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COMPARISON OF EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHART WITH HOMOGENEOUSLY WEIGHTED MOVING AVERAGE CONTROL CHARTS AND ITS APPLICATION

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ABSTRACT

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The Exponentially Weighted Moving Average (EWMA) control chart is a widely used memory-type control chart known for detecting small shifts in process means. The recently developed Homogeneously Weighted Moving Average (HWMA) control chart modifies the weighting scheme of EWMA, giving more weight to the latest data and distributing smaller weights evenly to past data to further improve sensitivity. This paper compares the performance of EWMA and HWMA control charts on an iron pipe production process dataset. The methodology involves a two-phase analysis: Phase I for establishing in-control process limits (with normality testing, parameter estimation, and determination of optimal smoothing weights) and Phase II for monitoring new data using the established charts. The performance of each chart is evaluated using the Average Run Length (ARL) metric – specifically, the ability to quickly detect small shifts (ARL_1) while maintaining a low false alarm rate (ARL_0). The results indicate that the HWMA chart consistently achieves a smaller ARL_1 than the EWMA chart for small mean shifts without sacrificing in-control ARL, implying higher sensitivity to subtle process changes. Consequently, the HWMA control chart can detect small deviations in the iron pipe length more rapidly than the EWMA chart. These findings align with recent literature and demonstrate practical significance for quality control: the HWMA chart would enable earlier detection of process issues, allowing for quicker corrective actions in manufacturing. We conclude that the HWMA control chart outperforms the EWMA chart in this application, and we recommend its use for processes where small shifts in the mean are of critical concern. Additionally, we suggest further validation through Monte Carlo simulation and comparisons with other control chart methods (such as CUSUM or extended EWMA variants) to reinforce these conclusions for broader contexts.

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1. INTRODUCTION

Quality control in the industrial field is significant to ensure that the products produced by the company have met the standards and consumer expectations [1]. The quality control tool commonly used in the industrial field is the control chart. According to [2], the control chart is a tool for visually monitoring and assessing whether a process is in statistical quality control, enabling problem-solving and quality-improving outcomes. The control chart was first developed by Dr. Walter Andrew Shewhart in 1942 and became known as the Shewhart control chart [3]. The Shewhart control chart is a memoryless control chart that only regards the current data and does not consider previous data. Shewhart control charts help detect significant process mean shifts [4]. However, Shewhart control charts are unable to detect small process mean shifts. Therefore, the memory-type control chart provides an alternative to overcome the shortcomings of the Shewhart control chart. A memory control chart is designed to identify small shifts in the process mean by considering both previous and current data [5]. An exponentially Weighted Moving Average (EWMA) control chart is one example of a memory-type control chart.

Roberts first introduced the EWMA control chart in 1959. Research on the EWMA control chart has been widely conducted, including [6], [7], [8], [9], [10], [11], and [12]. The most recent data is connected to the earlier data in the EWMA control chart, and each data is assigned a weight. However, according to EWMA plotting statistics, the weight value drops exponentially from the most recent data to the oldest data, giving more weight to the current data and less weight to the earlier data [7]. To improve the EWMA control chart's weight value distribution, Nasir Abbas invented the Homogeneously Weighted Moving Average (HWMA) control chart in 2018. The HWMA control chart uses an optimized weighting system to improve the performance of classical memory-type control charts for process parameter monitoring. HWMA control charts are, therefore, classified as homogeneous memory-type control charts [13]. HWMA control charting statistics assign larger weight values to the most recent data, and smaller weight values are homogeneously distributed over all previous data [14]. The distribution of HWMA weights can improve the performance ability of HWMA control charts compared to other control charts in detecting shifts in process mean [15].

PT Pacific Angkasa Abadi is a company that produces iron pipes in Gresik, East Java. One of the iron pipes produced by PT Pacific Angkasa Abadi is a type of 50×50 mm square black pipe with a target length of 6008 mm. The iron pipe produced requires a high level of precision according to the specifications set by the company, for example, the length of the iron pipe, which is expected to deliver results according to the size the company wants. However, the iron pipe-cutting process is prone to errors that can be caused by machine settings or machine damage. Therefore, it is necessary to supervise and monitor or control the length of iron pipes produced by PT Pacific Angkasa Abadi to maintain product quality and consumer confidence. Based on this description, in this study, the authors research the comparison of the EWMA control chart and the HWMA control chart in detecting shifts in the mean process on PT. Pacific Angkasa Abadi iron pipe production data.

2. RESEARCH METHODS

2.1 Literature Review

2.1.1 Normality Test

A normality test is a statistical test useful for ensuring that the data used is a sample from a normally distributed population [16]. The Kolmogorov-Smirnov test is one technique for testing statistical normality; it incorporates a nonparametric test, meaning that using it doesn't necessitate making any assumptions about the distribution of the data being evaluated. Normality testing using the Kolmogorov-Smirnov test is as follows [17]. The hypothesis:

H_0 : Data is distributed normally

H_1 : Data is not distributed normally

Test Statistics:

$$D_{count} = \max|S_n(x) - F_0(x)| \tag{1}$$

with D_{count} is the largest value of the absolute difference $S_n(x)$ and $F_0(x)$, $S_n(x)$ is empirical cumulative frequency distribution, and $F_0(x)$ is a theoretical cumulative frequency distribution.

Testing Criteria:

If the value is $D_{count} < D_{table\ (\alpha;n)}$ $(\alpha = 0.05)$, then H_0 it is accepted, which means the data is normally distributed.

2.1.2 Exponentially Weighted Moving Average Control Chart

Roberts first presented the Exponentially Weighted Moving Average (EWMA) control chart in 1959, and is an alternative to the Shewhart control chart in terms of detecting small shifts. EWMA control charts can be used for individual samples or subgroups with a sample size of $n > 1$. EWMA works by taking the last sample as the observed point and adding information to the line of previous sample values to create a dot plot that will later be visible on the chart [18].

The EWMA control chart can be defined as follows [3]:

$$Z_i = \lambda \bar{X}_i + (1-\lambda) Z_{i-1} \tag{2}$$

with Z_i is EWMA statistic value, λ is the weighting value chosen between 0 and 1 $(0 < \lambda \leq 1)$ and $\bar{X}_i$ is the average observation of each sample $i = 1,2, \dots n$.

The initial value of EWMA is Z_0 can be obtained from the average target value set by the manufacturer, so $Z_0 = \mu_0$ or equal to the average of the observed process, so it becomes $Z_0 = \bar{\bar{X}}$. The control limits of the EWMA control chart are as follows [3]:

$$LCL_i = \mu_0 - L \frac{\sigma}{\sqrt{m}} \sqrt{\frac{\lambda}{(2-\lambda)} [1 - (1-\lambda)^{2i}]} \tag{3}$$

$$CL = \mu_0 \tag{4}$$

$$UCL_i = \mu_0 + L \frac{\sigma}{\sqrt{m}} \sqrt{\frac{\lambda}{(2-\lambda)} [1 - (1-\lambda)^{2i}]} \tag{5}$$

with L is the the control limit width. In the formula, μ_0 represents the target process mean. The symbol σ is the population standard deviation, while m indicates the number of observations in each sample. The value λ is a weighting parameter chosen between 0 and 1, and i refers to the sample index.

Smaller values of λ will be more sensitive to small shifts, while larger values of λ will be more sensitive to large shifts [14]. Generally, weighting factor values that work well, especially for small process mean shifts, are in the interval $0{,}05 \leq \lambda \leq 0{,}25$. The most common or most used values in this interval are $0.05, 0.10,$ and 0.20. The rule of thumb is to look for smaller shifts with smaller weighting factor values [9]. In addition, to determine the weighting factor value to be used for a particular data, it can be done by finding the optimum weighting value. [19] concluded that the optimum weight value is the weight with the highest number of out-of-control observations because it is considered more sensitive in detecting process shifts.

