure 2**.

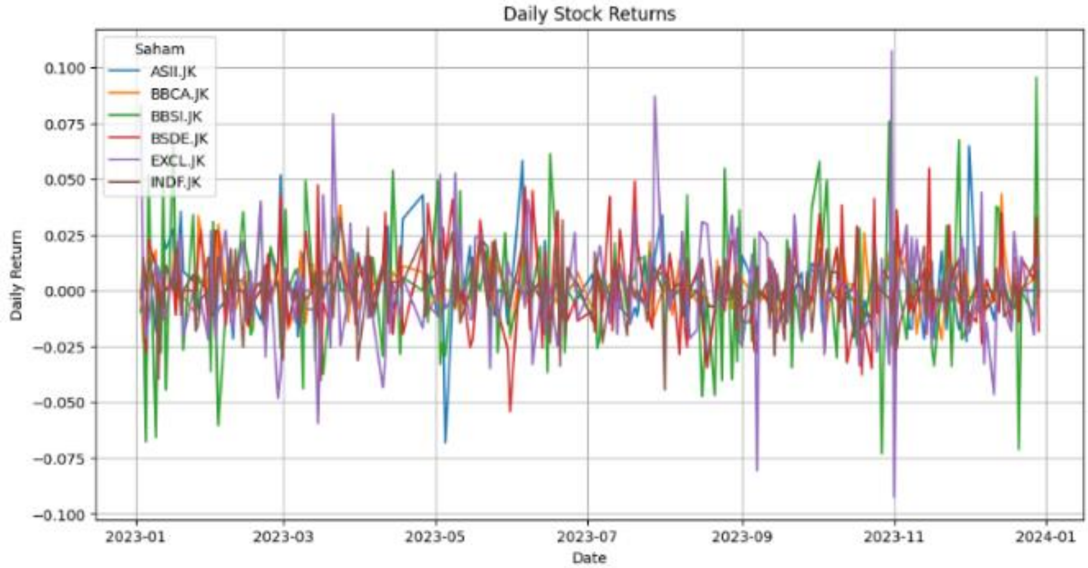

Figure 2. Daily Stock Return

3.8.3 Portfolio Optimization with SMPC-PSO

This study focuses on portfolio optimization using the Stochastic Model Predictive Control (SMPC) algorithm to determine the optimal asset allocation weight. The SMPC framework is first utilized to establish these optimal weights by leveraging historical data and predictive models to forecast future allocations. The results from the multi-objective optimization conducted with the SMPC algorithm reveal significant variations in asset allocation weights across different prediction horizons.

In the initial prediction horizon, **Table 2** outlines the optimal portfolio allocation. The highlighted funds are adjusted for each subsequent prediction horizon, leading to the final allocation shown in **Table 3**.

Table 2. Optimal Portfolio Allocation at The Initial Prediction Horizon

BBCA.JK	INDF.JK	ASII.JK	EXCL.JK	BBSI.JK	BSDE.JK	Risk Free
0.1283	0.0740	0.2394	0.1238	0.1940	0.1311	0.1094

Table 3. Final Portfolio Allocation After Adjustments Across Prediction Horizons

BBCA.JK	INDF.JK	ASII.JK	EXCL.JK	BBSI.JK	BSDE.JK	Risk Free
0.2476	0.1073	0.1496	0.0118	0.0108	0.1253	0.3476

The proportion of optimal allocation obtained is then used to estimate the wealth prediction, the wealth trend demonstrates significant growth from Horizon 1 to Horizon 252 shown in **Table 4**.

Tabel 4. Optimal Allocation

Horizon	BBCA.JK	INDF.JK	ASII.JK	EXCL.JK	BBSI.JK	BSDE.JK	Risk Free
1	0.128347	0.073990	0.239414	0.123791	0.193955	0.131123	0.109378
2	0.270935	0.014732	0.085564	0.223689	0.159444	0.014495	0.231141
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
251	0.166946	0.183312	0.182030	0.151261	0.093784	0.061704	0.160962
252	0.247554	0.107252	0.149635	0.011755	0.010824	0.125348	0.347628

The portfolio begins with an initial capital of IDR 100,000,000, and wealth increases from IDR 100,532,500 at Horizon 1 to IDR 328,633,100 at Horizon 252, indicating that optimal asset allocation yields stable wealth growth despite return fluctuations. **Figure 3** illustrates the significant change in wealth over the analysis period.

