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OPTIMIZATION MODEL FOR MULTI-DEPOT ELECTRIC VEHICLE ROUTING PROBLEM WITH SOFT TIME WINDOWS WITH SCENARIO-BASED ANALYSIS

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ABSTRACT

The adoption of electric vehicles has increased due to their cost-efficiency and environmental impact. However, limited battery capacity requires careful route planning to ensure vehicles complete deliveries efficiently. This study focuses on the Multi-Depot Electric Vehicle Routing Problem with Soft Time Windows (MDEVRPSTW), where electric vehicles can depart from and return to multiple depots, while serving customers within predefined time windows that allow limited violations with penalty costs. The model is formulated using Mixed Integer Linear Programming (MILP) and solved using the exact branch-and-bound method in Lingo 20.0. Two operational scenarios are considered: (1) vehicles must return to their original depot, and (2) vehicles are allowed to return to any depot. Hypothetical data is used to simulate delivery routes with varied time windows and battery capacity constraints. Results show that both scenarios produce feasible, cost-minimizing solutions. Allowing flexible depot return (scenario 2) consistently reduces total travel cost, highlighting the practical benefit of depot flexibility in real-world logistics. This model contributes to the EV routing literature by integrating multiple depots—both fixed and flexible return options—soft time windows, and battery constraints into a single formulation. However, it assumes constant travel speeds and does not account for charging durations, which presents an opportunity for future research.

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1. INTRODUCTION

The adoption of electric vehicles (EVs) is being actively promoted by many countries through various strategic policies. Likewise, the United States government offers financial incentives for electric vehicle buyers. These initiatives have led to a substantial increase in electric vehicle sales [1]. By mid-2023, global electric vehicle sales increased by 40%, reaching 4.27 million electric vehicles and 1.76 million hybrid vehicles. The total number of electric vehicles worldwide has reached 40 million units [2]. This upward trend highlights the immense potential of EVs to replace conventional fossil fuel-based vehicles, whether for personal, public, or corporate transportation purposes.

In general, electric vehicles offer significant advantages in terms of lower energy consumption compared to fossil fuel-based vehicles [3]. Additionally, electric vehicles also contribute to reducing greenhouse gas emissions. According to [4], electric vehicles produce significantly lower air pollution emissions, nearly zero, compared to internal combustion engine (ICE) vehicles. Despite their advantages, electric vehicles also have limitations, one of which is that they have shorter ranges compared to conventional vehicles. With a fully charged battery, electric vehicles can typically travel only 200 to 350 km [1]. To address this limitation, electric vehicles need to recharge at battery charging stations to continue their journey. Consequently, optimal route planning is essential to maximize the efficiency of electric vehicle usage. The goal of this planning is to identify the shortest route that allows electric vehicles to effectively complete their operational targets. This issue can be modeled as a Vehicle Routing Problem (VRP).

The Vehicle Routing Problem (VRP) is an optimization problem that aims to determine the shortest route a vehicle can take from a depot, visiting all customers, and then returning to the depot. VRP was first introduced by Dantzig and Ramser in 1959 [5] as an extension of the Travelling Salesman Problem (TSP). Over time, VRP has evolved into various variations, including the Electric VRP (EVRP), where electric vehicles are used as the fleet. EVRP presents a more complex modeling challenge compared to the traditional VRP, primarily due to the limited driving range of electric vehicles. Unlike fuel-based vehicles, electric vehicles require careful route planning to ensure that they do not deplete their battery charge during operation. As a result, the EVRP must incorporate constraints related to energy consumption, battery capacity, and the availability of charging stations along the route. EVRP has further developed into multiple variants, such as EVRP with different batteries and load capacities [6], EVRP with multiple trips and a heterogeneous fleet [7], EVRP with partial charging [8], and EVRP with soft time windows [9], soft time windows can be violated within a certain tolerance, but incur a penalty cost if violated [10].

Another variant of the EVRP is the Multi-Depot EVRP (MDEVRP). The MDEVRP addresses the routing of electric vehicles departing from multiple depots and allows them to return to any depot [11]. The MDEVRP is more realistic because, in real-world conditions, a company may have multiple depots to expand its sales area. Therefore, studying the MDEVRP model is important for improving a company's cost efficiency. [12] Proposed an MDVRP model with heterogeneous vehicles for its operations. [13] Developed a collaborative multi-depot pickup and delivery VRP with split loads and time windows. [14] Proposed an MDVRP model with time windows, considering both delivery and installation vehicles. The multi-depot approach can also be applied to the EVRP model, and several studies have explored this variation. [15] Formulated a multi-depot EVRP with fuzzy time windows and pickup/delivery constraints. [16] Proposed a multi-depot half-open time-dependent electric vehicle routing problem model. [17] Proposed a multi-depot EVRP model with time windows using Mixed Integer Linear Programming (MILP) and solved it using adaptive large neighborhood search. The previous research has not incorporated soft time windows and has not explored scenarios involving fixed and flexible final depots. Therefore, this article focuses on the Multi-Depot Electric Vehicle Routing Problem with Soft Time Windows (MDEVRPSTW) with two scenarios, fixed and flexible final depots.