2.1.3 Homogeneously Weighted Moving Average Control Chart

A new type of memory control chart that has gained widespread use is the Homogeneous Weighted Moving Average (HWMA) control chart, which is simpler and more effective than the Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) control charts. It is better at detecting small shifts in a process. HWMA assigns a larger weight to the current sample or most recent process value, and a smaller weight is distributed homogeneously or evenly to all previous samples. e.g. $X_{ij} \sim N(\mu, \sigma^2)$ it is a quality characteristic that will be controlled with $i = 1,2,3, \dots, n$ and $j = 1,2,3, \dots, m$. If the population parameters μ and σ are known i.e. yaitu $\mu = \mu_0$ and $\sigma^2 = \sigma_0^2$. The plotting statistics for the HWMA control chart are as follows [14]:

$$H_i = \lambda \bar{X}_i + (1-\lambda)\bar{\bar{X}}_{i-1} \tag{6}$$

where $\bar{X}_i$ is the sample average for the i^{th}. The weighting value (λ) is selected between 0 and 1, meaning $0 < \lambda \leq 1$. The average of the previous sample averages $(i-1)$ is calculated using the formula:

$$\bar{\bar{X}}_{i-1} = \frac{\sum_{i=1}^{i-1} \bar{X}_i}{i-1} \tag{7}$$

From **Equation (2)**, H_i is obtained when $i = 1$ that is

$$\begin{aligned} H_1 &= \lambda \bar{X}_1 + (1-\lambda)\bar{\bar{X}}_{1-1} \quad = \lambda \bar{X}_1 + (1-\lambda)\bar{\bar{X}}_0 \quad = \lambda \bar{X}_1 + (1-\lambda)\frac{\sum_{i=1}^{1-1} \bar{X}_i}{1-1} \\ &= \lambda \bar{X}_1 + (1-\lambda)\frac{\sum_{i=1}^{0} \bar{X}_i}{0} \end{aligned}$$

Since the expression $\bar{\bar{X}}_0$ is undefined, it cannot be used to determine the general form of H_i for $i = 1$. The value of $\bar{\bar{X}}_0$ is set as the same as the target mean of X, which μ_0. Thus, the expression applies for $i = 2,3,\ldots,n$ as follows:

When $i = 2$

$$\begin{aligned} H_2 &= \lambda \bar{X}_2 + (1-\lambda)\bar{\bar{X}}_{2-1} \\ &= \lambda \bar{X}_2 + (1-\lambda)\bar{\bar{X}}_1 \\ &= \lambda \bar{X}_2 + \left[(1-\lambda)\left(\frac{\bar{X}_1}{1}\right)\right] \\ &= \lambda \bar{X}_2 + (1-\lambda)\bar{X}_1 \end{aligned}$$

When $i = 3$

$$\begin{aligned} H_3 &= \lambda \bar{X}_3 + (1-\lambda)\bar{\bar{X}}_{3-1} \\ &= \lambda \bar{X}_3 + (1-\lambda)\bar{\bar{X}}_2 \\ &= \lambda \bar{X}_3 + (1-\lambda)\left[\frac{\bar{X}_1}{2} + \frac{\bar{X}_2}{2}\right] \\ &= \lambda \bar{X}_3 + \left[(1-\lambda)\left(\frac{\bar{X}_1}{2}\right) + (1-\lambda)\left(\frac{\bar{X}_2}{2}\right)\right] \end{aligned}$$

When $i = 4$

$$\begin{aligned} H_4 &= \lambda \bar{X}_4 + (1-\lambda)\bar{\bar{X}}_{4-1} \\ &= \lambda \bar{X}_4 + (1-\lambda)\bar{\bar{X}}_3 \\ &= \lambda \bar{X}_4 + (1-\lambda)\left[\frac{\bar{X}_1}{3} + \frac{\bar{X}_2}{3} + \frac{\bar{X}_3}{3}\right] \\ &= \lambda \bar{X}_4 + \left[(1-\lambda)\left(\frac{\bar{X}_1}{3}\right) + (1-\lambda)\left(\frac{\bar{X}_2}{3}\right) + (1-\lambda)\left(\frac{\bar{X}_3}{3}\right)\right] \\ &\vdots \\ H_n &= \lambda \bar{X}_n + \left[(1-\lambda)\left(\frac{\bar{X}_1}{n-1}\right) + (1-\lambda)\left(\frac{\bar{X}_2}{n-1}\right) + \cdots + (1-\lambda)\left(\frac{\bar{X}_{n-1}}{n-1}\right)\right] \end{aligned}$$

H_i can be expressed as follows:

$$H_i = \lambda \bar{X}_i + \left[(1-\lambda)\left(\frac{\bar{X}_1}{i-1}\right) + (1-\lambda)\left(\frac{\bar{X}_2}{i-1}\right) + \cdots + (1-\lambda)\left(\frac{\bar{X}_{i-1}}{i-1}\right)\right] \tag{8}$$

It can be seen from the description of the HWMA statistical value in **Equation (8)** that there is an average movement of each i. This average movement is called a moving average. In addition, the weight value, namely $(1-\lambda)$, is homogeneously or evenly distributed on the previous subgroup value. Therefore, H_i is called Homogeneously Weighted Moving Average.

Next, the control limit of the HWMA control chart is determined. To get the control limit, the mean and variance are required. Meanwhile, the mean of the HWMA statistics (H_i) can be determined as follows:

$$E(H_i) = E(\lambda\bar{X}_i + (1-\lambda)\bar{\bar{X}}_{i-1}) \qquad = \lambda(E(\bar{X}_i)) + (1-\lambda)(E(\bar{\bar{X}}_{i-1}))$$

Determine $E(\bar{X}_i)$

$$E(\bar{X}_i) = E\left(\frac{1}{m}\sum_{j=1}^{m} X_{ij}\right) = \frac{1}{m}((EX_{i1}) + E(X_{i2}) + \cdots + E(X_{im}))$$

Since X_{ij} is distributed as normal with mean μ and variance σ^2 then

$$E(\bar{X}_i) = \frac{1}{m}(\mu + \mu + \cdots + \mu) = \frac{1}{m}(m\mu) = \mu$$

Determine $E(\bar{\bar{X}}_{i-1})$

$$E(\bar{\bar{X}}_{i-1}) = E\left(\frac{1}{i-1}\sum_{k=1}^{i-1} \bar{X}_k\right) \quad = \frac{1}{i-1}(E(\bar{X}_1) + E(\bar{X}_2) + \cdots + E(\bar{X}_{i-1}))$$

Since $E(\bar{X}_i) = \mu$ then

$$E(\bar{\bar{X}}_{i-1}) = \frac{1}{i-1}(\mu + \mu + \cdots + \mu) \quad = \mu$$

Based on $E(\bar{X}_i)$ and $E(\bar{\bar{X}}_{i-1})$ that have been obtained, $E(H_i)$ is

$$E(H_i) = \lambda E(\bar{X}_i) + (1-\lambda)E(\bar{\bar{X}}_{i-1}) \qquad = \mu \tag{9}$$

The variance of H_i is determined as follows:

$$\begin{aligned} Var(H_i) &= Cov(Hi, Hi) \\ &= Cov(\lambda\bar{X}_i + (1-\lambda)\bar{\bar{X}}_{i-1}, \lambda\bar{X}_i + (1-\lambda)\bar{\bar{X}}_{i-1}) \\ &= Cov(\lambda\bar{X}_i, \lambda\bar{X}_i) + Cov\big(\lambda\bar{X}_i, (1-\lambda)\bar{\bar{X}}_{i-1}\big) + Cov\left((1-\lambda)\bar{\bar{X}}_{i-1}, \lambda\bar{X}_i,\right) \\ &\quad + Cov((1-\lambda)\bar{\bar{X}}_{i-1}, (1-\lambda)\bar{\bar{X}}_{i-1}) \end{aligned}$$

Since

$$Cov(\lambda\bar{X}_i, \lambda\bar{X}_i) = Var(\lambda\bar{X}_i)$$
$$Cov\left((1-\lambda)\bar{\bar{X}}_{i-1}, (1-\lambda)\bar{\bar{X}}_{i-1}\right) = Var((1-\lambda)\bar{\bar{X}}_{i-1})$$
$$Cov\big(\lambda\bar{X}_i, (1-\lambda)\bar{\bar{X}}_{i-1}\big) = Cov\left((1-\lambda)\bar{\bar{X}}_{i-1}, \lambda\bar{X}_i,\right)$$

then

$$\begin{aligned} Var(H_i) &= Var(\lambda\bar{X}_i) + Var\left((1-\lambda)\bar{\bar{X}}_{i-1}\right) + 2Cov\big(\lambda\bar{X}_i, (1-\lambda)\bar{\bar{X}}_{i-1}\big) \\ &= \lambda^2(Var(\bar{X}_i)) + (1-\lambda)^2(Var(\bar{\bar{X}}_{i-1})) + 2(\lambda)(1-\lambda)Cov\big(\bar{X}_i, \bar{\bar{X}}_{i-1}\big) \end{aligned}$$

Determine $Var(\bar{X}_i)$

$$Var(\bar{X}_i) = Var\left(\frac{1}{m}\sum_{j=1}^{m} X_{ij}\right) = \frac{1}{m^2}(Var(X_{i1}) + Var(X_{i2}) + \cdots + Var(X_{im}))$$

Since X_{ij} is distributed as normal with mean μ and variance σ^2 then

$$Var(\bar{X}_i) = \frac{1}{m^2}(\sigma^2 + \sigma^2 + \cdots + \sigma^2) = \frac{\sigma^2}{m}$$

Determine $Var(\bar{\bar{X}}_{i-1})$

$$\begin{aligned} Var(\bar{\bar{X}}_{i-1}) &= Var\left(\frac{1}{i-1}\sum_{k=1}^{i-1}\bar{X}_k\right) \\ &= \frac{1}{(i-1)^2}(i-1)\frac{\sigma^2}{m} \\ &= \frac{\sigma^2}{m(i-1)} \end{aligned}$$

Determine $Covar(\bar{X}_i, \bar{\bar{X}}_{i-1})$

$$Covar(\bar{X}_i, \bar{\bar{X}}_{i-1}) = E(\bar{X}_i\bar{\bar{X}}_{i-1}) - E(\bar{X}_i)E(\bar{\bar{X}}_{i-1})$$

