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DUAL RECIPROCITY BOUNDARY ELEMENT METHOD FOR SOLVING TIME-DEPENDENT WATER INFILTRATION PROBLEMS IN IMPERMEABLE CHANNEL IRRIGATION SYSTEMS

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ABSTRACT

The mathematical model of water infiltration in a furrow irrigation channel with an impermeable layer in homogeneous soil is formulated as a Boundary Value Problem (BVP) with the Modified Helmholtz Equation as the governing equation and mixed boundary conditions. The purpose of this study is to solve the infiltration problem using the Dual Reciprocity Boundary Element Method (DRBEM). The results show that the highest values of suction potential and water content are located beneath the permeable channel, while the lowest values are found at the soil surface outside the channel and beneath the impermeable layer. The values of suction potential and water content increase over time t and converge, indicating stability in the infiltration process. These findings align well with real-world scenarios, demonstrating that the developed mathematical model and its numerical solution using DRBEM accurately illustrate the time-dependent water infiltration process in impermeable furrow irrigation channels.

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1. INTRODUCTION

Water is one of the most essential resources for all life on Earth [1]. Human civilization has continually advanced and flourished due to the availability of adequate water resources. Without water, the processes of life cannot occur properly. Thus, it is imperative to manage water resources carefully for domestic, irrigation, and industrial purposes. Water with sufficient volume, quality, and appropriate location is crucial for humans and other living beings. Among its various uses, water plays a key role in irrigation. Irrigation refers to a water distribution system designed to channel water to specific areas, such as agricultural or plantation lands, to enhance plant growth [2]. This system must be efficient in its use of water resources, particularly in regions with limited water availability.

The fundamental classification of irrigation methods is divided into four types: surface irrigation, sprinkler irrigation, subsurface irrigation, and drip irrigation. Surface irrigation is the oldest and most widely applied method [3]. Although this method can be used for a variety of crops, it is not recommended for soils with high permeability or areas with steep slopes. Surface irrigation itself is further categorized into several methods. Among these, furrow irrigation is one of the most used and efficient methods for water distribution [4]. In this approach, water infiltrates the soil through the bottom and walls of the furrow, seeping into the ground.

The mathematical model for water infiltration problems in furrow irrigation channels is represented by a modified Helmholtz equation, which is a Partial Differential Equation (PDE) with Boundary Value Problems (BVP) [5]. One approach to solving this problem is through the numerical method known as the Dual Reciprocity Boundary Element Method (DRBEM). DRBEM is an extension or development of the Boundary Element Method (BEM) [6], [7].

BEM is a numerical method used to solve PDEs commonly encountered in mathematics and engineering physics. These include the Laplace Equation, Helmholtz Equation, Convection-Diffusion Equation, Potential Flow and Viscosity Flow Equations, Electrostatic and Electromagnetic Equations, and Linear Elasticity and Elastodynamic Equations [8]. The main idea behind the Boundary Element Method is that the solution to a PDE is expressed as a boundary integral equation containing the fundamental solution of the PDE [9]. One of the main advantages of DRBEM is that a single set of collocation points located on the domain boundaries can be used to obtain the solution at all points within the domain. This is different from the Finite Element Method (FEM) and the Finite Difference Method (FDM), where the solution is obtained only at the collocation points [10].

Several studies have applied DRBEM to analyze infiltration in furrow irrigation channels. For example, [11] used DRBEM to study six channel shapes: flat channel, rectangular channel, semicircular channel, trapezoidal channel, rectangular impermeable channel, and trapezoidal impermeable channel in time-independent furrow irrigation channels without vegetation. Their findings indicated that impermeable irrigation channels exhibited the highest efficiency, as evidenced by the lower water content outside the plant root zone. [12] extended this study by applying DRBEM to the same six-channel shapes but in time-independent furrow irrigation channels with vegetation. Their results showed that impermeable channels were more efficient than non-impermeable channels in terms of the amount of water absorbed by plants. Further studies have focused on time-dependent infiltration cases. For instance, [13] investigated time-dependent infiltration in trapezoidal irrigation channels, while [14] examined time-dependent infiltration in semicircular channels.

The motivation for this study is to improve water use efficiency in irrigation systems, particularly in arid and semi-arid regions. Time-dependent infiltration provides a more realistic representation of water movement during irrigation compared to steady-state conditions. The presence of impermeable layers also significantly affects infiltration dynamics, yet this aspect remains underexplored in literature. Therefore, this study aims to fill this gap and offer a more accurate understanding of the infiltration process in trapezoidal furrow irrigation channels with impermeable layers.

The configuration chosen in this study reflects real field conditions, particularly in furrow irrigation systems that use impermeable lining materials such as plastic or geomembrane at the bottom of the channel [15]. This practice is commonly applied to prevent excessive vertical infiltration losses, especially in regions with limited water availability such as Southeast Asia, India, and the Middle East [16], [17]. The use of impermeable linings has also been proven to improve irrigation efficiency and water distribution along the channel [18]. In addition, trapezoidal cross-sections are often used in irrigation channel design due to their ability to optimize water flow and ease of construction, making them popular among farmers [19].