The MDERVPTW is formulated as a MILP model and solved using the branch-and-bound method to achieve a globally optimal solution. Two models with different approaches are considered: one where vehicles must return to their initial depot and another where they can return to any depot. The need for these two approaches arises from different business requirements. In a delivery company, vehicles typically need to return to the same depot they started from to maintain inventory and scheduling consistency. However, in electric vehicle rental services—such as shared e-scooters or roadside EV rental hubs—vehicles do not need to return to their starting depot. Instead, they are typically returned to the nearest available depot for convenience and operational efficiency. The MDEVRPSTW model is applied to two implementation examples, each involving different vehicle battery capacities and customer time windows.

2. RESEARCH METHODS

This part will discuss the MDERVPSTW mathematical model in MILP form. The multi-depot constraints are adapted from [12]. The soft time window constraints are adapted from [9], while the battery level constraint is based on [18] with modifications to indices and variables to fit the current model.

2.1 Assumptions and Notations

The following assumptions are made to simplify the calculations and to define the scope of the MDEVRPTW model.

a. The model has several depots and electric vehicles that can be operated anytime.
b. The vehicles depart from and return to the depot.
c. Each customer is visited exactly once by an electric vehicle.
d. Customers can be visited from any depot.
e. Service must be performed within the customer's time windows. However, time windows can be violated within a specified tolerance limit.
f. All vehicles charge at battery swap charging stations, so the charging time for each vehicle is the same.
g. All vehicles run at the same average speed.
h. The battery consumption rate is only affected by the vehicle's travel distance. The rate of energy consumption per unit distance is constant.
i. The condition of the vehicle's battery while at the depot and charging station is full.
j. No obstacles during operation.

An example of an MDEVRPTW route is illustrated in Figure 1.

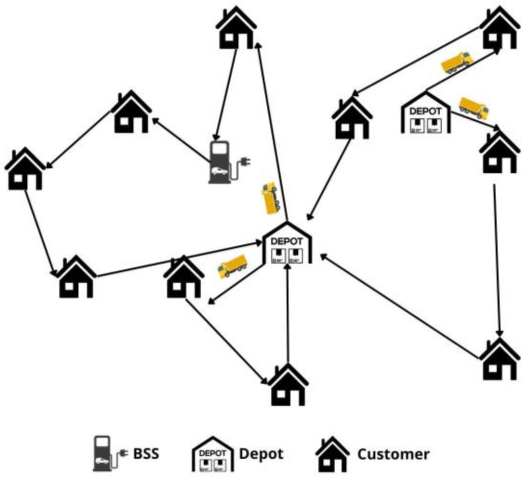

Figure 1. Example of an MDEVRPTW Route with 2 Depots and 10 Customers

To support this model, the following set notation, indices, parameters, and decision variables are introduced. D and D_E represent the set of initial and final depots, respectively. I represent the set of customers, F represent the set of charging stations, and $\mathbb{K}$ represent the set of vehicles. There are also sets that are unions of the previous sets. V_0 represents the set of initial depots, customers, and charging stations $\{V_0 = D \cup I \cup F\}$, V_E represents the set of final depots, customers, and charging stations $\{V_E = D_E \cup I \cup F\}$, V represents the set of all nodes $\{V = D \cup D_E \cup I \cup F\}$, and R represents the set of customers and charging stations $\{R = I \cup F\}$. Index i and j represent the vehicle's location where $i \neq j$, k represent the vehicle, and d represent the origin depot. The parameters used in this model are shown in Table 1 below:

Table 1. **Model Parameter**

Parameter	Description	Unit
J_{ij}	Distance from node i to node j	km
q_i	Demand of node i	kg
t_{ij}	Travel time from node i to node j	minute
C_k	Battery capacity of vehicle k	kWh
Q_k	Load capacity of vehicle k	kg
r	Vehicle energy consumption rate	kWh/km
p	Time windows tolerance	minute
v	Electric vehicle velocity	km/minute
s_i	Service time at node i	minute
$[a_i, b_i]$	*Time window* at node i	minute
c_f	Vehicle fixed cost	Rupiah
c_t	Vehicle travel cost	Rupiah/minute
c_r	Vehicle charging cost	Rupiah
c_{pa}	Penalty cost of earliness	Rupiah
c_{pb}	Penalty cost of tardiness	Rupiah

To record the vehicle's route, remaining battery, and vehicle arrival time, the following decision variables are defined. $x_{ijkd} = 1$ if vehicle k that departs from depot d travels from i to j, and $x_{ijkd} = 0$ otherwise. u_{ijkd} is remaining battery of the vehicle k from depot d at node j after departing from node i. y_{ij} is remaining load in the vehicle at node j after departing from node i. τ_{ikd} is arrival time of the vehicle k from depot d at node i.

