BAREKENG: Journal of Mathematics and Its Applications
December 2025 Volume 19 Issue 4 Page 2727–2738
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol19iss4pp2727-2738

THE LOCATING RAINBOW CONNECTION NUMBERS OF LOLLIPOP AND BARBELL GRAPHS

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Article History:

Received: 18th February 2025
Revised: 25th April 2025
Accepted: 28th May 2025
Available online: 1st September 2025

Keywords:

Barbell Graph;
Locating Rainbow Coloring;
Locating Rainbow Connection Number;
Lollipop Graph.

ABSTRACT

The concept of the locating rainbow connection number of a graph is an innovation in graph coloring theory that combines the concepts of rainbow vertex coloring and partition dimension on graphs. This concept aims to determine the smallest positive integer k such that there exists a locating rainbow k-coloring on the graph, ensuring that every vertex has a unique rainbow code. In this study, we investigate the locating rainbow connection number of the lollipop graph $L(m,n)$ and barbell graph $B(K_n)$. Using a literature study method, hypotheses were formulated and proven through theoretical analysis. The results show that $rvcl(L(m,n)) = max\{m,n\}$ and $rvcl(B(K_m)) = m$.

How to cite this article:

A. W. Bustan, T. Talib, N. S. Laamena, L. R. Saputra, and Nurhayati, "THE LOCATING RAINBOW CONNECTION NUMBERS OF LOLLIPOP AND BARBELL GRAPHS", *BAREKENG: J. Math. & App.*, vol. 19, iss. 4, pp. 2727-2738, December, 2025.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · Open Access

1. INTRODUCTION

The concept of the locating rainbow connection number of a graph is novel in graph coloring theory, introduced by Bustan et al. in 2021 [1]. This concept combines the ideas of partition dimension and rainbow vertex coloring of a graph. The partition dimension concept in graphs is related to the minimum size of a partition of the vertex set such that each vertex can be uniquely identified based on its distance vector to each part in the partition [2]. Meanwhile, the rainbow vertex coloring concept involves coloring the vertices of a graph in such a way that every two distinct vertices in the graph are connected by a rainbow vertex path [3]. This concept is an extension of the rainbow coloring concept introduced by Chartrand et al. in 2008 [4]. The rainbow coloring of a graph is one of the NP-hard problems [5]. Consequently, many researchers are interested in developing this concept further. Several results related to the rainbow connection number in graphs can be found in [6] and [7]. In a related line of research, the locating chromatic number has also been studied as a vertex-coloring parameter that combines identifying codes with proper coloring ([8], [9], [10]).

Let $G = (V(G), E(G))$ be a finite connected graph, and let k be a positive integer. A k-rainbow vertex coloring of G is a mapping $c: V(G) \to [1, k]$ such that for every two distinct vertices u and v in $V(G)$, there exists a path connecting them whose internal vertices have distinct colors. A path P in G whose internal vertices have distinct colors is called a rainbow vertex path. The rainbow vertex connection number of a graph G, denoted as $rvc(G)$, is the smallest positive integer k such that there exists a k-rainbow vertex coloring of G. The concept of *rainbow vertex coloring* is also classified as an NP-hard problem ([11], [12]). Some of the latest studies on the rainbow vertex connection number for certain graph classes can be found in [13]. Additionally, studies on the rainbow vertex connection number of graphs resulting from operations can be found in [14], [15], and [16].

For $i \in [1, k]$, let R_i be the set of vertices assigned a color i, and let $\Pi = R_1, R_2, \dots, R_k$ be an ordered partition of $V(G)$. The rainbow code of a vertex $v \in V(G)$ with respect to Π, denoted as $rc_\Pi(v)$, is the ordered k-tuple defined as $rc_\Pi(v) = \big(d(v, R_1), d(v, R_2), \dots, d(v, R_k)\big)$ with $d(v, R_i) = \{min\, d(v, y) | y \in R_i\}$ for every $i \in [1, k]$. If each vertex in G has a distinct rainbow code, then the coloring c is called a k-locating rainbow coloring of G. The locating rainbow connection number of G, denoted as $rvcl(G)$, is defined as the smallest positive integer k for which there exists a k-locating rainbow coloring of G. To simplify notation, the term "entry" is used to denote the distance of a vertex to a color set [17].