Since $\bar{X}_i$ and $\bar{\bar{X}}_{i-1}$ are independent of each other so that

$$\begin{aligned} Covar(\bar{X}_i, \bar{\bar{X}}_{i-1}) &= E(\bar{X}_i)E(\bar{\bar{X}}_{i-1}) - E(\bar{X}_i)E(\bar{\bar{X}}_{i-1}) \\ &= (\mu)(\mu) - (\mu)(\mu) \\ &= \mu^2 - \mu^2 \\ &= 0 \end{aligned}$$

Based on $Var(\bar{X}_i)$, $Var(\bar{\bar{X}}_{i-1})$ and $Covar(\bar{X}_i, \bar{\bar{X}}_{i-1})$ that have been obtained, $Var(H_i)$ is
For $i > 1$

$$\begin{aligned} Var(H_i) &= \lambda^2(Var(\bar{X}_i)) + (1-\lambda)^2(Var(\bar{\bar{X}}_{i-1})) + 2(\lambda)(1-\lambda)Cov(\bar{X}_i, \bar{\bar{X}}_{i-1}) \\ &= \lambda^2\left(\frac{\sigma^2}{m}\right) + (1-\lambda)^2\frac{\sigma^2}{m(i-1)} + 2(\lambda)(1-\lambda)(0) \\ Var(H_i) &= \frac{\lambda^2\sigma^2}{m} + (1-\lambda)^2\frac{\sigma^2}{m(i-1)} + 0 \\ &= \frac{\lambda^2\sigma^2}{m} + (1-\lambda)^2\frac{\sigma^2}{m(i-1)} \end{aligned} \tag{10}$$

For $i = 1$

$$\begin{aligned} Var(H_i) &= \lambda^2(Var(\bar{X}_i)) + (1-\lambda)^2(Var(\bar{\bar{X}}_{i-1})) + 2(\lambda)(1-\lambda)Cov(\bar{X}_i, \bar{\bar{X}}_{i-1}) \\ &= \lambda^2(Var(\bar{X}_1)) + (1-\lambda)^2(Var(\bar{\bar{X}}_{1-1})) + 2(\lambda)(1-\lambda)Cov(\bar{X}_1, \bar{\bar{X}}_{1-1}) \\ &= \lambda^2(Var(\bar{X}_1)) + (1-\lambda)^2(Var(\bar{\bar{X}}_0)) + 2(\lambda)(1-\lambda)Cov(\bar{X}_1, \bar{\bar{X}}_0) \end{aligned}$$

Since $Var(\bar{X}_i) = \frac{\sigma^2}{m}$, $\bar{\bar{X}}_0$ is a constant, and $\bar{X}_1$ with $\bar{\bar{X}}_0$ are independent of each other, then

$$Var(H_i) = \lambda^2\left(\frac{\sigma^2}{m}\right) + (1-\lambda)^2(Var(\bar{\bar{X}}_0)) + 2(\lambda)(1-\lambda)(0) = \frac{\lambda^2\sigma^2}{m} \tag{11}$$

Based on the mean for H_i in **Equation (9)** and the variance for H_i in **Equation (10)** and **Equation (11)**, the HWMA control limit is

Lower Control Limit:
$LCL_i = E(H_i) - L\sqrt{V(H_i)}$

$$LCL_i = \begin{cases} \mu - L\sqrt{\dfrac{\lambda^2\sigma^2}{m}} & , if\ i = 1 \\ \mu - L\sqrt{\dfrac{\lambda^2\sigma^2}{m} + (1-\lambda)^2\dfrac{\sigma^2}{m(i-1)}} & , if\ i > 1 \end{cases} \tag{12}$$

Central Limit:

$$CL = E(H_i) = \mu \tag{13}$$

Upper Control Limit:

$$UCL_i = E(H_i) + L\sqrt{V(H_i)}$$

$$UCL_i = \begin{cases} \mu + L\sqrt{\dfrac{\lambda^2\sigma^2}{m}} & , if\ i = 1 \\ \mu + L\sqrt{\dfrac{\lambda^2\sigma^2}{m} + (1-\lambda)^2\dfrac{\sigma^2}{m(i-1)}} & , if\ i > 1 \end{cases} \tag{14}$$

If μ neither σ is known and must be estimated then the control limits are μ_0 replaced by $\hat{\mu}$ and σ_0 replaced by $\hat{\sigma}$. The predictors of μ and σ are [3]:
Estimator for μ

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\bar{X}_i \tag{15}$$

where

$$\bar{X}_i = \frac{1}{m}\sum_{j=1}^{m}X_{i,j} \tag{16}$$

with $\hat{\mu}$ is estimator for μ, n is many samples, m is any observations, $\bar{X}_i$ is average of each sample, and $X_{i,j}$ is observed quality characteristic data. Estimator for σ:

$$\hat{\sigma} = \frac{1}{n}\sum_{i=1}^{n}s_i \tag{17}$$

where

$$s_i = \sqrt{\frac{1}{m-1}\sum_{j=1}^{m}\left(X_{i,j} - \bar{X}_i\right)^2} \tag{18}$$

with $\hat{\sigma}$ is estimator for σ and s_i is standard deviation for each sample.

2.1.4 Average Run Length

One approach for evaluating how well control charts identify process shifts is to look at their average run length. On a control chart, the average number of measurement points (ARL) that must be plotted before a point indicates an out-of-control condition [18]. The ARL value consists of ARL_0 (ARL in control) and ARL_1 (ARL out of control). With these two types of ARL, the best control chart can be selected, for ARL_0, the best control chart that has the largest ARL value, while for ARL_1, the best control chart with the smallest ARL value [20]. In general, ARL_0 and ARL_1 are expressed as [3]:

$$ARL_0 = \frac{1}{P(reject\ H_0 \mid H_0\ is\ true)} = \frac{1}{\alpha} \quad (19)$$

$$ARL_1 = \frac{1}{P(fail\ to\ reject\ H_0 \mid H_0\ is\ false)} = \frac{1}{1-\beta} \quad (20)$$

where H_0 is conditions where the process is in control or statistically controlled, α is probability of type I error, dan β is probability of type II error.

2.2 Data

The data used in this research are sourced from the final project publication entitled "Comparison of CUSUM and EWMA Control Charts in Quality Control of Iron Pipe Production at PT. Pacific Angkasa Abadi". The data is in production data for the length of iron pipe type black square pipe 50 ×50 mm and the target length is 6008 mm. Data collection was carried out for 100 samples every 3 minutes with 4 observations at each i^{th} sample collection in the period 13 October 2016. The variable used in this research was the length of the 50×50 mm square black iron pipe type produced by PT Pacific Angkasa Abadi.

2.3 Analysis Steps

The data analysis process carried out in this study is as follows:

a. The Phase I analysis stage involves creating a control chart using data from 70 samples with 4 observations each sample. The steps taken in Phase I are as follows:
   i. Perform normality testing on the 50×50 mm black iron pipe production data of Phase I.
   ii. Estimating the parameters μ and or σ^2 .
   iii. Determining the optimum weight of the Phase I control chart.
   iv. Form an EWMA Phase I control chart for some optimum weight values by calculating the Z_i, LCL, CL, and UCL values.
   v. Form a HWMA Phase I control chart for some optimum weight values by calculating the H_i, LCL, CL, and UCL values.
   vi. Revise the control chart if the process from the EWMA and HWMA Phase I control charts shows an out-of-control state.

b. Subsequently, carry out the Phase II analysis stage, which is the monitoring step, employing data from 30 samples with 4 observations each sample. The steps taken in Phase II are as follows:
   i. Calculate the EWMA statistical value (Z_i) for Phase II data.
   ii. Form a Phase II EWMA control chart based on the Z_i value of Phase II data as well as the LCL, CL, and UCL values of the controlled Phase I EWMA.
   iii. Calculates the HWMA statistical value (H_i) for Phase II data.
   iv. Form a Phase II HWMA control chart based on the H_i value of Phase II data as well as the LCL, CL, and UCL values of the controlled Phase I HWMA.

c. Evaluate the EWMA and HWMA control chart that have been formed based on the Average Run Length value.

3. RESULTS AND DISCUSSION

3.1 Phase I Analysis Stages

Phase I is the process of building or forming EWMA and HWMA control charts using information from historical data. The data used in Phase I was the first 70 samples, with 4 observations for each sample. Phase I is carried out to estimate the controlled process parameters, which help determine the EWMA and HWMA control limit values and are used in Phase II.

3.1.1 Normality Test

Because the HWMA control chart implies that the data is normally distributed with parameters μ and σ^2, the first step that was done was to do a normality test on the Phase I data. Testing normality on Phase I iron pipe production data using the Kolmogorov Smirnov test is as follows:

Hypothesis:

H_0 : Data is distributed normally

H_1 : Data is not distributed normally

Testing Criteria:

If the value is $D_{count} < D_{table\,(\alpha;n)}$ $(\alpha = 0.05)$, then H_0 it is accepted, which means the data is normally distributed.