To date, previous studies [11], [12], [13], [14] have not addressed the case of time-dependent flow in irrigation channels with impermeable layers. Building upon this gap, the present study investigates time-dependent infiltration in trapezoidal furrow irrigation channels under impermeable conditions. The DRBEM numerical method will be employed to solve the resulting mathematical model. The numerical solutions will then be used to determine water content and water uptake by plants. These results can provide insights into the characteristics of time-dependent infiltration and evaluate the effectiveness of trapezoidal impermeable irrigation channels.

2. RESEARCH METHODS

1. Model Construction

In this phase, the researcher constructs the mathematical model for time-dependent water infiltration in furrow irrigation channels based on the assumptions used in model formulation. The model is represented as a differential equation, accompanied by boundary and initial conditions, known in mathematics as IBVP (Initial and Boundary Value Problems). The model consists of:

   a. The governing equation: a modified Helmholtz equation.
   b. Boundary conditions: Neumann and Robin boundary conditions.

2. Solving the Model Using DRBEM

The model is solved numerically using the Dual Reciprocity Boundary Element Method (DRBEM) through the following steps:

   a. Establish the reciprocal relationship between the solution of the Helmholtz Equation and the fundamental solution of the Laplace Equation using the Poisson Equation’s reciprocal relationship.
   b. Formulate the boundary integral equation for the Helmholtz Equation by utilizing the reciprocal relationship between the Helmholtz solution and the fundamental Laplace solution, along with domain modifications.
   c. Approximate the domain integral in the boundary integral equation using a linear combination of radial basis functions centered at collocation points.
   d. Derive the boundary integral equation for the Helmholtz Equation in the form of a line integral using the previously modified boundary integral equation and domain integral approximation.
   e. Solve the resulting boundary integral equation by substituting collocation points, yielding a system of linear equations (SLE).
   f. Solve the SLE, then substitute its solution back into the boundary integral equation to obtain an expression that can evaluate the Helmholtz Equation at any point in the domain.

3. Implementation in Computer Programming

The DRBEM-based model developed in the previous step is implemented in a computer program, which involves the following stages:

   a. Pre-Processing: Discretize the domain, define collocation points, segment lengths, and normal vectors.
   b. Processing: Construct the DRBEM model and derive equations to determine solutions for each domain.
   c. Post-Processing: Evaluate the numerical solution and visualize the results for further analysis.

4. Conclusion

The output of the developed computer program provides the distribution patterns of water content in the soil surrounding the irrigation channel over time. Based on this output, conclusions can be drawn regarding the characteristics of time-dependent water infiltration in furrow irrigation channels.

3. RESULTS AND DISCUSSION

3.1 Mathematical Model of Time-Dependent Infiltration in Impermeable Furrow Irrigation

The mathematical model for water infiltration in impermeable furrow irrigation channels is formulated as an Initial and Boundary Value Problem (IBVP) with mixed Neumann and Robin boundary conditions. The governing equation is expressed as a modified Helmholtz equation. This governing equation is derived from Richard's equation, which is transformed using the Kirchhoff transformation and dimensionless variables.

Richard's equation is inherently a nonlinear differential equation, making it challenging to solve directly. Therefore, transformations are applied to simplify the equation into a form that is more manageable for numerical or analytical solutions. The mathematical representation of time-dependent infiltration processes remains rooted in Richard's equation, describing the dynamics of water movement in unsaturated soil.

$$\frac{\partial \theta}{\partial T} = \frac{\partial}{\partial X}\left(K\frac{\partial \psi}{\partial X}\right) + \frac{\partial}{\partial Z}\left(K\frac{\partial \psi}{\partial Z}\right) - \frac{\partial K}{\partial Z} \tag{1}$$

where θ represents the water content, K denotes the hydraulic conductivity, and ψ is the suction potential.

Richard's equation will be transformed into the Helmholtz equation, a linear differential equation. The transformation process involves the following steps [20]:

1. Kirchhoff Transformation

Apply the Kirchhoff transformation using the proposed formula:

$$K = K_0 e^{\alpha\psi}, \alpha > 0$$

$$\Theta = \int_{-\infty}^{\psi} K(s)ds \tag{2}$$

where Θ is the Matric Flux Potential (MFP).

2. Exponential Model of Hydraulic Conductivity

The hydraulic conductivity is defined using an exponential model:

$$K = K_0 e^{\alpha\psi}, \alpha > 0 \tag{3}$$

where K_0 represents the hydraulic conductivity in saturated soil, and α is a parameter characterizing the soil's properties.

3. Transformation into Dimensionless Variables

Transform the governing equation into a dimensionless form as follows:

$$x = \frac{\alpha}{2}X, z = \frac{\alpha}{2}Z, \Phi = \frac{\pi\Theta}{v_0 L}, t = \frac{\alpha^2 d}{4}T,$$

$$u = \frac{2\pi}{v_0 \alpha L}U, v = \frac{2\pi}{v_0 \alpha L}V, f = \frac{2\pi}{v_0 \alpha L}F, \tag{4}$$

Here, v_0 is the initial flux, L is half the length of the irrigation channel,

4. Laplace Transformation

Applying the Laplace transformation, the variable Φ can be transformed to t as follows:

$$\Phi^*(x,z,s) = \int_0^{\infty} e^{-st}\Phi(x,z,t)dt, \tag{5}$$

where the initial condition is assumed to be $\Phi(x,z,0) = 0$.

