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FORECASTING STATIONARY CLIMATE DATA USING AUTOREGRESSIVE MODELS AND HIGH-ORDER FUZZY

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ABSTRACT

Forecasting is essential for improving aviation safety, with air humidity being a critical factor influenced by air temperature. This study analyzes daily humidity data from I Gusti Ngurah Rai Airport, one of Indonesia's busiest air stations, using two time series modeling approaches: Autoregressive (AR) and high-order fuzzy modeling. The objective is to evaluate and compare their forecasting accuracy. Historical daily data from the Meteorology, Climatology, and Geophysics Agency of Indonesia were used to build the forecasting models. The optimal linear AR model served as the foundation for constructing the AR high-order fuzzy model, which incorporates linguistic rules to capture nonlinear patterns. Both models were implemented and evaluated using the Mean Squared Error (MSE) metric. Results show that the AR(2) model outperforms the AR high-order fuzzy model, achieving a lower MSE of 13.23. This suggests that the AR(2) model provides more accurate humidity forecasts over the observed period. These findings offer practical insights for policymakers and decision-makers in forecasting daily humidity levels and supporting aviation operations. While the study confirms the effectiveness of traditional AR modeling, it also highlights limitations of the fuzzy approach, particularly its sensitivity to parameter tuning and data sparsity. The integration of high-order fuzzy modeling represents a novel contribution to this domain, though further refinement is needed to enhance its forecasting performance.

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1. INTRODUCTION

I Gusti Ngurah Rai Airport is located in Tuban Village, Kuta District, Badung Regency, which lies in the central region of Indonesia. Astronomically, Tuban Village is positioned between 8°44'14"LS - 8°45'28"LS and 115°9'8"E - 115°11'9"E, characterized by a lowland physical environment [1]. As one of Indonesia's three busiest airports, maintaining the composition of air pressure and temperature is crucial. Air pressure and temperature are critical factors for ensuring flight safety, particularly during takeoff and landing. Among the key parameters associated with temperature is atmospheric humidity, which plays a significant role in influencing aviation conditions. Air humidity refers to the concentration of moisture present in the atmosphere, represented by the amount of water vapor contained in the air [2]. The air humidity factor also contributes to the airport's runway [3]. Air humidity data, collected from various meteorological and climatological stations, typically exhibit characteristics of minimal abrupt changes. The data distribution tends to fluctuate around a stable average, maintaining a relatively constant pattern, which suggests stationarity. Stationarity is a crucial assumption in the analysis of time series data [4].

The time series method is an analytical approach designed for forecasting future data trends. One widely used method within this framework is the Box-Jenkins method, which focuses solely on the dependent variable using historical data while disregarding independent variables. The key advantage of the Box-Jenkins method is its flexibility, as it does not require the data to consistently exhibit stationary patterns and is also applicable to datasets with seasonal fluctuations. The method comprises various models, including Autoregressive (AR), Moving Average (MA), and Autoregressive Moving Average (ARMA) for stationary data, as well as Autoregressive Integrated Moving Average (ARIMA) and Seasonal Autoregressive Integrated Moving Average (SARIMA) for non-stationary and seasonally patterned data [5].

This method has been extensively studied. For instance, [2] utilized the ARIMA model to predict average air temperature and daily humidity, while [4] applied the Box-Jenkins method to forecast traffic accident data in Semarang. Another forecasting technique suitable for stationary data is the high-order fuzzy Autoregressive (AR) method, which integrates the AR(p) model from Box-Jenkins with fuzzy regression concepts for time series data.

Fuzzy AR models have consistently garnered attention, with significant advancements made in recent years. Notable contributions to fuzzy time series research include studies by Chen and Chang [6], Cai et al. [7], Chen and Chen [8], Yolcu et al. [9], Ye et al. [10], and Chen and Jian [11]. Additionally, for trend data, Sulandari and Yudhanto employed a hybrid approach combining simple moving averages with weighted fuzzy time series to enhance forecasting accuracy [12]. Meanwhile, recent advancements in the Weighted Fuzzy Time Series (WFTS) optimized with Particle Swarm Optimization (PSO), have demonstrated improved predictive accuracy in modeling air temperature trends [13]. Given the importance of air temperature in regulating air humidity, which in turn affects flight safety, exploring robust forecasting techniques remains a critical area of study.

