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ENERGY OF NON-COPRIME GRAPH ON MODULO GROUP

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ABSTRACT

A graph is a mathematical structure consisting of a non-empty set of vertices and a set of edges connecting these vertices. In recent years, extensive research on graphs has been conducted, with one of the intriguing topics being the representation of graphs within algebraic structures, particularly groups. This approach bridges two areas of mathematics: graph theory and algebra. This study focuses on graph representation, specifically non-coprime graphs in the group of integers modulo $\mathbb{Z}_n$, *where* $n = p^k$, p *is a prime number, and* k *is a non-negative integer. The non-coprime graph of a group* G *is defined as a graph with the vertex set* $G/\{e\}$, *where* e *is the identity element of* G. *Two distinct vertices* r *and* s *are connected by an edge if* $gcd(|r|,|s|) \neq 1$. *Specifically, this research investigates the Sombor energy, the Degree Sum energy, the Degree Exponent Sum energy, the Laplacian energy, the Distance Laplacian energy, and the Distance Signless Laplacian energy of a non-coprime graph on a modulo group.*

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1. INTRODUCTION

A graph is a mathematical structure consisting of a non-empty set of vertices and edges connecting those vertices. A graph is considered complete if every vertex is adjacent to every other vertex via an edge [1]. Research on graphs has been extensively conducted in recent years, with mathematicians studying various representations of graphs, such as commuting and non-commuting graphs, cycle graphs, identity graphs, and zero divisor graphs. One particularly interesting topic is the representation of graphs in algebraic structures, specifically groups, as it combines two fields of mathematics: graph theory and algebra.

The research by [2] introduced the concept of the non-coprime graph on finite groups. The non-coprime graph represents the relationship between the elements of a group in algebra. For a given group G, the non-coprime graph of G is defined as the set of vertices $G\backslash\{e\}$, where two distinct vertices x and y are adjacent if and only if $\gcd(|x|,|y|) \neq 1$. In addition, [3]studied the representation of non-coprime graphs, focusing on the structure and properties of the non-coprime graph of the quaternion group. In a related study, [4] examined the neighbor energy and total degree energy of the non-coprime graph associated with the dihedral group. One of the algebraic structures examined in this study is the modulo group, which is a finite group with the operation of addition modulo n, denoted as $\mathbb{Z}_n$ [5]. In addition, [6] and [7] also investigated the graph representation of the group of integers modulo n and its subgroups of order n,where n is a prime power.

Graph theory finds diverse applications across both scientific and real-life contexts. For instance, [8] explores its use in determining the shortest paths between destinations. In contrast, [9] applies graph models to optimize data transfer processes, and [10] demonstrates the integration of graphs within an information system framework. Notably, in chemistry, a novelty and compelling application of graph theory is the concept of graph energy, which closely parallels the energy levels of π-electrons in conjugated carbon molecules, as discovered through the Hückel theory [11]. This correspondence reveals a unique intersection between mathematics and chemistry, where mathematical structure, namely, graphs, offers insights into chemical behavior. The novelty lies in this interdisciplinary bridge, where the abstract notion of graph energy serves as a predictive tool for understanding molecular properties, thereby expanding the role of graph theory beyond its traditional domains.

In this study, we aim to calculate various graph energies, including the Sombor energy, the Degree Sum energy, the Degree Exponent Sum energy, the Laplacian energy, the distance Laplacian energy, and the distance signless Laplacian energy, derived from the representation of non-coprime graphs on modulo groups.

2. RESEARCH METHODS

This research is a quantitative study using a literature review of previous studies. The research begins with a literature review, followed by deriving the general formula for the Sombor energy, the Degree Sum energy, the Degree Exponent Sum energy, the Laplacian energy, the distance Laplacian energy, and the distance signless Laplacian energy of non-coprime graph on modulo group, generalized for several cases of n. Subsequently, a conjecture is formulated, and the conjecture is proven. If the conjecture is validated, it is then established as a theorem.

3. RESULTS AND DISCUSSION

In this section, we will determine the Sombor energy, the Degree Sum energy, the Degree Exponent Sum energy, the distance Laplacian energy, and the Laplacian energy of non-coprime graph on modulo group with order p^k, p prime numbers, $k \in \mathbb{Z}^+$.

3.1 Preliminary

The binary operation of addition modulo n plays a crucial role in abstract algebra and is defined as follows:

Definition 1. [5] *The group of integers modulo n is a finite set $\{0,1,2,\dots,n-1\}$ equipped with the modulo addition operation. The group is denoted by $\mathbb{Z}_n$.*

The order of an element of groups is defined as follows.

Definition 2. [6] *If G is a group with identity element e and $x \in G$, the order of x is the power of a natural number such that $x^k = e$ is denoted by $|x| = k$.*

A non-coprime graph of the group of integers modulo n is a graph representation where the set of vertices includes all elements of the group except the identity element, denoted as $\mathbb{Z}_n \backslash \{e\}$. Two vertices x and y are adjacent if $\gcd(|x|, |y|) \neq 1$ [2]. Fundamentals in graph theory and connectivity are described by the concepts of vertex degree, paths, and distances.

The vertex degree of the graph is defined as follows.

Definition 3. [12] *(Vertex Degree). Let Γ be a graph, where $V(\Gamma)$ represents the set of vertices. The degree of $v_i \in V(\Gamma)$ is defined as the number of edges connected to v_i is denoted by d_i.*

A complete graph Γ is a simple graph in which every vertex is adjacent to all other vertices. In a complete graph with n vertices, each vertex has degree $n - 1$ [13]

The path of the graph is defined as follows.

Definition 4. [13] *A path is a route in which the vertices and edges traversed must not repeat.*

The distance of the graph will be defined below

Definition 5. [15] *The distance in a graph between two vertices v_i and v_j is defined as the length of the shortest path connecting them, denoted by $d\{v_i, v_j\}$.*

The energy of the graph will be defined below

Definition 6. [11] *If Γ is a graph and Φ is an eigenvalue of the graph matrix of Γ, which is an algebraic representation that encodes information about the relationships between vertices, then the energy of Γ is defined as*

$$E(\Gamma) = \sum_{i=1}^{n} |\Phi_i|. \tag{1}$$

Specifically, for the Laplacian energy and distance Laplacian energy of a graph, based on the upper and lower bounds of the energy of a graph, according to research by [16] [17] on the bounds of graph energy, the Laplacian energy of a graph is defined as follows:

Definition 7. [18] *If Γ is a graph with vertex set $V(\Gamma)$ with $|V(\Gamma)| = n$, edge set $E(\Gamma)$ with $|E(\Gamma)| = m$ and Φ is an eigenvalue of the graph matrix of Γ then the Laplacian energy of Γ denoted by $LE(\Gamma)$ is defined as*

$$LE(\Gamma) = \sum_{i=1}^{n} \left|\Phi_i - \frac{2m}{n}\right|. \tag{2}$$

The distance Laplacian energy of a graph is defined as follows:

Definition 8. [19] [20] *If Γ is a graph with $V(\Gamma) = \{v_1, v_2, \dots, v_n\}$ where $|V(\Gamma)| = n$, the distance degree of the vertex v_i is denoted by D_i, is given by $D_i = \sum_{j=1}^{n} d\{v_i, v_j\}$ and Φ is an eigenvalue of the graph matrix of Γ then the distance Laplacian energy and distance signless Laplacian energy of Γ denoted by $DLE(\Gamma)$ is defined as*

$$DL(\Gamma) = DSL(\Gamma) = \sum_{i=1}^{n} \left|\Phi_i - \frac{1}{n}\sum_{j=1}^{n} D_j\right|. \tag{3}$$

The concept of the Sombor energy, the Degree Sum energy, the Degree Exponent Sum energy, the Laplacian energy, the distance Laplacian energy, and the distance signless Laplacian energy of graphs is often associated with matrix representations. The determinant of a unique matrix can be calculated using methods derived from the following.

Lemma 1. [21] *If we have a matrix of order* p, *and* α, β *are scalars then*

$$\begin{vmatrix} \alpha & \beta & \cdots & \beta \\ \beta & \alpha & \cdots & \beta \\ \vdots & \vdots & \ddots & \vdots \\ \beta & \beta & \cdots & \alpha \end{vmatrix} = (\alpha-\beta)^{p-1}[\alpha-(p-1)\beta] \tag{4}$$

3.2 Sombor Energy

The Sombor energy is a recent development in spectral graph theory, incorporating vertex degrees into the graph structure. This matrix-based formulation serves as the foundation for defining Sombor energy, a novel graph invariant with promising applications in structural analysis.

Definition 9. [22] Let Γ be a graph, with $V(\Gamma)=\{v_1,v_2,\dots,v_n\}$ is the vertex set of the graph Γ, then its Sombor matrix of Γ is $SM(\Gamma)=[s_{ij}]$, where

$$s_{ij}=\begin{cases}\sqrt{d_i^2+d_j^2} & \mathit{if}\ \{v_i,v_j\}\ \mathit{adjacency\ in}\ \Gamma, \\ 0 & \mathit{else}.\end{cases} \tag{5}$$

Theorem 1. *Let* $\mathbb{Z}_{p^k}$ *be a modulo group, where* p *is a prime number,* $k \in \mathbb{Z}^+$, *then Sombor energy of* $\Gamma_{\mathbb{Z}_{p^k}}$ *is*

$$E_{SM}\left(\Gamma_{\mathbb{Z}_{p^k}}\right)=2\left(\left(p^k-2\right)^2\sqrt{2}\right) \tag{6}$$

Proof. Let $\mathbb{Z}_{p^k}=\{0,1,2,\dots,p^k-1\}$, where p is a prime number and $k\in\mathbb{Z}^+$. The order of any elements $x_i \in \mathbb{Z}_{p^k}$ is of the form p^m, where $m\in\mathbb{N}$. As a result, for all $x_i,x_j\in\mathbb{Z}_{p^k}$ with $i\neq j$, then $\gcd(|x_i|,|x_j|)\neq 1$. Based on the definition of a non-coprime graph, for all $x_i,x_j\in\mathbb{Z}_{p^k}$ are adjacent in Γ, forming a complete graph. Since the number of elements in $\mathbb{Z}_{p^k}$ is p^k and the non-coprime graph has $\mathbb{Z}_{p^k}\backslash\{e\}$ vertices, the number of vertices in Γ is p^k-1 with the degree of each vertex being p^k-2. Thus, the Sombor matrix of Γ will have an order of $(p^k-1)\times(p^k-1)$. Based on **Definition 9**, the Sombor matrix of $\Gamma_{\mathbb{Z}_{p^k}}$ is

$$SM\left(\Gamma_{\mathbb{Z}_{p^k}}\right)=\begin{pmatrix} 0 & (p^k-2)\sqrt{2} & \cdots & (p^k-2)\sqrt{2} \\ (p^k-2)\sqrt{2} & 0 & \cdots & (p^k-2)\sqrt{2} \\ \vdots & \vdots & \ddots & \vdots \\ (p^k-2)\sqrt{2} & (p^k-2)\sqrt{2} & \cdots & 0 \end{pmatrix} \tag{7}$$

The eigenvalues of the Sombor matrix are determined by solving the corresponding characteristic equation:

$$\left|SM\left(\Gamma_{\mathbb{Z}_{p^k}}\right)-\Phi I\right|=\left|\Phi I-SM\left(\Gamma_{\mathbb{Z}_{p^k}}\right)\right|=0 \tag{8}$$

$$\begin{vmatrix} \Phi & -\left((p^k-2)\sqrt{2}\right) & \cdots & -\left((p^k-2)\sqrt{2}\right) \\ -\left((p^k-2)\sqrt{2}\right) & \Phi & \cdots & -\left((p^k-2)\sqrt{2}\right) \\ \vdots & \vdots & \ddots & \vdots \\ -\left((p^k-2)\sqrt{2}\right) & -\left((p^k-2)\sqrt{2}\right) & \cdots & \Phi \end{vmatrix}=0 \tag{9}$$

Based on **Lemma 1**,

$$\left(\Phi-\left[-\left((p^k-2)\sqrt{2}\right)\right]\right)^{(p^k-1)-1}\left(\Phi-[(p^k-1)-1]\left[-\left((p^k-2)\sqrt{2}\right)\right]\right)=0 \tag{10}$$

Thus, we have

$$\left[\Phi+(p^k-2)\sqrt{2}\right]^{(p^k-2)}\left(\Phi+\left(p^k-2\right)^2\sqrt{2}\right)=0$$

$$\left[\Phi+(p^k-2)\sqrt{2}\right]^{(p^k-2)}=0 \text{ or } \left(\Phi+\left(p^k-2\right)^2\sqrt{2}\right)=0$$

As a result, we obtain $\Phi = -(p^k - 2)\sqrt{2}$ with multiplicity $p^k - 2$ and $\Phi = \left(p^k - 2\right)^2\sqrt{2}$ with multiplicity 1. Using **Definition 6**, the Sombor energy of the graph can be calculated as follows

$$\begin{aligned} E_{SM}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) &= \sum_{i=1}^{n} |\Phi_i| \\ &= \left(p^k - 2\right)\left|-(p^k - 2)\sqrt{2}\right| + \left|\left(p^k - 2\right)^2\sqrt{2}\right| \\ &= \left(p^k - 2\right)^2\sqrt{2} + \left(p^k - 2\right)^2\sqrt{2} \\ &= 2\left(\left(p^k - 2\right)^2\sqrt{2}\right) \blacksquare \end{aligned}$$