2.2 Mathematical Model

The objective function of this model is to minimize the total cost. The total cost consists of fixed costs, travel costs, charging costs, and penalty costs for violating time windows. Therefore, the objective function of this model can be written as **Equation (1)**.

$$\min z = \sum_{i\in D}\sum_{j\in V_E}\sum_{k\in K}\sum_{d\in D} c_f x_{ijkd} + \sum_{i\in V_0}\sum_{j\in V_E}\sum_{k\in K}\sum_{d\in D} c_t x_{ijkd} t_{ij} + \sum_{i\in V_0}\sum_{j\in V_E}\sum_{k\in K}\sum_{d\in D} c_t x_{ijkd} t_{ij} + \sum_{i\in I}\sum_{k\in K}\sum_{d\in D}\left(c_{pa}\max\{0, a_i - \tau_{ikd}\} + c_{pb}\max\{0, \tau_{ikd} - b_i\}\right) \quad (1)$$

Equation (1) shows the calculation of the total cost. The first component is the fixed cost, which is calculated by multiplying the fixed cost per vehicle by the number of vehicles used. The second component is the travel cost, which depends on the total distance or duration the vehicle travels. The third component is the charging cost, which accumulates when a vehicle visits a charging station. The final component is the penalty cost, incurred when a vehicle arrives either too early or too late with respect to the defined time windows. However, this equation is not linear because it includes two piecewise functions, which are $\max\{0, a_i - \tau_{ikd}\}$ and $\max\{0, \tau_{ikd} - b_i\}$. To linearize this equation, two additional variables, m_{ikd} and n_{ikd} are introduced, where:

$$m_{ikd} \geq 0, \qquad \forall i \in I, \forall k \in K, \forall d \in D \quad (2)$$

$$n_{ikd} \geq 0, \qquad \forall i \in I, \forall k \in K, \forall d \in D \quad (3)$$

$$m_{ikd} \geq a_i - \tau_{ikd}, \qquad \forall i \in I, \forall k \in K, \forall d \in D \quad (4)$$

$$n_{ikd} \geq \tau_{ikd} - b_i, \qquad \forall i \in I, \forall k \in K\ \forall d \in D \quad (5)$$

So, **Equation (1)** can be rewritten as **Equation (6)**.

$$\min z = \sum_{i\in D}\sum_{j\in V_E}\sum_{k\in K}\sum_{d\in D} c_f x_{ijkd} + \sum_{i\in V_0}\sum_{j\in V_E}\sum_{k\in K}\sum_{d\in D} c_t x_{ijkd} t_{ij} + \sum_{i\in V_0}\sum_{j\in V_E}\sum_{k\in K}\sum_{d\in D} c_t x_{ijkd} t_{ij} + \sum_{i\in I}\sum_{k\in K}\sum_{d\in D}(c_{pa} m_{ikd} + c_{pb} n_{ikd}) \quad (6)$$

The MDEVRPSTW model minimizes **Equation (6)** subject to the constraints below:

$$\sum_{i\in V_0}\sum_{k\in K}\sum_{d\in D} x_{ijkd} = 1, \qquad \forall j \in I \quad (7)$$

$$\sum_{i\in V_0}\sum_{k\in K}\sum_{d\in D} x_{jikd} = 1, \qquad \forall j \in I \quad (8)$$

$$\sum_{i\in V_0} x_{ijkd} = \sum_{i\in V_E} x_{jikd}, \qquad \forall j \in R, \forall k \in K, \forall d \in D \quad (9)$$

$$x_{d_1 ikd_2} = 0, \qquad \forall i \in V_E, \forall k \in K, \forall d_1, d_2 \in D, d_1 \neq d_2 \quad (10)$$

$$x_{ijkd} = 0, \qquad \forall i,j \in D \cup D_E, \forall k \in K, \forall d \in D \quad (11)$$

$$x_{iikd} = 0, \qquad \forall i \in V, \forall k \in K, \forall d \in D \quad (12)$$

$$\sum_{i\in V_0}\sum_{k\in K}\sum_{d\in D} x_{ijkd} \geq 0, \qquad \forall j \in F \quad (13)$$

$$\sum_{i\in V_0}\sum_{k\in K}\sum_{d\in D} x_{jikd} \geq 0, \qquad \forall j \in F \quad (14)$$

$$\sum_{j\in V} x_{ijkd} \leq 1, \qquad \forall i,d \in D, \forall k \in K \quad (15)$$

$$\sum_{i\in D}\sum_{j\in V_E} y_{ij} = \sum_{j\in V_E} q_j \quad (16)$$

$$\sum_{i\in V_0} y_{ij} - \sum_{i\in V_E} y_{ji} = q_j, \qquad \forall j \in R \quad (17)$$

$$y_{ij} \leq \sum_{d\in D}\sum_{k\in K}(Q_k - q_i) x_{ijkd}, \qquad \forall i \in V_0, \forall j \in V_E \quad (18)$$

$$y_{ij} \geq \sum_{d\in D}\sum_{k\in K} q_j x_{ijkd}, \qquad \forall i,j \in R \quad (19)$$