While classical models such as AR and ARMA are effective and statistically interpretable, they may lack the flexibility of fuzzy systems in modeling nonlinear relationships and handling imprecise or ambiguous data, which are often present in meteorological forecasting [14]. Therefore, this study proposes a high-order fuzzy AR approach, aiming to integrate the temporal modeling strength of classical methods to enhance forecasting performance in dynamic environments such as airport humidity.

The high-order AR fuzzy model was developed to address the limitations found in conventional high-order fuzzy AR methods [7]. By incorporating principles from classical autoregressive models, it offers improved practicality and broader applicability. In a notable contribution, Kocak (2017) introduced a high-order fuzzy ARMA(p,q) model designed to enhance forecasting accuracy, even in scenarios where the MA component is not explicitly applied [15]. A major issue underlined in that study is the prevailing reliance on fuzzy AR variables in most fuzzy time series models, which often overlook the importance of MA components. Unlike these models, classical time series approaches, AR, MA, and ARMA, are tailored to fit the data's underlying patterns, and neglecting essential MA terms in fuzzy models can result in specification errors and diminished forecasting accuracy. To overcome these issues, researchers have designed fuzzy ARMA models that incorporate both fuzzy AR and MA elements [16], [17], [18], [19].

Building on this foundation, the present study conducts a comparative analysis between the Box-Jenkins method and the high-order fuzzy AR model for forecasting air humidity at I Gusti Ngurah Rai Airport. Accurate humidity forecasts are essential for ensuring flight safety, especially in tropical regions. In this study, the significant Box-Jenkins model is utilized as a benchmark for determining the appropriate order of the high-order fuzzy AR model. The performance of both models is evaluated based on forecasting accuracy, with the preferred model identified through the smallest residual errors. Although high-order fuzzy AR models provide the advantage of blending statistical structure with fuzzy logic, their application in forecasting air humidity, particularly in tropical airport settings, remains underexplored. This study aims to fill that gap by comparing both approaches and identifying the most effective method for aviation-related humidity forecasting.

2. RESEARCH METHODS

2.1 Box Jenkins Method

The Box-Jenkins time series models, including AR, MA, and ARMA, are widely used for forecasting and require data to be stationary for effective application. These models rely on identifying and modeling patterns based on autocorrelation structures within the time series.

1. Model Identification
   Based on [20] the AR limited coefficient parameter value is set between $-1 < \varphi < 1$ for the process of AR(1) whilst for the process of AR(2), the coefficient parameter value are $-2 < \varphi_1 < 2$ and $-1 < \varphi_2 < 1$. Generally, the AR(p) model can be written as follows:
   $$Z_t = \varphi_1 Z_{t-1} + \varphi_2 Z_{t-2} + \cdots + \varphi_p Z_{t-p} + a_t \tag{1}$$
   According to [21], the coefficient value for the MA(1) process parameter is limited to $|\theta_1| < 1$ while for the MA(2) process parameter are $\theta_1 + \theta_2 < 1, \theta_2 - \theta_1 < 1$, and $-1 < \theta_2 < 1$. The MA parameters for a q order process are written as follows.
   $$Z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q} \tag{2}$$
   The ARMA process combines both AR and MA models. Generally, the equation for the ARMA(p,q) model is expressed as:
   $$Z_t = \varphi_1 Z_{t-1} + \cdots + \varphi_p Z_{t-p} + a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \tag{3}$$
   If the autocorrelation function (ACF) decreases exponentially and the partial autocorrelation function (PACF) cuts off at the p-th lag, the time series can be modeled using the AR(p) process. If the PACF decreases exponentially and the ACF cuts off at the q-th lag, the time series is suitable for the MA(q) process. If both the ACF and PACF decrease exponentially, the ARMA model should be chosen.
2. Stationarity Test
   To assess stationarity in the mean, this analysis employs a unit root test to determine whether the data contains a unit root. The unit root test used in this study is the Phillips-Perron Root Test. To evaluate stationarity in variance, the Box-Cox Transformation is applied, and any non-stationary variance is addressed through data transformation. The stationarity of the data can also be observed through the ACF and PACF plots, by checking if the patterns follow an exponential decay. If the data is stationary, a temporary model can be identified based on the ACF and PACF plots.
   a. ACF
      The autocorrelation coefficient shows the correlation between the time series and the time series itself with a lag difference of 0, 1, 2 periods or more. The covariance between Z_t and Z_{t+k} is as follows [20].
      $$\gamma_k = Cov(Z_t, Z_{t+k}) = E(Z_t - \mu)(Z_{t+k} - \mu) \tag{4}$$
      Correlation between Z_t and Z_{t+k} is
      $$\rho_k = \frac{Cov(Z_t, Z_{t+k})}{\sqrt{Var(Z_t)}\sqrt{Var(Z_{t+k})}} = \frac{\gamma_k}{\gamma_0} \tag{5}$$
   b. PACF
      Used to measure the degree of closeness between Z_t and Z_{t+k} after the effect of Z_{t+1}, …, $Z_{t+k} - 1$ is removed [21]. The function is demonstrated as follows in (6).