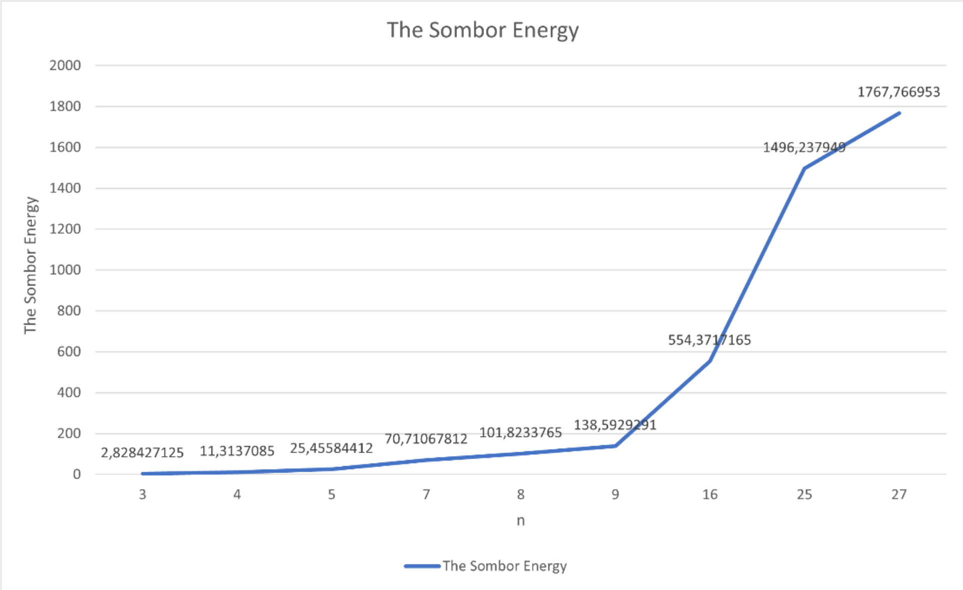

Figure 1. The Example Values of The Sombor Energy

After obtaining the Sombor energy theorem for the non-coprime graph on a modulo group of order p^k, where p is a prime number, and $k \in \mathbb{Z}^+$, an example case is presented for a simple value of n.

Example 1. Let Γ be a non-coprime graph of the group $\mathbb{Z}_n$ with $n = 3$. Calculate the Sombor energy of $\Gamma_{\mathbb{Z}_n}$

For $n = 3$, the Sombor energy $\Gamma_{\mathbb{Z}_3}$ is

$$\begin{aligned} E_{SM}\left(\Gamma_{\mathbb{Z}_3}\right) &= 2(3-2)^2\sqrt{2} \\ &= 2\sqrt{2} \\ E_{SM}\left(\Gamma_{\mathbb{Z}_3}\right) &\approx 2.828427125. \end{aligned}$$

3.3 Degree Sum Energy

The Degree Sum energy is a recent development in spectral graph theory, integrating vertex degrees directly into its structure. The associated matrix forms the basis for defining the Degree Sum energy, a novel graph invariant with promising applications in structural analysis.

Definition 10. [23] Let Γ be a graph, with $V(\Gamma) = \{v_1, v_2, \dots, v_n\}$ is the vertex set of the graph Γ, then its Degree Sum matrix of Γ is $DS(\Gamma) = [ds_{ij}]$, where

$$ds_{ij} = \begin{cases} d_i + d_j & \text{if } \{v_i, v_j\} \text{ adjacency in } \Gamma, \\ 0 & \text{else}. \end{cases} \quad (11)$$

Theorem 2. *Let $\mathbb{Z}_{p^k}$ be a modulo group, where p is a prime number, $k \in \mathbb{Z}^+$, then Degree Sum energy of $\Gamma_{\mathbb{Z}_{p^k}}$ is*

$$E_{DS}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = 4\left(\left(p^k - 2\right)^2\right) \quad (12)$$

Proof. Let $\mathbb{Z}_{p^k} = \{0,1,2, \dots, p^k - 1\}$, where p is a prime number and $k \in \mathbb{Z}^+$. The order of any elements $x_i \in \mathbb{Z}_{p^k}$ is of the form p^m, where $m \in \mathbb{N}$. As a result for all $x_i, x_j \in \mathbb{Z}_{p^k}$ with $i \neq j$, then $\gcd(|x_i|, |x_j|) \neq 1$. Based on the definition of a non-coprime graph, for all $x_i, x_j \in \mathbb{Z}_{p^k}$ are adjacent in Γ, forming a complete graph. Since the number of elements in $\mathbb{Z}_{p^k}$ is p^k and the non-coprime graph has $\mathbb{Z}_{p^k} \backslash \{e\}$ vertices, the number of vertices in Γ is $p^k - 1$ with the degree of each vertex being $p^k - 2$. Thus, the Degree Sum matrix of Γ will have an order of $(p^k - 1) \times (p^k - 1)$. Based on **Definition 10**, the Degree Sum matrix of $\Gamma_{\mathbb{Z}_{p^k}}$ is

$$DS\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = \begin{pmatrix} 0 & 2(p^k - 2) & \cdots & 2(p^k - 2) \\ 2(p^k - 2) & 0 & \cdots & 2(p^k - 2) \\ \vdots & \vdots & \ddots & \vdots \\ 2(p^k - 2) & 2(p^k - 2) & \cdots & 0 \end{pmatrix} \tag{13}$$

The eigenvalues of the Degree Sum matrix are determined by solving the corresponding characteristic equation:

$$\left| DS\left(\Gamma_{\mathbb{Z}_{p^k}}\right) - \Phi I \right| = \left| \Phi I - DS\left(\Gamma_{\mathbb{Z}_{p^k}}\right) \right| = 0 \tag{14}$$

$$\begin{vmatrix} \Phi & -\left(2(p^k - 2)\right) & \cdots & -\left(2(p^k - 2)\right) \\ -\left(2(p^k - 2)\right) & \Phi & \cdots & -\left(2(p^k - 2)\right) \\ \vdots & \vdots & \ddots & \vdots \\ -\left(2(p^k - 2)\right) & -\left(2(p^k - 2)\right) & \cdots & \Phi \end{vmatrix} = 0 \tag{15}$$

Based on **Lemma 1**,

$$\left(\Phi - \left[-\left(2(p^k - 2)\right)\right]\right)^{(p^k - 1) - 1} \left(\Phi - [(p^k - 1) - 1]\left[-\left(2(p^k - 2)\right)\right]\right) = 0 \tag{16}$$