$$y_{ij} = 0, \qquad \forall i \in R, \forall j \in D_E \quad (20)$$

$$y_{ij} = 0, \qquad \forall i,j \in D \cup D_E \quad (21)$$

$$y_{ii} = 0, \qquad \forall i \in R \quad (22)$$

$$u_{ijkd} \leq C_k, \qquad \forall i \in V_0, \forall j \in V_E, \forall k \in \mathbb{K}, \forall d \in D \quad (23)$$

$$C_k(x_{ijkd} - 1) \leq u_{ijkd} - (C_k - rJ_{ij}) \leq C_k(1 - x_{ijkd}), \forall i \in D \cup F, \forall j \in V_E, \forall k \in K, \forall d \in D \quad (24)$$

$$C_k(x_{ijkd} - 1) \leq u_{ijkd} - (u_{likd} - rJ_{ij}) \leq C_k(1 - x_{ijkd}), \quad \forall i \in I, \forall l \in V_0, \forall j \in V_E, \forall k \in K, \forall d \in D \quad (25)$$

$$a_i - p \leq \tau_{ikd} \leq b_i + p, \qquad \forall i \in V_E, \forall k \in K, \forall d \in D \quad (26)$$

$$\tau_{jkd} \geq \tau_{ikd} + s_i + t_{ij} - M(1 - x_{ijkd}), \qquad \forall i \in V_0, \forall j \in V_E, \forall k \in K, \forall d \in D \quad (27)$$

$$x_{ijkd} \in \{0,1\}, \qquad \forall i,j \in V, \forall k \in K, \forall d \in D \quad (28)$$

$$y_{ij} \geq 0, \qquad \forall i,j \in V \quad (29)$$

$$\tau_{ikd} \geq 0, \qquad \forall i \in V, \forall k \in K, \forall d \in D \quad (30)$$

$$u_{ijkd} \geq 0, \qquad \forall i,j \in V, \forall k \in K, \forall d \in D \quad (31)$$

Equation (7) - **Equation (8)** ensure that each customer is visited exactly once. **Equation (9)** ensures the vehicle must leave the customers and charging station to the next destination. **Equation (10)** ensures the vehicle only departs from its origin depot. **Equation (11)** and **Equation (12)** guarantee that there's no route between depots and looping routes. Equation (13) - Equation (14) guarantee that the charging station can be visited or not at all. Equation (15) shows that not all vehicles at each depot are required to operate. Equation (16) ensures the total load departing from the depot must equal to the total customer demand along the route taken by the vehicle. Equation (17) ensures the difference between the load entering and leaving a customer or charging station must equal to the customer's demand. Equation (18) ensures the vehicle load afterwards doesn't exceed the remaining vehicle load at the previous customer. Equation (19) guarantees that there is at least enough amount left to serve the next customer. Equation (20) ensures the remaining load must be 0 when returning to the depot. Equation (21) and Equation (22) guarantee that there is no load carried between depots and the same customer. Equation (23) ensures the remaining battery doesn't exceed the battery capacity. When $x_{ijkd} = 1$, Equation (24) and Equation (25) yield $u_{ijkd} = C_k - rJ_{ij}$ and $u_{ijkd} = u_{likd} - rJ_{ij}$, respectively. So, these equations show the calculation of battery reduction when departing from depot or charging station and customer, respectively. Equation (26) ensures the vehicle arrives at the specified time interval. Equation (27) shows the accumulated time when the vehicle arrives at a node. Equation (28) - Equation (31) refer to the binary and non-negativity of the decision variables.

The MDEVRPSTW model can be expressed as minimizing Equation (6) by satisfying the constraints on Equation (2) - Equation (5) and Equation (7) - Equation (31). This model is executed on AMD Ryzen 3 5300U with 8 GB of RAM using Lingo 20.0. LINGO 20.0 was chosen because it offers a user-friendly interface and integrated modeling environment that simplifies the formulation and solving of various optimization problems, making it accessible for both beginners and practitioners. While it provides cost-effective and efficient solutions for small to medium-sized problems, it may lack the scalability and advanced features found in enterprise-level solvers like CPLEX or Gurobi.

3. RESULTS AND DISCUSSION

In this section, the implementation of the MDEVRPSTW model will be discussed. There are two scenarios that will be discussed: the first scenario is that the vehicles are only allowed to return to their origin depot, and the second scenario is that the vehicles are permitted to return to any depot. In each scenario, two examples with longer time windows and shorter time windows with small battery capacity will be implemented. Both examples will be used to validate the model. The vehicle considered in this study is an electric delivery truck. All data used here is hypothetical and generated for modeling purposes, except for the supporting data. The battery capacity of each example is 125 kWh and 75 kWh, respectively. The load capacity of each example is 125 kg. Time windows of each example are [0,300] and [0,240] respectively. The time window tolerance is 5 minutes.

