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MODIFIED STATISTICAL-BASED VALUE AT RISK FOR MULTI-OBJECTIVE OPTIMAL-BASED PORTFOLIO ANALYSIS OF INDONESIAN STOCK RETURN DISTRIBUTION

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ABSTRACT

Basically, all stock investments aim to obtain maximum profit with low risk. The formation of a stock investment portfolio is always accompanied by measuring returns and risks that show its performance. Portfolio risk measurement is often faced with the challenge that returns are not normally distributed, so that measurements using the normality assumption cannot be applied. This study proposes the development of a modification of stock portfolio risk measurement so that it is not limited to the normality assumption. The development is carried out by modifying the calculation of Value at Risk (VaR) to consider the skewness and kurtosis values (hereinafter referred to as modified VaR), so that the normal distribution assumption can be eliminated. As a method for compiling a stock portfolio, the Multi-Objective Optimization technique was chosen because it can modify risk averse so that the risk can be adjusted to the risk profile of each investor and is able to stabilize the mean return value. For its implementation, this paper uses real stock data which of course has returns that are not normally distributed, namely the four Indonesian stocks based on the largest capitalization recorded in January 2025 (blue chip), namely BREN, BBCA, BYAN, and BBRI obtained through finance.yahoo.com. The analysis method is divided into three steps, including multi-objective optimization completion, portfolio return calculation, and finally modified VaR estimation. The results of the study show that BBCA has the largest weight with a portion of more than 40% of the four stocks, so BBCA will be the priority stock for this portfolio. The portfolio formed using multi-objective optimization is proven to have a stable mean return because the portfolio mean return is between several of its constituent stocks (vice versa) which is around 0.01%, and the smallest estimated value of the portfolio modified VaR is 1.67%. Thus, a portfolio based on multi-objective optimization is not only able to create a portfolio that provides a small risk in risk measurement without assuming a normal distribution, but at the same time multi-objective optimization is also able to provide competitive returns with its constituent stocks.

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1. INTRODUCTION

Investment is one way to develop assets owned to get profits in the future. In general, investment can be made in various forms, both real asset investment and financial asset investment [1]. In addition, based on the objectives, investment can also be divided into two categories, namely short-term investment and long-term investment [2]. Investment goals often influence decision making. The decisions taken are used to maximize the profits that will be obtained [3]. Investments that guarantee high profits are stocks. Stocks with stable returns and little amounts of liabilities are targets that are always sought by investors [4]. Stocks that have these characteristics are referred to as blue chip stocks [5]. This type of stock is usually stocks that are well known on the Stock Exchange because they have products that have become market leaders [6]. Although blue chip stocks have stable profits, they also have high investment risks [7].

Investment risk is a condition where investors fail to get their capital because the issuer of shares is declared bankrupt, or the shares are not sold on the Stock Exchange [8]. To minimize investment risk, investors can form an optimal portfolio by using diversification. A portfolio is defined as a collection of stocks or securities with a certain level of return and risk. Portfolio risk analysis is usually faced with the abnormality of the distribution of returns on the constituent assets, so that the risk measures obtained may not be appropriate. The portfolio is made as efficient as possible, which can provide the greatest possible expected return value with minimal risk. [9]. There are several methods to form an optimal portfolio. Those forms are maximizing profits, minimizing volatility, targeting profits, targeting volatility, or maximizing profits and minimizing volatility. The greater the volatility of each investment, the greater the possibility of profit or loss [10].

Value at Risk (VaR) is a metric commonly used in statistical disciplines to measure the maximum risk of loss on a value movement with a certain level of confidence [11]. The variance-covariance method and the historical simulation method are the most frequently used approaches in the Value at Risk method [12]. In the variance-covariance method, it is assumed that the volatility of an investment return or portfolio is normally distributed. While the historical simulation method is assumed that the return from an investment or portfolio is not normally distributed [13]. However, VaR is basically developed under the assumption of normal distribution, thus requiring the data or returns analyzed to be normally distributed. On the other hand, real return data almost always does not follow a normal distribution. Therefore, it is necessary to develop its statistical distribution so that it can be more flexible and not bound by the necessity of assuming a normal distribution. In related research, Gunjan and Bhattacharyya generally provide a review related to portfolio optimization, one of which is based on value at risk [14]. Next, Zhou conducted research in optimizing portfolios for financial data with a mean-variance approach, which was then analyzed for risk using conditional value at risk [15]. In addition, multi-object-based optimization can also be used to make optimal stock portfolios [16].