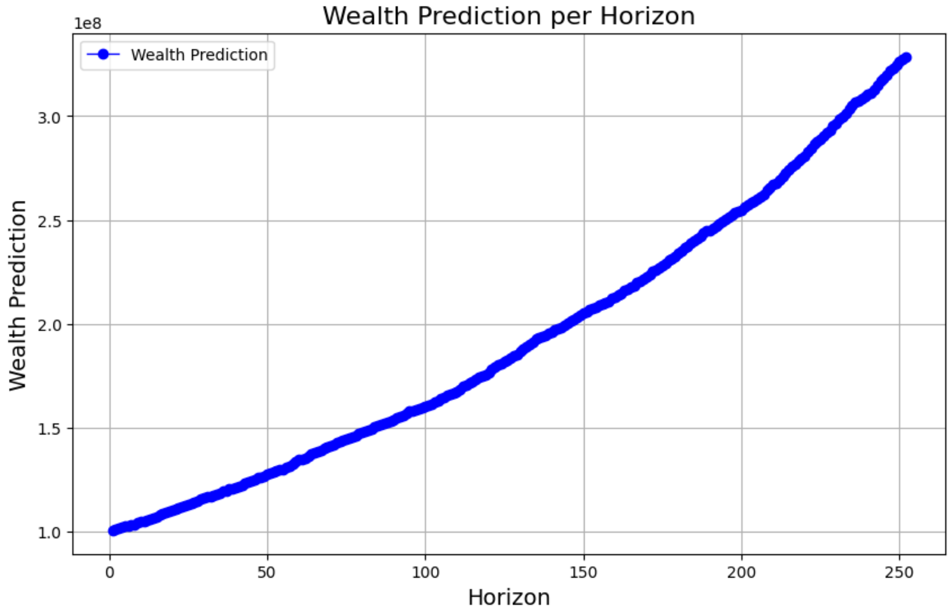

Figure 3. Wealth Prediction Per Horizon

CVaR values, which are presented in **Figure 4**, decrease significantly from Horizon 1 to Horizon 5 (0.0063 to 0.0080), indicating the reduced risk as the portfolio is optimized. However, from Horizon 248 to Horizon 250, small spikes in CVaR (0.0029 to 0.0010) suggest a decrease in risk, likely due to market volatility or unpredictability in the later stages of the simulation.

Furthermore, in **Figure 4**, we visualize CVaR fluctuations, showing a trend of rising extreme risk, which shows a decreasing trend of risk that gradually becomes lower in the final horizons.

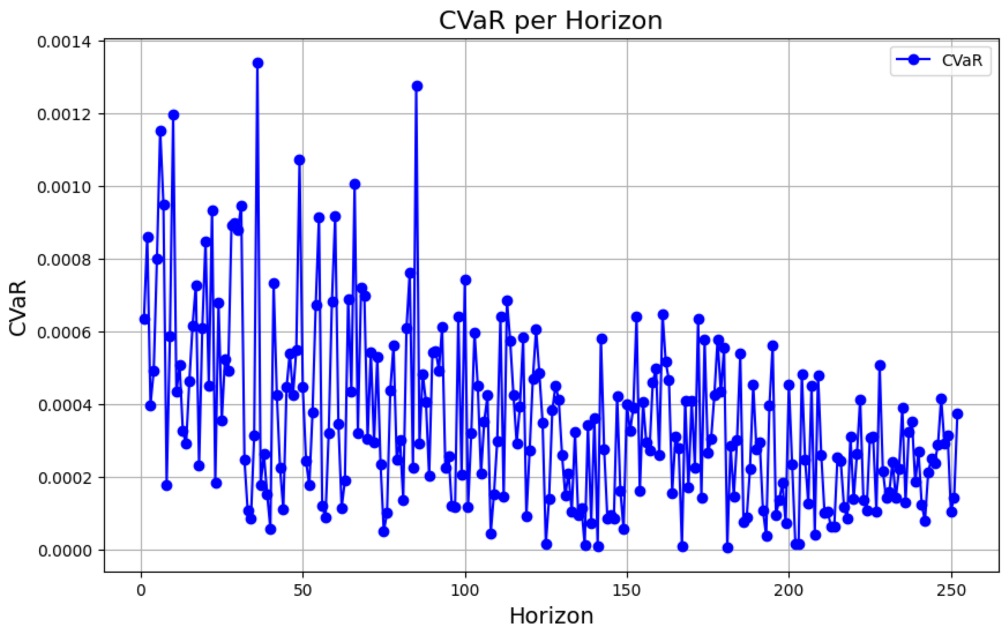

Figure 4. CVaR Per Horizon

Evaluation of the optimization results involves a comparison between MOMPC and the Multi-Objective Particle Swarm Optimization (PSO) algorithms, as discussed in the next section.

3.8.4 Portfolio Optimization with MOPSO

This section applies the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm to optimize the stock portfolio. Results include daily returns, wealth projections, and risk assessments using Conditional Value-at-Risk (CVaR) over 248 days, with evaluations based on the optimal allocation weights in **Table 5**.