In each example, there are 2 depots, 1 battery swap station (BSS) type charging station, 10 customers, and 2 vehicles per depot. The coordinates of the depot, charging station, and customer, and data regarding the amount of demand and time windows of each location are shown in Table 2. The coordinates of the depot, charging station, and customer are used to calculate the distance of each location with the Euclidean distance formula. The depot and charging station can be visited anytime, so the time windows of the depot and charging station are the entire operating time.

Table 2. Coordinate, Demand, Service Duration, and Time Windows of Each Node

Location	Coordinate	Demand (kg)	Service Duration (minutes)	Time Windows of Example 1	Time Windows of Example 2
D1	(35,35)	0	0	[0,300]	[0,240]
D2	(50,50)	0	0	[0,300]	[0,240]
BSS	(29,47)	0	10	[0,300]	[0,240]
P1	(41,49)	10	10	[30,240]	[0,60]
P2	(35,17)	15	10	[0,120]	[0,60]
P3	(55,45)	23	10	[0,120]	[50,80]
P4	(55,20)	30	10	[60,240]	[120,240]
P5	(15,30)	26	10	[60,240]	[120,240]
P6	(25,30)	13	10	[30,120]	[0,60]

Location	Coordinate	Demand (kg)	Service Duration (minutes)	Time Windows of Example 1	Time Windows of Example 2
P7	(20,50)	15	10	[0,240]	[60,240]
P8	(10,43)	33	10	[0,240]	[180,240]
P9	(55,60)	18	10	[30,240]	[30,240]
P10	(30,60)	17	10	[60,240]	[90,120]

The notations D1 and D2 represent depot 1 and depot 2, respectively. The notations P1 to P10 represent customers, while BSS denotes the charging station. In addition to location data, supporting data is also required as parameters. The supporting data is sourced from [19] and [20]. The data is presented in Table 3.

Table 3. Supporting Data

Parameter	Description	Value	Unit	References
c_f	Vehicle fixed cost	30.000,00	Rupiah	[20]
c_t	Vehicle travel cost	1.800,00	Rupiah/minute	[19]
c_r	Vehicle charging cost	4.400,00	Rupiah	[19]
c_{pa}	Penalty cost of earliness	2.200,00	Rupiah	[19]
c_{pb}	Penalty cost of tardiness	2.200,00	Rupiah	[19]
r	Vehicle energy consumption rate	1	kWh/km	[20]
v	Electric vehicle velocity	1	km/minute	[20]

3.1 Scenario 1: Vehicle Must Return to Its Origin Depot

In this scenario, the vehicles are required to return to their starting depot after completing their route. This scenario is selected because, in real-world applications, there are cases where vehicles are required to return to the initial depot—for example, in delivery companies. To ensure this, an additional constraint is required. The additional constraint is shown in Equation (32). This equation ensures that vehicles departing from d_1 will not return to d_2, and vice versa with $d_1 \neq d_2$. Since the vehicle must return to the depot, it will definitely return to its origin depot.

$$x_{id_1kd_2} = 0, \qquad \forall i \in V_0, \forall k \in K, \forall d_1 \in D_E, \forall d_2 \in D, d_1 \neq d_2 \tag{32}$$

In the example discussed in this section, a vehicle from depot 1 should not return to depot 2, and vice versa. So, the constraint in Equation (32) can be written as Equation (33) and Equation (34).

$$x_{i1k2} = 0, \qquad \forall i \in V_0, \forall k \in K \tag{33}$$

$$x_{i2k1} = 0, \qquad \forall i \in V_0, \forall k \in K \tag{34}$$

a. Scenario 1 – Example 1

The optimal solution from the MDEVRPSTW model with time windows of example 1 and additional Equation (33)– Equation (34) is Rp920,370.10 with 7.60 minutes of computation time. The route generated for the vehicle from depot 1 is D1−P6−P5−P8−P7−P10−P1−D1 and D1−P4−P2−D1, while the route for the vehicle from depot 2 is D2−P9−P3−D2. In this example, a total of 3 vehicles are used, with 2 vehicles from depot 1 and 1 vehicle from depot 2. The formed routes can be seen in Figure 2.

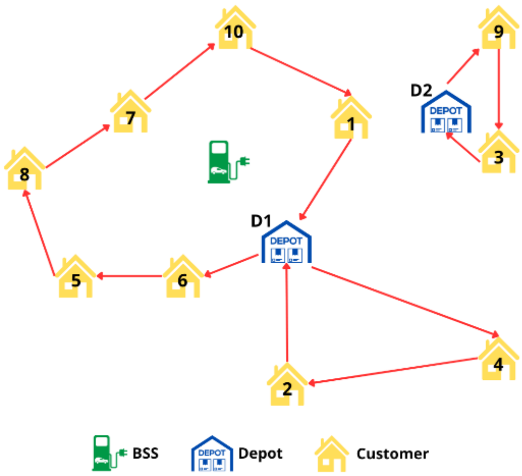

Figure 2. Route of Example 1 of Scenario 1

In this example, the total distance of vehicles from depot 1 is 92.24 km and 63.22 km, while the vehicle from depot 2 is 33.25 km. Because the vehicle mileage is equal to the number of vehicle batteries used ($r =$ 1 kWh/km), the battery usage doesn't exceed 125 kWh, so there is no need to visit a charging station. The time windows in example 1 are quite long, so there is no violation of the customer's time windows. The result of example 1 of this scenario can be further seen in Table 4.