This paper uses the Multi-Objective Optimization technique in stock portfolio optimization. Next, it proposes the development of a statistical distribution in risk analysis based on Value at Risk that does not require the assumption of a normal distribution. For its real implementation, this paper applies it to analyze the risk of a portfolio of several Indonesian stocks. The Indonesian Stock Portfolio used comes from leading stocks. Meanwhile, the order of the remaining parts of this paper is arranged as follows. In the second part, we propose a literature review including Indonesian Blue-Chip Stocks, Portfolio, Multi-Objective Optimization, and Value at Risk. The next section, methodology used in this research, includes data, completion of multi-objective optimization, calculating portfolio returns, and VaR estimation. Next, the main section presents the results and some discussion details. At the end of the paper, it is closed to a conclusion.

2. RESEARCH METHOD

2.1 Indonesian Blue-Chip Stocks

Investors pay attention to one of the essential things in investing in the market capitalization value, which is the value of a company's shares outstanding in the capital market. Based on market capitalization, stocks are divided into three types, namely blue-chip, middle-tier, and small-chips shares. The difference is that blue-chip shares have a market capitalization value of above ten trillion, medium-cap shares have a market capitalization value of between one trillion to five trillion, and small-chip shares have a market capitalization value of under 1 trillion [17]. Not only does it have a large market size, but blue-chip stocks also have other advantages, such as shares of publicly owned companies that have a high percentage called liquify, shares circulating in the capital market have a long term and stable company performance **[18]**.

In Indonesia, most investors only want high returns without paying attention to the risks involved **[19]**. Market capitalization data has a significant influence in determining the interest of investors to be used as investment instruments [20]. The most superior stocks on the stock exchange are blue-chip stocks. Blue-chip stocks are very profitable for investors because they are marker leaders and can make stable dividend payments [21].

2.2 Portfolio

Stocks are letters that show proof of the ownership of a company. A combination of several stocks is called a portfolio. Portfolios are suitable for investors who have a large amount of capital, so that the capital can be allocated to many stocks. The goal of a portfolio is to generate more profits than just one stock [22]. Every investor must want maximum profit; therefore, a portfolio analysis is needed that aims to maximize profits while minimizing risk at a certain level of confidence. Portfolio profit level and portfolio risk level are two things that need to be considered in portfolio analysis [23]. The percentage split for each stock in the portfolio is key to getting an optimal portfolio [24], [25].

If there are n stocks in a portfolio, each stock is given a weighting to form a portfolio symbolized by vector $\mathbf{x}, \mathbf{x} = [x_1, x_2, \dots, x_n]^T$. It means that the assets in the portfolio have their respective weights so that x has a value from zero to one hundred percent. Each stock in the portfolio has an expected return, an expected return is shown in the vector $\mathbf{r}, \mathbf{r} = [r_1, r_2, \dots, r_n]^T$, with r_i as expected or mean of the stock return i-th. Just as each stock has an expected return, the portfolio also has an expected return. The portfolio's expected return is formulated as in Eq. (1) [26]:

$$x_p = \mathbf{r}^T\mathbf{x} = \sum_{i=1}^{n} x_i r_i . \tag{1}$$

In addition, several individual stocks that are combined will form a variance or covariance which can describe the risky conditions implicitly. These values are presented in the form of a matrix as following Eq. (2):

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{pmatrix}, \tag{2}$$

with σ_{ii} is the variance of asset i and $\sigma_{ij} = \sigma_{ji}$ is the covariance between asset i and j. The portfolio variance is expressed as following Eq. (3):

$$\sigma_p^2 = \mathbf{x}^T\mathbf{\Sigma}\mathbf{x} = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} , \tag{3}$$

where n is the number of assets in the portfolio.

2.3 Multi-Objective-Based Optimization

The French-Italian economist Pareto is considered to be the inventor who first introduced the concept of multi-objective-based optimization [27]. Furthermore, this optimization is called multi-objective optimization which is an integral part of optimization activities [28]. It has tremendous practical importance because almost all real-world optimization problems can be modeled using multiple conflicting objectives. The traditional way to solve such a problem is mainly focused on scaling multiple goals into a single goal. In contrast, the evolutionary way is to solve the multi-objective optimization problem [29].

