BAREKENG: Journal of Mathematics and Its Applications
December 2025 Volume 19 Issue 4 Page 3033–3046
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol19no4pp3033-3046

FUZZY MULTI-OBJECTIVE OPTIMIZATION FOR THE PLACEMENT OF REVERSE VENDING MACHINE IN URBAN WASTE MANAGEMENT SYSTEM

Chawis Boonmee[1*], **Nopphamart Khuankaew**[2], **Phavika Mongkolkittaveepol**[3]

[1,2,3]*Department of Industrial Engineering, Faculty of Engineering, Chiang Mai University Chiang Mai, 50200, Thailand*

*Corresponding author's e-mail: * chawis.boonmee@cmu.ac.th*

Article History:

Received: 7th April 2025
Revised: 3rd June 2025
Accepted: 16th June 2025
Available online: 1st September 2025

Keywords:

Fuzzy optimization;
Multi-objective;
Reverse Vending Machines;
Waste management;
Facility location problem.

ABSTRACT

This study proposes a fuzzy multi-objective optimization model for strategically placing Reverse Vending Machines (RVMs) within urban waste management systems. The research follows a structured methodology comprising seven key stages. First, a conceptual model was designed to address the challenges of post-consumer waste collection. Second, a mathematical model was formulated to optimize two conflicting objectives: maximizing recyclable waste collection and minimizing transportation distances. Third, the model was reformulated using fuzzy parameters—specifically, triangular membership functions—to account for uncertainties in waste generation rates, disposal demand, and transportation costs. Fourth, data were collected from a Lampang Province, Thailand case study covering 15 communities and 17 candidate RVM locations. Fifth, the fuzzy model was solved using the Weighted Sum Method and implemented via exact optimization in LINGO software. Sixth, results were analyzed, showing that five RVMs can be optimally installed under a 5,000,000 THB budget, achieving 23,911.50 kilograms of waste collection with a minimized transportation distance of 179.90 kilometers. Sensitivity analyses on distance, budget, and objective weights revealed key trade-offs between operational efficiency and environmental performance. Finally, the study concludes with implications for policy and planning, emphasizing the potential of fuzzy optimization in enhancing real-world recycling infrastructure. The proposed framework supports data-driven, sustainable decision-making for urban waste systems. Future research may explore dynamic waste generation patterns, behavioral modeling, and the use of metaheuristic algorithms for large-scale implementation.

How to cite this article:

C. Boonmee, N. Khuankaew and P. Mongkolkittaveepol., "FUZZY MULTI-OBJECTIVE OPTIMIZATION FOR THE PLACEMENT OF REVERSE VENDING MACHINE IN URBAN WASTE MANAGEMENT SYSTEM," *BAREKENG: J. Math. & App.*, vol. 19, iss. 4, pp. 3033-3046, December, 2025.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · Open Access

1. INTRODUCTION

The exponential growth of plastic waste presents a critical global challenge, hindering progress toward the United Nations Sustainable Development Goals (SDGs), especially Goal 12 (Responsible Consumption and Production) and Goal 13 (Climate Action). According to the Pollution Control Department, 25.7 million tons of municipal solid waste were generated in Thailand in 2022, with only 11% (2.83 million tons) recycled. This low recycling rate highlights inefficiencies in current waste management systems and the urgent need for innovative solutions. Improper plastic waste management leads to environmental, social, and economic consequences, including marine pollution, ecosystem disruption, and increased greenhouse gas emissions. Moreover, microplastics from degraded waste contaminate air, water, and food, posing serious public health risks, such as immune and reproductive system damage.

The current linear economic model of "produce, consume, and dispose" has proven unsustainable, necessitating a transition to a circular economy. Reverse vending machines (RVMs) have emerged as an innovative, automated solution for collecting and recycling waste directly from consumers. However, the strategic deployment of these machines is crucial to achieving both sustainability goals and operational efficiency. The absence of systematic approaches for selecting optimal RVM locations remains a significant barrier to improving waste collection efficiency and minimizing environmental impacts. Addressing this issue is essential for Thailand's progress toward achieving the SDGs and establishing sustainable waste management systems.