Test Statistics:

Based on **Equation (1)**, the values of D_{hitung} and D_{tabel} with $\alpha = 0.05$ are:

Table 1. Kolmogorov Smirnov Phase I Test

Value	Count	Table
Deviation	0.076	0.081

Table 1 shows that the value $D_{count} = 0.076 < D_{table} = 0.081$ is accepted, which means that H_0 the Phase I iron pipe length data is normally distributed.

3.1.2 Determination of Optimum Weighting Values

The HWMA Phase I control chart is created to determine the optimum weighting values once a normality test on the iron pipe production data from PT Pacific Angkasa Abadi reveals that the data is distributed normally. However, to form HWMA control chart, process parameter estimation is first carried out. PT Pacific Angkasa Abadi has set a target value for the length of iron pipe or the expected value in production, namely 6008 mm so that the value $\mu_0 = 6008$. However, the value σ is not yet known so it must be estimated first. Based on **Equation (17)**, it is obtained $\hat{\sigma} = 2.827$.

To determine the optimum weighting value, several weighting values ranging from 0.01 to 1 were tried to see the HWMA control chart, which identified shifting the process mean in PT iron pipe production data. Pacific Angkasa Abadi. A control chart that can identify the most significant number of points out of control is used to determine the optimum weighting value. After forming the HWMA control chart for $\lambda = 0.01$ to $\lambda = 1$, the number of points outside the control limits for each weighting value will be obtained as follows:

Table 2. Number of Out-of-Control HWMA Control Charts based on Weighting

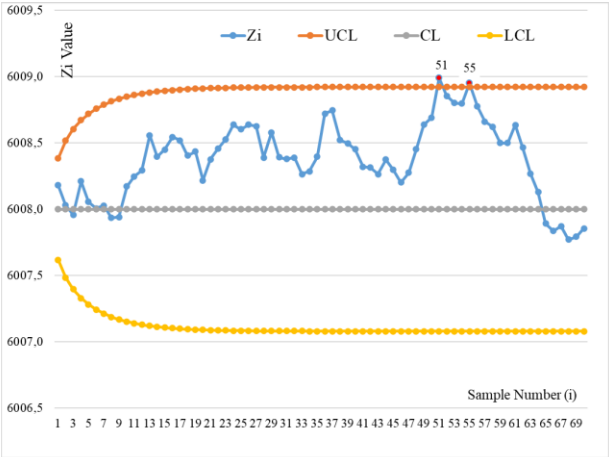
No	Weight (λ)	Many Out-of - Control Points	No	Weight (λ)	Many Out-of- Control Points
1	0.01	1	19	0.19	2
2	0.02	1	⋮	⋮	⋮
3	0.03	3	62	0.62	2
⋮	⋮	⋮	63	0.63	1
8	0.08	3	64	0.64	1
9	0.09	4	65	0.65	1
10	0.1	4	66	0.66	1
11	0.11	4	67	0.67	0
12	0.12	3	68	0.68	0
⋮	⋮	⋮	⋮	⋮	⋮
18	0.18	3	100	1	0

Data Source: data processed

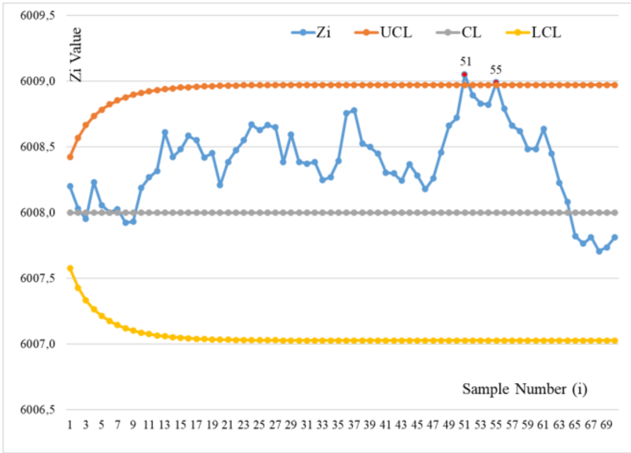
Table 2 show that the HWMA control chart has the highest quantity of points that are out of control is weighting values of $\lambda = 0.09, \lambda = 0.10$, and $\lambda = 0.11$, respectively, which is equal to four out-of-control points. Thus, the optimum weighting values for the HWMA control chart were obtained from PT Pacific Angkasa Abadi iron pipe production data, and they were $\lambda = 0.09, \lambda = 0.10$, and $\lambda = 0.11$, respectively.

3.1.3 Phase I Exponentially Weighted Moving Average Control Chart

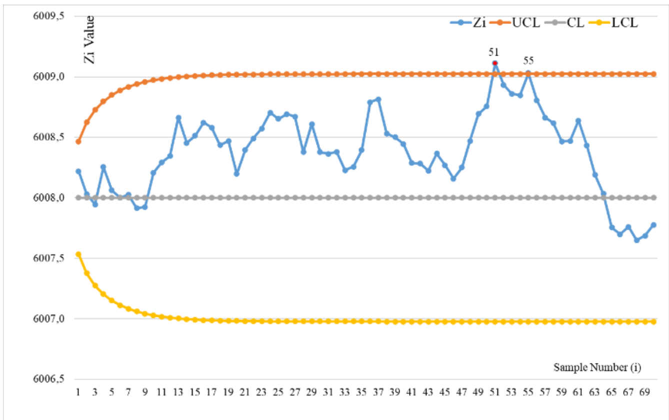
After obtaining the optimum weight values, the next is to form the Phase I control chart. First, the EWMA control chart is formed because a comparison will be made between the EWMA control chart and HWMA control charts to determine the control chart with the best performance-detecting process shifts. The EWMA Phase I control chart for the optimum weight values of $\lambda = 0.09, \lambda = 0.10$ and $\lambda = 0.11$ with $\mu_0 = 6008$ and $\hat{\sigma} = 2.827$ is as follows:



(a)



(b)



(c)

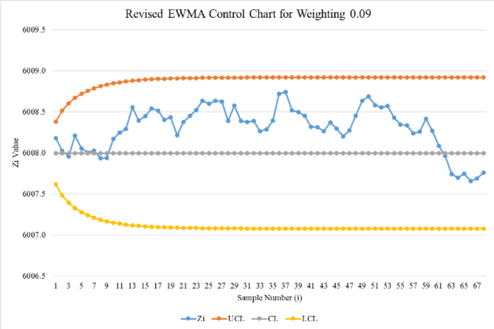
Figure 1. EWMA Phase I Control Chart,

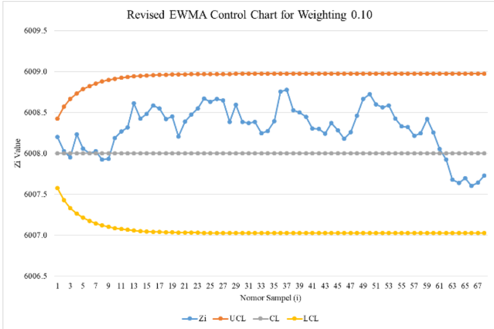
(a) EWMA for $\lambda = 0.09$, (b) EWMA for $\lambda = 0.10$, (c) EWMA for $\lambda = 0.11$

Figure 1 shows that the EWMA control chart for $\lambda = 0.09$, $\lambda = 0.10$, and $\lambda = 0.11$ has 2 out of control points, namely the 51st and 55th sample points, respectively. These results indicate that PT Pacific Angkasa Abadi's iron pipe production data is not statistically controlled. The Phase I control chart must be in control because it is used for monitoring and controlling processes in the future. Therefore, it is necessary to improve the EWMA control chart and recalculate the process parameter values and control limits until a control chart is obtained with all points in control.

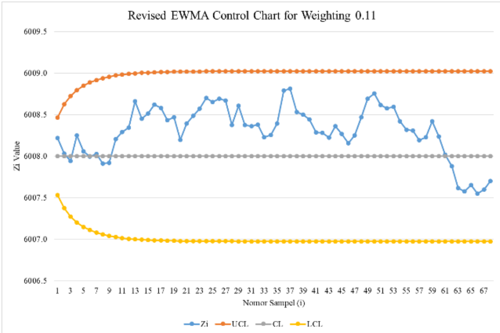
3.1.4 Revised Phase I Exponentially Weighted Moving Average Control Chart

Removing the sample points 51st and 55th from the control data is used to revise the EWMA control chart. The remaining data used to form the Phase I EWMA control chart is 68 data. Then the revised Phase I EWMA control chart is as follows:





(a) **(b)**



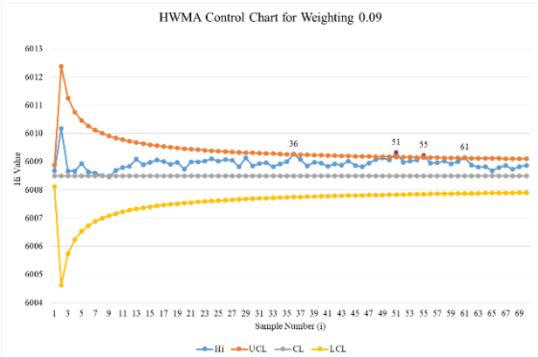
(c)

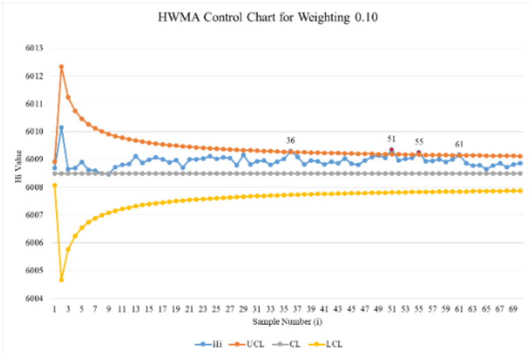
Figure 2. Revised HWMA Phase I Control Chart
(a) EWMA for $\lambda = 0.09$, (b)EWMA for $\lambda = 0.10$, (c) EWMA for $\lambda = 0.11$

Figure 2 show that the revised EWMA control chart for $\lambda = 0.09, \lambda = 0.10$, and $\lambda = 0.11$ respectively and it is found that there are no out of control data so that it can be claimed that since the production process is under control, Phase II process control can be applied.