$$\hat{\varphi}_{k+1,k+1} = \frac{\hat{\rho}_{k+1} - \sum_{j=1}^{k} \hat{\varphi}_{kj}\hat{\rho}_{k+1-j}}{1 - \sum_{j=1}^{k} \hat{\varphi}_{kj}\hat{\rho}_{j}} \tag{6}$$

In time series analysis, a key aspect is the identification and configuration of the model based on the available data. The principle of parsimony applies to model identification, which suggests using as few parameters as necessary to achieve an effective model.

3. Parameter Estimation
The next step involves estimating the AR and MA parameters. The method used for this estimation is the least squares method. Significant parameter test is to find out the significance of the φ and θ parameter using t_{test}.

$$t_{test} = \frac{\hat{\varphi}}{se(\hat{\varphi})} \ or \ t_{test} = \frac{\hat{\theta}}{se(\hat{\theta})} \tag{7}$$

If the absolute value of the t-statistic exceeds the critical value from the t-distribution at a chosen significance level (e.g., $\alpha = 0.05$), the null hypothesis is rejected, implying that the parameter is significant.

4. Diagnostic Check
The following step is examining the model in order to know whether it is good enough to be used.
   a. Residual Normality Assumption Test
   Residual normality can be assessed by examining the normality plot. If the residuals align closely with the diagonal line, it indicates that they are normally distributed.
   b. Residual Independent Test
   This test is conducted to detect the independence of residuals across time lags using the Ljung–Box test, which assesses whether autocorrelations of the residuals are significantly different from zero [22].

2.2 AR High-Order Fuzzy Method

At this stage, the calculation process is carried out using the AR high-order fuzzy methods. The steps involved in the calculation are as follows:

1. Fuzzy Time Series Process
Fuzzy time series is a forecasting method that leverages fuzzy principles as its foundational framework. This method is designed to identify and capture patterns from historical data, enabling it to model trends and behaviours over time [23][24].
   a. The universe of discourse U

$$U = [Xmin - D_1, Xmax + D_2]$$

   The u_i interval is a sub interval of the universal set U [15].
   b. Sub-interval A_i

$$A_i = \frac{fA_i(u_1)}{u_1} + \frac{fA_i(u_2)}{u_2} + \cdots + \frac{fA_i(u_b)}{u_b} \tag{8}$$

$$fA_i(u_1) = \begin{cases} 1 & k = i \\ 0{,}5 & k = i-1 \ and \ i+1 \\ 0 & otherwise \end{cases}$$

   c. Determine fuzzy logic relations for AR(p) model
   For example, when the fuzzy logic relations for fuzzy AR(2) model are as

$$A2, A3 \rightarrow A3, A2, A3 \rightarrow A3, A2, A3 \rightarrow A5$$

   are found out to be

$$A2, A3 \rightarrow A3, A3, A5$$

   d. Forecast high-order fuzzy
   Classifying the FLR that has been obtained from the third stage into groups to form a Fuzzy Logical Relationship Group (FLRG) and combining the same relationship.

2. Defuzzification Calculation Process

a. Defuzzification forms the prediction
By using the middle value in the set U, the following formula obtained is:

$$\hat{x}(t) = \frac{a \times m_j + b \times m_k + c \times m_l}{a + b + c} \tag{9}$$

b. Determining residual
Determination of residuals in the model can use the following formula.

$$e(t) = x(t) - \hat{x}(t) \tag{10}$$

2.3 Criteria for Selection of the Method

Mean Square Error (MSE) is one of the criteria used to select the better model based on the residual forecasting results.

$$MSE = \frac{\sum_{t=1}^{n} (x_t - \hat{x}_t)^2}{n} \tag{11}$$

2.4 Data Source

This study employs secondary data obtained from the Meteorology, Climatology, and Geophysics Agency (BMKG), specifically covering the period from December 2020 to January 2021. This two-month period was chosen as it represents the peak of the wet season in Indonesia, where fluctuations in air humidity are more dynamic and can significantly impact aviation safety, particularly in tropical regions. The dataset consists of daily average air humidity recorded at I Gusti Ngurah Rai Airport.