Thus. we have

$$[\Phi + 2(p^k - 2)]^{(p^k - 2)} \left(\Phi + 2\left(p^k - 2\right)^2\right) = 0$$

$$[\Phi + 2(p^k - 2)]^{(p^k - 2)} = 0 \text{ or } \left(\Phi + 2\left(p^k - 2\right)^2\right) = 0$$

As a result, we obtain $\Phi = -2(p^k - 2)$ with multiplicity $p^k - 2$ and $\Phi = 2\left(p^k - 2\right)^2$ with multiplicity 1. Using **Definition 6**, the Degree Sum energy of the graph can be calculated as follows

$$\begin{aligned} E_{DS}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) &= \sum_{i=1}^{n} |\Phi_i| \\ &= \left(p^k - 2\right)\left|-2(p^k - 2)\right| + \left|2\left(p^k - 2\right)^2\right| \\ &= 2\left(p^k - 2\right)^2 + 2\left(p^k - 2\right)^2 \\ &= 4\left(\left(p^k - 2\right)^2\right) \end{aligned}$$

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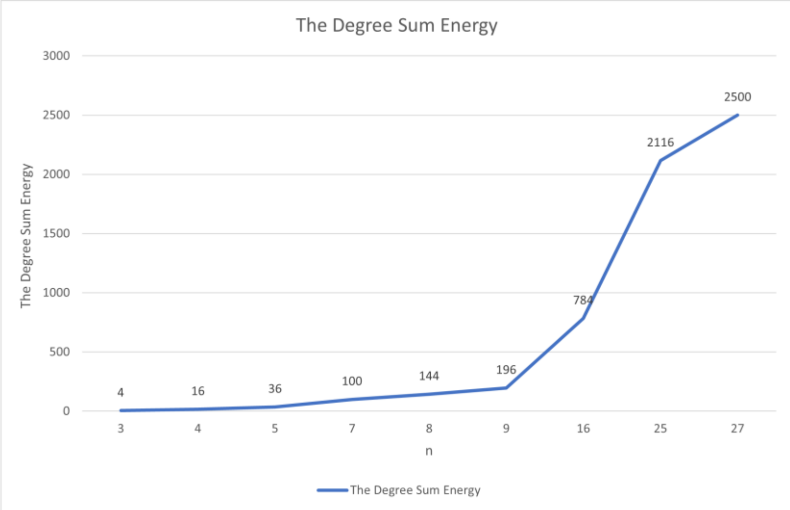

Figure 2. The Example Values of The Degree Sum Energy

After obtaining the Degree Sum energy theorem for the non-coprime graph on a modulo group of order p^k,where p is a prime number, and $k \in \mathbb{Z}^+$, an example case is presented for a simple value of n.

Example 2. Let Γ be a non-coprime graph of the group $\mathbb{Z}_n$ with $n = 4$. Calculate the Degree Sum energy of $\Gamma_{\mathbb{Z}_n}$. For $n = 4$, the Degree Sum energy $\Gamma_{\mathbb{Z}_4}$ is

$$\begin{aligned} E_{DS}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) &= 4(4-2)^2 \\ &= 16. \end{aligned}$$

3.4 Degree Exponent Sum Energy

The Degree Exponent Sum energy represents a recent extension of spectral graph theory, incorporating vertex degrees raised to a power into its structural analysis. The resulting matrix also contributes to the definition of the Degree Exponent Sum energy, a distinctive graph invariant with valuable applications in structural and mathematical modeling.

Definition 11. [24] *Let Γ be a graph, with $V(\Gamma) = \{v_1, v_2, \dots, v_n\}$ is the vertex set of the graph Γ, then its Degree Exponent Sum matrix of Γ is $DES(\Gamma) = [des_{ij}]$, where*

$$des_{ij} = \begin{cases} d_i^{d_j} + d_i^{d_j} & if\ i \neq j, \\ 0 & else. \end{cases} \tag{17}$$

Theorem 3. *Let $\mathbb{Z}_{p^k}$ be a modulo group, where p is a prime number, $k \in \mathbb{Z}^+$, then Degree Exponent Sum energy of $\Gamma_{\mathbb{Z}_{p^k}}$ is*

$$\mathrm{E_{DES}}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = 4\left((\mathrm{p}^k - 2)^{(\mathrm{p}^k - 1)}\right) \tag{18}$$

Proof. Let $\mathbb{Z}_{p^k} = \{0,1,2,\dots,p^k - 1\}$, where p is a prime number and $k \in \mathbb{Z}^+$. The order of any elements $x_i \in \mathbb{Z}_{p^k}$ is of the form p^m,where $m \in \mathbb{N}$. As a result, for all $x_i, x_j \in \mathbb{Z}_{p^k}$ with $i \neq j$, then $\gcd(|x_i|, |x_j|) \neq 1$. Based on the definition of non-coprime graph, for all $x_i, x_j \in \mathbb{Z}_{p^k}$ are adjacent in Γ, forming a complete graph. Since the number of elements in $\mathbb{Z}_{p^k}$ is p^k and the non-coprime graph has $\mathbb{Z}_{p^k}\backslash\{e\}$ vertices, the number of vertices in Γ is $p^k - 1$ with the degree of each vertex being $p^k - 2$. Thus, the Degree Exponent Sum matrix of Γ will have an order of $(p^k - 1) \times (p^k - 1)$. Based on **Definition 11**, the Degree Exponent Sum matrix of $\Gamma_{\mathbb{Z}_{p^k}}$ is

$$DES\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = \begin{pmatrix} 0 & 2\left((p^k-2)^{(p^k-2)}\right) & \cdots & 2\left((p^k-2)^{(p^k-2)}\right) \\ 2\left((p^k-2)^{(p^k-2)}\right) & 0 & \cdots & 2\left((p^k-2)^{(p^k-2)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ 2\left((p^k-2)^{(p^k-2)}\right) & 2\left((p^k-2)^{(p^k-2)}\right) & \cdots & 0 \end{pmatrix} \tag{19}$$

The eigenvalues of the Degree Exponent Sum matrix are determined by solving the corresponding characteristic equation:

$$\left| DES\left(\Gamma_{\mathbb{Z}_{p^k}}\right) - \Phi I\right| = \left|\Phi I - DES\left(\Gamma_{\mathbb{Z}_{p^k}}\right)\right| = 0 \tag{20}$$

$$\begin{vmatrix} \Phi & -\left(2\left((p^k-2)^{(p^k-2)}\right)\right) & \cdots & -\left(2\left((p^k-2)^{(p^k-2)}\right)\right) \\ -\left(2\left((p^k-2)^{(p^k-2)}\right)\right) & \Phi & \cdots & -\left(2\left((p^k-2)^{(p^k-2)}\right)\right) \\ \vdots & \vdots & \ddots & \vdots \\ -\left(2\left((p^k-2)^{(p^k-2)}\right)\right) & -\left(2\left((p^k-2)^{(p^k-2)}\right)\right) & \cdots & \Phi \end{vmatrix} = 0 \tag{21}$$