5. Substitution and Assumptions

$$\Phi^* = \phi e^{z} \tag{6}$$

By the transformation process into the transformed governing **Equation (1)**, the equation can be reformulated into the following:

$$(1+s)\phi = \frac{\partial^2 \phi}{\partial^2 x} + \frac{\partial^2 \phi}{\partial^2 z} \tag{7}$$

Equation (7) is referred to as the Modified Helmholtz Equation, which represents the water infiltration process in unsaturated irrigation channels.

Next, the boundary conditions will be established. It is assumed that water enters only through the permeable walls of the channel and that, at infinite depth, the rate of the Matric Flux Potential (MFP) approaches zero. The domain R and the incoming flux can be illustrated in **Figure 1**.

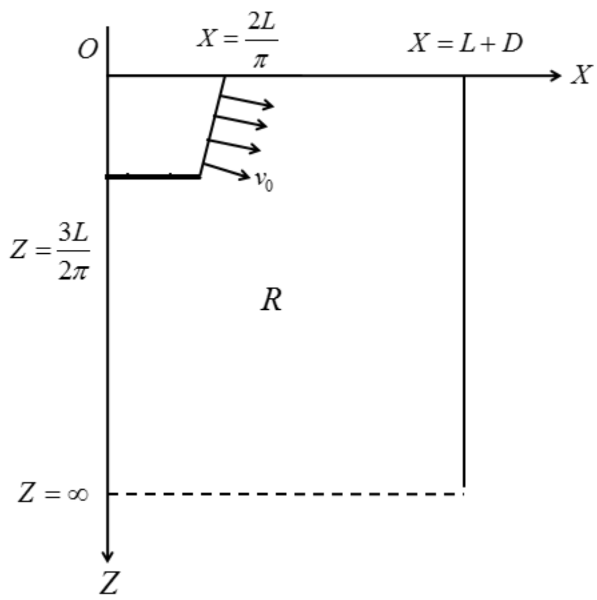


Figure 1. Domain of the Infiltration Problem in a Channel with Impermeable Layers

Based on this assumption, the boundary conditions are formulated as follows:

$$F = \begin{cases} -v_0, & \text{on permeable boundaries,} \\ 0, & \text{on impermeable boundaries,} \\ 0, & \text{on the soil surface outside the channel,} \\ 0, & \text{for } X = 0 \text{ and } Z \geq 0, \\ 0, & \text{for } X = L + D, \text{and } Z \geq 0, \end{cases} \tag{8}$$

Applying the same transformation techniques as used for the governing equations.

$$\frac{\partial \phi}{\partial n} = \begin{cases} \frac{2\pi}{\alpha L} e^{-z} + \phi n_2, & \text{on permeable boundaries,} \\ \phi n_2, & \text{on impermeable boundaries,} \\ -\phi, & \text{on the soil surface outside the channel,} \\ 0, & \text{for } x = 0, \text{and } z \geq 0, \\ 0, & \text{for } x = \frac{\alpha}{2}(L+D), \text{and } z \geq 0, \\ -\phi, & \text{for } 0 \leq x \leq \frac{\alpha}{2}(L+D), \text{and } z = \infty \end{cases} \tag{9}$$

Next, the line segments can be defined as follows:

C_1 : on permeable boundaries,

C_2 : on impermeable boundaries,

C_3 : on the soil surface outside the channel,

C_4 : $x = 0$ and $z \geq 0$,

$$C_5 : x = \frac{\alpha}{2}(L + D) \text{ and } 0 \leq z \leq c,$$

$$C_6 : 0 \leq x \leq \frac{\alpha}{2}(L + D) \text{ and } z = c. \tag{10}$$

Assume that the domain R is closed and bounded by the curve C, where $C = C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6$. The mathematical model for water infiltration in the irrigation channel in dimensionless variables is as follows:

$$\frac{\partial^2 \phi(x,z,t)}{\partial^2 x} + \frac{\partial^2 \phi(x,z,t)}{\partial^2 z} = (1 + s(x,z,t))\phi(x,z,t), (x,y,z) \in R\ , t \geq 0,$$

with boundary conditions

$$\frac{\partial \phi}{\partial n} = \begin{cases} \frac{2\pi}{\alpha L} e^{-z} + \phi n_2, & \text{on } C_1, \\ \phi n_2, & \text{on } C_2, \\ -\phi, & \text{on } C_3, \\ 0, & \text{on } C_4, \\ 0, & \text{on } C_5, \\ -\phi, & \text{on } C_6. \end{cases} \tag{11}$$

3.2 Solution Using Dual Reciprocity Boundary Element Method

The inverse Laplace transform will be computed to determine the time-dependent values of Matric Flux Potential ($\Phi(x,z,t)$). Using **Equation (6)**, numerical values for Φ^* will be obtained by applying the Stehfest formula for the inverse Laplace transform. According to this formula, if $\Phi^*(x,z,s)$ is the Laplace transform of a function $\Phi(x,z,t)$, then the inverse transform can be calculated as follows **[14]**:

$$\Phi(x,z,t) \simeq \frac{\log(2)}{t} \sum_{p=1}^{2P} K_n \Phi^*(x,z,s_p), \tag{12}$$

Where:

$$s_p = p\frac{\log(2)}{t},$$

$$K_p = (-1)^{(p+P\,)} \sum_{m=(p+1)/2}^{\min(p,P)} \frac{m^P (2m)!}{(P-m)!\, m!\, (m-1)!\, (p-m)!\, (2m-p)!}$$

Here, P is an even positive integer.