Data analysis was conducted using Minitab 16, Microsoft Excel, and R Studio. The research is structured into three main stages: forecasting using the Box-Jenkins method, forecasting using the high-order fuzzy autoregressive (AR) method, and evaluating the forecasting accuracy by comparing the results based on the highest level of accuracy achieved.

1. Forecasting using the Box-Jenkins method
   The steps for forecasting using the Box-Jenkins method are as follows:
   a. Creating a time plot of the average air humidity data.
   b. Conducting data exploration by plotting the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF). Then, testing for stationarity of the data using the Phillips-Perron (PP) test to determine whether the data is stationary in both mean and variance.
   c. Identifying the time series model based on the ACF and PACF plots.
   d. Estimating the parameters of the selected time series model.
   e. Performing diagnostic checking to determine whether the model meets assumptions, namely the residual independence test and residual normality test. If the model meets the assumptions, proceed to the next step.
   f. Calculating the forecast values using the Box-Jenkins method.
2. Forecasting using the high-order AR fuzzy method
   The steps for forecasting using the high-order AR fuzzy method are as follows [15]:
   a. Determining the order of the high-order AR fuzzy model based on the model obtained in step 1.f.
   b. Defining the fuzzy interval.
   c. Determining the universe of discourse (U) and dividing it into several intervals of equal length.
   d. Establishing the fuzzy set A_i.
   e. Constructing the Fuzzy Logical Relationship (FLR) $A_i \rightarrow A_j$.
   f. Formulating the Fuzzy Logical Relationship Group (FLRG).
   g. Conducting the defuzzification process and calculating the forecast values.
3. Comparing the accuracy of both forecasting methods
   The steps for comparing the forecasting accuracy are as follows:
   a. Calculating the residuals of both methods using the Mean Square Error (MSE) calculation.
   b. Determining the best forecasting model based on the smallest residual value.

3. RESULTS AND DISCUSSION

3.1 Descriptive Statistics the Average Air Humidity Data

This section presents the descriptive analysis of the average air humidity data used in the study. The dataset comprises 62 daily observations, and the descriptive statistics are shown in Table 1.

Table 1. Descriptive Statistics of the Average Air Humidity Data

Mean	Minimum	Median	Maximum
83.742	74	85	91

Based on Table 1, the average air humidity during the observation period was 83.74%, with values ranging from 74% to 91%, indicating generally stable and humid conditions. This moderate variability supports the assumption of stationarity in the data.

3.2 Modelling with Box-Jenkins Method

The development of the average air humidity forecasting model is conducted using the Box-Jenkins method, based on 62 days of data. The modeling process involves several sequential steps, as outlined below:

1. Model Identification
   At this stage, the stationarity of the data is checked using the actual data plot (see Fig. 1), the ACF and PACF plots (Fig. 2), and the unit root test.

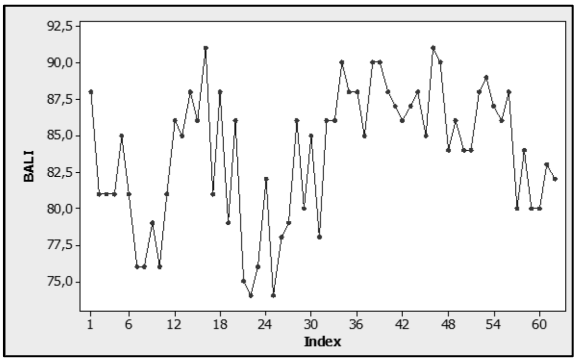
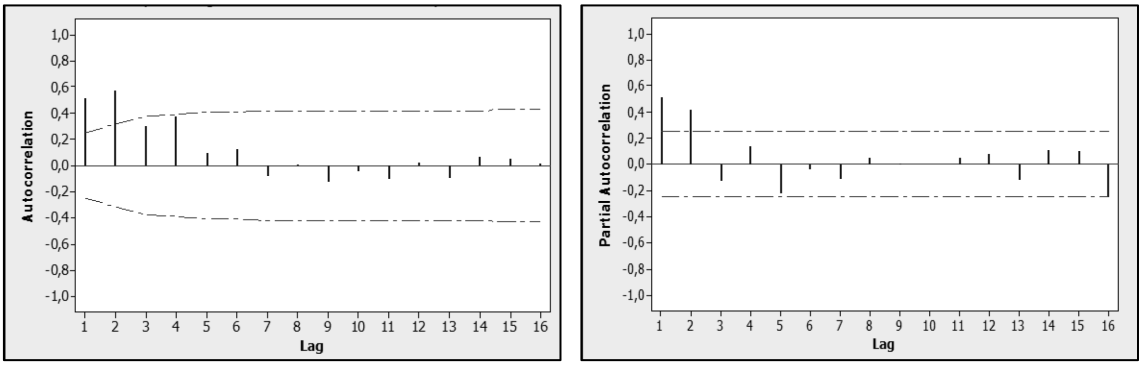