Based on **Lemma 1**,

$$\left(\Phi - \left[-\left(2\left((p^k-2)^{(p^k-2)}\right)\right)\right]\right)^{(p^k-1)-1}\left(\Phi - [(p^k-1)-1]\left[-\left(2\left((p^k-2)^{(p^k-2)}\right)\right)\right]\right) = 0 \tag{22}$$

Thus, we have

$$\left[\Phi + 2\left((p^k-2)^{(p^k-2)}\right)\right]^{(p^k-2)}\left(\Phi + 2\left((p^k-2)^{(p^k-1)}\right)\right) = 0$$

$$\left[\Phi + 2\left((p^k-2)^{(p^k-2)}\right)\right]^{(p^k-2)} = 0 \text{ or } \left(\Phi + 2\left((p^k-2)^{(p^k-1)}\right)\right) = 0$$

As a result, we obtain $\Phi = -2\left((p^k-2)^{(p^k-2)}\right)$ with multiplicity p^k-2 and $\Phi = -2\left((p^k-2)^{(p^k-1)}\right)$ with multiplicity 1. Using **Definition 6**, the Degree Exponent Sum energy of the graph can be calculated as follows

$$\begin{aligned} E_{DES}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) &= \sum_{i=1}^{n} |\Phi_i| \\ &= (p^k-2)\left|-2\left((p^k-2)^{(p^k-2)}\right)\right| + \left|-2\left((p^k-2)^{(p^k-1)}\right)\right| \\ &= 2\left((p^k-2)^{(p^k-1)}\right) + 2\left((p^k-2)^{(p^k-1)}\right) \\ &= 4\left((p^k-2)^{(p^k-1)}\right) \blacksquare \end{aligned}$$

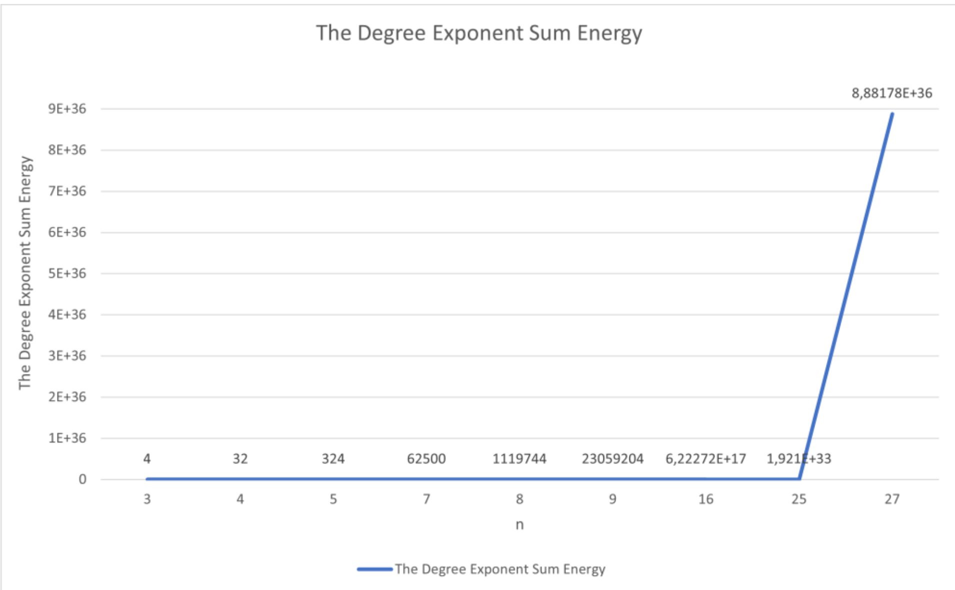

Figure 3. The Example Values of The Degree Exponent Sum Energy

After obtaining the Degree Exponent Sum energy theorem for the non-coprime graph on a modulo group of order p^k, where p is a prime number, and $k \in \mathbb{Z}^+$, an example case is presented for a simple value of n.

Example 3. Let Γ be a non-coprime graph of the group $\mathbb{Z}_n$ with $n = 5$. Calculate the Degree Exponent Sum energy of $\Gamma_{\mathbb{Z}_n}$

For $n = 5$, the Degree Exponent Sum energy $\Gamma_{\mathbb{Z}_5}$ is

$$\begin{aligned} E_{DES}(\Gamma_{\mathbb{Z}_5}) &= 4(5-2)^{(5-1)} \\ &= 4(3)^{(4)} \\ &= 324. \end{aligned}$$

3.5 Laplacian Energy

The Laplacian energy is a recent addition to spectral graph theory, as it incorporates vertex degrees into its structure. This matrix serves as the foundation for defining Laplacian energy, a novel graph invariant with potential applications in structural analysis

Definition 12. [25] *Let Γ be a graph, with $V(\Gamma) = \{v_1, v_2, \dots, v_n\}$ is the vertex set of the graph Γ, then its Laplacian matrix of Γ is $L(\Gamma) = [l_{ij}]$, where*

$$l_{ij} = \begin{cases} -1 & if\ \{v_i, v_j\}\ adjacency\ in\ \Gamma, \\ 0 & if\ \{v_i, v_j\}\ adjacency\ in\ \Gamma, \\ d_i & if\ i = j. \end{cases} \tag{23}$$

Theorem 4. *Let $\mathbb{Z}_{p^k}$ be a modulo group, where p is a prime number, $k \in \mathbb{Z}^+$, then Laplacian energy of $\Gamma_{\mathbb{Z}_{p^k}}$ is*

$$\mathrm{E_L}\left(\Gamma_{\mathbb{Z}_{\mathrm{p^k}}}\right) = 2(\mathrm{p^k} - 2) \tag{24}$$