The value of P is chosen as 3 in this study. Based on the selected P, different values of s are evaluated, corresponding to $s = 1,2,3,4,5,6$. Subsequently, **Equation (6)**, along with $s = 1,2,3,4,5,6$ and boundary **Equation (11)** will be solved using the DRBEM to obtain the solution Φ for a specific t. A more complete process of the numerical solution of DRBEM can be seen in **[21]**.

After obtaining the value of Φ, it can be transformed back to its original form to yield ψ, representing the suction potential during the infiltration process in irrigation channels:

$$\psi = \frac{1}{\alpha}\, ln\left(\frac{\alpha \Phi v_0 L}{\pi K_0}\right) \tag{13}$$

Water content is defined as the ratio of the volume of water contained in the soil to the total volume of soil. The relationship between suction potential and water content can be expressed as **[22]**:

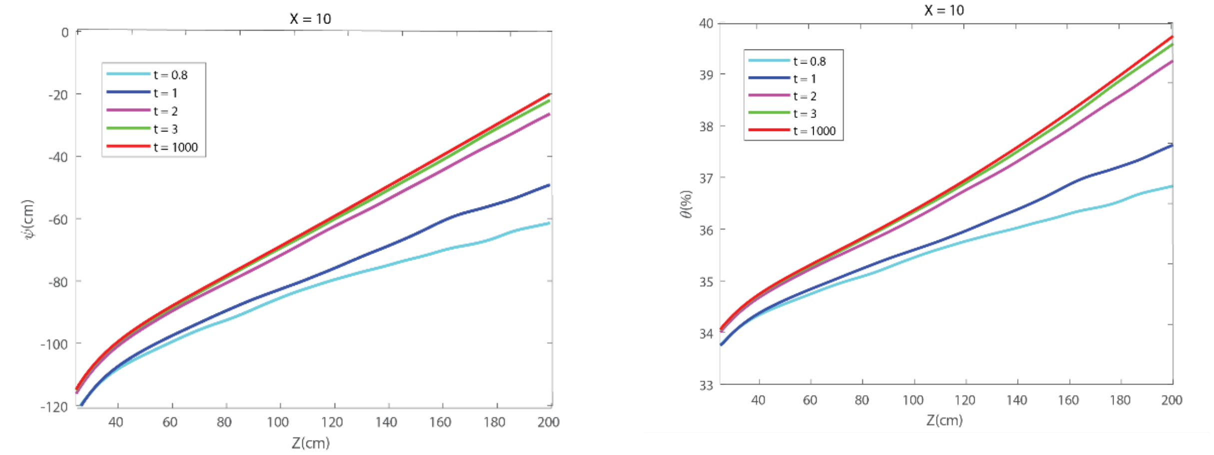
$$\theta = \left(\frac{1}{1 + (\alpha\psi)^n}\right)^m (\theta_s - \theta_r)\theta_r \tag{14}$$

where, θ_r is the residual water content, θ_s is the saturated water content, α, m and n are empirical parameters that depend on the soil type.

This research employs the Dual Reciprocity Boundary Element Method (DRBEM) to solve the problem of time-dependent water infiltration in trapezoidal irrigation channels. The soil type studied is Pima

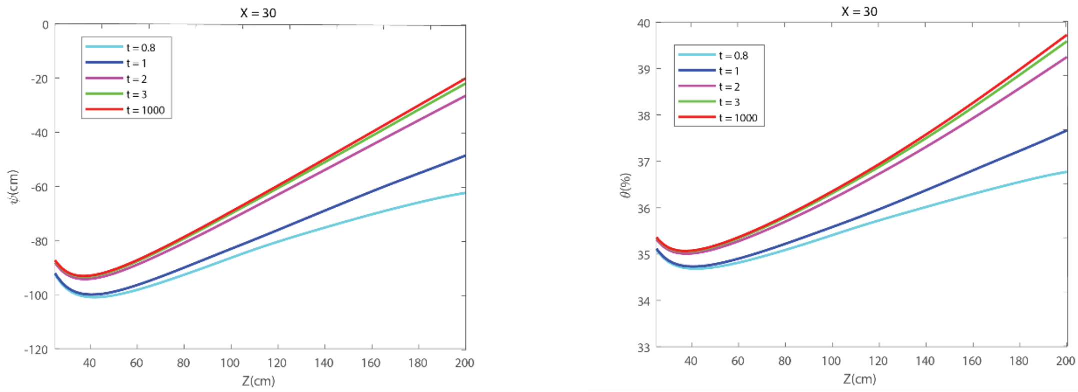
Clay Loam, which is characterized by the following parameters: $\alpha = 0.014\, cm^{-1}$, $K_0 = 1.115 \times 10^{-4}\, cm/s = 9.9\, cm/day$, $\theta_r = 0.095$, $\theta_s = 0.41$, $n = 1.13$ [15], [16]. The depth domain is set to $c = 4$, as previous studies indicate that this value provides a good numerical approximation without significant influence on ϕ [25]. $L = 50$ cm and $D = 50$ cm represents half the width of the channel and the exterior soil surface, respectively. The number of boundary segments N for discretizing the boundary domain is chosen as $N = 400$, while the number of interior points M for domain discretization is selected as $M = 900$.