The multi-objective approach is formed by weighting multiple objectives, denoted as $f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_n(\mathbf{x})$, into one objective function. The solution method that can be implemented is minimizing the sum of the positive weighted convexities of the objectives using a single-objective approach, as following Eq. (4) [30]:

$$\text{Minimize } F(\mathbf{x}) = \sum_{i=1}^{n} a_i f_i(\mathbf{x}), \quad a > 0, \quad i = 1,2, \dots, n . \tag{4}$$

where $\mathbf{x}$ is the initial weight vector of the assets in the portfolio (usually equally distributed for each asset) and $\mathbf{x}^*$ is the optimal weight of the assets in the portfolio. Basically, Pareto optimality represents a multi-objective optimization, where the vector $\mathbf{x}^*$ is called Pareto optimal with the only condition that there is no $\mathbf{x}$ such that $f_i(\mathbf{x}) < f_i(\mathbf{x}^*)$ for all $i = 1,2,\ldots,n$. Besides that, $\mathbf{x}^*$ is a Pareto point if $f_i(\mathbf{x}^*)$ reaches its minimum value.

2.4 Value at Risk

The Value at Risk (VaR) method is a metric in risk assessment that is able to explain the minimum loss that may occur for an investor in an asset or a collection of assets at a predefine level of significance [31], [32]. There is an insignificance and predetermined probability τ that the actual loss will be more significant [33]. In the context of returns, it is usually possible to consider VaR estimation. Formally, we express by r_t the return on the asset at time t. On a certain day, forward risk value for a long trading position at the τ significance level, recorded $VaR_\tau(r_t)$, is conditionally anticipated as the set of information, F_t, available at time t determined by the formula as following Eq. (5) [34]-[37] :

$$P(r_{t+1} \leq VaR_\tau(r_t)|F_t) = \tau. \tag{5}$$

Such that,

$$\tau = \int_{-\infty}^{VaR_\tau(r_t)} F_t\,(r_t)dr_t. \tag{6}$$

This expression means that VaR is the τ-th conditional quantile of the return distribution.

An alternative way to calculate VaR for the distribution of non-normal returns must use the Cornish-Fisher approach [38]. Start by defining a standard portfolio return with Eq. (7):

$$z_{t+1} = \frac{R_{PF,t+1}}{\sigma_{PF,t+1}} \sim IIDN(0,1), \tag{7}$$

where R_{PF} is return, σ_{PF} is the standard deviation of the return, and $D(0,1)$ shows a distribution that has a mean of zero and a variance equal to 1. Modified VaR with coverage level τ can then be estimated with the following Eq. (8) [38]:

$$VaR_{p,t+1} = -\sigma_{PF,t+1} CF_\tau^{-1}, \tag{8}$$

with:

$$CF_\tau^{-1} = \Phi_\tau^{-1} + \frac{\zeta_1}{6}[(\Phi_\tau^{-1})^2 - 1] + \frac{\zeta_2}{24}[(\Phi_\tau^{-1})^3 - 3\Phi_\tau^{-1}] - \frac{\zeta_1^2}{36}[2(\Phi_\tau^{-1})^3 - 5\Phi_\tau^{-1}], \tag{9}$$

where Φ_τ^{-1} is the inverse value of the cumulative distribution function at τ percent, ζ_1 and ζ_2 are respectively the excess kurtosis skewness of the standardized returns z_t.

The steps of this study consists of:

1. Collecting research data

   The data in the research is daily closing price data from several stocks in Indonesia that have large market capitalization, so that they can represent other stocks in Indonesia.

2. Calculating the return of each stock

   After the stock data is collected, the return calculation is carried out on each stock. The return in question is the daily return which is the return value against the previous day.

3. Forming a stock portfolio

   Next, by utilizing the average information from the stock returns obtained and their covariance, it can be used to form a stock investment portfolio. Portfolio formation is carried out by solving multi-objective optimization problems by considering the mean return, covariance, and risk averse values of investors. After the problem is solved, the portion equation for each stock in the portfolio is obtained. Of course, the total number of all portions is limited to a maximum of 1 or 100% so that it can reflect the investment in an instrument.

4. Calculating the return on the stock portfolio

If the stock portfolio has been formed, the next step is to calculate the return on the stock portfolio in the same way as the previous calculation of the return on each stock.

5. Calculating the mean return of the stock portfolio and each of its constituent stocks

   After obtaining the return of the stock portfolio, an illustration is made to show the location of the mean return of the stock portfolio relative to the mean return of each of its constituent stocks. So that it is expected to be able to describe the location of the comparison of the mean return of the stock portfolio with the mean return of each of its constituent stocks.