The locating rainbow connection number is useful in building security systems by optimizing the placement of biometric scanners. In this system, doors represent graph vertices, and edges denote hallways. Assigning the same scanner type to all doors poses a security risk-if one is compromised, all rooms become vulnerable. A more secure yet cost-effective solution is to minimize the number of scanner types while ensuring secure access, which can be achieved using the rainbow vertex connection concept. Additionally, assigning unique codes to doors based on scanner types enhances security by allowing quick identification of compromised access points [17].

The locating rainbow connection number of a graph has been studied in trees and bipartite graphs by providing characterizations of their locating rainbow connection numbers [17]. Additionally, it has been examined in amalgamation graphs, particularly in the amalgamation of complete graphs [18]. Furthermore, Bustan et al. [19] have also investigated the locating rainbow connection number in several classes of vertex-transitive graphs, including cycle graphs.

Graph coloring, particularly the locating rainbow connection number, is an intriguing topic of study. The novelty of this concept implies that, for many classes of graphs, their locating rainbow connection numbers remain undetermined. Bustan et al. demonstrated that the locating rainbow connection number of a complete graph is equal to its order. Therefore, we aim to determine the locating rainbow connection number for graphs that contain a complete graph, including the lollipop graph and the barbell graph, and analyze its relationship with the locating rainbow connection number of the complete graph itself. The lollipop graph, denoted as $L(m, n)$, is a graph obtained by connecting a complete graph K_m of order m to a path graph P_n of order n using a bridge. The barbell graph, denoted as $B(K_m)$, is a simple graph obtained by connecting two copies of a complete graph K_m with a bridge.

2. RESEARCH METHODS

The research employs a literature study method, following these stages:

1. Literature review

   At this stage, an in-depth examination is conducted on facts, observations, lemmas, and theorems related to the concept of the locating rainbow connection number of a graph, as well as the characteristics of both the lollipop graph and the barbell graph. Additionally, an analysis is performed to determine the most appropriate proof methods for the lemmas and theorems that will be established.

2. Formulating hypotheses

   Based on the literature review, a hypothesis is proposed regarding the value of the locating rainbow connection number for the lollipop graph and the barbell graph. This hypothesis is then formulated into a lemma or theorem.

3. Theorem proof

   This stage involves proving the hypothesis concerning the locating rainbow connection number for both the lollipop and barbell graphs. The proof consists of two main steps: Lower bound proof: Established using contradiction and direct proofs involving lemmas or factual statements. Upper bound proof: Established by defining an appropriate coloring function.

4. Conclusion

   Once the hypothesis is successfully proven, it is formally stated as a theorem.

3. RESULTS AND DISCUSSION

To simplify notation, we define $[a,b] = \{x \in \mathbb{Z} |\ a \le x \le b\}$. We divide the results of this study into two subsections: Subsection 3.1 discusses the locating rainbow connection number of the lollipop graph, while Subsection 3.2 focuses on the locating rainbow connection number of the barbell graph.

The following are some previous research results related to the locating rainbow connection number of a graph.

Lemma 1 [1] *Let m be a positive integer with $m \ge 3$. If G be a connected graph of order m, then $2 \le rvcl(G) \le m$.*

Lemma 2 [1] *Let c be a locating rainbow coloring of G, and let u and v be two distinct vertices in G. If $d(u,x) = d(v,x)$ for all $x \in V(G) - \{u,v\}$ then $c(u) \ne c(v)$.*

Lemma 3 [20] *If p is the number of cut vertices in a graph G, then $rvcl(G) \ge p$.*

Lemma 4 [17] *Let G is a connected graph of order $m \ge 3$. $rvcl(G) = m$ if and only if G is isomorphic to complete graph.*

Lollipop and barbell graphs are two classes of graphs that both contain a clique, which is a set of vertices forming a complete subgraph. Therefore, before discussing the main results, we present a lemma concerning the coloring rules of a graph that contains a clique whose locating rainbow connection number is smaller than the number of vertices in the maximum clique of the graph.

