BAREKENG: Journal of Mathematics and Its Applications
March 2026 Volume 20 Issue 1 Page 0413–0426
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol20iss1pp0413-0426

BAYESIAN ESTIMATION OF THE SCALE PARAMETER OF THE WEIBULL DISTRIBUTION USING THE LINEX AND ITS APPLICATION TO STROKE PATIENT DATA

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Article Info

Article History:
Received: 29th March 2025
Revised: 30th May 2025
Accepted:17th June 2025
Available online: 24th November 2025

Keywords:
Bayesian;
Linear exponential;
Stroke;
Type III censored;
Weibull distribution.

ABSTRACT

Survival analysis is used to study the timing of an event, such as recovery or death, in the context of medical data. One of the diseases that many people suffer from is stroke. Based on the survey results, the number of stroke sufferers in Indonesia reached 8.3% of 1000 people in Indonesia continues to increase every year, especially among the elderly. The research conducted aims to model the estimation of the type III censored Weibull distribution parameters with the Bayesian Linear Exponential Loss Function (LINEX) method. This study uses secondary data on stroke patients in the period January-November 2024 with a sample of 62 patients at the Haji Surabaya Regional General Hospital. Weibull distribution model with Bayesian approach using Linear Exponential Loss Function (LINEX) was applied to estimate the distribution parameters and survival function. The estimation results show that the parameter α is 6.32342 with an average hospitalization time of 5.9151646 days. MSE value is 0.000270555, which indicates that the estimation model is more accurate in predicting data for the length of hospitalization for stroke patients at the Haji Surabaya Regional General Hospital. The probability value of the survival function of stroke patients who have been hospitalized on the 5th day shows a probability of 82.4% so that no further hospitalization is needed, which indicates that the patient's health condition is improving. In addition, the hazard function analysis shows that the longer a patient is hospitalized, the greater the risk of the patient not recovering.

How to cite this article:

T. R. Rahmanita, A. Kurniawan, E. Ana, Sediono and D. Amelia., "BAYESIAN ESTIMATION OF THE SCALE PARAMETER OF THE WEIBULL DISTRIBUTION USING THE LINEX AND ITS APPLICATION TO STROKE PATIENT DATA", *BAREKENG: J. Math. & App.*, vol. 20, iss. 1, pp. 0413-0426, Mar, 2026.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · Open Access

1. INTRODUCTION

Survival analysis is a statistical method used to study the time from a defined starting point to until the occurrence of a certain event, such as death or recovery from disease [1]. This analysis is widely used in the medical field, especially in assessing the survival of patients against a disease [2]. In addition, survival analysis is also applied in various studies to improve product quality and develop modern treatment methods [3].

In the healthcare context, data on the length of time a patient stays in a hospital is one of the important variables that can be analyzed to evaluate the efficiency of hospital services as well as provide insight into the severity of certain diseases [4]. However, length of stay data is often incomplete due to censored observations. Censoring is a condition where information is incomplete, such as patients who are discharged from the hospital before they are fully recovered or when the study is stopped before all patients have been treated [5]. This type of censoring is often used in research on patient survival data by a disease is type III censored. Type III censored is an observation that is limited by the time of each patient entering the observation at different times because some patients very rarely enter the observation at the same time as the censored observation due to a predetermined time limit.

In survival data analysis, various distribution models can be applied, one of which is the Weibull distribution [2]. This distribution is a continuous random variable distribution that is often used to evaluate system reliability and is known for its flexibility. The flexibility is seen in its ability to transform into other distributions, such as the exponential distribution, depending on changes in its scale and shape parameters. According to research [6], this distribution has flexibility by allowing a more diverse form of hazard function and has a more precise estimate of the time of failure or important events in reliability studies or age analysis. After determining the appropriate data distribution, the next step is to estimate its parameters. Parameter estimation is the process of estimating or guessing population characteristics based on samples taken, which is part of statistical inference. In estimation theory, there are two main approaches to estimating population parameters from data distributions, namely the classical approach and the Bayesian approach [7].

In the Bayesian method, the parameter is treated as a random variable that has a probability distribution [8]. The probability distribution for the unknown parameters, which is chosen based on two approaches, namely subjective and based on previous research, is called the prior distribution. There are two types of prior distributions: informative priors and non-informative priors. An informative prior is a prior distribution that contains information related to parameter α, while a non-informative prior is a prior distribution that has no information about parameter α, one of which is in the form of a conjugate prior [9]. In both prior distributions, if used in reference to the Bayesian theorem, it will produce a posterior distribution. The non-informative prior has less significant on the posterior distribution, but it will still affect the analysis results. The analysis results of the non-informative prior distribution are more influenced by the data, so it will provide a more neutral estimate. This prior distribution is considered more flexible because it does not require strong prior knowledge. This makes the non-informative prior distribution appropriate for data with very limited or no prior information to avoid unwanted bias. The results of the posterior distribution will be used for further processing by following the rules of Bayesian theory with the aim of obtaining parameter estimates.

Some research related to parameter estimation related to the Bayesian approach is research by [10] explaining the use of Bayesian methods in estimating Weibull distribution parameters on type II and type I censored data with the Maximum Likelihood Estimation (MLE) estimation method. The result of this research is that the greater the t time, the small the chance of maintaining remission time. Furthermore, in research by [11] which describes the use of the bootstrap method on type III censored survival data with the confidence interval of the Weibull distribution parameters with the Maximum Likelihood Estimation (MLE) estimation method. The result of this research is the probability of lung cancer patients at RSUD Dr. Kariadi Semarang to survive more than 5 days is 99.2%.