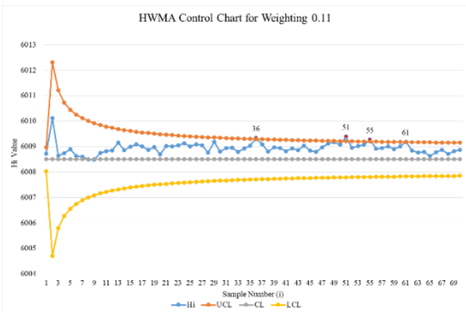
3.1.5 Phase I Homogeneously Weighted Moving Average Control Chart

From the optimum weighting values that have been obtained, the HWMA Phase I control charts for $\lambda = 0.09$, $\lambda = 0.10$, and $\lambda = 0.11$ with $\mu_0 = 6008$ and $\hat{\sigma} = 2{,}827$ are as follows:





(a) **(b)**



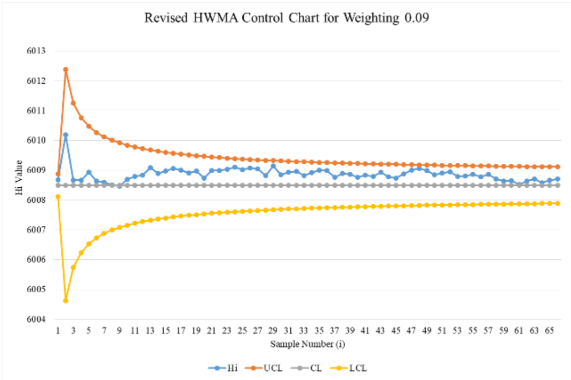
(c)

Figure 3. HWMA Phase I Control Chart
(a) HWMA for $\lambda = 0.09$, (b)HWMA for $\lambda = 0.10$, (c) HWMA for $\lambda = 0.11$

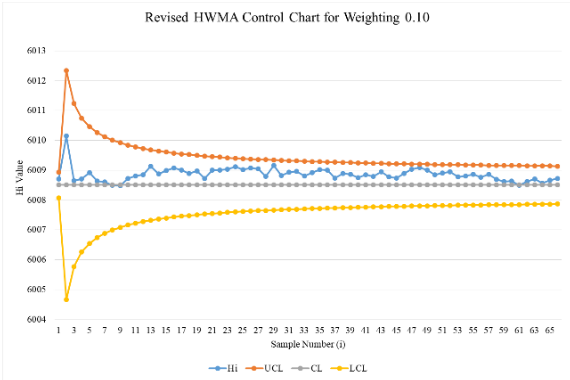
Figure 3 shows that the HWMA control chart for $\lambda = 0.09, \lambda = 0.10$, and $\lambda = 0.11$ respectively, there are 4 points that are out of control, namely the 36th, 51st, 55th, and 61st sample points. This indicates that PT Pacific Angkasa Abadi's iron pipe production data is not statistically controlled. The Phase I control chart must be in control because it is used for monitoring and controlling processes in the future. Therefore, it is necessary to improve the HWMA and recalculate the process parameter values and control limits until a control chart is obtained with all points in control.

3.1.6 Revised Phase I Homogeneously Weighted Moving Average Control Chart

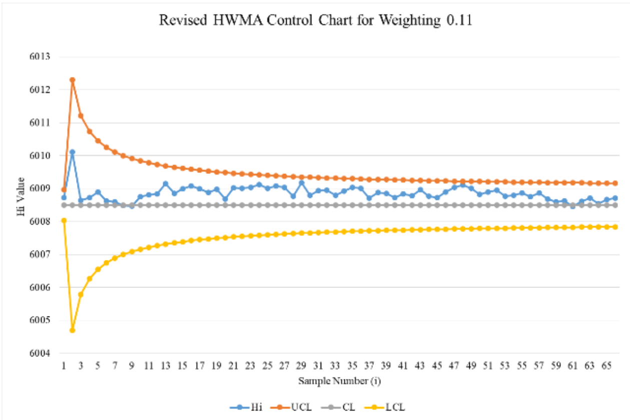
Revised to the HWMA control chart were carried out by removing data that was out of control, namely sample points 36th, 51st, 55th, and 61st. The remaining data used to form the Phase I HWMA control chart was 66 data. So, the revised HWMA Phase I control chart is obtained as follows:



(a)



(b)



(c)

Figure 4. Revised HWMA Phase I Control Chart

(a) HWMA for $\boldsymbol{\lambda = 0.09}$, $(\boldsymbol{b})$HWMA for $\boldsymbol{\lambda = 0.10}$, (c) HWMA for $\boldsymbol{\lambda = 0.11}$

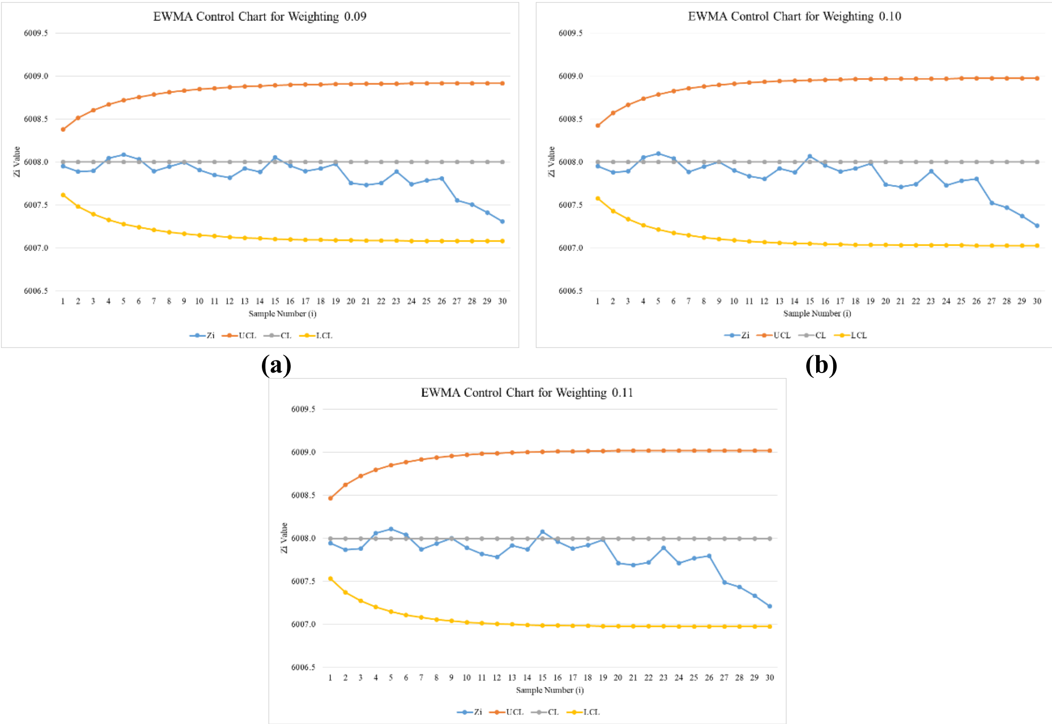
Figure 4 shows the revised HWMA control chart for $\lambda = 0.09, \lambda = 0.10$, and $\lambda = 0.11$ respectively and it is found that there are no out of control data. Therefore, it can be said that the production process is in control or has been statistically controlled.

3.2 Phase II Analysis Stage

Phase II is carried out to monitor or monitor the process using new data based on control limits obtained from Phase I. The data used in Phase II is 30 samples with 4 observations for each sample.