Figure 1. The Plot of Average Air Humidity Data
(Source: Minitab 16)

Based on Fig. 1, it is evident that the data does not exhibit a stationary pattern, as a trend is observed towards the end of the dataset. As a result, the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots will be examined to further assess the characteristics of the data (see Fig. 2).



(a) (b)

Figure 2. The Plot ACF (a) and PACF (b) of Average Air Humidity Data
(Source: Minitab 16)

The graphs shown in Fig. 2 indicates that the data may be stationary, based on the stable autocorrelation structure and absence of apparent trend or seasonal patterns.

2. Stationarity Test of Data in Mean
The stationarity of the average air humidity data was tested using the Phillips-Perron Unit Root Test [25]. This test evaluates the null hypothesis $H_0 : \rho = 0$, which indicates the presence of a unit root and suggests that the variable is not stationary. The alternative hypothesis is $H_1 : \rho \neq 0$, meaning that the variable does not have a unit root and is therefore stationary. Based on the test results for the data, the Dickey-Fuller value is -38.363, with a truncation lag parameter of 3 and a $p - value$ of 0.01. Since the $p - value$ (0.01) less than the significance level $\alpha = 0.05$, we reject the null hypothesis. This implies that there is no unit root in the data, indicating that the average air humidity variable is stationary. To evaluate stationarity in variance, a Box-Cox transformation was applied. The transformation effectively stabilized the variance over time, as indicated by a relatively constant rolling standard deviation. This confirms the assumption of homoskedasticity required for the application of the Box-Jenkins model.
3. Parameter Estimation
Based on the ACF and PACF plots, the potential time series models identified are ARMA(2,2), ARMA(2,1), ARMA(1,1), AR(2), MA(2), AR(1), and MA(1). These models are considered based on the observed patterns in the autocorrelations and partial autocorrelations of the data.

Table 2. Parameter Estimation and Significance Test of Time Series Model

Model	Parameter	Parameter Estimated Value	Standard Error	t_{test} Value	*p-value*	Significance
ARMA(2,2)	$\emptyset_1$	-0.1496	0.1404	-1.07	0.291	not significant
	$\emptyset_2$	0.7286	0.1408	5.18	0.000	significant
	θ_1	-0.5687	0.2009	-2.83	0.006	significant
	θ_2	0.1299	0.2009	0.65	0.521	not significant
	C	35.2686	0.652	54.09	0.000	significant
ARMA(2,1)	$\emptyset_1$	-0.2031	0.141	-1.44	0.155	significant
	$\emptyset_2$	0.6565	0.1002	6.55	0.000	not significant
	θ_1	-0.6709	0.1653	-4.06	0.000	significant
	C	45.7985	0.7526	60.86	0.000	significant
ARMA(1,1)	$\emptyset_1$	0.8272	0.1191	6.95	0.000	significant
	θ_1	0.4224	0.191	2.21	0.031	significant
	C	14.466	0.2789	51.86	0.000	significant
AR(2)	$\emptyset_1$	0.2942	0.118	2.49	0.016	significant
	$\emptyset_2$	0.4247	0.1179	3.6	0.001	significant
	C	23.5483	0.4621	50.96	0.000	significant
AR(1)	$\emptyset_1$	0.5174	0.1105	4.68	0.000	significant
	C	40.4361	0.5037	80.27	0.000	significant
MA(2)	θ_1	-0.4004	0.118	-3.39	0.001	significant
	θ_2	-0.423	0.1181	-3.58	0.001	significant
	C	83.8075	0.8814	95.09	0.000	significant
MA(1)	θ_1	-0.2774	0.1241	-2.24	0.029	significant
	C	83.7560	0.6962	120.31	0.000	significant