The computational program is implemented iteratively by varying the infiltration time t, where: $t = 0.8, 1, 2, 3, \infty$. For each time step t, the values of suction potential ψ and water content θ are evaluated at several points along the lines $X = 10\, cm$, $X = 30\, cm$, $X = 50\, cm$, $X = 70\, cm$, and $X = 90\, cm$, for $0 \leq Z \leq 200\, cm$. The results for ψ and θ are plotted as graphs to illustrate variations with depth Z. Moreover, based on these graphs, the relationship between ψ and θ for each t can be determined. The graphs are displayed in Figure 2 to Figure 6.



(a) (b)

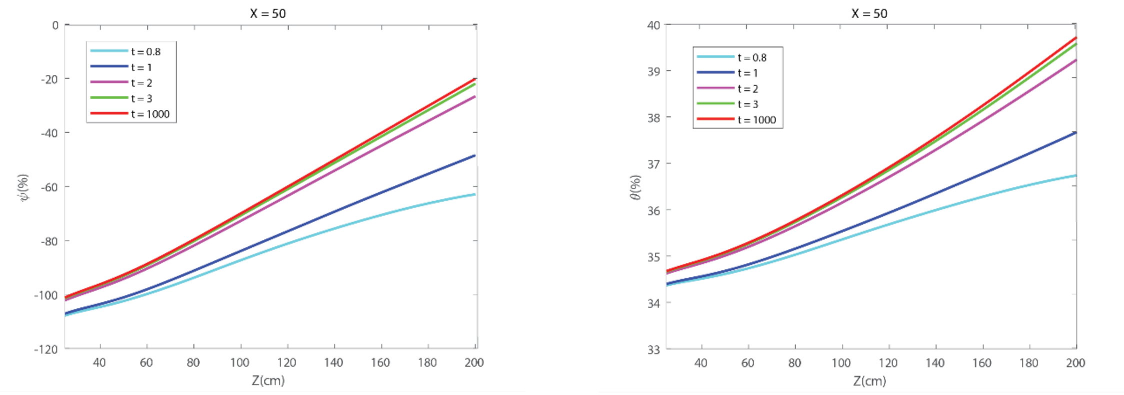
Figure 2. (a) Suction Potential and (b) Water Content for Each Time Step t along $X = 10$ cm



(a) (b)

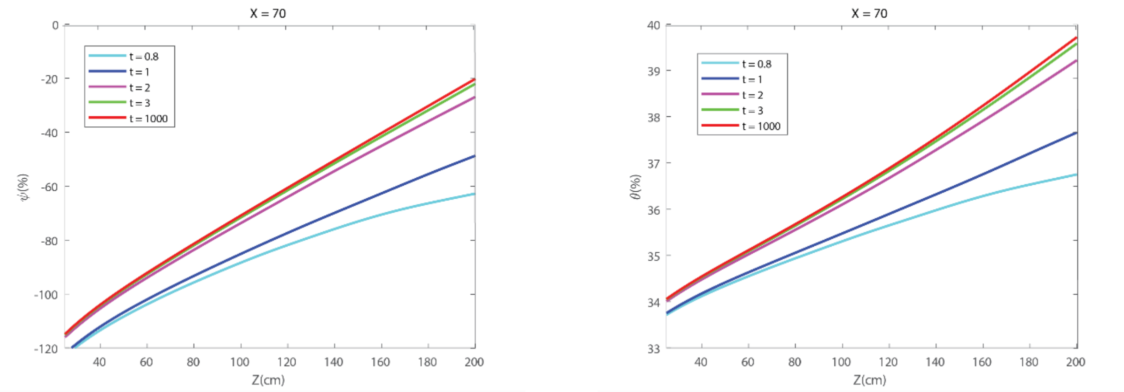
Figure 3. (a) Suction Potential and (b) Water Content for Each Time Step t along $X = 30$ cm

Figure 4 and Figure 5 represent the graphs of $\boldsymbol{\psi}$ and $\boldsymbol{\theta}$, respectively, along lines $\boldsymbol{X} = \mathbf{10\ cm}$ and $\boldsymbol{X} = \mathbf{30\ cm}$. Line $\boldsymbol{X} = \mathbf{10\ cm}$ is located beneath the flat side of the trapezoidal channel, while the line $\boldsymbol{X} = \mathbf{30\ cm}$ is on its sloping side. Based on Figure 4, the values of $\boldsymbol{\psi}$ and $\boldsymbol{\theta}$ increase from the smallest to the largest as depth increases. This indicates that the lowest Suction Potential and Water Content values occur directly beneath the impermeable channel and increase with depth. Similarly, based on Figure 5, the values of $\boldsymbol{\psi}$ and $\boldsymbol{\theta}$ initially decrease slightly and then increase as depth increases. This suggests that on the sloping side of the trapezoidal channel, which is permeable, the Suction Potential and Water Content decrease slightly at first and then increase with depth. Both observations align with the assumption that water infiltration occurs solely from the channel.