Lemma 5. *If G is a connected graph with a maximum clique K, and $rvcl(G) = r < |K|$, then the vertices in K must be colored using at most $r - 1$ colors.*

Proof. Suppose that there exists a locating rainbow coloring of G using exactly r colors, and all vertices in a maximum clique $K \subseteq V(G)$ are colored using these r colors. Since K is a clique, every pair of distinct vertices in K is adjacent, and thus K forms a complete subgraph. Assume that $|K| > r$, since there are only r available colors, by the pigeonhole principle, at least two vertices in K must receive the same color. In this context, the pigeonhole principle asserts that if a set of $|K|$ vertices is assigned labels from a set of only r colors with $|K| > r$, then at least one color must be assigned to more than one vertex. Let the two vertices be v and w such that $c(v) = c(w) = b$. For each $i \ne b$, we have $d(v, R_i) = d(w, R_i) = 1$. Therefore, $rc_\Pi(v) = rc_\Pi(w)$, contradicting the assumption that the coloring is a locating rainbow coloring. Thus, it is not possible for all r colors to be used in K if $|K| > r$, and the vertices in K must be colored using at most $r - 1$ colors. ■

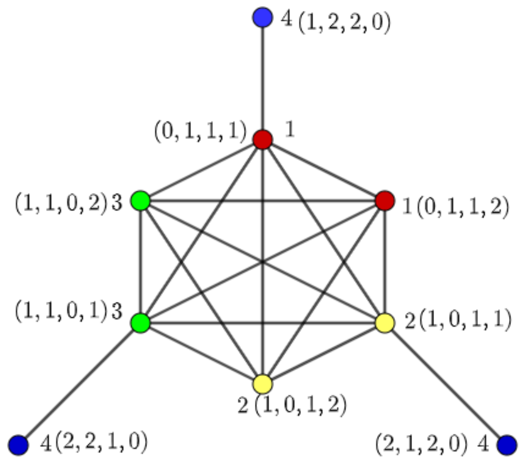

Figure 1. A graph G with Locating Rainbow 4-Coloring

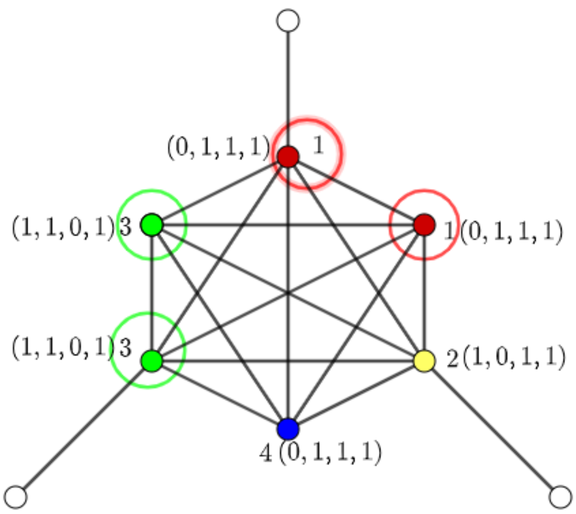

Figure 2. A graph G with Vertex 4-Coloring

Observe **Figure 1** and **Figure 2**, where $rvcl(G) = 4$. Both figures illustrate the same graph and are colored using four colors, but in different ways. In **Figure 1**, it is shown that if one of the colors is not used in the clique, then the coloring with four colors is a locating rainbow coloring. Conversely, if all colors are used in the clique, then there exist at least two vertices with the same rainbow code, as shown in **Figure 2**. Therefore, such coloring is not a locating rainbow coloring.

3.1 Locating Rainbow Connection Number of Lollipop Graphs

Let the vertex set and edge set of the lollipop graph be defined, respectively, as follows: $V(L(m,n)) = \{v_i | i \in [1,m]\} \cup \{w_j | j \in [1,n]\}$ and $E(L(m,n)) = \{v_i v_j | i,j \in [1,m], i \neq j\} \cup \{w_j w_{j+1} | j \in [1, n-1]\}$. (See **Figure 3** for illustration)

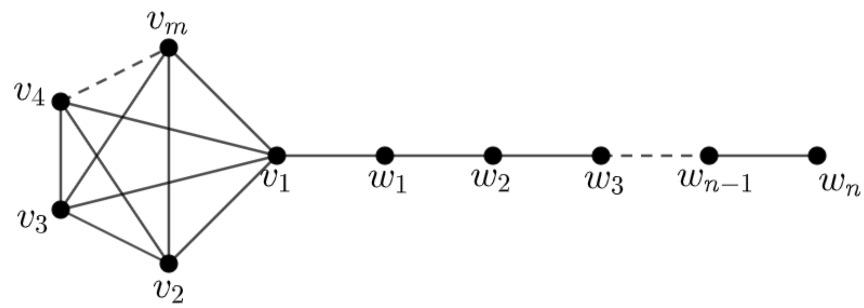

Figure 3. $L(m, n)$

As explained in the introduction chapter, one of the lower bounds of the locating rainbow connection number is the rainbow vertex connection number of a graph. Therefore, before presenting the main results in this subsection, we first provide findings related to the rainbow vertex connection number of the lollipop graph.