Table 5. The Optimal Allocation Weight

BBCA.JK	INDF.JK	ASII.JK	EXCL.JK	BBSI.JK	BSDE.JK	Risk Free
0.0002	0.0000	0.0052	0.9888	0.0002	0.0057	0.0000

Optimal allocation weights are used to predict daily portfolio wealth. Figure 5 shows an increase from IDR 100,000,000 to IDR 100,002,841 in the first horizon, consistent with negative returns in the initial phase. Moreover, the fluctuation of wealth across prediction horizons results in the investor's wealth amounting to IDR 99,993,802 by the conclusion of the simulation.

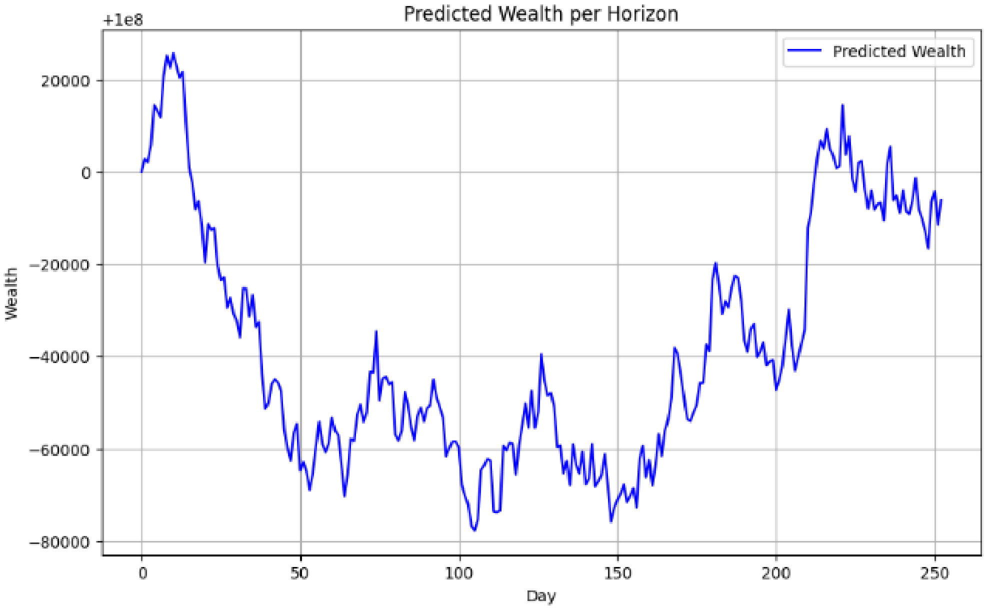

Figure 5. The Wealth Per Horizon

Subsequently, Figure 6 illustrates the risk simulation across the prediction horizon. At the start of the horizon, the risk associated with the wealth is quantified at 0.002. The risk exhibits fluctuations throughout the horizon, ultimately reaching a value of 0.007 by the end of the prediction horizon.

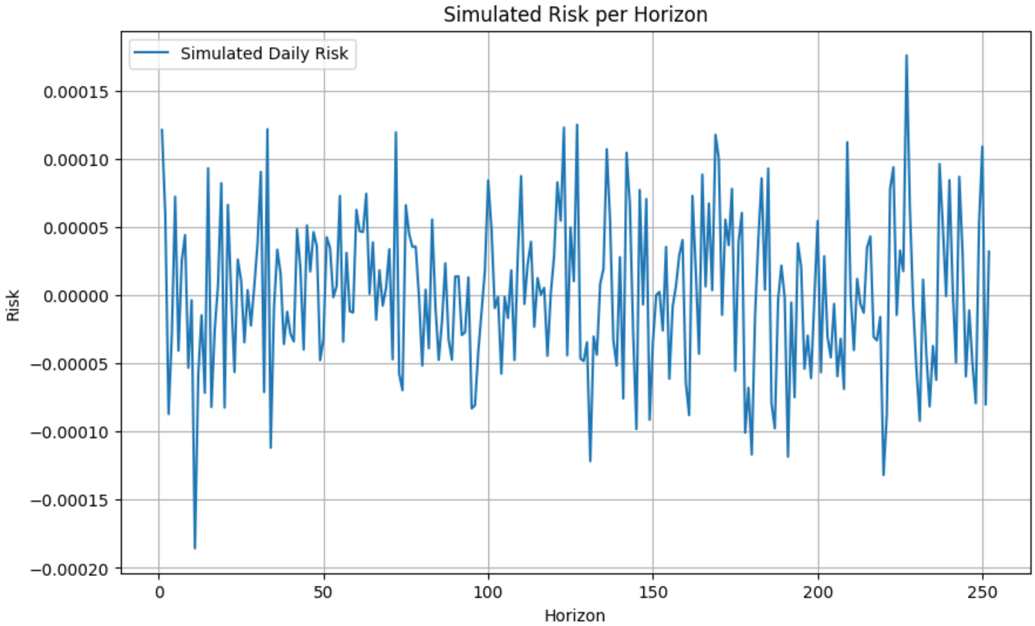

Figure 6. Simulated Risk Per Horizon

3.8.5 Results Analysis

Despite fluctuations in the early simulation stages, the MOPSO algorithm demonstrates solid performance in optimizing portfolio allocations, successfully converging to maximize wealth and minimize risk. However, the MOMPC algorithm outperforms MOPSO in several key areas. Given an initial capital of