6. Testing the normality distribution of the stock portfolio return

   Before measuring the risk of the stock portfolio, first test the normality distribution of the portfolio return. Testing is the basis for determining the method used to calculate investment risk using VaR. This part is often missed, even though VaR requires the assumption of a normal distribution of returns. If the return of the stock portfolio is not normally distributed, then the calculation of the size of the portfolio risk is done using modified VaR.

7. Calculating the size of the risk of the stock portfolio and each of its constituent stocks

   The final part of the analysis is closed by calculating the size of the stock portfolio risk and providing an illustration of the comparison of the risk value of the stock portfolio with the risk of each of its constituent stocks.

2.5 Data

This research takes four Indonesian stocks that have the largest market capitalization in January 2025, namely PT Barito Renewables Energy Tbk (BREN), PT Bank Central Asia Tbk (BBCA), PT. Bayan Resources Tbk (BYAN), and PT. Bank Rakyat Indonesia (Persero) Tbk. (BBRI). The research was conducted using daily closing price data from October 9th, 2023 (considering BREN's Initial Public Offering) to February 25th, 2025, obtained through finance.yahoo.com. The entire analysis was carried out using R software, especially the Performance Analytics packages, to calculate the Value at Risk (VaR).

2.6 Analysis

The analysis step is divided into three steps, including solving Multi-Objective Optimization, calculating portfolio return, and last is estimate Value at Risk (VaR).

2.6.1 Solve Multi-Objective Optimization

Based on Research Method 2.2., a specific formulation for portfolio optimization can be determined by recognizing the two objectives of minimizing portfolio risk at Eq. (2) and maximizing expected return portfolio at Eq. (1). That statement is equivalent to minimizing the negative portfolio expected return and portfolio risk. The multi-objective optimization can be accomplished using the Lagrange multiplier as follows:

$$L(\mathbf{x}) = -\mathbf{r}^T\mathbf{x} + \mu\mathbf{x}^T\mathbf{\Sigma}\mathbf{x} + \lambda(\mathbf{1}^T\mathbf{x} - 1), \tag{10}$$

where $\mathbf{1}^T$ is a vector with element 1 so $\mathbf{1}^T = [1,1,\dots,1]$. Set $\frac{\partial L(\mathbf{x})}{\partial \mathbf{x}}$ so that:

$$\frac{\partial L(\mathbf{x})}{\partial \mathbf{x}} = -\mathbf{r} + 2\mu\mathbf{\Sigma}\mathbf{x} + \lambda\mathbf{1} = 0 \tag{11}$$

Such that,

$$\mathbf{x} = \frac{1}{2\mu}(\mathbf{\Sigma}^{-1})(\mathbf{r} - \lambda\mathbf{1}). \tag{12}$$

The idea behind the solution is to use Lagrange multipliers λ, substitute Eq. (12) into the constraint $\mathbf{1}^T\mathbf{x} = 1$:

$$\lambda = \frac{\mathbf{1}^T\mathbf{\Sigma}^{-1}\mathbf{r} - 2\mu}{\mathbf{1}^T\mathbf{\Sigma}^{-1}\mathbf{1}}. \tag{13}$$

Let $a_1 = \mathbf{1}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}$ and $a_2 = \mathbf{1}^T\boldsymbol{\Sigma}^{-1}\mathbf{r}$ both of which are scalars, (13) can be written as:

$$\lambda = \frac{a_2 - 2\mu}{a_1}. \tag{14}$$

Thus, the solution of the objective function that has been described for the portfolio weight vector $\mathbf{x}$ ($\mathbf{x}^*$) is written as follows:

$$\mathbf{x}^* = \frac{1}{2\mu}\boldsymbol{\Sigma}^{-1}\mathbf{r} - \frac{\boldsymbol{\Sigma}^{-1}}{2\mu}\left(\frac{a_2 - 2\mu}{a_1}\right)\mathbf{1}, \tag{15}$$

where μ represents the risk aversion index with the minimum of zero and there is no maximum value. Risk aversion index determination can be based on historical analysis of actual investment decisions, so that investors can freely adjust according to their respective risk preferences.

2.6.2 Calculate Portfolio Return

The rate of return is the ratio of changes in the value of assets that depend on the last time. The asset return formula can be stated as follows:

$$R_{asset,t} = \frac{V_{asset,t} - V_{asset,t-1}}{V_{asset,t-1}}, \tag{16}$$

where $V_{asset,t-1}$ is the price of an asset in period t.