Proof. Let $\mathbb{Z}_{p^k} = \{0,1,2, \dots, p^k - 1\}$, where p is a prime number and $k \in \mathbb{Z}^+$. The order of any elements $x_i \in \mathbb{Z}_{p^k}$ is of the form p^m, where $m \in \mathbb{N}$. As a result, for all $x_i, x_j \in \mathbb{Z}_{p^k}$ with $i \neq j$, then $\gcd(|x_i|, |x_j|) \neq 1$. Based on the definition of non-coprime graph, for all $x_i, x_j \in \mathbb{Z}_{p^k}$ are adjacent in Γ, forming a complete graph. Since the number of elements in $\mathbb{Z}_{p^k}$ is p^k and the non-coprime graph has $\mathbb{Z}_{p^k} \backslash \{e\}$ vertices, the number of vertices in Γ is $p^k - 1$ with the degree of each vertex being $p^k - 2$. Thus, the Laplacian matrix of Γ will have an order of $(p^k - 1) \times (p^k - 1)$. Based on **Definition 12**, the Laplacian matrix of $\Gamma_{\mathbb{Z}_{p^k}}$ is

$$L\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = \begin{pmatrix} p^k - 2 & -1 & \cdots & -1 \\ -1 & p^k - 2 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & p^k - 2 \end{pmatrix} \tag{25}$$

The eigenvalues of the Laplacian matrix are determined by solving the corresponding characteristic equation:

$$\left| L\left(\Gamma_{\mathbb{Z}_{p^k}}\right) - \Phi I \right| = \left| \Phi I - L\left(\Gamma_{\mathbb{Z}_{p^k}}\right) \right| = 0 \tag{26}$$

$$\begin{vmatrix} \Phi - (p^k - 2) & 1 & \cdots & 1 \\ 1 & \Phi - (p^k - 2) & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & \Phi - (p^k - 2) \end{vmatrix} = 0 \tag{27}$$

Based on **Lemma 1**,

$$\left(\Phi - (p^k - 2) - 1\right)^{(p^k-1)-1} \left(\Phi - (p^k - 2) - [(p^k - 1) - 1]1\right) = 0 \tag{28}$$

Thus, we have

$$[\Phi - (p^k - 1)]^{(p^k-2)} \left(\Phi - 2(p^k - 2)\right) = 0$$

$$[\Phi - (p^k - 1)]^{(p^k-2)} = 0 \text{ or } \left(\Phi - 2(p^k - 2)\right) = 0$$

As a result, we obtain $\Phi = p^k - 1$ with multiplicity $p^k - 2$ and $\Phi = 2(p^k - 2)$ with multiplicity 1. Using **Definition 7**, with $|V(\Gamma)| = p^k - 1$ and $|E(\Gamma)| = \frac{(p^k-2)(p^k-1)}{2}$ the Laplacian energy of the graph can be calculated as follows

$$E_L\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = \sum_{i=1}^{n} \left| \Phi_i - \frac{2\left(\frac{(p^k-2)(p^k-1)}{2}\right)}{(p^k - 1)} \right|$$

$$\begin{aligned} &= (p^k - 2)|(p^k - 1) - (p^k - 2)| + |2(p^k - 2) - (p^k - 2)| \\ &= (p^k - 2) + (p^k - 2) \\ &= 2(p^k - 2) \blacksquare \end{aligned}$$

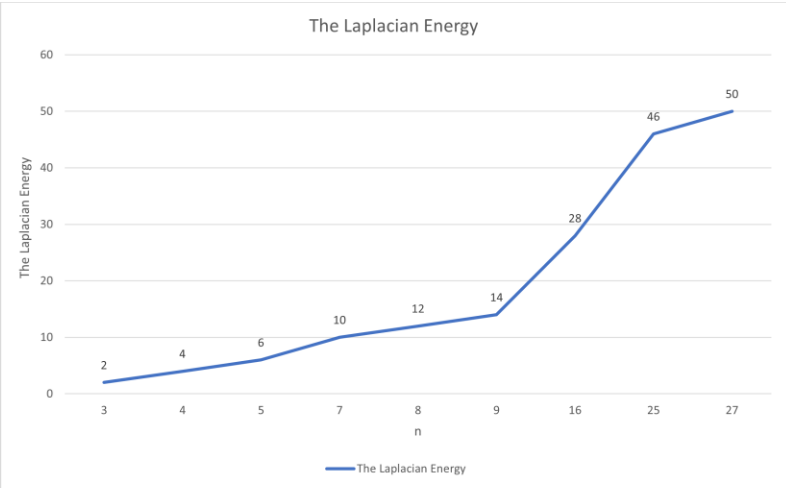

Figure 4. The Example Values of The Laplacian Energy

After obtaining the Laplacian energy theorem for the non-coprime graph on a modulo group of order p^k, where p is a prime number, and $k \in \mathbb{Z}^+$, an example case is presented for a simple value of n.

Example 4. Let Γ be a non-coprime graph of the group $\mathbb{Z}_n$ with $n = 7$. Calculate the Laplacian energy of $\Gamma_{\mathbb{Z}_n}$

For $n = 7$, the Laplacian energy $\Gamma_{\mathbb{Z}_7}$ is

$$\begin{aligned} E_L(\Gamma_{\mathbb{Z}_7}) &= 2(7 - 2) \\ &= 10. \end{aligned}$$

3.6 Distance Laplacian Energy

The Distance Laplacian energy is a recent addition to spectral graph theory, as it incorporates vertex degrees into its structure. This matrix serves as the foundation for defining Distance Laplacian energy, a novel graph invariant with potential applications in structural analysis.

Definition 13. [19] *Let Γ be a graph, with $V(\Gamma) = \{v_1, v_2, \ldots, v_n\}$ is the vertex set of the graph Γ, then its distance Laplacian matrix of Γ is $DL(\Gamma) = T_r(\Gamma) - D(\Gamma)$.*

Where $T_r(\Gamma)$ denotes the transmission matrix of Γ and $D(\Gamma)$ denotes the distance matrix of Γ, with definitions given as follows.

Definition 14. [26] *Let Γ be a graph, with $V(\Gamma) = \{v_1, v_2, \dots, v_n\}$ is the vertex set of the graph Γ, then its transmission matrix of Γ is $T_r(\Gamma)$ has elements in the i-th row and i-th column that represent the sum of distances from v_i to all other vertices.*

Definition 15. [27] *Let Γ be a graph, with $V(\Gamma) = \{v_1, v_2, \dots, v_n\}$ is the vertex set of the graph Γ, then its distance matrix of Γ is $D(\Gamma)$, where the element in the (i,j)-th position is the distance from v_i to v_j in Γ denoted $d\{v_i, v_j\}$.*

Theorem 5. *Let $\mathbb{Z}_{p^k}$ be a modulo group, where p is a prime number, $k \in \mathbb{Z}^+$, then Distance Laplacian energy of $\Gamma_{\mathbb{Z}_{p^k}}$ is*

$$E_{DL}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = 2(p^k - 2) \tag{29}$$