Theorem 1. *Let $m \geq 3$ and $n \geq 1$ be integers. If $L(m,n)$ be a lollipop graph of order $m+n$, then $rvc(L(m,n)) = n$.*

Proof. It is known that the number of cut vertices in the graph $L(m,n)$ is n. Therefore, based on **Lemma 3**, we have $rvc(L(m,n)) \geq n$. Furthermore, by coloring all cut vertices with n distinct colors, it follows that any two vertices in $L(m,n)$ are connected by a rainbow vertex path. Thus, $rvc(L(m,n)) = n$. ■

Figure 4 illustrates a rainbow vertex coloring for the lollipop graph $L(m,n)$ for any m and n.

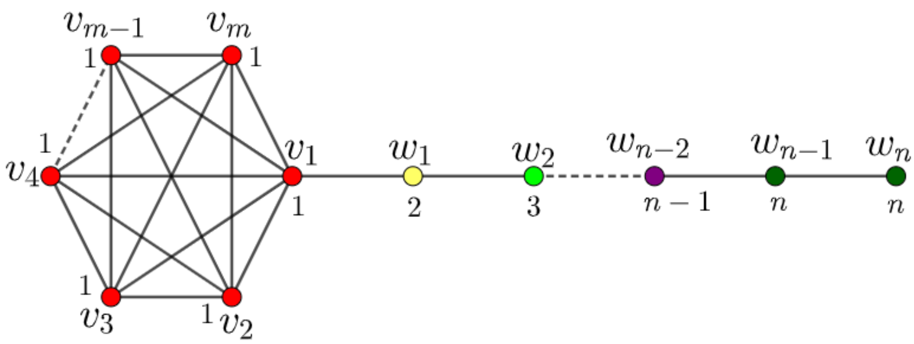

Figure 4. A rainbow Vertex Coloring of $L(6, 5)$.

Theorem 2. *Let $m \geq 3$ and $n \geq 1$ be integers. If $L(m,n)$ be a lollipop graph of order $m+n$, then $rvcl(L(m,n)) = max\{m,n\}$*

Proof. The proof of the lower bound is divided into two cases as follows:

Case 1, m ≥ n.

Suppose that $rvcl(L(m,n)) = m - 1$. If all colors are used in the subgraph K_m, based on **Lemma 5**, there must exist at least two distinct vertices v_i and v_j such that $rc_\Pi(v_i) = rc_\Pi(v_j)$ which contradicts the definition of a locating rainbow coloring. Conversely, if not all colors are used in the subgraph K_m, without loss of generality, suppose that the vertices in K_m use only $m-2$ colors. Since only $m-2$ colors are used; there must be at least two distinct vertices v_i and v_j (with $i, j \neq 1$) that share the same color. Given that the distances from these two vertices to all other vertices in $L(m,n)$ are identical, **Lemma 2** implies that they have the same rainbow code, which leads to a contradiction. Thus, we conclude that $rvcl(L(m,n)) \geq m$.

Case 2, $m < n$.

Observe that the graph $L(m,n)$ has n cut vertices. Thus, by **Lemma 3**, it follows that $rvcl(L(m,n)) \geq n$.

From the two cases above, we obtain $rvcl(L(m,n)) \geq max\{m,n\}$.

Next, we establish that $rvcl(L(m,n)) \leq max\{m,n\}$ by defining a coloring rule as follows: $c(V(L(m,n)) \to [1, max\{m,n\}]$.

$$c(v_i) = \begin{cases} 1, for\ i = 1 \\ i-1, \text{for } i \in [2, m] \end{cases}$$
$$c(w_j) = \begin{cases} j+1, for\ j \in [1, n-1] \\ max\ \{m, n\}, for\ j = n \end{cases}$$

Based on the coloring defined above, it is evident that all cut vertices have distinct colors. Furthermore, apart from adjacent vertices, any two vertices in $L(m,n)$ are always connected by a path whose internal vertices are all cut vertices, as shown in the proof of the upper bound in **Theorem 1**. This ensures that every pair of vertices in $L(m,n)$ is connected by a rainbow vertex path.

The following table presents the rainbow vertex $\boldsymbol{u-v}$ paths, demonstrating that for every pair of vertices in $L(m,n)$, there always exists a rainbow vertex path connecting them.