Several studies have investigated waste management, site selection, and recycling system optimization, offering valuable insights but revealing significant gaps. The literature on waste management and facility location optimization has made significant strides. However, it continues to exhibit enduring gaps in three critical areas: facility location optimization using mathematical models, handling uncertainties in waste management, and the strategic deployment of RVMs. These domains are vital for advancing sustainable and efficient urban waste management solutions.

Facility location optimization has long been central to improving waste management systems [1], [2]. [3] developed a bi-objective integer programming model to optimize kitchen waste transfer stations, integrating cost minimization and environmental impact reduction using an enhanced NSGA-II algorithm. [4] introduced a multi-objective location-allocation model for municipal solid waste management, addressing economic, environmental, and social equity objectives. Similarly, [5] applied a mixed-integer programming approach to optimize waste-to-energy facility locations, balancing energy generation, emissions reduction, and costs. While these studies present sophisticated models, they are primarily based on deterministic assumptions, which limit their applicability in real-world, variable conditions. Furthermore, their frameworks fail to integrate emerging technologies such as RVMs. [6] employed Greedy Randomized Adaptive Search Procedure (GRASP) with Adaptive Large Neighborhood Search (ALNS) to optimize the location-routing of infectious waste collection in Northeast Thailand. While effective for routing infectious waste, the study lacked fuzzy parameters or uncertainty modeling essential for real-world municipal waste management. [7] developed a hybrid MCA model for infectious waste disposal, focusing on cost minimization and location suitability. Unlike their emphasis on hazardous waste, our research addresses urban recyclable waste, integrating fuzzy multi-objective optimization for RVM placement. This approach prioritizes environmental sustainability, circular economy principles, and real-world uncertainty, filling significant research gaps. Furthermore, [8] focused on conventional waste disposal methods in Chiang Mai and Lamphun provinces, neglecting advanced recycling systems and multi-objective optimization frameworks. These limitations highlight the need for dynamic models that incorporate technological advancements and real-time variability in waste flows.

Uncertainty is an inherent challenge in waste management systems, affecting parameters such as waste production rates, treatment costs, and facility capacities. [9] explored optimization techniques, including fuzzy, stochastic, and interval programming, to address these uncertainties, emphasizing the growing use of fuzzy-stochastic methods and minimax regret optimization. While insightful, Singh's research did not address advanced systems like RVMs or their spatial deployment, missing an opportunity to enhance the practical application of these techniques. [10] employed a fuzzy analytical hierarchy process (FAHP) and goal programming for multi-objective site selection in infectious waste management. However, their focus on hazardous waste limits the model's relevance for urban recyclable waste application. Similarly, [11] proposed a scenario-based fuzzy-stochastic quadratic programming (SFQP) model for municipal solid waste (MSW) management, effectively managing dual uncertainties through probability distributions and fuzzy sets.

However, these studies do not extend to advanced technologies like RVMs or address real-time operational variability. Integrating uncertainty modeling with multi-objective optimization frameworks for RVM placement remains a critical research gap.

Reverse Vending Machines (RVMs) offer a promising solution to boost recycling by accepting beverage containers and encouraging public participation. [12] proposed a three-step optimization process to improve RVM sorting efficiency in limited spaces, while [13] developed a low-cost, durable RVM emphasizing affordability and community engagement. However, both studies overlook optimal location selection—a key factor in maximizing recycling efficiency. Existing research often lacks multi-objective optimization frameworks that consider transportation costs, environmental impacts, and waste collection efficiency, while failing to incorporate uncertainties in waste generation and user participation.

From these studies, it is evident that research focusing on RVMs remains scarce. Key limitations include the lack of emphasis on automated recycling systems, limited use of multi-objective frameworks, insufficient integration of fuzzy parameters to address real-world uncertainties, and the absence of localized applications in developing economies like Thailand. These limitations indicate a pressing need for research that integrates cutting-edge optimization techniques, uncertainty modeling, and practical applications to address the challenges of sustainable waste management.