Previous studies used the classical estimation method, namely Maximum Likelihood Estimation (MLE), which is optimal if the normal distribution assumption is met. Therefore, to overcome these shortcomings, Bayesian estimation methods are used. One of them is Linear Exponential Loss Function (LINEX) which is more suitable for data when estimation errors have asymmetric impacts and different consequences. LINEX provides more efficient estimation than MLE in the case of asymmetric error distribution, due to its ability to adjust the loss function to the imbalance between positive and negative errors [12].

In previous research, the estimation of continuous distribution parameters with a Bayesian approach to survival analysis has also been carried out, namely research by [13] which describes the use of the Bayesian method of LINEX loss function in estimating exponential distribution parameters on type I censored data using Jeffrey priors. The results of this research indicate that patient failure (death) for patients with chronic renal failure with an initial cause of diabetic disease is higher than patients with an initial cause of non-diabetic disease. Furthermore, research by [14] describes the use of the Bayesian method LINEX loss function in estimating two exponential distribution parameters on type I censored data using Jeffrey priors. The results showed that after working for a long period of time, employees will usually experience attrition. The longer an employee works for a company, the less likely they are to stay with the company. One of the most commonly used Bayes approach parameter estimation methods is the LINEX loss function [15].

The LINEX loss function was introduced by Varian in 1975. The use of LINEX loss function in parameter estimation is very useful in risk analysis and statistical modeling where asymmetry in distribution or different risk levels in opposite directions are important considerations. Parameter estimation using the LINEX loss function method is done by minimizing the average risk or expected value of the LINEX loss function. The Bayesian approach with the LINEX loss function method requires a Likelihood function, prior distribution, and posterior distribution. The Bayesian approach allows the use of conjugate priors to improve the accuracy of parameter estimation. In addition, the use of linear exponential loss function (LINEX) in this method provides an alternative way to assess estimation error by considering bias and variance simultaneously. The Linear Exponential Loss Function method will be chosen as the loss function to optimize parameter estimation, given the importance of accuracy in the context of healthcare.

According to data from the Ministry of Health in 2023, the prevalence of stroke reached 8.3% per 1000 population in Indonesia continues to increase every year, especially among the elderly. Stroke is one of the leading causes of death and disability worldwide, including in Indonesia. With a percentage of 11.2% of total disability and 18.5% of total mortality. This disease is a big burden for the health care system because stroke treatment requires intensive care and long-term therapy [16]. Risk factors such as hypertension, diabetes, dyslipidaemia, cardiac disorders, and unhealthy lifestyles are the main causes of the increasing prevalence of stroke [17]. Data shows that the incidence of stroke continues to increase along with population growth and changes in people's lifestyles [18]. Haji Surabaya Regional General Hospital, as one of the referral hospitals in Surabaya, is a service center for stroke patients with various demographic backgrounds and health conditions. With the increasing incidence of stroke, it is important to analyze the length of time patients stay in the hospital. In this context, survival analysis is an appropriate method to evaluate the survival time data of stroke patients.

Based on the description above, on this occasion the author wants to know the model of the censored Weibull distribution parameter estimation type III with the Bayesian Linear Exponential Loss Function (LINEX) method. In this research, the prior distribution used is the EXP (1) distribution with novelty using data on the length of time stroke patients are hospitalized. This research is expected to contribute to the development of health data analysis methods, especially those related to censored data, and become a reference for further research in the field of health.

2. RESEARCH METHODS

2.1 Weibull Distribution

According to [2], the Weibull distribution is a distribution that describes extreme events such as the life time of living things. This distribution is most often applied in life time distribution models. Suppose a continuous random variable X is Weibull distributed, with α as the scale parameter and β as the shape parameter, then the probability density function (PDF) of the distribution is

$$f(x) = \begin{cases} \frac{\beta}{\alpha}\left(\frac{X}{\alpha}\right)^{\beta-1} exp\left[-\left(\frac{X}{\alpha}\right)^{\beta}\right], for\ x > 0 \\ 0, for\ others \end{cases} . \quad (1)$$

If $\beta = 1$ then $f(x) = \frac{1}{\alpha} e^{-\frac{1}{\alpha}x} \sim exp(\alpha)$

The CDF of the Weibull distribution is

$$F(x) = 1 - exp\left[-\left(\frac{X}{\alpha}\right)^{\beta}\right], for\ x > 0. \quad (2)$$

The survival function of the Weibull distribution based on the formula in the equation is

$$S(x) = exp\left[-\left(\frac{X}{\alpha}\right)^{\beta}\right], for\ t > 0. \quad (3)$$

The hazard function of the Weibull distribution based on the formula in the equation is

$$h(x) = \frac{\beta}{\alpha}\left(\frac{X}{\alpha}\right)^{\beta-1}. \quad (4)$$

After obtaining the probability density function, the average formula of the Weibull distribution can be defined as follows:

$$E(x) = \alpha\gamma\left(\frac{1}{\beta} + 1\right). \quad (5)$$

To evaluate the accuracy of the estimation model used in predicting the length of hospitalization for stroke patients, we need to calculate the Mean Squared Error (MSE). Here is the formula to calculate MSE:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(f(x_i) - \hat{f}(x_i)\right)^2 \quad (6)$$

2.2 Type III Censored Data

Type III censoring occurs when objects drop out of the study at different times within a predetermined observation period. During this period, some objects may fail or die before the end of the observation period, which allows their survival time to be known precisely. On the other hand, some other objects may remain alive until the end of the observation period or drop out of the study, but the time of failure is not fully recorded [19]. It is said to be type III censored if individuals or objects enter the study at different times over a period of time.