2.6.3 Estimate Modified Value at Risk

Before calculating the value at risk, we first tested the distribution of returns; we used the Kolmogorov-Smirnov test based on the standard normal distribution. We utilize R software to calculate Value at Risk using the VaR function in the PerformaceAnalytics packages so that if the return is not normally distributed, the VaR calculation uses the "*modified*" method (Cornish-Fisher Approximation); otherwise, we use the "*historical*" method.

3. RESULTS AND DISCUSSION

3.1 Descriptive Statistics of Stocks

Data used daily closing price data for four Indonesian stocks with the largest market capitalization, namely PT Barito Renewables Energy Tbk (BREN), PT Bank Central Asia Tbk (BBCA), PT. Bayan Resources Tbk (BYAN), and PT. Bank Rakyat Indonesia (Persero) Tbk. (BBRI). The data are taken from October 9th, 2023, to February 25th, 2025, as a sample are presented in descriptive statistics as follows, presented in Table 1.

Table 1. Descriptive Statistics of The Daily Closing Price of Stock

Stock	BREN	BBCA	BYAN	BBRI
Sample	331	331	331	331
Mean	7425	9737	18649	4984
Median	7300	9775	19000	4920
Minimum	975	8600	15175	3800
Maximum	11900	10950	20700	6400
Standard Deviation	2034.49	543.54	1295.23	628.28
Skewness	-0.1593163	-0.1257472	-0.5244079	0.2565711
Kurtosis	2.847986	2.130600	2.280344	2.260756

Based on Table 1, information is obtained that each stock has a different price range. However, it does not hinder this research because the analysis tends to be more influenced by fluctuations and the risk of price movements of each stock. The daily closing price movement of each stock is presented in the form of a time-series plot, as shown in Fig. 1.