3.2.1 Phase II Exponentially Weighted Moving Average Control Chart

The Phase II EWMA control chart for each weight value based on the control limit value of the Phase I EWMA control chart in a controlled state is:



(a) **(b)**

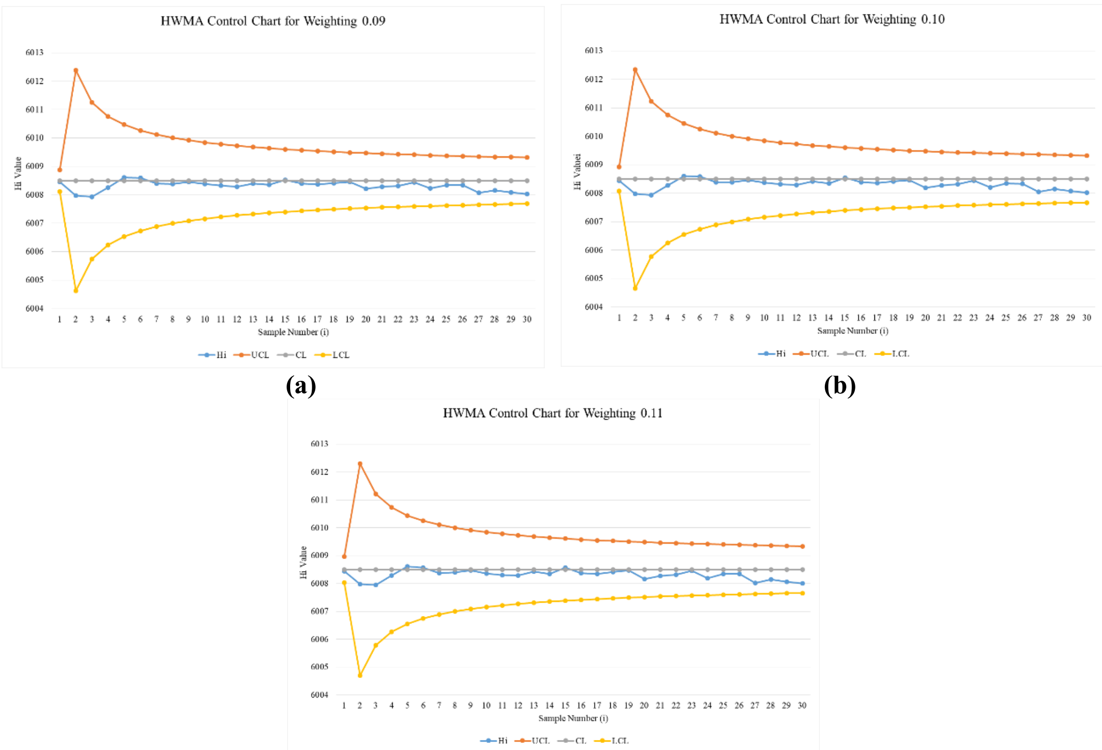
(c)

Figure 5. EWMA Phase II Control Chart
(a) EWMA for $\lambda = 0.09$, (b)EWMA for $\lambda = 0.10$, (c) EWMA for $\lambda = 0.11$

Figure 5 shows that in the Phase II EWMA control chart for $\lambda = 0.09, \lambda = 0.10$, and $\lambda = 0.11$ respectively, there are no out of control data is found, so it can be said that the production process is in control or has been statistically controlled.

3.2.2 Phase II Homogeneously Weighted Moving Average Control Chart

Based on the control limit values of the Phase I HWMA control chart in the control state, the Phase II HWMA control chart for each weight value is:



(a) **(b)**

(c)

Figure 6. HWMA Phase II Control Chart
(a) HWMA for $\lambda = 0.09$, (b)HWMA for $\lambda = 0.10$, (c) HWMA for $\lambda = 0.11$

Figure 6 shows that on the HWMA Phase II control chart for $\lambda = 0.09$, $\lambda = 0.10$ and $\lambda = 0.11$ respectively, there are no out of control data is found, so it can be said that the production process is in control or has been statistically controlled.

3.3 Performance Comparison of Exponentially Weighted Moving Average Control Chart and Homogeneously Weighted Moving Average Control Chart

The performance of the control chart cannot be ascertained by the HWMA control chart that has been created. As a result, the sensitivity level is determined using the ARL number. The control chart with the lesser ARL value is the optimal one, according to the ARL_1 value [12] [14].

3.3.1 Average Run Length Value of Exponentially Weighted Moving Average Control Chart

Determine the ARL_1 value by using the mean and variance of the EWMA control chart before computing the ARL value. The EWMA control chart's ARL1 value formula is described as follows:

$$
\begin{aligned}
ARL_1 &= \frac{1}{1-\beta} \\
&= \frac{1}{1 - Pr(LCL \leq Z_i \leq UCL \mid \mu = \mu_0 + k\sigma)} \\
&= \frac{1}{1 - Pr\left(\frac{LCL - E(Z_i)}{\sqrt{Var(Z_i)}} \leq \frac{Z_i - E(Z_i)}{\sqrt{Var(Z_i)}} \leq \frac{UCL - E(Z_i)}{\sqrt{Var(Z_i)}} \middle| \mu = \mu_0 + k\sigma\right)} \\
&= \frac{1}{1 - Pr\left(\frac{LCL - \mu}{\sqrt{\frac{\sigma^2\lambda}{(2-\lambda)m}}} \leq Z \leq \frac{UCL - \mu}{\sqrt{\frac{\sigma^2\lambda}{(2-\lambda)m}}} \middle| \mu = \mu_0 + k\sigma\right)} \\
&= \frac{1}{1 - Pr\left(\frac{LCL - (\mu_0 + k\sigma)}{\sqrt{\frac{\sigma^2\lambda}{(2-\lambda)n}}} \leq Z \leq \frac{UCL - (\mu_0 + k\sigma)}{\sqrt{\frac{\sigma^2\lambda}{(2-\lambda)n}}}\right)} \\
&= \frac{1}{1 - \left[\Phi\left(\frac{UCL - (\mu_0 + k\sigma)}{\sqrt{\frac{\sigma^2\lambda}{(2-\lambda)m}}}\right) - \Phi\left(\frac{LCL - (\mu_0 + k\sigma)}{\sqrt{\frac{\sigma^2\lambda}{(2-\lambda)m}}}\right)\right]}
\end{aligned} \tag{21}
$$

In Table 3, the ARL values of the EWMA control chart for a shift of 0 to 0.5 are shown based on Equation (21):

Table 3. ARL Value of EWMA Control Chart

k	ARL Value of EWMA		
	$\lambda = 0.09$	$\lambda = 0.10$	$\lambda = 0.11$
0	370.398	370.398	370.398
0.02	316.563	321.530	325.698
0.04	216.039	226.490	235.711
0.06	135.330	146.217	156.266
0.08	83.939	92.935	101.526
0.10	52.997	59.832	66.531
0.12	34.355	39.394	44.436
0.14	22.919	26.603	30.353
0.16	15.741	18.438	21.225

k	ARL Value of EWMA		
	$\lambda = 0.09$	$\lambda = 0.10$	$\lambda = 0.11$
0.18	11.127	13.115	15.195
0.20	8.092	9.571	11.135
0.30	2.459	2.855	3.290
0.40	1.327	1.456	1.602
0.50	1.057	1.095	1.144

Table 3 shows that the greater the shift value used, the smaller the ARL value of the EWMA control chart for each weighting. This means that fewer in-control or controlled quantity samples are needed until out-of-control or uncontrolled samples are obtained.

3.3.2 Average Run Length Value of Homogeneously Weighted Moving Average Control Chart

Similar to the EWMA control chart, find the ARL_1 value formula based on the mean and variance of the HWMA control chart before computing the ARL value. The following is a description of the HWMA control chart's ARL_1 value formula:

$$\begin{aligned}
ARL_1 &= \frac{1}{1-\beta} \\
&= \frac{1}{1 - Pr(LCL \le H_i \le UCL \mid \mu = \mu_0 + k\sigma)} \\
&= \frac{1}{1 - Pr\left(\frac{LCL - E(H_i)}{\sqrt{Var(H_i)}} \le \frac{H_i - E(H_i)}{\sqrt{Var(H_i)}} \le \frac{UCL - E(H_i)}{\sqrt{Var(H_i)}} \middle| \mu = \mu_0 + k\sigma\right)} \\
&= \frac{1}{1 - Pr\left(\frac{LCL - \mu}{\sqrt{\frac{\lambda^2\sigma^2}{m}}} \le Z \le \frac{UCL - \mu}{\sqrt{\frac{\lambda^2\sigma^2}{m}}} \middle| \mu = \mu_0 + k\sigma\right)} \\
&= \frac{1}{1 - Pr\left(\frac{LCL - (\mu_0 + k\sigma)}{\sqrt{\frac{\lambda^2\sigma^2}{m}}} \le Z \le \frac{UCL - (\mu_0 + k\sigma)}{\sqrt{\frac{\lambda^2\sigma^2}{m}}}\right)} \\
&= \frac{1}{1 - \left[\Phi\left(\frac{UCL - (\mu_0 + k\sigma)}{\sqrt{\frac{\lambda^2\sigma^2}{m}}}\right) - \Phi\left(\frac{LCL - (\mu_0 + k\sigma)}{\sqrt{\frac{\lambda^2\sigma^2}{m}}}\right)\right]}
\end{aligned} \tag{22}$$

In **Table 4**, the ARL values of the EWMA control chart for a shift of 0 to 0.5 are shown based on **Equation (22)**:

Table 4. ARL Value of HWMA Control Chart

k	ARL Value of HWMA		
	$\lambda = 0.09$	$\lambda = 0.10$	$\lambda = 0.11$
0	370.398	370.398	370.398
0.02	178.986	200.075	218.598
0.04	57.367	71.552	86.073
0.06	20.922	27.821	35.528
0.08	9.024	12.383	1.361
0.10	4.580	6.303	8.429

k	ARL Value of HWMA		
	$\lambda = 0.09$	$\lambda = 0.10$	$\lambda = 0.11$
0.12	2.707	3.646	4.840
0.14	1.837	2.377	3.080
0.16	1.407	1.726	2.156
0.18	1.189	1.378	1.646
0.20	1.080	1.189	1.356
0.30	1.000	1.001	1.007
0.40	1.000	1.000	1.000
0.50	1.000	1.000	1.000

Table 4 shows that the greater the shift value used, the smaller the ARL value of the HWMA control chart for each weighting, which means that the fewer the in control or controlled quantity samples needed until out of control or uncontrolled is obtained.