Likelihood function is a function of the parameter α and is denoted by $L(\alpha)$. If X_i is a random sample containing observational data (x_i, δ_i), i = 1,2, … , n and assumed to be independent of each other with probability density function $f(x_i)$ and survival function $S(x_i)$, then the Likelihood function on type III censored data can be expressed as follows.

$$L(\alpha) = \prod_{i=1}^{n} f(x_i)^{\delta_i} S(x_i)^{1-\delta_i}, \quad (7)$$

with δ_i the censoring indicator, which is 1 if the data is not censored and 0 if the data is censored. The value of x_i is obtained from min (X_i, L_i) where X_i is the life time of individual i, and L_i is the censoring time of individual i, with $i = 1, 2, \ldots, n$.

2.3 Linear Exponential Loss Function

Linear Exponential Loss Function (LINEX) is one of the methods used in the Bayesian approach. This method is applied in the analysis of statistical estimation and prediction problems that show an exponential increase on one side of zero and almost linear on the other side of zero **[20]**. LINEX loss function can be explained as follows:

$$L(\hat{\alpha}, \alpha) = \exp\left(a\,(\hat{\alpha} - \alpha)\right) - a(\hat{\alpha} - \alpha) - 1\ ,\ \ a \neq 0. \quad (8)$$

With α is a parameter to control the size of the difference between the actual value and the estimated value. The value of a will be more accurate in measuring the estimation results if the value is higher. According to **[21]**, the value of a can be positive or negative.

The posterior expectation for the parameter α denoted by E_α from the LINEX loss function can be written as follows:

$$E_\alpha\left(L(\hat{\alpha}, \alpha)\right) = E_\alpha\left[e^{a\,(\hat{\alpha}-\alpha)} - a(\hat{\alpha} - \alpha) - 1\right]. \quad (9)$$

With E_α representing the posterior expected value, in the LINEX loss function, the Bayes estimator of α which can be denoted as $\hat{\alpha}_L$, is as follows:

$$\hat{\alpha}_L = -\frac{1}{a} ln\left(E_\alpha \left(e^{-a\alpha}\right)\right) \tag{10}$$

By using the condition $E_\alpha \left(e^{-a\alpha}\right)$

2.4 Prior Conjugate

A non-informative prior distribution is a prior distribution that contains no information about the parameters. One approach to non-informative priors is to use the conjugate method. The Conjugate Method in Bayesian is an approach where the prior and the Likelihood distribution have a mathematical structure that produces a posterior in the same form as the prior. If the prior is chosen from a family of distributions that are conjugate to the Likelihood, then the analysis process becomes analytically simpler. Non-informative prior distributions provide little information relative to the experiment, while conjugate methods produce posterior distributions that belong to the same family of probability distributions. The combination of non-informative priors with conjugate methods can provide more flexible results in Bayesian analysis.

2.5 Data Analysis Method

The data used in this research are secondary data obtained from medical records of the Haji Surabaya Regional General Hospital (RSUD) for stroke patients. The data includes information on 60 stroke patients who have complete medical records during the period January to November 2024. The following is an explanation of the research variables in Table 1.

Table 1. Research Variables

Variable Name	Scale	Description
Length of Hospitalization Time	Continuous	Day
Age	Continuous	Year
Gender	Nominal	0 = Female 1 = Male
Employment status	Nominal	0 = Private Employee 1 = Self-employed 2 = Housewife 3 = Civil Servant 4 = Other

Based on Table 1, it can be seen that the data to be analyzed is the duration of hospitalization of stroke patients. To calculate the estimation of Weibull distribution parameters on type III censored data using the Linear Exponential Loss Function (LINEX) method, the steps that need to be followed are as follows:

1. Assumes that as many as n random samples $x_1, x_2, \dots, x_n$ are live test data originating from the Weibull distribution with parameter α.
2. Perform data censoring by determining the classification of the patient's discharge condition or censoring time to obtain type III censored data.
3. Determine the form of the probability density function (pdf) for the Weibull distribution.
4. Determine the survival function of the Weibull distribution.
5. Determine the form of the Likelihood function $L(\alpha)$ f or the Weibull distribution with type III censored data.
6. Taking the EXP (1) distribution as the prior distribution.
7. Form the posterior distribution $P(\alpha|X_i)$.
8. Parameter estimation using the Linear Exponential Loss Function (LINEX) method is based on the posterior distribution that has been obtained, following these steps:
   a. Determine the form of the Linear Exponential loss function $L(\hat{\alpha}, \alpha)$ which can be written as $E_\alpha[L(\hat{\alpha}, \alpha)]$.
   b. Minimize the posterior expected value of the LINEX loss function $E_\alpha[L(\hat{\alpha}, \alpha)]$ with respect to α, thus obtaining the Bayes estimator parameter denoted by $\hat{\alpha}_L$.
   c. Determining the value of E_α [exp $(-a\alpha)$].
   d. Insert the result from step 3 into the equation $\hat{\alpha}_L$.