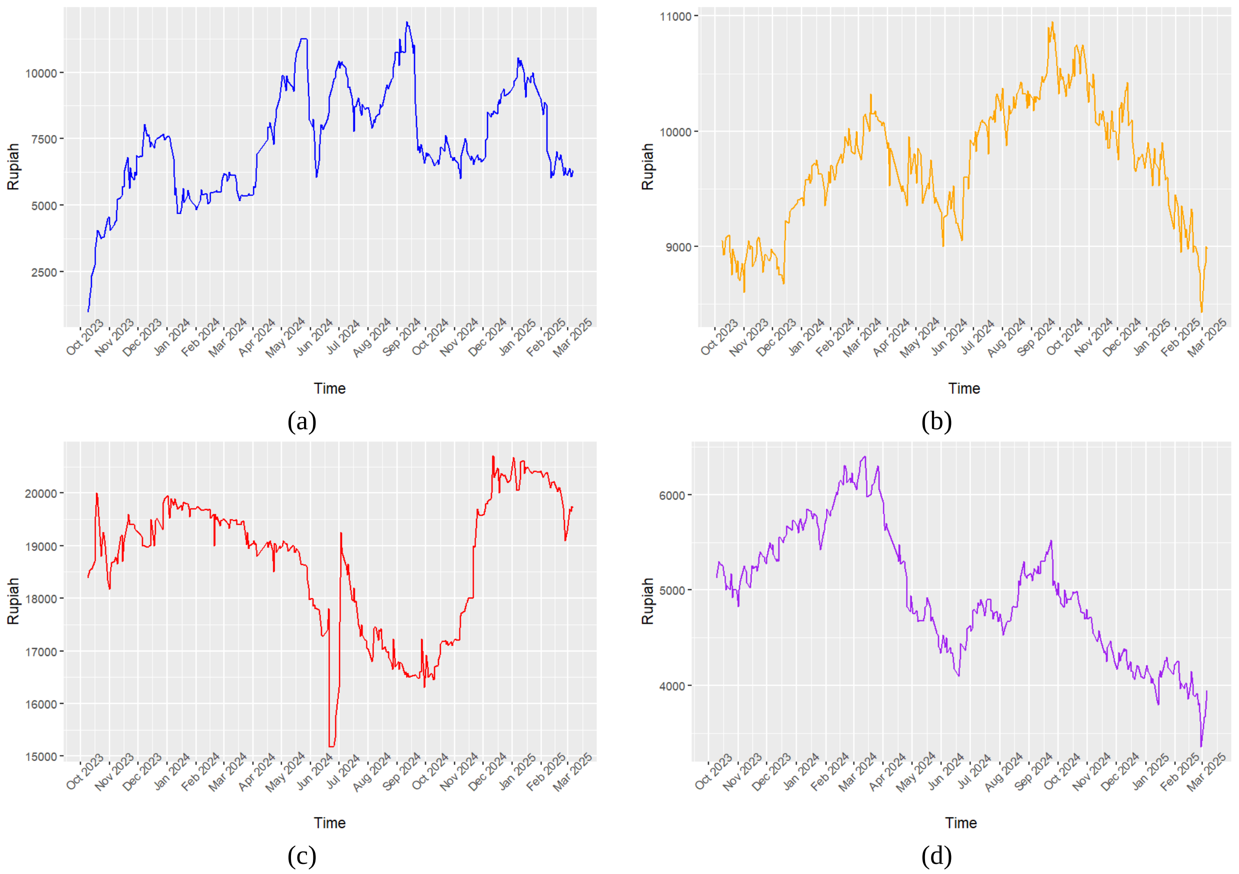

Figure 1. Illustrations of The Daily Closing Price for (a) BREN, (b) BBCA, (c) BYAN, (d) BBRI

It can be seen in Fig. 1 that the price of each share fluctuates greatly. Thus, investors are faced with price drops at any time, which can cause losses if they only invest in one stock. Therefore, we suggest forming an optimal portfolio to minimize risk and maximize returns.

3.2 Mean Return and Variance-Covariance of Stock

The calculation weight of each stock in the portfolio requires the mean return and variance-covariance value. The result of calculating mean returns Table 2.

Table 2. Mean Return of Stock Price

Stock	Mean Return (%)
BREN	0.7384974
BBCA	0.0025795
BYAN	0.0303218
BBRI	-0.07266919

The variance-covariance matrix has been obtained as follows

$$\Sigma = \begin{pmatrix} 0.0036989320 & 0.0000185102 & 0.0000443217 & 0.0000358161 \\ 0.0000185102 & 0.0002054987 & 0.0000137691 & 0.0001255492 \\ 0.0000443217 & 0.0000137691 & 0.0000250898 & -0.0000036198 \\ 0.0000358161 & 0.0001255492 & -0.0000036198 & 0.0003597005 \end{pmatrix}$$

where sequentially for rows and columns from 1 to 4 represent BREN, BBCA, BYAN, and BBRI stocks.

3.3 Optimized Portfolio Weight

The calculation of the Optimized Portfolio Weight is highly dependent on the risk aversion index, denoted by μ. We assume and simulate a risk aversion of 100 to 2500 as a simulation so that the weight calculation of portfolio results based on the risk aversion index are presented in Fig. 2.