Based on the results from Table 2, the models with significant parameters are ARMA(1,1), AR(1), AR(2), MA(1), and MA(2). To ensure the adequacy of a significant model, a diagnostic check must be performed to verify that the residuals exhibit white noise characteristics and follow a normal distribution.
4. Diagnostic Check

Table 3. Residual Independence Test of Time Series

Model	Lag	Q-Ljung Box	*p-value*	Residual Independence
ARMA(1,1)	12	27.5	0.001	not independent
	24	37.8	0.014	not independent
	36	43.5	0.104	independent
	48	5.5	0.207	independent
AR(2)	12	8.2	0.511	independent
	24	25.4	0.232	independent
	36	34.8	0.381	independent
	48	44	0.516	independent

Model	Lag	Q-Ljung Box	*p-value*	Residual Independence
AR(1)	12	45.7	0	not independent
	24	55	0	not independent
	36	60.4	0.003	independent
	48	69.6	0.014	independent
MA(2)	12	15	0.091	independent
	24	29.4	0.104	independent
	36	39.9	0.191	independent
	48	49	0.317	independent
MA(1)	12	43.6	0	not independent
	24	49.9	0.001	not independent
	36	64.8	0.001	not independent
	48	74.4	0.005	not independent

As shown in Table 3, the independent models based on the residual independence test are the AR(2) and MA(2) models. The Q-Ljung Box test results indicate that the AR(2) and MA(2) models have independent residuals across multiple lags, as their p-values are consistently higher than the significance threshold (typically 0.05). In contrast, models such as AR(1), MA(1), and ARMA(1,1) show signs of residual dependence, meaning they do not fully meet the white noise assumption. Given that residual independence is a key requirement for a well-fitted time series model, AR(2) and MA(2) are considered suitable candidates for further evaluation. However, to fully determine the best-performing model, another diagnostic check, the residual normality test, is conducted, as shown in Table 4. This test examines whether the residuals of the selected models follow a normal distribution, which is essential for accurate forecasting and inference.

Table 4. Residual Normality Test

Model	Residual Normality
AR(2)	fulfilled
MA(2)	not fulfilled

According to the results of the parameter significance test presented in Table 4, the AR(2) model demonstrates the best performance, with significant residual independence and residual normality. This model has an MSE value of 13.23, indicating its suitability for forecasting.

3.3 Modelling with AR(2) High-Order Fuzzy Method

1. Fuzzy Time Series Process
   a. Determining the Universal Set U with an Interval Length of 5
   The first step in forecasting average air humidity using the high-order fuzzy AR method is to define the universal set U. The minimum and maximum humidity values are 74 and 91, respectively, resulting in an average-based interval of 5. An interval length of 5 was chosen to partition the universal set into equal segments, facilitating clear linguistic interpretation (e.g., "low", "medium", "high" humidity). This approach aligns with environmental fuzzy time series applications [26]. Thus, the universal set is $U = [71,95]$, partitioned into 5 intervals. The order used in this model is 2, where $D1$ and $D2$ represent the first and second lags of the historical data, respectively. Table 5 presents the interval partitions and their middle values.

Table 5. Universal Set U

Interval		Middle Value	
U1	[71,75]	m1	73
U2	[75,79]	m2	77
U3	[79,83]	m3	81
U4	[83,87]	m4	85
U5	[87,91]	m5	89
U5	[91,95]	m6	93

b. Fuzzy Set Determination

After determining the membership values for each $u_i(i = 1,2,\dots,5)$ in the fuzzy set A_i from the universal set U based on the defined partition intervals, the next step is fuzzification of historical data. The results of the fuzzification process are presented in Table 6.

Table 6. Fuzzification

Date	$x(t)$	Fuzzification	Affecting
01-12-2020	88	A5	-
02-12-2020	81	A3	-
03-12-2020	81	A3	A5,A3
04-12-2020	81	A3	A3,A3
05-12-2020	85	A4	A3,A3
06-12-2020	81	A3	A3,A4
⋮	⋮	⋮	⋮
30-01-2021	83	A4	A3,A3
31-01-2021	82	A3	A3,A4

Table 6 presents the fuzzification results of daily average air humidity values $x(t)$. Each value is mapped to its corresponding fuzzy set $(A_1 - A_5)$ based on predefined intervals. The Affecting column indicates the fuzzy sets that influence the current state in the formation of fuzzy logical relationships, which are essential for developing the high-order fuzzy AR model.