Proof. Let $\mathbb{Z}_{p^k} = \{0,1,2,\dots,p^k - 1\}$, where p is a prime number and $k \in \mathbb{Z}^+$. The order of any elements $x_i \in \mathbb{Z}_{p^k}$ is of the form p^m, where $m \in \mathbb{N}$. As a result, for all $x_i, x_j \in \mathbb{Z}_{p^k}$ with $i \neq j$, then $\gcd(|x_i|, |x_j|) \neq 1$. Based on the definition of non-coprime graph, for all $x_i, x_j \in \mathbb{Z}_{p^k}$ are adjacent in Γ, forming a complete graph. Since the number of elements in $\mathbb{Z}_{p^k}$ is p^k and the non-coprime graph has $\mathbb{Z}_{p^k}\backslash\{e\}$ vertices, the number of vertices in Γ is $p^k - 1$ with the degree of each vertex being $p^k - 2$. Thus, the distance Laplacian matrix of Γ will have an order of $(p^k - 1) \times (p^k - 1)$. Based on **Definition 12**, **Definition 13**, **Definition 14**, the distance Laplacian matrix of $\Gamma_{\mathbb{Z}_{p^k}}$ is

$$DL\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = \begin{pmatrix} p^k - 2 & -1 & \cdots & -1 \\ -1 & p^k - 2 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & p^k - 2 \end{pmatrix} \tag{30}$$

The eigenvalues of the distance Laplacian matrix are determined by solving the corresponding characteristic equation:

$$\left| DL\left(\Gamma_{\mathbb{Z}_{p^k}}\right) - \Phi I \right| = \left| \Phi I - DL\left(\Gamma_{\mathbb{Z}_{p^k}}\right) \right| = 0 \tag{31}$$

$$\begin{vmatrix} \Phi - (p^k - 2) & 1 & \cdots & 1 \\ 1 & \Phi - (p^k - 2) & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & \Phi - (p^k - 2) \end{vmatrix} = 0 \tag{32}$$

Based on **Lemma 1**,

$$(\Phi - (p^k - 2) - 1)^{(p^k - 1) - 1}\left(\Phi - (p^k - 2) - [(p^k - 1) - 1]1\right) = 0 \tag{33}$$

Thus, we have

$$[\Phi - (p^k - 1)]^{(p^k - 2)}\left(\Phi - 2(p^k - 2)\right) = 0$$

$$[\Phi - (p^k - 1)]^{(p^k - 2)} = 0 \text{ or } \left(\Phi - 2(p^k - 2)\right) = 0$$

As a result, we obtain $\Phi = p^k - 1$ with multiplicity $p^k - 2$ and $\Phi = 2(p^k - 2)$ with multiplicity 1. Using **Definition 8**, with $\sum_{j=1}^{n} D_j = (p^k - 1)(p^k - 2)$ the distance Laplacian energy of the graph can be calculated as follows

$$
\begin{aligned}
E_{DL}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) &= \sum_{i=1}^{n}\left|\Phi_i - \frac{1}{(p^k-1)}\sum_{j=1}^{n} D_j\right| \\
&= (p^k - 2)\left|(p^k-1) - \left(\frac{(p^k-1)(p^k-2)}{(p^k-1)}\right)\right| + \left|2(p^k-2) - \left(\frac{(p^k-1)(p^k-2)}{(p^k-1)}\right)\right| \\
&= (p^k - 2) + (p^k - 2) \\
&= 2(p^k - 2)\blacksquare
\end{aligned}
$$

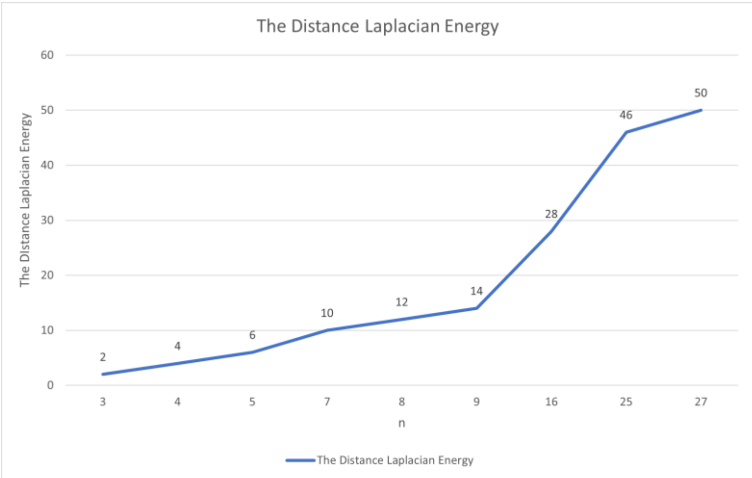

Figure 5. The Example Values of The Distance Laplacian Energy

After obtaining the distance Laplacian energy theorem for the non-coprime graph on a modulo group with of p^k, where p is a prime number, and $k \in \mathbb{Z}^+$, an example case is presented for a simple value of n.

Example 5. Let Γ be a non-coprime graph of the group $\mathbb{Z}_n$ with $n = 8$. Calculate the distance Laplacian energy of $\Gamma_{\mathbb{Z}_n}$

For $n = 8$, the Distance Laplacian energy $\Gamma_{\mathbb{Z}_8}$ is

$$
\begin{aligned}
E_L\left(\Gamma_{\mathbb{Z}_7}\right) &= 2(8-2) \\
&= 12.
\end{aligned}
$$

3.7 Distance Signless Laplacian Energy

The Distance Signless Laplacian energy is a recent addition to spectral graph theory, as it incorporates vertex degrees into its structure. This matrix serves as the foundation for defining Distance Signless Laplacian energy, a novel graph invariant with potential applications in structural analysis.

Definition 16. [28] *Let Γ be a graph, with $V(\Gamma) = \{v_1, v_2, \dots, v_n\}$ is the vertex set of the graph Γ, then its distance signless Laplacian matrix of Γ is $DSL(\Gamma) = T_r(\Gamma) + D(\Gamma)$.*

Where $T_r(\Gamma)$ denotes the transmission matrix of Γ and $D(\Gamma)$ denotes the distance matrix of Γ, with definitions given as follows **Definition 14** and **Definition 15**.