The application of the estimation results to real data can be done with the following steps:

1. Determine the real data to be applied.
2. Test the exponential distribution of real data based on the Kolmogorov-Smirnov test.
3. Import the data into Mathematica software.
4. Calculating the estimation of the α parameter using the LINEX method can be done using Mathematica software.
5. Interpreting the result of step d.

3. RESULTS AND DISCUSSION

3.1 Analysis of Bayes Estimation

Assuming X is s a Weibull distributed random variable, the observation data of variable X is obtained as many as n $(x_1, x_2, \dots, x_n)$. Thus, the probability density function (PDF) of the variable X can be expressed as follows:

$$f(x) = \frac{\beta}{\alpha}\left(\frac{X}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{X}{\alpha}\right)^{\beta}\right], for\ x > 0 \tag{11}$$

$$0, for\ others$$

The cumulative distribution function (CDF) of the variable X is represented as follows:

$$F(x) = 1 - e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}, for\ x > 0 \tag{12}$$

In censored data to form the Likelihood function, the survival function of the Weibull distribution is required which can be explained as follows:

$$\begin{aligned} S(x) &= 1 - F(x) \\ &= 1 - \left(1 - e^{-\left(\frac{X}{\alpha}\right)^{\beta}}\right) \\ &= e^{-\left(\frac{X}{\alpha}\right)^{\beta}} \end{aligned} \tag{13}$$

After obtaining the survival function, the Likelihood function of random variables in censored data can be explained as follows:

$$\begin{aligned} L(\alpha) &= \prod_{i=1}^{n} f(x_i)^{\delta_i} S(x_i)^{1-\delta_i} \\ &= \prod_{i=1}^{n} \left(\left(\frac{\beta}{\alpha}\right)\left(\frac{X_i}{\alpha}\right)^{\beta-1} . e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}\right)^{\delta_i} . \left(e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}\right)^{1-\delta_i} \\ &= \prod_{i=1}^{n} \left(\frac{\beta}{\alpha}\right)^{\delta_i} . \left(\frac{X_i}{\alpha}\right)^{\delta_i(\beta-1)} . e^{-\delta_i\left(\frac{X_i}{\alpha}\right)^{\beta}} . e^{-(1-\delta_i)\left(\frac{X_i}{\alpha}\right)^{\beta}} \\ &= \prod_{i=1}^{n} \left(\frac{\beta}{\alpha}\right)^{\delta_i} . \left(\frac{X_i}{\alpha}\right)^{\delta_i(\beta-1)} e^{-\delta_i\left(\frac{X_i}{\alpha}\right)^{\beta}} . e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}} . e^{\delta_i\left(\frac{X_i}{\alpha}\right)^{\beta}} \\ &= \prod_{i=1}^{n} \left(\frac{\beta}{\alpha}\right)^{\delta_i} . \left(\frac{X_i}{\alpha}\right)^{\delta_i\beta} . \left(\frac{X_i}{\alpha}\right)^{-\delta_i} . e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}} \\ &= \prod_{i=1}^{n} \left(\frac{\beta}{\alpha}\right)^{\delta_i} . \left(\frac{X_i}{\alpha}\right)^{\delta_i\beta} . \left(\frac{\alpha}{X_i}\right)^{\delta_i} . e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}} \end{aligned}$$

$$= e^{\sum_{i=1}^{n} -\left(\frac{X_i}{\alpha}\right)^{\beta}} \left(\frac{\beta}{\alpha}\right)^{\sum_{i=1}^{n}\delta_i} . \prod_{i=1}^{n}\left(\frac{X_i}{\alpha}\right)^{\delta_i\beta} . \prod_{i=1}^{n}\left(\frac{\alpha}{X_i}\right)^{\delta_i} \quad (14)$$

Based on the likelihood function above, the posterior distribution can be calculated using the following formula:

$$P(\alpha|X_i) = \frac{L(\alpha).P(\alpha)}{\int_0^{\infty} L(\alpha).P(\alpha)\,d\alpha}$$

With the assumption: $\alpha \sim Exp\ (1)$

$$P(\alpha) = e^{-\alpha} \quad (15)$$

$$for\ \alpha > 0$$

After the prior distribution and likelihood function are obtained, to find the prior distribution both will be combined which can be explained as follows:

$$\begin{aligned} P(\alpha|X_i) &= \frac{L(\alpha).P(\alpha)}{\int_0^{\infty} L(\alpha).P(\alpha)\,d\alpha} \\ &= \frac{e^{\sum_{i=1}^{n} -\left(\frac{X_i}{\alpha}\right)^{\beta}} \left(\frac{\beta}{\alpha}\right)^{\sum_{i=1}^{n}\delta_i} . \prod_{i=1}^{n}\left(\frac{X_i}{\alpha}\right)^{\delta_i\beta} . \prod_{i=1}^{n}\left(\frac{\alpha}{X_i}\right)^{\delta_i} .(e^{-\alpha})}{\int_0^{\infty} e^{\sum_{i=1}^{n} -\left(\frac{X_i}{\alpha}\right)^{\beta}} \left(\frac{\beta}{\alpha}\right)^{\sum_{i=1}^{n}\delta_i} . \prod_{i=1}^{n}\left(\frac{X_i}{\alpha}\right)^{\delta_i\beta} . \prod_{i=1}^{n}\left(\frac{\alpha}{X_i}\right)^{\delta_i} .(e^{-\alpha})\,d\alpha} \end{aligned} \quad (16)$$