Table 1. Rainbow Vertex $u - v$ Paths

u	v	Condition	Rainbow Vertex $u - v$ Path
v_i	v_j	$i, j \in [1, m], i \neq j$	$v_i v_j$
v_i	w_j	$i \in [1, m], j \in [1, n]$	$v_i v_1 w_1 w_2 w_3 \ldots . w_{j-1} w_j$
w_i	w_j	$i, j \in [1, n], i \neq j$	$w_i w_{i+i} w_{i+2}, \ldots . w_j$

Additionally, from the given coloring, several conditions regarding the rainbow code can be derived as follows.

1. $c(v_1) = c(v_2) = 1$, but $d(v_1, R_z) < d(v_2, R_z)$ with $z = max\{m, n\}$.
2. Every vertex in the subgraph K_m except v_1 is assigned a unique color. As a result, although they have the same distance to other vertices in $L(m,n)$ all these vertices have distinct rainbow codes.
3. Every vertex in the subgraph P_n except w_n is assigned a unique color, ensuring that these vertices have distinct rainbow codes.
4. For $n < m$, $c(w_n) = c(w_{n-1}) = n$, but $d(w_n, R_1) > d(w_{n-1}, R_1)$. Otherwise, the color of w_n and w_{n-1} are different. Therefore, these two vertices have different rainbow codes.
5. Every vertex in the subgraph K_m is at distance one from a set of vertices colored with m different colors for $m \geq 3$ whereas every vertex in the subgraph P_n is at most distance one from at most two different color sets. Therefore, $rc_\Pi(v_i) \neq rc_\Pi(w_j)$.

Based on the five conditions mentioned above, it follows that every vertex in $L(m,n)$ has a unique rainbow code. Therefore, the bound has been established, leading to the conclusion that $rvcl(L(m,n)) = max\{m,n\}$. ∎

Figure 5 illustrates a locating rainbow coloring of $L(m,n)$ for $m \geq n$, while **Figure 6** illustrates a locating rainbow coloring of $L(m,n)$ for $m < n$.