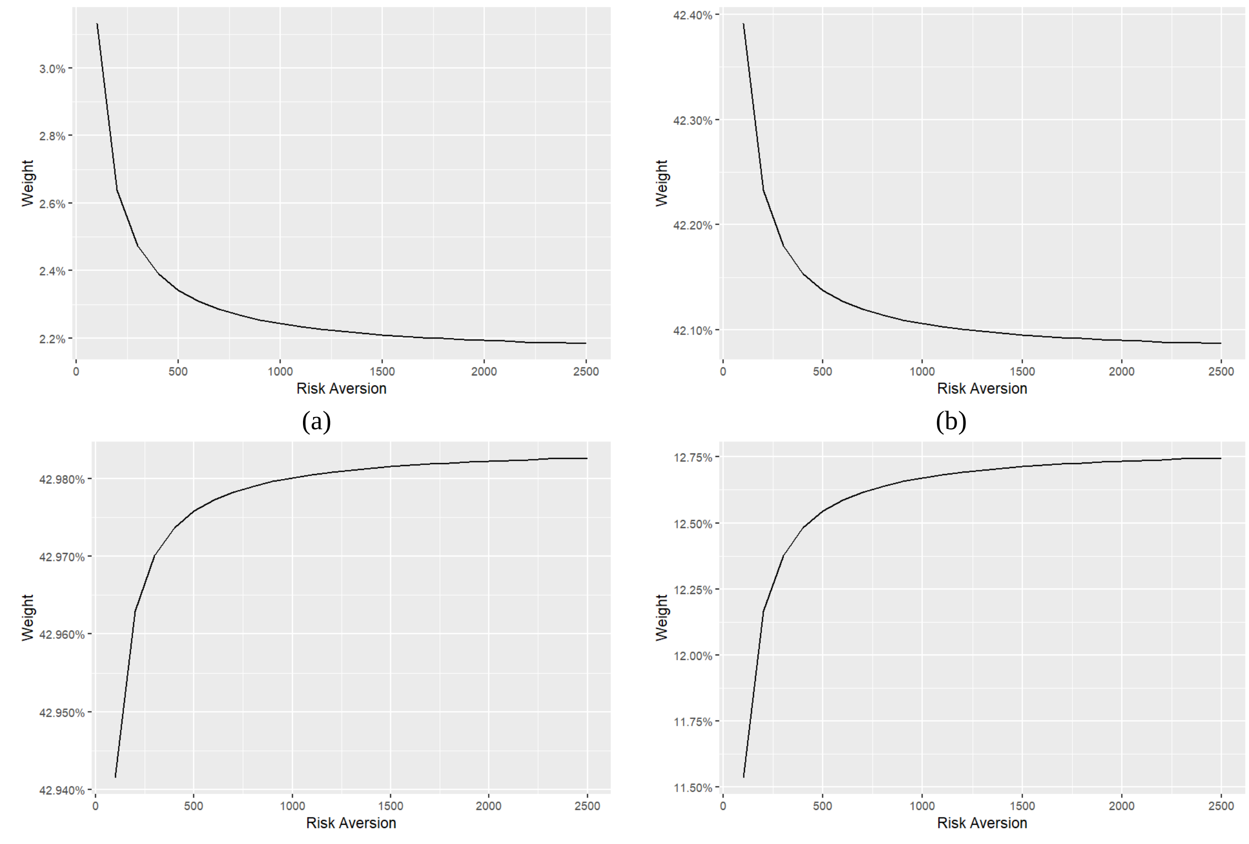

Figure 2. The Stock Weight in Portfolio for (a) BREN, (b) BBCA, (c) BYAN, and (d) BBRI

The consideration of risk over expected return by an investor can be expressed through the risk aversion index. Risk aversion is usually specific, so multi-objective optimization is considered suitable for this problem. Multi-objective optimization provides investors with an optimal asset allocation strategy that can simultaneously maximize expected return and minimize risk based on their respective risk aversion index values. When the weight of each stock is added up at the same risk aversion value, it will be worth one which is in accordance with the concept of the portfolio itself. Based on Fig. 2, the difference in risk aversion affects the weight of each stock. Stocks with high volatility tend to give a rising graph in proportion to risk aversion, but not always, depending on return analysis. In this portfolio, based on the calculation, the priority stock is BBCA, which is suggested to have the most considerable weight.

3.4 Portfolio Return

The calculation of portfolio return is intended to compare each stock's mean return with the portfolio's mean return. The calculation results, which are also influenced by the risk aversion index μ, are presented in Fig. 3.

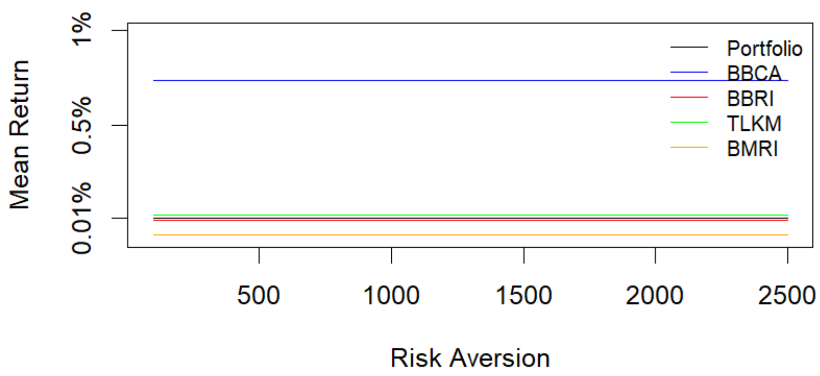

Figure 3. Comparison of The Portfolio's Average Return with Constituent Stocks

Based on Fig. 3, the multi-objective optimization approach can stabilize the mean return value so that it is always between the mean returns of several constituent stocks, not the smallest or vice versa. Thus, investors do not need to worry about differences in mean returns based on risk aversion.

3.5 Value at Risk

Finally, we compare portfolio risk with individual stocks. Risk estimation is measured using Value at Risk (VaR). Before estimating Value at Risk, we must first test the normality of the return distribution of the portfolio and each stock. The test was carried out using the Kolmogorov-Smirnov test, and the results were obtained as shown in Table 3.

Table 3. The Result of Normality Kolmogorov-Smirnov Test of Each Stock and Portfolio

Instrument Investment	Kolmogorov-Smirnov Statistic	P-value
Portfolio	0.48164	$< 2.2 \times 10^{-16}$
BREN	0.44502	$< 2.2 \times 10^{-16}$
BBCA	0.48489	$< 2.2 \times 10^{-16}$
BYAN	0.47726	$< 2.2 \times 10^{-16}$
BBRI	0.47816	$< 2.2 \times 10^{-16}$

With a significant level of 0.05 (or 5%), it can be concluded that all stock returns, including portfolios, do not follow the normal distribution. Due to the return that is not normally distributed, the estimation is carried out using a "*modified*" approach (Cornish-Fisher Approximation).