After the posterior distribution is obtained, the next step is to estimate the previously unknown parameter α. In this research, the estimation of parameter α will be performed using the Bayesian Linear Exponential Loss Function method, which is defined as follows:

$$L(\hat{\alpha},\alpha) = \exp\big(a\ (\hat{\alpha}-\alpha)\big) - a(\hat{\alpha}-\alpha) - 1,\ \ a \neq 0 \quad (17)$$

The Bayesian LINEX estimate of α in the Weibull distribution is obtained by minimizing the posterior expectation value of the loss function above which can be described as follows:

Calculating the posterior expectation value of the loss function LINEX:

$$\begin{aligned} E_{\alpha}\big(L(\hat{\alpha},\alpha)\big) &= E_{\alpha}\left[e^{a\,(\hat{\alpha}-\alpha)} - a(\hat{\alpha}-\alpha) - 1\right] \\ &= E_{\alpha}\left(e^{a\,(\hat{\alpha}-\alpha)}\right) - E_{\alpha}\left(a(\hat{\alpha}-\alpha)\right) - 1 \\ &= E_{\alpha}\left(e^{a\hat{\alpha}}\right).E_{\alpha}\,(e^{-a\alpha}) - E_{\alpha}(a\hat{\alpha}) + E_{\alpha}(a\alpha) - 1 \\ &= e^{a\hat{\alpha}}.E_{\alpha}\,(e^{-a\alpha}) - a\hat{\alpha} + E_{\alpha}(a\alpha) - 1 \\ &= e^{a\hat{\alpha}}.E_{\alpha}\,(e^{-a\alpha}) - a\big(\hat{\alpha} + E_{\alpha}(\alpha)\big) - 1 \end{aligned} \quad (18)$$

Furthermore, to minimize the equation, we find the first derivative of $\hat{\alpha}$ and equate the result to zero. Thus, the Bayesian LINEX estimator for the α parameter is obtained as follows:

$$\begin{aligned} \frac{\partial\left(E_{\alpha}\big(L(\hat{\alpha},\alpha)\big)\right)}{\partial\alpha} &= 0 \\ \frac{\partial\left(e^{a\hat{\alpha}}.E_{\alpha}\,(e^{-a\alpha}) - a\big(\hat{\alpha}+E_{\alpha}(\alpha)\big) - 1\right)}{\partial\alpha} &= 0 \\ a.e^{a\hat{\alpha}}.E_{\alpha}\left(e^{-a\hat{\alpha}}\right) - a &= 0 \\ a.e^{a\hat{\alpha}}.E_{\alpha}\left(e^{-a\hat{\alpha}}\right) &= a \\ e^{a\hat{\alpha}}.E_{\alpha}\left(e^{-a\hat{\alpha}}\right) &= \frac{a}{a} \\ e^{a\hat{\alpha}}.E_{\alpha}\left(e^{-a\hat{\alpha}}\right) &= 1 \end{aligned}$$

$$\begin{aligned} ln(e^{a\hat{\alpha}}) &= \ln\left(\frac{1}{E_\alpha\,(e^{-a\alpha})}\right) \\ a\hat{\alpha} &= \ln(1) - ln\big(E_\alpha\,(e^{-a\alpha})\big) \\ \hat{\alpha} &= \frac{\ln(1) - ln\big(E_\alpha\,(e^{-a\alpha})\big)}{a} \\ \hat{\alpha} &= \frac{-ln\big(E_\alpha\,(e^{-a\alpha})\big)}{a} \\ \hat{\alpha} &= -\frac{1}{a}\Big(ln\big(E_\alpha\,(e^{-a\alpha})\big)\Big) \end{aligned} \quad (19)$$

From this estimator, the next step is to calculate the value of $(E_\alpha(e^{-a\alpha}))$ which is the posterior expectation value of $e^{-a\alpha}$. This expectation calculation process can be explained as follows:

$$\begin{aligned} E_\alpha(e^{-a\alpha}) &= \int_0^\infty e^{-a\alpha}.P(\alpha|X_i)d\alpha \\ &= \int_0^\infty e^{-a\alpha}.\left(\frac{(e^{-\alpha}).\ e^{\sum_{i=1}^n -\left(\frac{X_i}{\alpha}\right)^\beta}\left(\frac{\beta}{\alpha}\right)^{\sum_{i=1}^n \delta_i}.\prod_{i=1}^n\left(\frac{X_i}{\alpha}\right)^{\delta_i\beta}.\prod_{i=1}^n\left(\frac{\alpha}{X_i}\right)^{\delta_i}}{\int_0^\infty e^{\sum_{i=1}^n -\left(\frac{X_i}{\alpha}\right)^\beta}\left(\frac{\beta}{\alpha}\right)^{\sum_{i=1}^n \delta_i}.\prod_{i=1}^n\left(\frac{X_i}{\alpha}\right)^{\delta_i\beta}.\prod_{i=1}^n\left(\frac{\alpha}{X_i}\right)^{\delta_i}.(e^{-\alpha})\,d\alpha}\right)d\alpha \\ &= \frac{\int_0^\infty e^{-\alpha(a+\alpha)}.\ e^{\sum_{i=1}^n -\left(\frac{X_i}{\alpha}\right)^\beta}\left(\frac{\beta}{\alpha}\right)^{\sum_{i=1}^n \delta_i}.\prod_{i=1}^n\left(\frac{X_i}{\alpha}\right)^{\delta_i\beta}.\prod_{i=1}^n\left(\frac{\alpha}{X_i}\right)^{\delta_i}d\alpha}{\int_0^\infty e^{\sum_{i=1}^n -\left(\frac{X_i}{\alpha}\right)^\beta}\left(\frac{\beta}{\alpha}\right)^{\sum_{i=1}^n \delta_i}.\prod_{i=1}^n\left(\frac{X_i}{\alpha}\right)^{\delta_i\beta}.\prod_{i=1}^n\left(\frac{\alpha}{X_i}\right)^{\delta_i}.(e^{-\alpha})\ d\alpha} \end{aligned} \quad (20)$$

Thus, the estimator of the Weibull distribution parameters with the Bayesian LINEX method is as follows:

$$\hat{\alpha} = -\frac{1}{a}\left(ln\left(\frac{\int_0^\infty e^{-\alpha(a+\alpha)}.\ e^{\sum_{i=1}^n -\left(\frac{X_i}{\alpha}\right)^\beta}\left(\frac{\beta}{\alpha}\right)^{\sum_{i=1}^n \delta_i}.\prod_{i=1}^n\left(\frac{X_i}{\alpha}\right)^{\delta_i\beta}.\prod_{i=1}^n\left(\frac{\alpha}{X_i}\right)^{\delta_i}d\alpha}{\int_0^\infty e^{\sum_{i=1}^n -\left(\frac{X_i}{\alpha}\right)^\beta}\left(\frac{\beta}{\alpha}\right)^{\sum_{i=1}^n \delta_i}.\prod_{i=1}^n\left(\frac{X_i}{\alpha}\right)^{\delta_i\beta}.\prod_{i=1}^n\left(\frac{\alpha}{X_i}\right)^{\delta_i}.(e^{-\alpha})\ d\alpha}\right)\right) \quad (21)$$

3.2 Descriptive Statistics

Descriptive statistical analysis to determine the data characteristics of the data that has been obtained. The following are the results of the characteristics of the length of hospitalization time that have been displayed in Table 2 below.

Table 2. Length of Hospitalization Time Characteristics

	N	Minimum	Maximum	Mean	Std. Deviation
Length of Hospitalization Time	60	1	16	6.68	2.487

Based on Table 2, it can be seen that the average length of hospitalization is 6.68 days with a standard deviation value of 2.487. This data also has a minimum data value of 1 and a maximum of 16. In addition, the descriptive analysis results in Table 3 below are required.

Table 3. Descriptive Statistics of Patient Data

Category		N	%
Gender	Male	31	51.7
	Female	29	48.3
Age	Adult (19-59 years)	30	50
	Elderly ($\geq$ **60** years)	30	50
Employment status	Private Employee	14	23.3
	Self-employed	1	1.7
	Housewife	6	10
	Civil Servant	2	3.3
	Others	37	61.3

Based on Table 3, the results show that of the 60 stroke patients, most were male, namely 31 people (51.7%) and 29 people (48.3%) were female. The number of adults aged 19 to 59 years and elderly aged more than 60 years is balanced, namely 30 stroke patients (50%). Most of them were male, 31 people (51.7%) and 29 people (48.3%) were female. The employment status of patients was mostly others, as many as 37 patients (61.7%). Followed by patients who worked as private employees as many as 14 patients (23.3%).

3.3 Implementation of Bayes Estimation Analysis

After the Weibull distribution parameter estimation is obtained using the Bayesian LINEX Loss Function method, the next stage is the application to real data. The data is used to apply the results of the Weibull distribution parameter estimation obtained through the Bayesian Exponential Linear Loss Function (LINEX) method based on the EXP (1) distribution as the prior distribution is secondary data in the form of data on the length of time of hospitalization of stroke patients affected by stroke attacks at the Haji Surabaya Regional General Hospital in 2024. The research took a sample of 62 stroke patients who had complete medical records during the period January to November 2024. After censoring from 62 stroke inpatients who can be observed are 60 patients. Data distribution testing is the first step that can be done before estimating parameters using actual data. This research uses the Weibull data distribution with the following hypothesis:

H_0 = Data follows the Weibull distribution
H_1 = Data does not follow the Weibull distribution

By using the test criterion α = 0.05, the decision will reject H_0 if the p-value < 0.05. The results of the data distribution suitability test show a p-value of 0.21856 for the length of hospitalization data of stroke patients. Because the p-value of the distribution suitability test is greater than α = 0.05, the decision taken is to accept H_0. Therefore, it can be concluded that the data on the length of hospitalization of stroke patients in January-November 2024 follows the Weibull distribution.