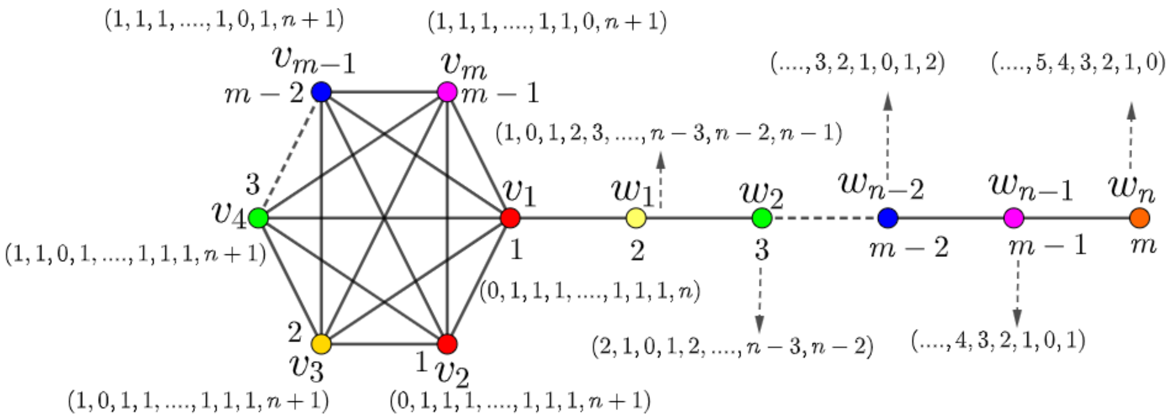

Figure 5. A Locating Rainbow Coloring of $L(m,n)$ for $m \geq n$

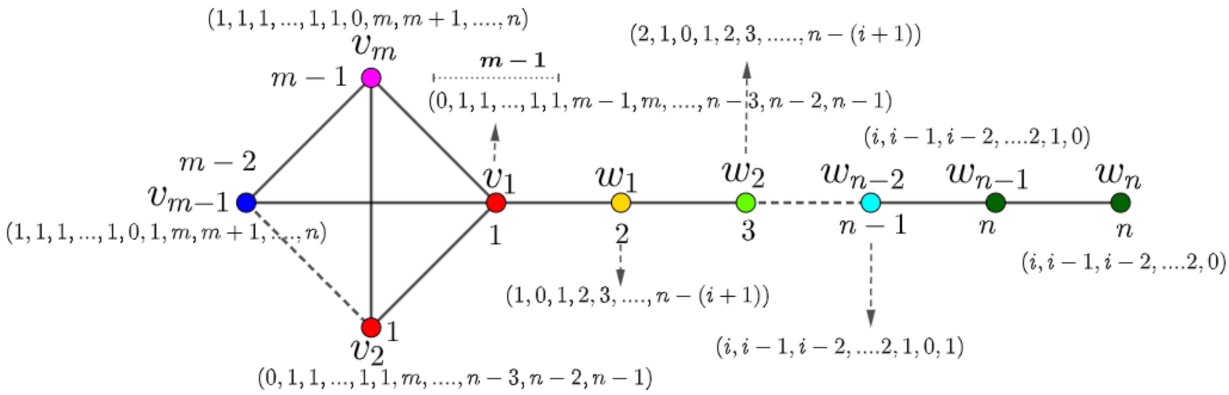

Figure 6. A Locating Rainbow Coloring of $L(m, n)$ for $m < n$

Based on **Theorem 1** and **Theorem 2**, it can be observed that the order of the complete graph does not affect the value of the rainbow vertex connection number of the lollipop graph and only depends on the number of cut vertices. In contrast, the locating rainbow connection number of a lollipop graph is influenced by the maximum of m and n. Additionally, in rainbow vertex coloring, each vertex v_i for $i \in [2, m]$ can be assigned the same color, whereas in locating rainbow coloring, each of these vertices must be assigned a distinct color.

3.2 Locating Rainbow Connection Number of Barbell Graphs

Let the vertex set and edge set of the barbell graph be defined, respectively, as follows: $V(B(K_m)) = \{v_i | i \in [1, m]\} \cup \{w_j | j \in [1, m]\}$ and $E(B(K_m)) = \{v_i v_k | i, k \in [1, m], \mathrm{i} \neq \mathrm{k}\} \cup \{\mathrm{v}_1 w_1\} \cup \{w_j w_l | j, l \in [1, m], j \neq l\}$. (See **Figure 7** for illustration).

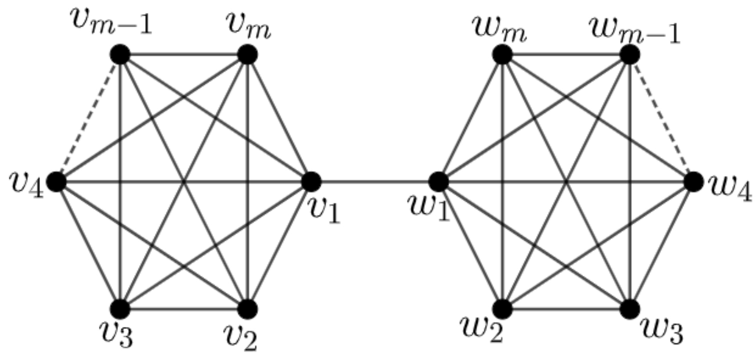

Figure 7. $B(K_m)$

Theorem 3. *Let m be a natural number with $m \geq 3$. If $B(K_m)$ be a barbell graph of order $2m$, then $rvc(B(K_m)) = 2$.*

Proof. Based on **Lemma 3**, we have $rvc(B(K_m)) \geq 2$. By assigning $c(v_i) = 1$ and $c(w_i) = 2$ we obtain $rvc(B(K_m)) = 2$ since, apart from the two adjacent vertices, all other vertices in the graph are connected by a rainbow vertex path whose internal vertices are v_1 and/or w_1. ■

Figure 8 illustrates a rainbow vertex coloring of the graph $B(K_m)$.

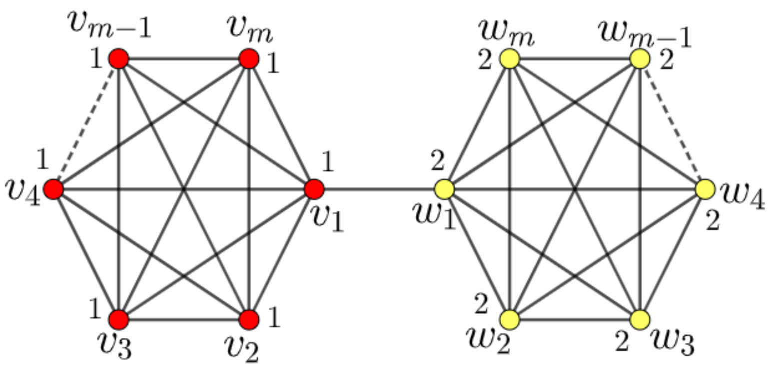

Figure 8. A Rainbow Vertex Coloring of $B(K_m)$

Theorem 4. *Let m be a natural number with $m \geq 3$. If $B(K_m)$ be a barbell graph of order $2m$, then $rvcl(B(K_m)) = m$.*