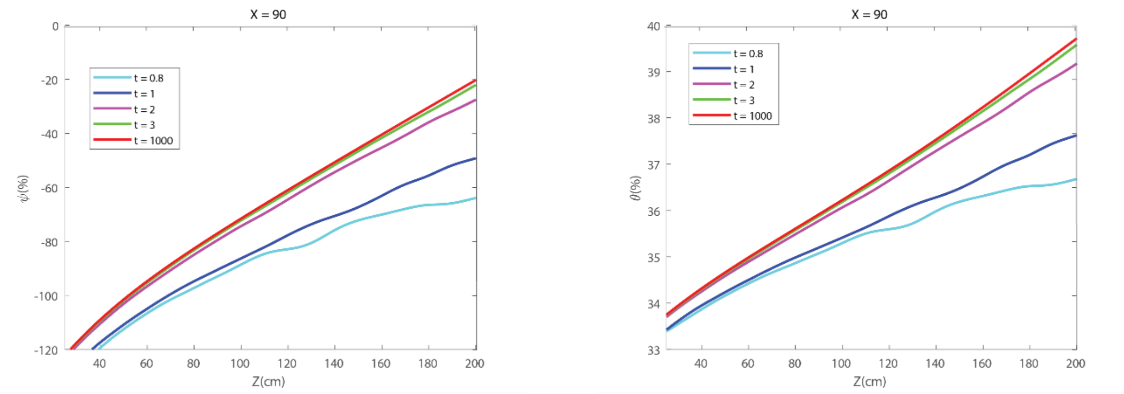
(a) (b)

Figure 4. (a) Suction Potential and (b) Water Content for Each Time Step t along $X = 50$ cm



(a) (b)

Figure 5. (a) Suction Potential and (b) Water Content for Each Time Step t along $X = 70$ cm



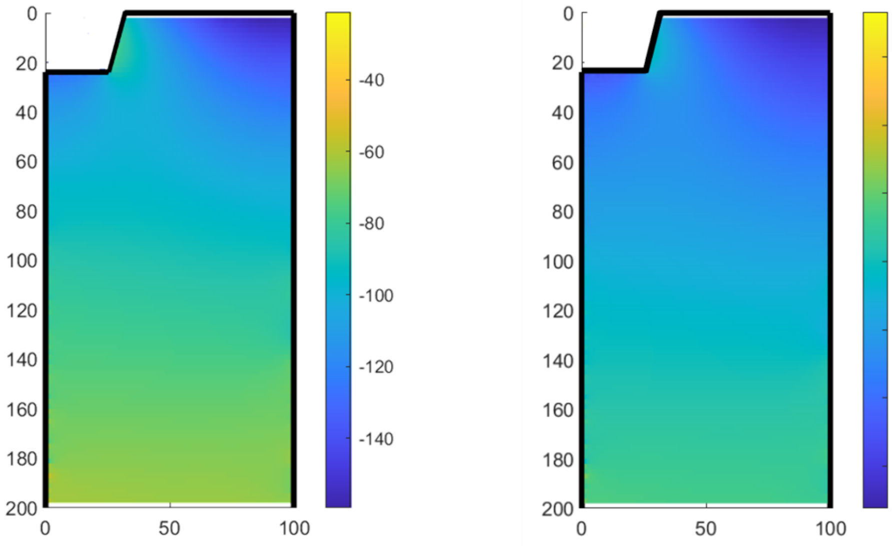
(a) (b)

Figure 6. (a) Suction Potential and (b) Water Content for each time step t along $X = 90$ cm

Based on Figure 4 to Figure 6, which depict the graphs of ψ and θ along lines $X = 50$ cm, $X = 70$ cm, and $X = 90$ cm , the values of ψ and θ in these sections correspond to areas not located beneath the channel. These figures show that the values of ψ and θ increase with depth and converge to a certain point. This indicates that suction potential is directly proportional to water content. The increase in θ with depth also suggests that the water content in shallow soil layers is lower than in deeper layers, particularly in areas outside the soil beneath the channel.

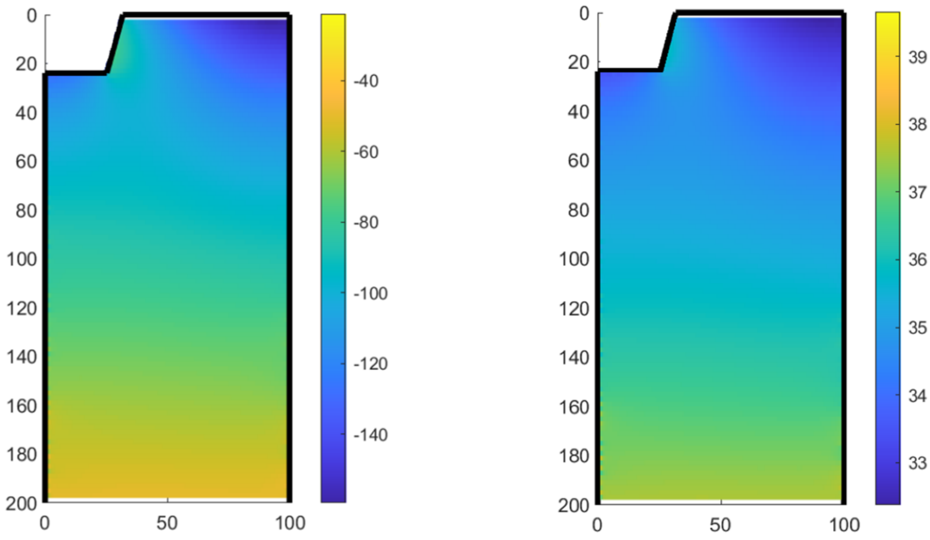
Based on Figure 2 to Figure 6, which illustrate the graphs of ψ and θ along lines $X = 10\ cm$, $X = 30$ cm, $X = 50$ cm, $X = 70$ cm, and $X = 90$ cm, for $0 \leq Z \leq 200\ cm$ for various t, it is evident that the observed values of ψ and θ are significantly influenced by time. The results demonstrate that increases in the values of ψ and θ correspond to larger t values. In other words, the longer the infiltration time, the deeper the water penetrates into the soil.