This research aims to address these gaps by proposing a fuzzy multi-objective mathematical model for the strategic placement of RVMs. The key contributions of this study include addressing sustainability issues by aligning with SDG 12 and SDG 13 to improve recycling efficiency and reduce environmental impacts. A multi-objective optimization framework is introduced, balancing objectives of maximizing recyclable waste collection and minimizing transportation distances through the Weighted Sum Method. By incorporating fuzzy logic, the model effectively addresses uncertainties in waste generation rates, machine capacities, and transportation distances, ensuring robustness and real-world applicability. Additionally, the research is validated through a case study in Mueang District, Lampang Province, providing actionable insights for addressing waste management challenges in developing countries. The findings are expected to assist policymakers and businesses in deploying RVMs strategically, contributing to a sustainable circular economy by enhancing recycling efficiency, reducing waste to landfills, and minimizing carbon footprints. This research offers a comprehensive approach to sustainable waste management by combining advanced optimization techniques with practical applications.

2. RESEARCH METHODS

The research methodology consists of seven systematic steps, as illustrated in Figure 1. The process begins with designing a conceptual model, followed by formulating a mathematical model with fuzzy parameters. The model is then reformulated using fuzzy logic to handle uncertainty. Next, data is collected from a case study in Lampang Province. The model is solved using an exact algorithm, and the results are analyzed through sensitivity analysis. Finally, the study presents conclusions and practical implications, offering insights for sustainable urban waste management and future research development.

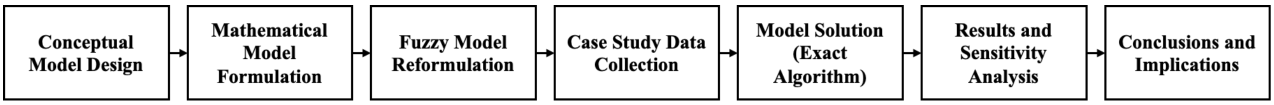


Figure 1. Overall Research Methodology Framework

2.1 Proposed Mathematical Model

2.1.1 Conceptual Model

This research develops a multi-objective mathematical model for selecting optimal locations for Reverse Vending Machines (RVMs) to address challenges in post-consumer waste management. The model is designed to achieve two main objectives: first, to maximize waste collection efficiency by placing RVMs near densely populated areas with high waste generation, thereby enhancing resource utilization and encouraging public participation; and second, to minimize transportation costs and environmental impact by reducing the distance between RVMs and recycling facilities. This dual-objective approach promotes operational efficiency and environmental sustainability, offering a practical framework that integrates economic and ecological factors for improved urban waste management planning. The model incorporates deterministic constraints and fuzzy parameters to address waste generation, distance, cost, and disposal rate uncertainties. It ensures that the solution meets operational, financial, and spatial feasibility while aligning with sustainability goals, such as the SDGs, particularly SDG 12 (Responsible Consumption and Production) and SDG 13 (Climate Action). Several key assumptions are made to operationalize the model and ensure its practical applicability. Waste generation is assumed to be predictable based on historical trends and varies depending on population density and the level of community engagement near each RVM. Each RVM has a fixed storage capacity, influencing maintenance frequency and operational efficiency. Transportation costs are directly proportional to the distance between RVMs and recycling facilities, highlighting the importance of route optimization. Due to budget limitations, the number of RVMs is restricted, requiring efficient allocation to maximize coverage. RVM placement is limited to accessible and regulation-compliant locations. Usage rates are influenced by proximity and the effectiveness of awareness campaigns. The model ensures equitable access across socio-economic groups and prioritizes reduced greenhouse gas emissions through optimized transport routes. Each community is assigned to only one RVM to streamline logistics. Lastly, fuzzy sets with triangular membership functions are employed to account for waste generation and transportation costs uncertainties.