Because the portion of each risk aversion is different, we choose the most significant VaR value from the portfolio so that it is displayed in the form of a bar plot in Fig. 4.

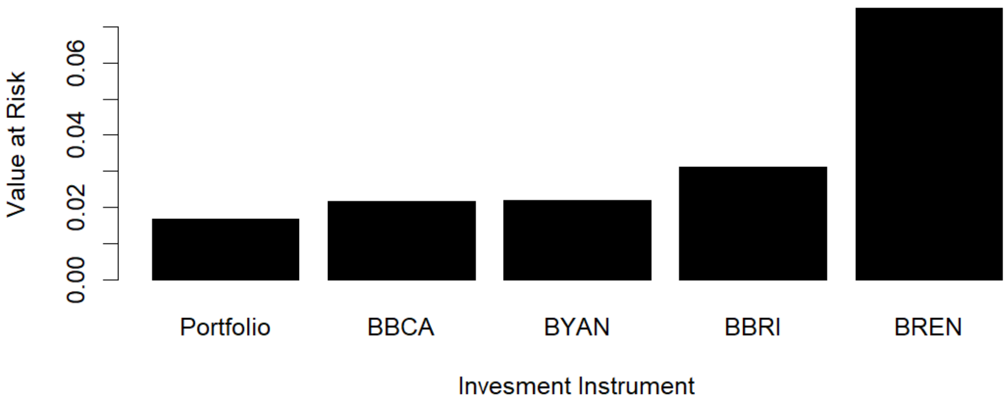

Figure 4. Comparison of The Portfolio's Value at Risk (VaR) with Constituent Stocks

In Fig. 4, the estimated var of the portfolio has the smallest value of all the estimated var of stocks. This result proves that the multi-objective optimization approach can minimize risk to the smallest possible degree while maximizing return. VaR of the portfolio of 1.667386% provides information that supposes we invest in a portfolio formed of IDR 1,000,000. There will be a five percent chance that the daily loss will equal or exceed IDR 16,673.86 (1.667386% of IDR 1,000,000), or it can be said that the portfolio has a 95 percent chance of being worth IDR 983,326.14 (1,000,000-16,673.86) or more tomorrow. The use of the multi-objective method has been proven to be able to adjust the portfolio portion to the amount of risk aversion of each investor, not only determining an optimal proportion based on maximum return as in existing research. Through the results that have been described, investors are able to adjust the amount of maximum loss that can still be borne daily. In addition, investors can also adjust the risk averse value to adjust their losses as well.

4. CONCLUSION

Portfolio risk analysis on Indonesian stocks based on Value at Risk (VaR) with Multi-Objective Optimization conducted in this study obtained several findings and conclusions. First, by using a risk aversion index of 100 to 2500. The results of the formation of a stock portfolio using the multi-objective optimization technique show that BBCA has the largest weight with a portion of more than 40% of the four stocks, so BBCA will be a priority stock for this portfolio. The multi-objective optimization approach also shows evidence that the formed stock portfolio has a stable mean return because the mean return of the portfolio is between several of its constituent stocks (and vice versa) which is around 0.01%. Second, the proposed development of a VaR-based risk measurement method to ignore the assumption of normality of returns provides a successful measurement showing the smallest estimated modified VaR value of the stock portfolio, which is 1.67%. Thus, it can be concluded that Multi-Objective Optimization can not only create a portfolio that provides a small risk, but at the same time, Multi-Objective Optimization can also provide competitive returns with its constituent stocks. In addition, the proposed development of modified VaR has successfully provided a calculation of risk value without requiring the assumption of normality distribution to be met.

Basically, this study is limited to avoiding the assumption of normality so that it does not pay attention to the distribution of returns. For future research, development can be carried out by modifying the VaR formula so that it can be specifically applied to one or several specific distributions, which can better guarantee the risk measure obtained is able to describe the actual investment risk value.

Author Contributions

Wisnowan Hendy Saputra: Conceptualization, Formal analysis, Methodology, Validation, Writing - original draft. Hasri Wiji Aqsari: Data curation, Resources, Software, Visualization, Writing - review & editing. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare no competing interest.

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