Table 4. The Result of Example 1 of Scenario 1

Depot	Location	Time Windows	Arrival Time (minutes)	Remaining Load (kg)	Remaining Battery (kWh)
1 (Vehicle 1)	D1	[60, 240]	35	45	125
	P4	[60, 240]	60	45	100
	P2	[0, 120]	120	15	79.776
	D1	[0, 300]	148	0	61.776
1 (Vehicle 2)	D1	[0, 300]	18.820	114	125
	P6	[30, 120]	30	114	113.82
	P5	[60, 240]	60	101	103.82
	P8	[0, 240]	83.928	75	89.891
	P7	[0, 240]	106.135	42	77.685
	P10	[60, 240]	130.277	27	63.542
	P1	[30, 240]	155.833	10	47.986
	D1	[0, 300]	181.065	0	32.755
2 (Vehicle 1)	D2	[0, 300]	18.820	41	125
	P9	[30, 240]	30	41	113.82
	P3	[0, 120]	120	23	98.82
	D2	[0, 300]	137.071	0	91.749

As shown in Table 4, the remaining load when the vehicles return to the depot is 0 kg, which is consistent with the defined constraints. In addition, the vehicle arrival times comply with the specified time windows in Example 1, indicating that no time window violations occur in this case.

b. Scenario 1 – Example 2

The MDEVRPTW in Example 2, which features shorter time windows, results in a higher optimal solution Rp1,192,661.00 with 18.83 minutes of computation time. The vehicle routes from depot 1 are D1−P10−BSS−P7−P8−P5−D1 and D1−P6−P2−D1, while the vehicle routes from depot 2 are D2−P9−P1−D2 and D2−P3−P4−D2. The illustration of the routes in example 2 for this case can be seen in Figure 3.

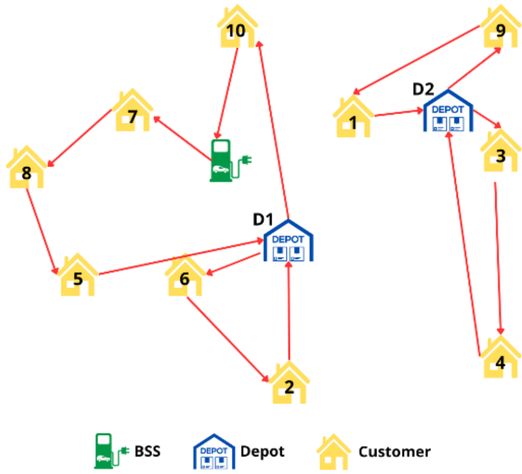

Figure 3. Route of Example 2 of Scenario 1

The total distance traveled by each vehicle from depot 1 is 94.77 km and 45.58 km, while the total distance traveled by each vehicle from depot 2 is 38.04 km and 62.48 km. Since the battery capacity of the vehicles in this example is only 75 kWh, a vehicle from depot 1 needs to visit a charging station. Detailed information regarding the arrival time, remaining load, and remaining vehicle battery can be found in Table 5.

Table 5. The Result of Example 2 of Scenario 1

Depot	Location	Time Windows	Arrival Time (minutes)	Remaining Load (kg)	Remaining Battery (kWh)
1 (Vehicle 1)	D1	[0, 240]	89.773	91	75
	P10	[90, 120]	115.268	91	49.505
	BSS	[0, 240]	138.307	74	75
	P7	[60, 240]	157.793	74	65.513
	P8	[180, 240]	180	59	53.307
	P5	[120, 240]	203.928	26	39.378
	D1	[0, 240]	234.544	0	18.763
1 (Vehicle 2)	D1	[0, 240]	0	23	75
	P6	[0, 60]	11.18	23	63.82
	P2	[0, 60]	37.582	15	47.418
	D1	[0, 240]	65.582	0	29.418
2 (Vehicle 1)	D2	[0, 240]	21.015	28	75
	P9	[30, 240]	32.195	28	63.82
	P1	[0, 60]	60	10	46.015
	D2	[0, 240]	79.055	0	36.96
2 (Vehicle 2)	D2	[0, 240]	72.929	53	75
	P3	[50, 80]	80	53	67.929
	P4	[120, 240]	115	30	42.929
	D2	[0, 240]	155.414	0	12.515

As shown in Table 5, there is a time window violation for P4. The time window for P4 is [120, 240], but it was visited at the 115th minute because the vehicle visited P9 at the 80th minute, before it became too late to visit. As a result, a penalty cost was incurred for arriving too early at P4. Additionally, the remaining load when the vehicles return to the depot is 0 kg, which is consistent with the defined constraints.