3.3.3 Comparison of Average Run Length Value of Exponentially Weighted Moving Average Control Chart and Homogeneously Weighted Moving Average Control Chart

A comparison between the ARL values of the EWMA and HWMA control charts determines which control chart is more effective in identifying shifts in the mean process. As seen in Figure 7, this is accomplished by using the ARL values that have been determined for each of the EWMA and HWMA control charts:

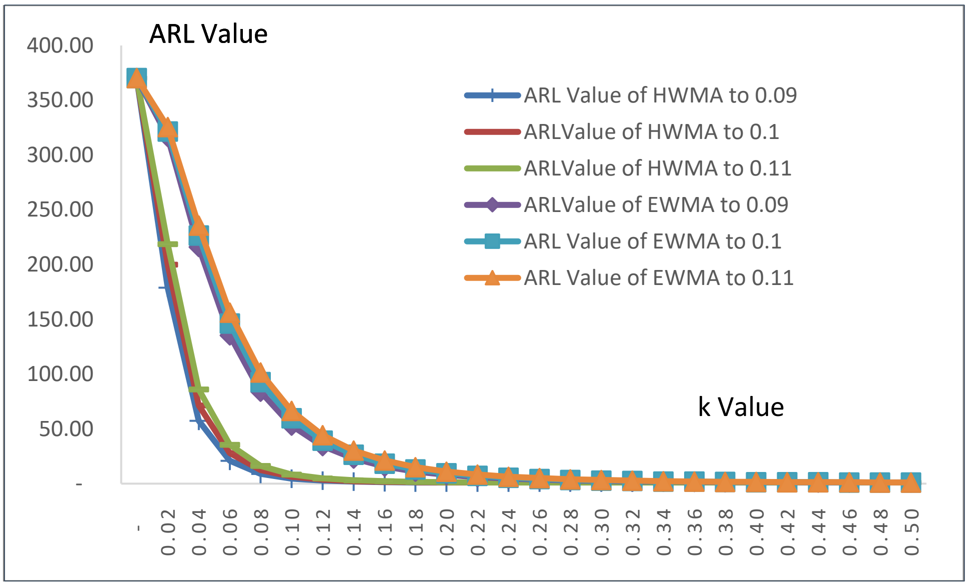


Figure 7. Comparison of ARL Values for EWMA and HWMA

For every weight value, Figure 7 shows that the ARL value of the HWMA control chart is smaller than that of the EWMA control chart. To put it another way, the control chart can identify changes in the manufacturing process more quickly since fewer samples are required until an out-of-control signal occurs. Consequently, it can be said that the HWMA control chart is more sensitive due to the lower ARL value. This paper shows that the HWMA control chart of PT Pacific Angkasa Abadi's iron pipe production has an LCL = 6007.35, CL = 6008.00, and UCL = 6008.65, and based on the ARL value shows that it is more sensitive than the EWMA control chart.

The comparative analysis of EWMA and HWMA control charts on the iron pipe production process has several practical and theoretical implications. In practical terms, our results suggest that implementing the HWMA chart for monitoring pipe lengths will enhance quality control by reducing the detection time for subtle process shifts. For the manufacturer (PT Pacific Angkasa Abadi), this means that if the cutting machine or process starts to drift slightly (perhaps due to tool wear, minor calibration issues, or environmental factors), the HWMA chart will alert operators sooner than an EWMA chart under the same conditions. Early detection of such small shifts can prevent the production of out-of-specification pipes, thereby reducing waste and rework and maintaining customer satisfaction with product quality. Given that the Phase II monitoring showed the process remained in control, one interpretation is that the control system is effective; however, should any small shift begin to develop, an HWMA chart would be a more vigilant guardian of quality.

Theoretically, our study reinforces existing literature on the performance of memory-type control charts. The superiority of HWMA over EWMA in detecting small mean shifts that we observed is in line with the findings of the original HWMA paper [14]. These studies also reported substantially lower ARL_1 for HWMA compared to EWMA for small shifts, often using simulation or mathematical analysis. Our work contributes to this body of knowledge by comparing real industrial data and showing the advantages in practice without artificial data generation. Moreover, we followed a two-phase control chart approach, which mirrors real-world implementation, including parameter estimation and chart revision, which provides a realistic basis for comparison. The outcome that HWMA signaled more points in Phase I suggests that in a historical dataset, HWMA can uncover more indications of instability (which could correspond to real small shifts or anomalies) that EWMA might miss. This indicates HWMA might be useful in monitoring and retrospectively analyzing process history for subtle issues.

It is also noteworthy to compare our findings with related studies on the same production process. A previous study compared CUSUM and EWMA charts on the iron pipe data and found that CUSUM was more sensitive than EWMA for small shifts [21]. This is expected since CUSUM, like EWMA, is designed for small shifts. Our study did not explicitly include CUSUM. However, given that HWMA outperforms EWMA and likely also would outperform CUSUM (based on literature claims), one could infer that HWMA might be an even better choice than CUSUM for this process.

Additionally, a recent study applied an extended EWMA chart to the same pipe data and reported improved performance over the standard EWMA [22] [23]. This trend of research – exploring enhanced versions of EWMA/CUSUM (whether through modified schemes like HWMA, extended EWMA, or hybrid charts) – underscores the importance of refining detection ability for quality control. Our results specifically highlight HWMA as a strong candidate in that continuum of improvements.

One concern that may arise is the complexity of implementing HWMA in an industrial setting. While EWMA is well-known and relatively straightforward to compute, HWMA's formula may appear more complex due to averaging past data. However, in practice, HWMA can be implemented recursively as well, and the additional computation (maintaining a running average of all past data) is trivial for modern computing systems or even programmable logic controllers. Thus, the improved performance does not come with an undue burden in calculation or understanding for quality engineers. Training may be needed to familiarize staff with interpreting HWMA charts, but the interpretation (points outside control limits indicate potential shifts) remains the same as any Shewhart-type chart.

Our analysis assumed normality and independence of observations. We verified normality for our dataset; however, if the process data were significantly non-normal, one might need to apply a normalizing transformation or use a robust control chart design. Independence is harder to verify but was assumed based on the sampling scheme (taking measurements at intervals likely larger than any process autocorrelation time). If autocorrelation were present, it could inflate false alarms for any control chart. Practitioners might use time-series models or adjust sampling to mitigate correlation in such cases. Our results are most directly applicable under the assumption that those conditions hold.

While we have demonstrated HWMA's superiority for small shifts, one should note that the actual benefit in a real production environment will also depend on the frequency of such shifts occurring. If the process is very stable and shifts are rare, an EWMA chart might suffice. However, in high-precision manufacturing like this, even rare small shifts can have a cost, so the extra safeguard of HWMA is justified. Another limitation is that we did not consider the effect of parameter estimation error on ARL. In Phase I, we estimated μ and σ from 66–68 samples; these estimates carry uncertainty, which can slightly affect the realized ARL in Phase II. For fairness, both charts used the same estimates, so the comparison remains fair, but the absolute ARL numbers might differ if the true parameters were known. Typically, ARL calculations assume known parameters. The fact that our Phase II had no false alarms suggests our estimated limits were appropriate.

One could perform a Monte Carlo simulation study to strengthen the evidence further. For example, many process runs can be simulated with a known small shift introduced at a random point, and the run length can be recorded until the EWMA and HWMA signal. Comparing the empirical ARL distributions from simulation would complement our theoretical ARL calculations and account for any real-world complexities (estimation error, non-normality, etc.). This was beyond the scope of our current paper, but we recommend it as future work to confirm the robustness of HWMA's advantage. Additionally, comparing HWMA with other advanced control chart methods (such as the CUSUM, adaptive EWMA, or the newer mixed-memory charts) on the same dataset would provide a more complete picture of how HWMA stands relative to all options. We focused on EWMA vs HWMA as that was our primary interest. Still, given the existence of "progressive" or "extended" EWMA methods, it would be insightful to see if HWMA still holds an edge or if combinations of methods could perform even better.