Additionally, to observe the distribution patterns of suction potential ψ and water content θ within the domain under the influence of time, surface plots were generated as shown in **Figure 7** to **Figure 11**. In this context, the domain represents the cross-sectional area of a channel in the soil, with a width of 100 cm and a depth of 200 cm.



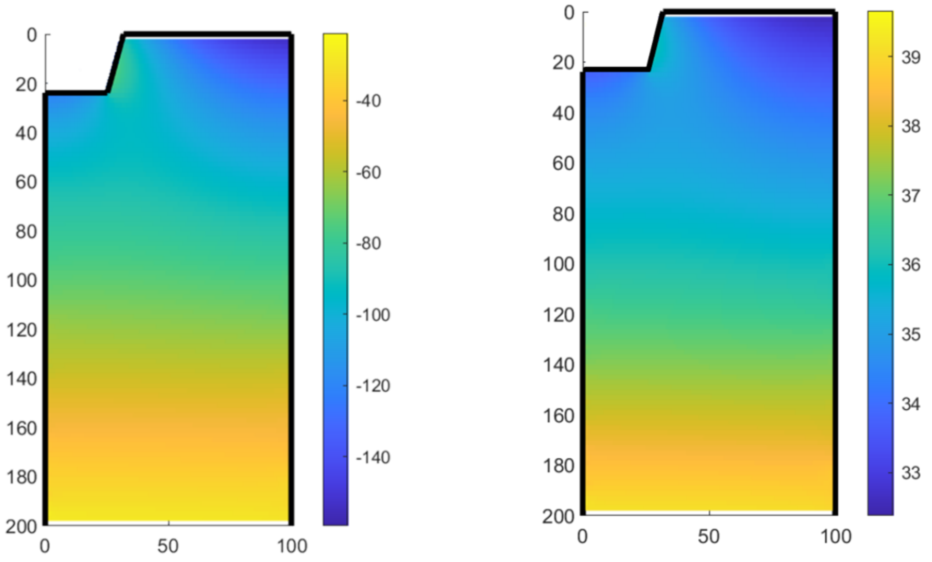
(a) **(b)**

Figure 7. (a) Suction Potential and (b) Water Content at $t = 0.8$



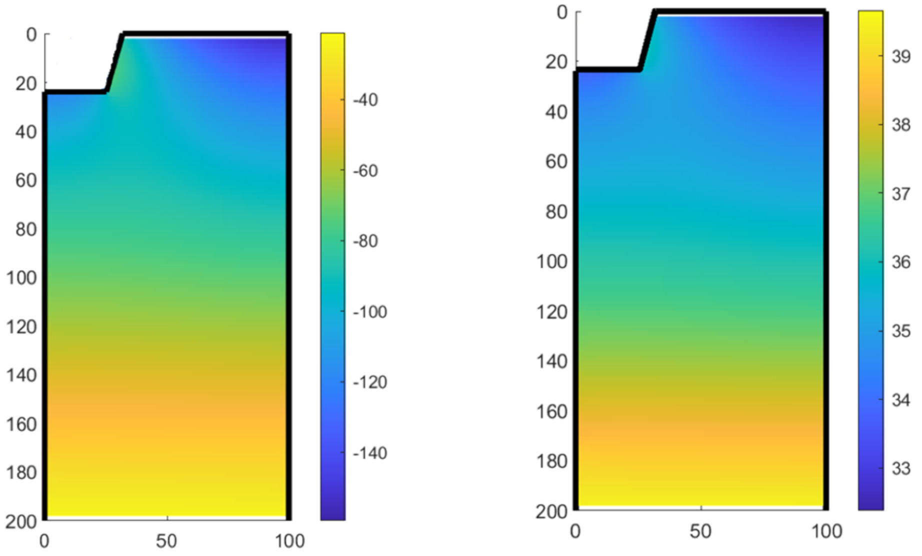
(a) **(b)**

Figure 8. (a) Suction Potential and (b) Water Content at $t = 1$



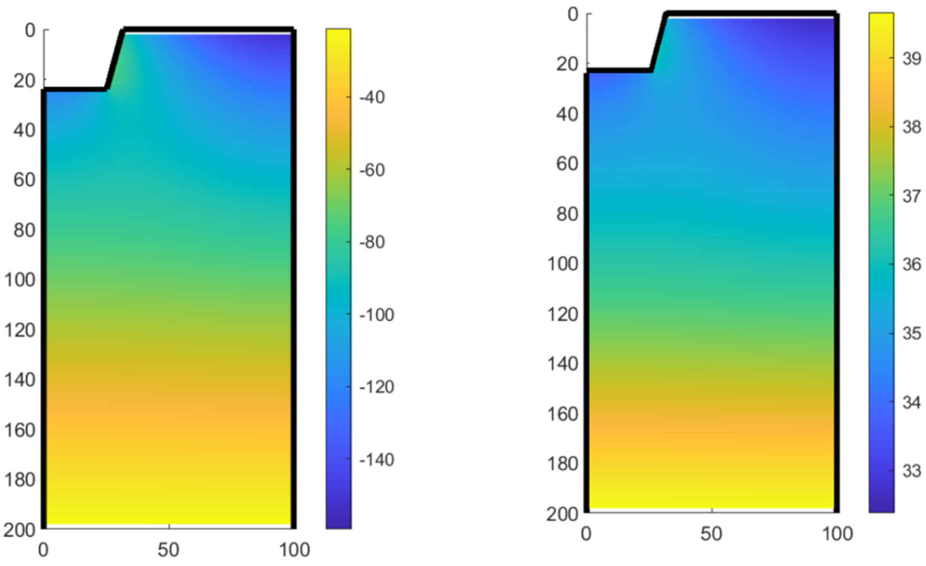
(a) (b)

Figure 9. (a) Suction Potential and (b) Water Content at $t = 2$



(a) (b)

Figure 10. (a) Suction Potential and (b) Water Content at $t = 3$



(a) **(b)**

Figure 11. (a) Suction Potential and (b) Water Content at $t = 1000$

The maximum and minimum values of suction potential ψ and water content θ across all t values within the domain are presented in **Table 1**.

Table 1. Minimum and Maximum Values of Suction Potential and Water Content Analyzed

No	t	Suction potential (cm)		Water content (%)	
		Min	Max	Min	Max
1	0.8	-157.5419	-56.1307	32.4409	37.1709
2	1	-159.3552	-41.9498	32.3808	38.1272
3	2	-152.1610	-27.1092	32.6230	39.2067
4	3	-151.3570	-22.9264	32.6507	39.5191
5	1000	-150.9144	-21.0491	32.6660	39.6593

Based on **Figure 7** to **Figure 11**, the largest suction potential $\boldsymbol{\psi}$ and water content $\boldsymbol{\theta}$ values in the upper soil layers are located directly beneath the permeable channel, while the smallest values are found at the soil surface outside the channel and beneath the impermeable channel for each $\boldsymbol{t}$. In contrast, the lower soil layers exhibit almost identical suction potential and water content values. Additionally, the distribution patterns of suction potential and water content for the five different $\boldsymbol{t}$ values appear to be similar.

From the results above, several conclusions can be drawn:

1. Suction potential and water content are directly proportional, resulting in similar graphical representations.
2. Water infiltrates the soil exclusively through the permeable layer, with no infiltration occurring through the impermeable layer, which aligns with the assumptions of the constructed model.
3. In the lower soil layers, suction potential and water content tend to stabilize, consistent with the assumption that the Matric Flux Potential (MFP) rate approaches zero in these layers.
4. Water entering through the permeable layer laterally percolates into the root zone, indicating that this irrigation channel design is effective in optimizing water usage.