After the shape of the data distribution is known, the next step is to calculate the estimated value of the α parameter in the stroke patient length of stay data. This calculation is done with the assumption that the parameter β is 7. Next, the value of a is determined based on the estimation results that have been obtained. This is done by taking the value of a that is closest to the average value of all the results of the calculation of $\hat{\alpha}$. The value of a has the condition that $a \geq 2$. The value of a that will be used is $a = 3, 4, 5, 6, 7, 8, 9, 10$. The average of all $\hat{\alpha}$ values is 6.08779125. So, the value of $\hat{\alpha}$ that is closest to the average value is $\hat{\alpha}$ using the value of a = 6. In addition, it can also be seen that the higher the value of a , the higher the value of $\hat{\alpha}$ will be. A high value of a will give riskier estimation results, because the equation of the actual value with the estimated value will be higher.

After knowing the value of a, the parameter estimation value α can be calculated. Based on the calculation results, the parameter estimation result α with a value of $a = 6$ and $\beta = 7$ on the data of the length of time of hospitalization of stroke patients at the Surabaya Hajj Regional General Hospital in January-November 2024 is 6.32342. By using Eq. (5), it can be found that the average length of hospitalization time for stroke patients at the Haji Surabaya Regional General Hospital is 5.9151646 days. This shows that stroke patients at RSUD Haji Surabaya tend to have recovered or been discharged from the hospital during a relatively short period of time. However, it also depends on the reasons that cause discharge from the hospital such as recovery, death, or outpatient care. Additionally, to assess the accuracy of the parameter estimation model used, the Mean Squared Error (MSE) was calculated. By using Eq. (6), the obtained MSE value is 0.000270555, which indicates the extent to which the model's predictions deviate from the actual observed data. A small MSE value indicates that the estimation model is more accurate in predicting the length of hospitalization for stroke patients at the Haji Surabaya Regional General Hospital.

The probability of patient recovery from hospital can be calculated through survival analysis. This process is done by substituting the parameter estimation results into the survival function in Eq. (13) resulting in the data presented in Table 4 as follows.

Table 4. Stroke Patient Opportunity Time Data

Length of Hospitalization	$S(t_i)$
1	1.000
2	1.000
3	0.995
4	0.960
5	0.824

Length of Hospitalization	$S(t_i)$
6	0.500
7	0.130
8	0.006
9	0.000
10	0.000
12	0.000
16	0.000

Here the author shows the graph that has been obtained based on Table 4 in Fig. 1 below.

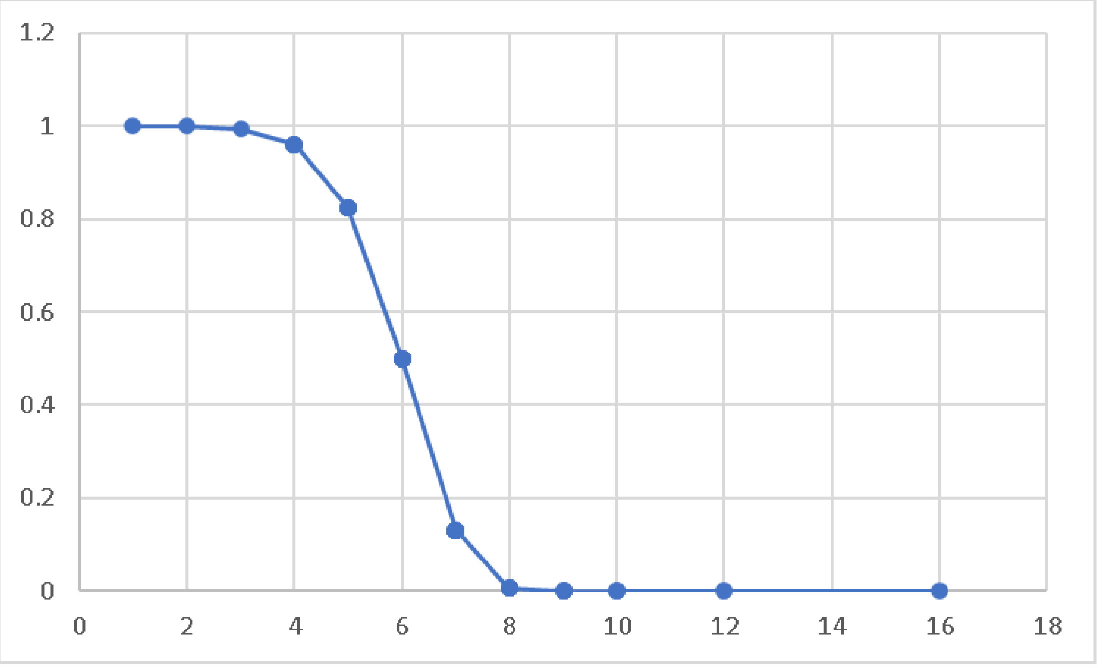

Figure 1. Survival Analysis Graph

Based on Table 4 and Fig. 1 above, it can be seen that the probability of survival of stroke patients who have been hospitalized on the 5th day is 0.824 or 82.4%. As the length of hospitalization of stroke patients at RSUD Haji Surabaya increases, the probability of the survival function decreases. The decrease in the probability of survival over time reflects that patients are getting closer to the point where they no longer need hospital care, indicating that their health condition is improving. Thus, on the 5th day, there is a 91.76% probability that the patient has recovered and no longer requires hospitalization.