2. Defuzzification Calculation Process

Table 7. Forecast Results and Error

Date	$x(t)$	Fuzzy Forecast $(F(t))$	Defuzzified Forecast $\hat{x}(t)$	Error
01-12-2020	88	-	-	-
02-12-2020	81	-	-	-
03-12-2020	81	A3,A4,A5,A1,A2,A3,A4,A5	82.5	-1.5
04-12-2020	81	A1,A2,A3,A4,A5,A1,A2,A3,A4,A5	81	0
05-12-2020	85	A1,A2,A3,A4,A5,A1,A2,A3,A4,A5	81	4
06-12-2020	81	A1,A2,A3,A4,A5,A2,A3,A4,A5,A6	83	-2
07-12-2020	76	A2,A3,A4,A5,A6,A1,A2,A3,A4,A5	83	-7
⋮	⋮	⋮	⋮	⋮
30-01-2021	83	A1,A2,A3,A4,A5,A1,A2,A3,A4,A5	81	2
31-01-2021	82	A1,A2,A3,A4,A5,A2,A3,A4,A5,A6	83	-1

Based on the results obtained from determining the universal set U with an interval length of 5, as shown in Table 5, fuzzification in Table 6, and the predicted values and errors in Table 7, the AR High Order Fuzzy model yielded an MSE value of 13.45.

3.4 Better Model Selection

Based on the calculations above, the model with the smallest residual error was selected, as shown in Table 8.

Table 8. Comparison MSE Value

	Box-Jenkins	AR High Order Fuzzy
MSE	13.23	13.45

Table 9. Average Humidity Forecasting Results

Date	Forecasted Value
February 1st 2021	82.9
February 2nd 2021	82.8
February 3rd 2021	83.1
February 4th 2021	83.1
February 5th 2021	83.3
February 6th 2021	83.4
February 7th 2021	83.4

Table 8 and Table 9 present the model comparison and the resulting forecasts. Table 8 shows that the AR(2) model from the Box-Jenkins method produced the lowest MSE (13.23), indicating it as the most accurate model. Based on this, Table 9 provides the 7-day forecast of average air humidity, offering valuable insights for meteorological planning and enhancing operational safety at I Gusti Ngurah Rai Airport.

The results indicate that the Box-Jenkins method, specifically the AR(2) model, outperforms the high-order fuzzy AR model in forecasting air humidity at I Gusti Ngurah Rai Airport, as evidenced by the lower MSE value. These findings are consistent with previous studies that have demonstrated the effectiveness of the Box-Jenkins method in time series forecasting, particularly in meteorological applications. For example, Alfitri and Purnami [2] utilized the ARIMA model to predict average air temperature and daily humidity, showing that statistical models can provide reliable forecasts for weather-related parameters. However, some studies have highlighted the potential advantages of fuzzy AR models in capturing nonlinear patterns in environmental data, suggesting that further optimization may enhance their performance in specific contexts.

4. CONCLUSION

Based on the results and time series analysis, the Box-Jenkins method demonstrates higher suitability with the AR(2) model. This conclusion is supported by its lower MSE value of 13.23, compared to the MSE of the AR high-order fuzzy method, which is 13.45. The corresponding equation for the AR(2) model is

$$Z_t = 23.5483 + 0.2942Z_{t-1} + 0.4247Z_{t-2} + a_t$$

The model has shown reliable forecasting performance in capturing humidity dynamics over the observation period. These findings can serve as a basis for policy-making and decision-making at meteorological and climatological stations. Considering the significant influence of air humidity on runway conditions, the results of air humidity forecasting provide valuable insights to support airport operations and safety measures.

Author Contributions

Alfien Diva Kayyisa: Conceptualization, Methodology, Data Curation, Resources, Formal Analysis, Draft Preparation, Writing-Original Draft, Software, Validation. Winita Sulandari: Conceptualization, Supervision, Writing-Review and Editing, Funding Acquisition. Isnandar Slamet: Data Curation, Visualization, Validation. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare no competing interest.

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