The results from Scenario 1 indicate that the proposed model is capable of generating a cost-efficient solution while satisfying all operational constraints. All vehicles are able to return to their origin depots in accordance with the return-to-depot requirement. The constructed routes largely comply with the specified time windows; however, minor violations are observed. These violations trigger penalty costs, thus validating the implementation of soft time windows within the model. The ability to tolerate such deviations while penalizing them appropriately reflects the model's flexibility in handling real-world delivery constraints.

3.2 Scenario 2: Vehicle Can Return to Any Depot

In this scenario, vehicles have the option to return to the nearest depot from their last destination. The additional constraint for this scenario is outlined in **Equation (35)**. This constraint ensures the final depot can be visited multiple times or not at all. The final depot can also be visited by vehicles departing from any customers or charging stations.

$$\sum_{i \in V_0} \sum_{j \in D_E} \sum_{k \in K} x_{ijkd} \geq 0, \qquad \forall d \in D \tag{35}$$

a. Scenario 2 – Example 1

The optimal solution from the MDEVRPSTW model with additional **Equation (35)**, for example 1 with longer time windows and larger vehicle load capacity, is Rp832,082.30. The vehicle route for depot 1 is D1−P2−P4−P3−P9−D2 and D1−P6−P5−P8−P7−P10−P1−D2, while the vehicle from depot 2 doesn't operate. All vehicles depart from depot 1 and return to depot 2. Depot 2 was chosen as the final depot due to its proximity to the last customer. A clearer view of the route can be seen in **Figure 4**.

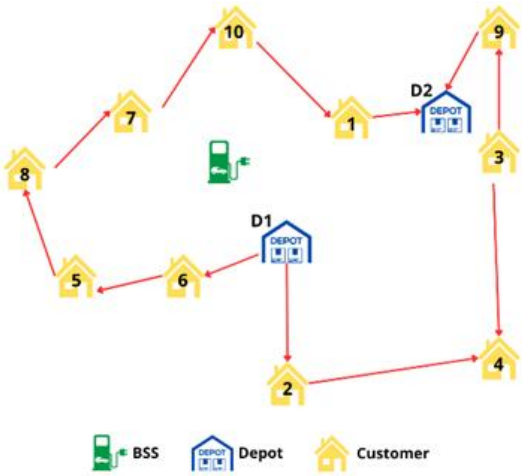

Figure 4. Route of Example 1 of Scenario 2

The optimal solution for example 1 in this scenario is cheaper because fewer vehicles are used, and the travel distances are shorter. The travel distances are 89.404 km and 86.07 km. Since the battery capacity is 125 kWh, there is no need for the vehicle to visit charging stations. Arrival time of each vehicle also complies with the time windows, so there is no penalty cost incurred. The computation time for this case is 17.16 seconds, which is significantly faster than previous examples. This is due to the more relaxed constraints, which result in fewer feasible solutions being explored. Detailed information on the arrival time, remaining load, and remaining vehicle battery at each location can be found in **Table 6**.

Table 6. The Result of Example 1 of Scenario 2

Depot	Location	Time Windows	Arrival Time (minutes)	Remaining Load (kg)	Remaining Battery (kWh)
1 (Vehicle 1)	D1	[60, 240]	36.776	86	125
	P2	[0, 120]	54.776	86	107
	P4	[60, 240]	85	71	86.776
	P3	[0, 120]	120	41	61.776
	P9	[30, 240]	145	18	46.776
	D2	[0, 300]	166.18	0	35.596
1 (Vehicle 2)	D1	[0, 300]	28.82	114	125
	P6	[30, 120]	40	114	113.82

Depot	Location	Time Windows	Arrival Time (minutes)	Remaining Load (kg)	Remaining Battery (kWh)
	P5	[60, 240]	60	101	103.82
	P8	[0, 240]	83.928	75	89.891
	P7	[0, 240]	106.135	42	77.685
	P10	[60, 240]	130.277	27	63.542
	P1	[30, 240]	155.833	10	47.986
	D2	[0, 300]	174.888	0	38.93

As shown in **Table 6**, the remaining load when the vehicles return to the depot is 0 kg, which is consistent with the defined constraints. In addition, the vehicle arrival times comply with the specified time windows in Example 1, indicating that no time window violations occur in this case.

b. Scenario 2 – Example 2

Example 2 of this scenario results in an optimal solution of Rp1,168,840.00. The routes of vehicles from depot 1 are D1−P6−P2−D1 and D1−P10−BSS−P7−P8−P5−D1, while the routes of vehicles from depot 2 are D2−P3−P4−D1 and D2−P9−P1−D2. All vehicles operate to meet the customer's tighter time windows. The travel distances of vehicles from depot 1 are 45.58 km and 94.77 km, while vehicles from depot 2 are 57.07 km and 38.04 km. A vehicle from depot 1 needs to visit a charging station because the battery capacity of this vehicle is only 75 kWh. What's interesting about this result is that a vehicle from depot 2 chooses to return to depot 1 because it's closer to P7. This causes the optimal solution for example 2 of this scenario to be cheaper than the previous scenario. An illustration of the route can be seen in **Figure 5**.