In addition, an analysis is needed to measure the level of risk of events at a certain time, assuming that the patient does not recover. If the hazard function is high at a certain time, it means that the risk of the patient not recovering at that time is greater. This can be shown in Fig. 2 below.

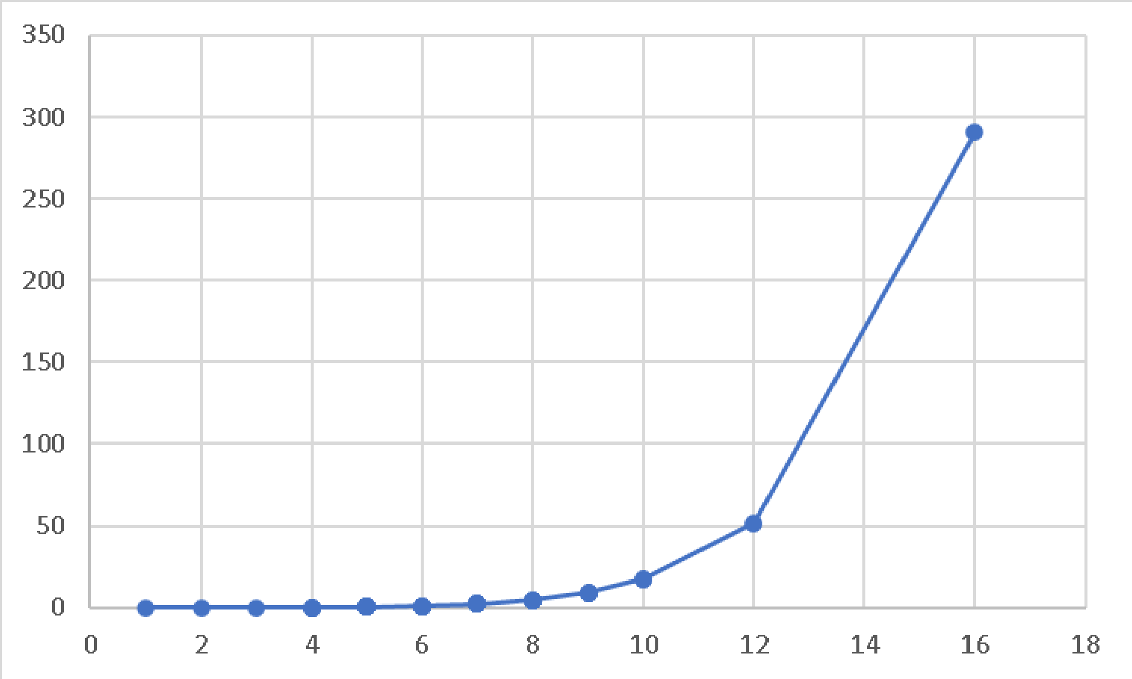

Figure 2. Hazard Function Graph

Based on Fig. 2, the following is a graph of the hazard function that continues to increase, which shows a very high peak on day 16, indicating a period of very high risk. This could mean that on day 16 of hospitalization, there is a high probability that the patient will not recover, either due to improvement in condition or even due to complications that force a change in patient status.

4. CONCLUSION

The estimation results were applied to secondary data, namely stroke patient data on the length of hospitalization time at the Haji Surabaya Regional General Hospital in January-November 2024. In this research, a sample of 62 stroke patients was taken with censorship limits, namely death or discharge at their own request. After setting the censoring limit, of the 62 stroke patients that could be observed, 60 stroke patients were observed. The result of the parameter estimation α is 6.32342. With an average value of 5.9151646 days. In addition, the accuracy value of the parameter estimation model used with the Mean Squared Error (MSE) calculation obtained a value of 0.000270555. A small MSE value indicates that the estimation model is more accurate in predicting the length of hospitalization of stroke patients at the Haji Regional General Hospital, Surabaya. Based on these results, it can be concluded that stroke patients at RSUD Haji Surabaya have recovered and no longer need hospitalization. Then, the estimated chance of stroke patients who have undergone hospitalization on the 5th day is 0.824 or 82.4%. The longer the stroke patient is hospitalized at RSUD Haji Surabaya, the lower the chance of survival function. If the probability value of the survival function decreases over time, it indicates that no further hospitalization is required, indicating that the patient's health condition is improving. This can be interpreted that on the 5th day of the patient in the hospital there is a 91.76% chance that the patient has recovered and no longer requires hospitalization. In addition, there is a hazard function used to measure the risk level of an event at a certain time, on a graph that continues to increase, it states that there is a high probability that the patient will not recover, either because of improving conditions or even because of complications that force a change in the patient's status.

Author Contributions

Tentri Ryan Rahmanita: Data Curation, Funding Acquisition, Investigation, Writing – Original Draft, Writing – Review and Editing. Ardi Kurniawan: Conceptualization, Funding Acquisition, Formal Analysis, Methodology, Software. Elly Ana: Project Administration, Resources, Methodology. Sediono,: Resources, Supervision, Validation. Dita Amelia: Project Administration, Software, Visualization. All authors discussed the results and contributed to the final manuscript.

Funding Statement

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Acknowledgment

The authors would like to thank the Statistics Study Program, Faculty of Science and Technology, Universitas Airlangga, as well as the Haji Surabaya Regional General Hospital for the opportunity and support in conducting this research as a medium for applying learning materials.

Declarations

The authors declare no competing interest.

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