4. CONCLUSIONS

After completing Phase I analysis, the iron pipe production process was confirmed to be under statistical control. Final estimates of the process mean and standard deviation were used to construct EWMA and HWMA control charts. In Phase II, applying these charts to new data showed no out-of-control signals, indicating that the process remained stable. ARL analysis revealed that the HWMA chart is more sensitive to small shifts in the process mean compared to the EWMA chart. For example, for a shift as small as 0.05σ, the HWMA chart signalled much faster. This advantage comes from HWMA's equal weighting of past data, unlike EWMA's exponentially decreasing weights. Importantly, this increased sensitivity did not lead to more false alarms, as both charts had similar ARL_0 values.

These findings support previous research and demonstrate that HWMA is a more effective tool for detecting small shifts in processes like pipe length. HWMA can be adopted to improve quality control, allowing for quicker responses to deviations and better protection of product quality. This study also offers a practical approach for implementing control charts—covering assumption testing, parameter optimization, and limit adjustment. Further validation through simulations (e.g., Monte Carlo studies) under different process conditions and across industries is recommended to strengthen the evidence and broaden HWMA's applicability.

In conclusion, the HWMA control chart is a superior alternative to the traditional EWMA chart for detecting small process shifts. By using a more balanced weighting scheme, HWMA provides quicker detection without increasing false alarms. Its successful application in this study highlights its practical value in enhancing modern quality control systems.

REFERENCES

[1] E. Prihastono, "PENGENDALIAN PROSES STATISTIK UNTUK MENINGKATKAN PRODUKTIVITAS DAN KUALITAS PADA INDUSTRI," *DinamikaTeknik*, vol. 6, no. 2, 2012.

[2] L. V. Hignasari, "TINJAUAN TEORITIS PENGENDALIAN KUALITAS PRODUK HASIL INDUSTRI DENGAN METODE STATISTIK," *Jurnal Ilmiah Vastuwidya*, vol. 3, no. 1, pp. 24–29, 2020.

[3] D. C. Montgomery, *INTRODUCTION TO STATISTICAL QUALITY CONTROL*, 8th ed. John Wiley & Sons, 2020.

[4] S. Sarliani and others, "PENGEMBANGAN BAGAN KENDALI TRIPLE HOMOGENEOUSLY WEIGHTED MOVING AVERAGE DALAM MENINGKATKAN DETEKSI PERGESERAN RATA-RATA PROSES= DEVELOPMENT OF THE TRIPLE HOMOGENEOUSLY WEIGHTED MOVING AVERAGE CONTROL CHART IN ENHANCING DETECTION OF PROCESS MEAN SHIFTS," Universitas Hasanuddin, 2024.

[5] N. A. Ajadi, O. Asiribo, and G. Dawodu, "Progressive mean exponentially weighted moving average control chart for monitoring the process location," *International Journal of Quality & Reliability Management*, vol. 38, no. 8, pp. 1680–1694, 2020.doi: https://doi.org/10.1108/IJQRM-05-2020-0138

[6] M. S. Saccucci and J. M. Lucas, "Average Run Lengths for Exponentially Weighted Moving Average Control Schemes Using the Markov Chain Approach," *Journal of Quality Technology*, vol. 22, no. 2, pp. 154–162, 1990, doi: https://doi.org/10.1080/00224065.1990.11979227.

[7] J. S. Hunter, "THE EXPONENTIALLY WEIGHTED MOVING AVERAGE," *Journal of Quality Technology*, vol. 18, no. 4, pp. 203–210, 1986, doi: https://doi.org/10.1080/00224065.1986.11979014.

[8] V. Alevizakos, A. Chatterjee, K. Chatterjee, and C. Koukouvinos, "THE EXPONENTIATED EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHART," *Statistical Papers*, vol. 65, no. 6, pp. 3853–3891, 2024, doi: https://doi.org/10.1007/s00362-024-01544-2.

[9] V. V Koshti and A. A. Kalgonda, "A STUDY OF ROBUSTNESS OF THE EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHART: A SIMULATION APPROACH," *International Journal of Advanced Scientific and technical research, 2 (1)*, pp. 519–525, 2011.

[10] I. Antono, R. Santoso, and Y. Wilandari, "KOMPUTASI METODE EXPONENTIALLY WEIGHTED MOVING AVERAGE UNTUK PENGENDALIAN KUALITAS PROSES PRODUKSIMENGGUNAKAN GUI MATLAB (STUDI KASUS: PT DJARUM KUDUS SKT BRAK MEGAWON III)," *Jurnal Gaussian*, vol. 5, no. 4, pp. 673–682, 2016.

[11] N. Nelwati, H. Yozza, and M. Maiyastri, "PETA KENDALI EXPONENTIALLY WEIGHTED MOVING AVERAGE (EWMA) UNTUK JUMLAH WISATAWAN YANG BERKUNJUNG KE SUMATERA BARAT," *Jurnal Matematika UNAND*, vol. 4, no. 4, pp. 83–90, 2019.doi: https://doi.org/10.25077/jmu.4.4.83-90.2015

[12] W. Febrina and W. Fitriana, "EXPONENTIAL WEIGHT MOVING AVERAGE (EWMA) CONTROL CHART FOR QUALITY CONTROL OF CRUDE PALM OIL PRODUCT," *International Journal of Management and Business Applied*, vol. 1, no. 1, pp. 19–27, 2022.doi: https://doi.org/10.54099/ijmba.v1i1.93

[13] M. Riaz, S. Ahmad, T. Mahmood, and N. Abbas, "ON REASSESSMENT OF THE HWMA CHART FOR PROCESS MONITORING," *Processes*, vol. 10, no. 6, p. 1129, 2022.doi: https://doi.org/10.3390/pr10061129

[14] N. Abbas, "HOMOGENEOUSLY WEIGHTED MOVING AVERAGE CONTROL CHART WITH AN APPLICATION IN SUBSTRATE MANUFACTURING PROCESS," *Comput Ind Eng*, vol. 120, pp. 460–470, 2018.doi: https://doi.org/10.1016/j.cie.2018.05.009

[15] Z. Rasheed, H. Zhang, S. M. Anwar, and B. Zaman, "HOMOGENEOUSLY MIXED MEMORY CHARTS WITH APPLICATION IN THE SUBSTRATE PRODUCTION PROCESS," *Math Probl Eng*, vol. 2021, no. 1, p. 2582210, 2021.doi: https://doi.org/10.1155/2021/2582210

[16] T. Cahyono, "STATISTIK UJI NORMALITAS," *Yayasan Sanitarian Banyumas, Banyumas, Indonesia*, 2015.

[17] A. Quraisy, "NORMALITAS DATA MENGGUNAKAN UJI KOLMOGOROV-SMIRNOV DAN SAPHIRO-WILK: STUDI KASUS PENGHASILAN ORANG TUA MAHASISWA PRODI PENDIDIKAN MATEMATIKA UNISMUH MAKASSAR," *J-HEST Journal of Health Education Economics Science and Technology*, vol. 3, no. 1, pp. 7–11, 2020.doi: https://doi.org/10.36339/jhest.v3i1.42

[18] D. T. Wijayanti, H. Helmi, and N. Imro'ah, "PERBANDINGAN KINERJA PETA KENDALI CUMULATIVE SUM DAN PETA KENDALI EXPONENTIALLY WEIGHTED MOVING AVERAGE," *Bimaster: Buletin Ilmiah Matematika, Statistika dan Terapannya*, vol. 9, no. 4, 2020.

[19] R. M. Pratiwi and W. Wibawati, "FUZZY UNIVARIATE CONTROL CHART UNTUK MONITORING KUALITAS KETEBALAN LEM LABELSTOCK di PT" XYZ"," *Jurnal Sains dan Seni ITS*, vol. 9, no. 2, p. 487961, 2021.doi: https://doi.org/10.12962/j23373520.v9i2.53549

[20] S. A. Abbasi, S. H. Nassar, M. M. Aldosari, and O. A. Adeoti, "EFFICIENT HOMOGENEOUSLY WEIGHTED DISPERSION CONTROL CHARTS WITH AN APPLICATION TO DISTILLATION PROCESS," *Qual Reliab Eng Int*, vol. 37, no. 8, pp. 3221–3241, 2021.doi: https://doi.org/10.1002/qre.2904

[21] M. Hakam, "PERBANDINGAN GRAFIK KENDALI CUSUM (CUMULATIVE SUM) DAN EWMA (EXPONENTIALLY WEIGHTED MOVING AVERAGE) DALAM PENGENDALIAN KUALITAS PRODUKSI PIPA BESI PADA PT. PACIFIC ANGKASA ABADI," *Surabaya: Tugas Akhir-Jurusan Matematika ITS Surabaya*, 2017.

[22] F. Marzeta and others, "PENGGUNAAN PETA KENDALI EXTENDED EXPONENTIALLY WEIGHTED MOVING AVERAGE PADA DATA PRODUKSI PIPA BESI PT. PACIFIC ANGKASA ABADI," Universitas Hasanuddin, 2024.

[23] A. M. Rajab and others, "PENERAPAN BAGAN KENDALI MODIFIED CUMULATIVE SUM DALAM MENDETEKSI PERGESERAN RATA-RATA PADA DATA PRODUKSI PIPA BESI PT. PACIFIC ANGKASA ABADI," Universitas Hasanuddin, 2023.