100,000,000, the optimal allocation for each asset and portfolio risk is presented in **Table 6**. A more detailed representation of this allocation can be observed in **Figure 7**. MOMPC achieves a final portfolio value of IDR 328,633,100, significantly higher than MOPSO's IDR 99,993,802, highlighting its effectiveness in asset allocation and maximizing gains amid market fluctuations. Additionally, MOMPC stabilizes expected returns faster, achieving stability by day 150, while MOPSO stabilizes closer to day 200, indicating MOMPC's superior adaptability to market volatility.

Table 6. The Optimal Allocation Weight

	BBCA.JK	INDF.JK	ASII.JK	EXCL.JK	BBSI.JK	BSDE.JK	RF	Wealth Prediction
MOMPC	0.2476	0.1073	0.1496	0.0118	0.0108	0.1253	0.3476	IDR 328,633,100,
MOPSO	0.0002	0.0000	0.0052	0.9888	0.0002	0.0057	0.0000	IDR 99,993,802,

Regarding risk management, MOMPC shows a more efficient reduction in Conditional Value-at-Risk (CVaR). By day 180, MOMPC's CVaR approaches zero, while MOPSO only reaches a similar value towards the end of the simulation. This demonstrates MOMPC's ability to dynamically adjust portfolio weights to minimize risk without sacrificing returns, making it a more effective algorithm for portfolio optimization in volatile market conditions.

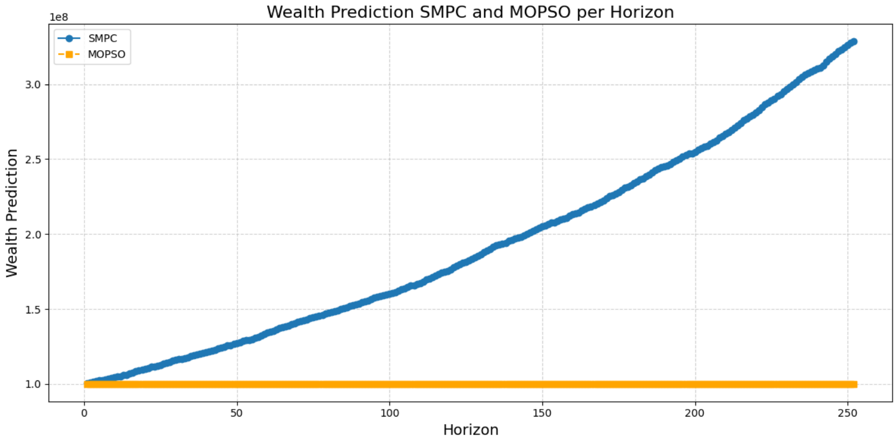

Figure 7. Wealth Prediction MOMPC and MOPSO Per Horizons

4. CONCLUSIONS

This study has empirically demonstrated the effectiveness of the Multi-Objective Model Predictive Control (MOMPC) algorithm in achieving the primary objective of optimizing stock portfolios by maximizing returns while minimizing risk in dynamic market environments. In response to the research problems, the results provide clear evidence that MOMPC significantly outperforms the Multi-Objective Particle Swarm Optimization (MOPSO) benchmark algorithm across multiple performance indicators. Specifically, MOMPC achieves a substantially higher final portfolio value (IDR 328,633,100 compared to MOPSO's IDR 99,993,802), accelerates the stabilization of returns (achieving convergence by day 150, whereas MOPSO requires up to day 200), and demonstrates superior risk control through a more rapid decline in Conditional Value-at-Risk (CVaR). These findings affirm MOMPC's enhanced capacity for wealth maximization, return stability, and effective risk management, indicating its suitability in volatile financial markets.

Nonetheless, the study is not without limitations. The optimization was conducted on a restricted selection of assets and utilized historical return data, which may not comprehensively represent real-time market complexities. Additionally, the simulation model does not incorporate certain practical constraints such as transaction costs, tax considerations, macroeconomic disruptions, or investor behavioral factors. Future research is therefore encouraged to address these aspects to enhance the applicability and robustness of the proposed optimization framework.

ACKNOWLEDGMENT

We thank the Ministry of Education, Culture, Research, and Technology, Indonesia, which supported the research through the Postgraduate Research Grant 2024.

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