Based on **Figure 7** to **Figure 11** and **Table 1**, it can be observed that suction potential and water content increase over time, indicating the progressive infiltration of water into the soil. Interestingly, the values of these two variables show minimal differences at $\boldsymbol{t = 2}$, $\boldsymbol{t = 3}$, and $\boldsymbol{t = 1000}$, suggesting that the infiltration process tends to stabilize shortly after water application. This result reflects real-world conditions in which soil reaches a quasi-steady infiltration state relatively quickly. Such insight is valuable for practical applications, particularly in the context of improving irrigation efficiency. Knowing the approximate time at which soil moisture and suction potential stabilize can inform decisions on when to stop irrigation without causing water loss due to percolation. Thus, the proposed approach not only models the infiltration process accurately but also provides a useful reference for optimizing water usage in agricultural practices.

4. CONCLUSION

1. The mathematical model of water infiltration in a furrow irrigation channel with an impermeable layer in homogeneous soil is formulated as a Boundary Value Problem (BVP) with the Modified Helmholtz Equation as the governing equation and mixed boundary conditions.
2. The highest values of suction potential and water content are located beneath the permeable channel, while the lowest values are found at the soil surface outside the channel and beneath the impermeable layer.
3. The implementation of various t values demonstrate that both suction potential and water content increase as t increases.
4. The solutions obtained align well with real-world scenarios, indicating that the mathematical model developed and its solution using the Dual Reciprocity Boundary Element Method (DRBEM) provide an accurate representation of the time-dependent water infiltration process in impermeable furrow irrigation channels.

AUTHOR CONTRIBUTIONS

Yanne Irene: Writing – Review and Editing, Supervision, Software, Resources, Project Administration, Methodology, Funding Acquisition, Formal Analysis, Conceptualization. Muhammad Manaqib: Writing – Original Draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis. Mochammad Rafli Alamsyah: Writing – Original Draft, Visualization, Validation, Software, Investigation, Formal Analysis. Madona Yunita Wijaya: Writing – Review and Editing, Supervision, Investigation, Formal Analysis. All authors discussed the results and contributed to the final manuscript.

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CONFLICT OF INTEREST

The authors declare no conflicts of interest.

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