Proof. Suppose that $rvcl(B(K_m)) = m - 1$. Without loss of generality, consider the vertices v_i for $i \in [1, m]$. Consequently, there exist at least two distinct vertices, $\boldsymbol{v_i}$ and $\boldsymbol{v_j}$, that share the same color. Consider the following possibilities:

1. Suppose that only $m - 2$ colors are used for all vertices v_i, $i \in [1, m]$. Then, by the pigeonhole principle, there exist at least two vertices u and v such that $c(u) = c(v)$. If u and v also have identical rainbow codes, then this contradicts **Lemma 2**, which states that any two vertices with identical codes must be assigned different colors. Hence, at least $m - 1$ colors are necessary.
2. If the vertices $\boldsymbol{v_i}$ use at most $m - 3$ distinct colors and $\boldsymbol{v_1}$ is not one of the two vertices sharing the same color, then this situation reduces to the first case.
3. If the vertices $\boldsymbol{v_i}$ use at most $m - 3$ distinct colors and v_1 is one of the two vertices sharing the same color, then there exist two other vertices, excluding v_1, that share the same color, reducing the situation again to the first case.

From these three possibilities, it is concluded that when using only $m - 1$ colors, there must exist at least two vertices in $B(K_m)$ with the same color and same rainbow codes, leading to a contradiction. Therefore, it must hold that $rvcl(B(K\_m) \geq m.$

Next, we establish that $rvcl(B(K_m) \leq m$ by defining a coloring rule as follows: $c(V(B(K\_m)) \rightarrow [1, m]$.

$$c(v_i) = \begin{cases} 1, \text{for } i = 1 \\ i - 1, \text{for } i \in [2, m] \end{cases}$$

$$c(w_j) = \begin{cases} 2, \text{for } j = 1 \\ j, \text{for } j \in [2, m] \end{cases}$$

Based on the coloring defined above, it is evident that v_1 and w_1 have distinct colors. Furthermore, apart from adjacent vertices, any two vertices in $B(K_m)$ are always connected by a path whose internal vertices are cut vertices, as shown in the proof of the upper bound in **Theorem 3**. So that every pair of vertices in $B(K_m)$ is connected by a rainbow vertex path.

The following table presents the rainbow vertex $\boldsymbol{u - v}$ paths, demonstrating that for every pair of vertices in $B(K_m)$, there always exists a rainbow vertex path connecting them.

Table 2. Rainbow Vertex $\boldsymbol{u - v}$ Paths

$\boldsymbol{u}$	$\boldsymbol{v}$	Condition	Rainbow Vertex $\boldsymbol{u - v}$ Path
v_i	v_j	$i, j \in [1, m], i \neq j$	$v_i v_j$
v_i	w_j	$i, j \in [1, m]$	$\mathrm{v_i v_1 w_1 w_j}$
w_i	w_j	$i, j \in [1, m], i \neq j$	$w_i w_j$

Additionally, from the given coloring, several conditions regarding the rainbow code can be derived as follows.

1. Color 1 appears exclusively at vertices v_1 and v_2., while color 2 is assigned only to vertices w_1, w_2, and v_3.
2. $c(v_1) = c(v_2) = 1$, but $d(v_1, R_m) < d(v_2, R_m)$.
3. $c(w_1) = c(w_2) = c(v_3) = 2$, but $d(w_1, R_1) = d(v_3, R_1) = 1$, $d(w_2, R_1) > 1$, and $d(w_1, R_m) < d(v_3, R_m)$.
4. Every vertex v_i for $i \in [2, m]$ is assigned a unique color. As a result, although they have the same distance to other vertices in $B(K_m)$ all these vertices have distinct rainbow codes.
5. Every vertex w_j for $j \in [2, m]$ is assigned a unique color. As a result, although they have the same distance to other vertices in $B(K_m)$ all these vertices have distinct rainbow codes.
6. In our construction, aside from colors 1 and 2, each remaining color is used exactly twice, once for a vertex in the set $\{v_i\}$ and once for a vertex in the set $\{w_j\}$. For any two vertices v_i and w_j such that $c(v_i) = c(w_j) = b$, there always exists a color $1 \neq b$, particularly color 1, such that $d(v_i, R_1) = 1$, while $d(w_j, R_1) = 2$. This ensures that their rainbow codes are distinct. As for the case where $c(v_i) \neq c(w_j)$, it is obvious that their rainbow codes differ since the vertex colors themselves are already different.