Theorem 6. *Let $\mathbb{Z}_{p^k}$ be a modulo group, where p is a prime number, $k \in \mathbb{Z}^+$, then Distance Signless Laplacian Laplacian energy of $\Gamma_{\mathbb{Z}_{p^k}}$ is*

$$E_{DSL}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = 2(p^k - 2) \tag{34}$$

Proof. Let $\mathbb{Z}_{p^k} = \{0,1,2,\dots,p^k - 1\}$, where p is a prime number and $k \in \mathbb{Z}^+$. The order of any elements $x_i \in \mathbb{Z}_{p^k}$ is of the form p^m, where $m \in \mathbb{N}$. As a result, for all $x_i, x_j \in \mathbb{Z}_{p^k}$ with $i \neq j$, then $\gcd(|x_i|, |x_j|) \neq 1$. Based on the definition of non-coprime graph, for all $x_i, x_j \in \mathbb{Z}_{p^k}$ are adjacent in Γ, forming a complete graph. Since the number of elements in $\mathbb{Z}_{p^k}$ is p^k and the non-coprime graph has $\mathbb{Z}_{p^k} \backslash \{e\}$ vertices, the number of vertices in Γ is $p^k - 1$ with the degree of each vertex being $p^k - 2$. Thus, the distance signless Laplacian matrix of Γ will have an order of $(p^k - 1) \times (p^k - 1)$. Based on **Definition 16**, **Definition 14**, **Definition 15**, the distance signless Laplacian matrix of $\Gamma_{\mathbb{Z}_{p^k}}$ is

$$DSL\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = \begin{pmatrix} p^k - 2 & 1 & \cdots & 1 \\ 1 & p^k - 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & p^k - 2 \end{pmatrix} \tag{35}$$

The eigenvalues of the distance signless Laplacian matrix are determined by solving the corresponding characteristic equation:

$$\left| DSL\left(\Gamma_{\mathbb{Z}_{p^k}}\right) - \Phi I \right| = \left| \Phi I - DSL\left(\Gamma_{\mathbb{Z}_{p^k}}\right) \right| = 0 \tag{36}$$

$$\begin{vmatrix} \Phi - (p^k - 2) & -1 & \cdots & -1 \\ -1 & \Phi - (p^k - 2) & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & \Phi - (p^k - 2) \end{vmatrix} = 0 \tag{37}$$

Based on **Lemma 1**,

$$\left(\Phi - (p^k - 2) - (-1)\right)^{(p^k-1)-1} \left(\Phi - (p^k - 2) - [(p^k - 1) - 1](-1)\right) = 0 \tag{38}$$

Thus, we have

$$[\Phi - (p^k - 3)]^{(p^k-2)} (\Phi) = 0$$

$$[\Phi - (p^k - 3)]^{(p^k-2)} = 0 \text{ or } (\Phi) = 0$$

As a result, we obtain $\Phi = p^k - 3$ with multiplicity $p^k - 2$ and $\Phi = 0$ with multiplicity 1. Using **Definition 8**, with $\sum_{j=1}^{n} D_j = (p^k - 1)(p^k - 2)$ the Distance Signless Laplacian energy of the graph can be calculated as follows

$$\begin{aligned} E_{DSL}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) &= \sum_{i=1}^{n} \left| \Phi_i - \frac{1}{(p^k - 1)} \sum_{j=1}^{n} D_j \right| \\ &= (p^k - 2) \left| (p^k - 3) - \left(\frac{(p^k - 1)(p^k - 2)}{(p^k - 1)} \right) \right| + \left| - \left(\frac{(p^k - 1)(p^k - 2)}{(p^k - 1)} \right) \right| \\ &= (p^k - 2) + (p^k - 2) \\ &= 2(p^k - 2) \end{aligned}$$

■

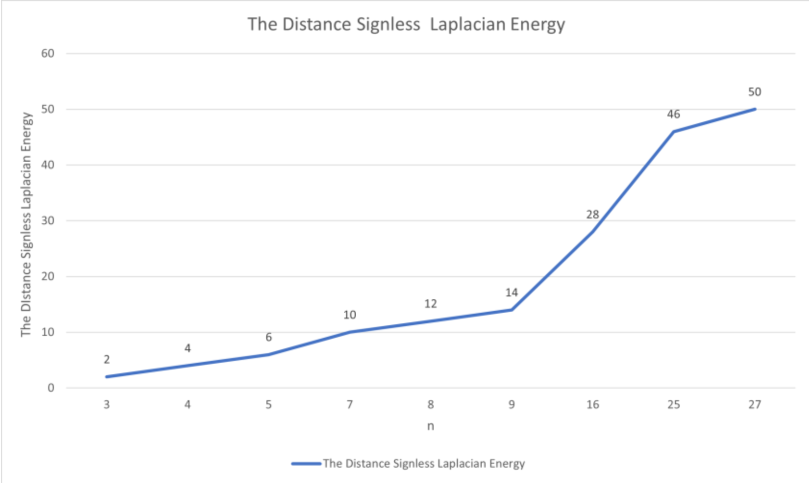

Figure 6. The Example Values of The Distance Signless Laplacian Energy

After obtaining the distance signless Laplacian energy theorem for the non-coprime graph on a modulo group with of p^k, where p is a prime number, and $k \in \mathbb{Z}^+$, an example case is presented for a simple value of n.

Example 6. Let Γ be a non-coprime graph of the group $\mathbb{Z}_n$ with $n = 9$. Calculate the Distance Signless Laplacian energy of $\Gamma_{\mathbb{Z}_n}$

For $n = 9$, the Distance Signless Laplacian energy $\Gamma_{\mathbb{Z}_9}$ is

$$\begin{aligned} E_L\left(\Gamma_{\mathbb{Z}_7}\right) &= 2(9-2) \\ &= 14. \end{aligned}$$

4. CONCLUSION

Based on the result of the discussion above, the Sombor energy, the Degree Sum energy, the Degree Exponet Sum energy, the Laplacian energy, the Distance Laplacian energy, and the Distance Signless Laplacian energy of the non-coprime graph on modulo group $\mathbb{Z}_n$, where $n = p^k$, p is a prime number, and k is a non-negative integer, are given repectively as follow:

1. Sombor Energy

$$E_{SM}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = 2\left((p^k - 2)^2\sqrt{2}\right)$$

2. Degree Sum Energy

$$E_{DS}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = 4\left((p^k - 2)^2\right)$$

3. Degree Exponent Sum Energy

$$E_{DES}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = 4\left((p^k - 2)^{(p^k-1)}\right)$$

4. Laplacian Energy

$$E_L\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = 2(p^k - 2)$$

5. Distance Laplacian Energy

$$E_{DL}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = 2(p^k - 2)$$

6. Distance Signless Laplacian Energy

$$E_{DSL}\left(\Gamma_{\mathbb{Z}_{p^k}}\right) = 2(p^k - 2)$$

There is a unique general form in which the Laplace energy and the Laplace energy share the same general form because the resulting graph is a simple complete graph.

AUTHOR CONTRIBUTIONS

Gusti Yogananda Karang: Conceptualization, Formal Analysis, Investigation, Methodology, Validation, Writing – Original Draft. I Gede Adhitya Wisnu Wardhana: Funding Acquisition, Project Administration, Resources, Supervision, Validation, Writing – Review and Editing. Manimaran Angamuthu: Formal Analysis, Investigation, Visualization, Writing – Review and Editing. All authors discussed the results and contributed to the final manuscript.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest to report.

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