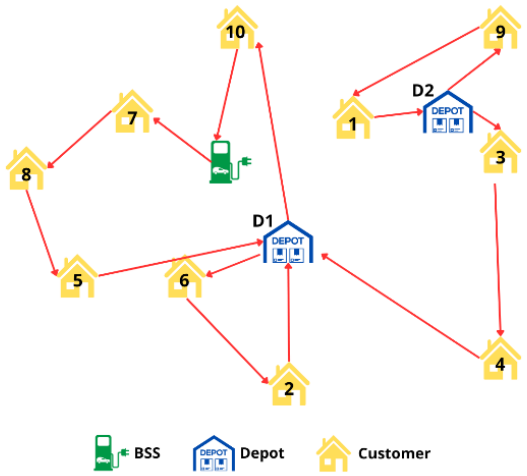

Figure 5. Route of Example 2 of Scenario 2

In this example, the computation time of example 2 in this scenario is longer than example 1, which is 14.48 minutes. This is due to changes in the time windows that have become shorter, and the battery capacity is smaller, so more feasible solutions need to be explored. However, this computing time is still faster than example 2 in scenario 1. Details regarding arrival times, remaining loads, and remaining vehicle batteries can be seen in **Table 7** below.

Table 7. The Result of Example 2 of Scenario 2

Depot	Location	Time Windows	Arrival Time (minutes)	Remaining Load (kg)	Remaining Battery (kWh)
1 (Vehicle 1)	D1	[0, 240]	0	28	75
	P6	[0, 60]	11.18	28	63.82
	P2	[0, 60]	37.582	15	47.418
	D1	[0, 240]	65.582	0	29.418
1 (Vehicle 2)	D1	[0, 240]	89.773	91	75
	P10	[90, 240]	115.268	91	49.505
	BSS	[0, 240]	138.307	74	75

Depot	Location	Time Windows	Arrival Time (minutes)	Remaining Load (kg)	Remaining Battery (kWh)
	P7	[60, 240]	157.793	74	65.513
	P8	[180, 240]	180	59	53.307
	P5	[120, 240]	203.928	26	39.378
	D1	[0, 240]	234.544	0	18.763
2 (Vehicle 1)	D2	[0, 240]	72.929	53	75
	P3	[50, 80]	80	53	67.929
	P4	[120, 240]	115	30	42.929
	D1	[0, 240]	150	0	17.929
2 (Vehicle 2)	D2	[0, 240]	18.82	28	75
	P9	[30,240]	30	28	63.82
	P1	[0, 60]	57.805	10	46.015
	D2	[0, 240]	76.86	0	36.96

As presented in **Table 7**, there is a time window violation at point P4, similar to what occurred in Scenario 1. Although the designated time window for P4 is between minutes 120 and 240, it was visited at minute 115. This early arrival happened because the vehicle prioritized visiting P9 at minute 80 to avoid being late. Consequently, a penalty cost was applied due to the early arrival at P4. On the other hand, the remaining load upon the vehicle's return to the depot is 0 kg, which aligns with the predefined constraints.

The results from Scenario 2 demonstrate that the model continues to produce cost-minimizing solutions while satisfying all imposed constraints. Vehicles are allowed to return to any available depot, providing greater routing flexibility. The generated routes align with the defined time windows; however, consistent violations are observed at the same node, which is P4. This indicates that such violations are strategically necessary to minimize the total cost. These findings highlight both the consistency and adaptability of the model, confirming its ability to accommodate different real-world scenarios through scenario-specific configurations. Furthermore, the comparison between the two examples shows that Scenario 2 incurs a lower total cost than Scenario 1, as selecting a closer final depot helps reduce travel expenses.

4. CONCLUSION

This study confirms that the proposed MDEVRPSTW model is flexible and adaptable to different real-world routing scenarios. The model performs well under both constraints: when vehicles are required to return to their original depot and when they are allowed to return to any depot. Notably, the scenario allowing vehicles to return to any depot yields lower total travel costs, as it provides greater routing flexibility and reduces unnecessary travel. In addition, the model effectively handles soft time windows, allowing for controlled violations that are penalized in the cost function. This makes the model suitable for applications where strict adherence to time constraints is not always feasible. For future work and real-world implementation, the model could be extended by incorporating additional realistic features such as charging duration, variable vehicle speeds, partial charging, pickup-and-delivery constraints, and split deliveries, to further enhance its applicability in electric vehicle logistics planning.

AUTHOR CONTRIBUTIONS

Elfina Tan: Conceptualization, Data Curation, Software, Methodology, Writing – Original Draft. Toni Bakhtiar: Resources, Supervision, Writing - Review and Editing. Jaharuddin: Supervision, Writing - Review and Editing. All authors discussed the results and contributed to the final manuscript.

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CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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