   Based on the five conditions mentioned above, it follows that every vertex in $B(K_m)$ has a unique rainbow code. Therefore, the bound has been established, leading to the conclusion that $rvcl(B(K_m)) = m$. ■

Figure 9 illustrates the locating rainbow coloring of the graph $B(K_m)$.

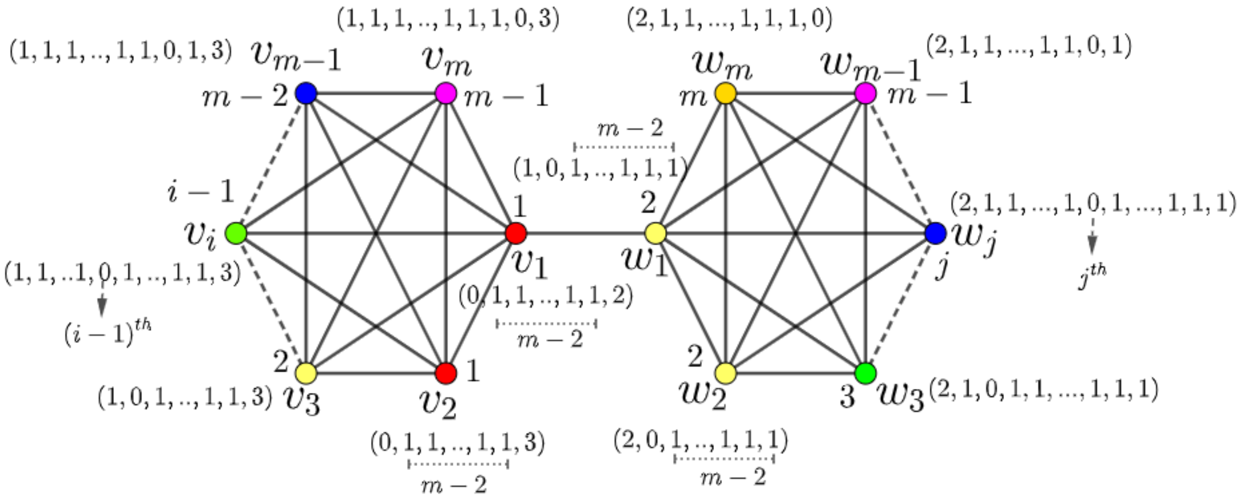

Figure 9. A Locating Rainbow Coloring of $B(K_m)$

Based on **Theorem 3** and **Theorem 4**, it is obtained that the rainbow vertex connection number of the barbell graph and the locating rainbow connection number of the barbell graph differ significantly. Specifically, $rvc(B(K_m)) = 2$, whereas the value of $rvcl(B(K_m))$ is directly proportional to the order of the complete graph.

4. CONCLUSION

In conclusion, we have determined the locating rainbow connection number for two specific types of graphs. It is observed that the locating rainbow connection number of the barbell graph is always equal to the order of the complete graph. In contrast, for the lollipop graph, this number depends on the maximum value of its two structural components. These findings contribute to a deeper understanding of locating rainbow coloring properties in structured graph classes.

AUTHOR CONTRIBUTIONS

Ariestha Widyastuty Bustan: Conceptualization, Methodology, Formal Analysis, Supervision, Validation, Writing Original Draft. Taufan Talib: Formal Analysis, Methodology, Supervision, Writing – Review and Editing. Novita Serly Laamena: Formal Analysis, Methodology. Lamanisa Rasid Saputra: Visualization. Nurhayati: Formal Analysis, Writing - Original Draft. All authors discussed the results and contributed to the final manuscript.

FUNDING STATEMENT

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

ACKNOWLEDGMENT

The authors conducted and completed this research independently without specific assistance from individuals or institutions that require acknowledgment.

CONFLICT OF INTEREST

The authors declare no conflict of interest related to the publication of this study. All findings and interpretations presented in this article are solely those of the authors.

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