2.1.2 Mathematical Model

The model includes several indices, parameters, and decision variables to represent the system effectively. Let *i* denote the index for communities (*i* = 1, 2, ..., *I*), *j* for potential RVM locations (*j* = 1, 2, ..., *J*), *k* for waste types (*k* = 1, 2, ..., *K*), and *m* for recycling facilities (*m* = 1, 2, ..., *M*). The parameters are defined as follows: $\widetilde{D}_{ij}$ represents the distance between community *i* and RVM *j* (in kilometers), and $\tilde{F}_{jm}$ is the distance between RVM *j* and recycling facility m. $\tilde{V}_k$ denotes the disposal rate of waste type *k*, while Cap_{jk} is the capacity of the machine at location *j* to handle plastic waste of type k. $\tilde{C}_j$ represents the installation cost of the machine at location *j*, and $\tilde{R}$ is the waste disposal demand rate. $\widetilde{H}_i$ indicates the number of households in community i, B is the total budget for machine installation, M is the maximum allowable distance from community areas to machine locations, and L is the maximum number of machines to be installed. The decision variables include W_{ik}, the volume of waste type *k* collected from community *i*; X_j, a binary variable equal to 1 if an RVM is installed at location *j*, and 0 otherwise; and Y_{ij}, which equals 1 if community *i* is assigned to RVM *j*, and 0 otherwise

Objective functions

$$Max\ Z1 = \sum_i \sum_k W_{ik} \tag{1}$$

$$Min\ Z2 = \sum_i \sum_j \widetilde{D}_{ij} Y_{ij} + \sum_j \sum_m \tilde{F}_{jm} X_j \tag{2}$$

Constraints

$$\sum_j Cap_{jk} X_j \geq \sum_i W_{ik} \qquad \forall k \tag{3}$$

$$W_{ik} \geq \tilde{V}_k \tilde{R} \widetilde{H}_i \qquad \forall i, k \tag{4}$$

$$\sum_j \tilde{C}_j X_j \leq B \tag{5}$$

$$\widetilde{D}_{ij} Y_{ij} \leq M \qquad \forall i, j \tag{6}$$

$$\sum_j X_j \leq L \tag{7}$$

$$Y_{ij} \leq X_j \qquad \forall i,j \tag{8}$$

$$\sum_j Y_{ij} = 1 \qquad \forall i \tag{9}$$

$$X_j \in \{0,1\}, Y_{ij} \in \{0,1\} \qquad \forall i,j \tag{10}$$

The mathematical model proposed in this research is designed to optimize the placement of RVMs to achieve two primary objectives: maximizing waste collection efficiency (**Equation (1)**) and minimizing transportation distances (**Equation (2)**). The first objective (**Equation (1)**) focuses on maximizing the total recyclable waste collected by strategically locating RVMs near communities with high waste generation and active engagement levels. The second objective (**Equation (2)**) seeks to minimize the distances between communities and RVMs, as well as between RVMs and recycling facilities, thereby reducing both transportation costs and greenhouse gas emissions. To ensure the model's feasibility, several constraints have been incorporated. The capacity constraint (**Equation (3)**) ensures that each installed RVM can accommodate the volume of waste collected across all recyclable materials. Waste generation constraints (**Equation (4)**) guarantee that the amount of waste collected from each community meets or exceeds the expected waste generation levels, which are based on household numbers and typical disposal patterns. A financial budget constraint (**Equation (5)**) limits the total installation costs of RVMs to stay within a predefined threshold. Similarly, a distance constraint (**Equation (6)**) ensures that the distance between communities and their assigned RVMs does not exceed a maximum allowable threshold. The model also incorporates a limit on the total number of RVMs (**Equation (7)**) that can be installed, reflecting logistical and budgetary restrictions. To enhance the efficiency of RVM utilization, **Equation (8)** ensures that communities can only be connected to operational RVMs, while **Equation (9)** ensures that each community is assigned to exactly one RVM. The decision-making framework relies on binary decision variables (defined in **Equation (10)**), which specify whether an RVM is installed at a particular location and whether a community is assigned to that RVM. By addressing these constraints and objectives, the model effectively balances the goals of maximizing waste collection and minimizing operational costs and environmental impacts. This integrated and holistic approach promotes sustainable and efficient waste management planning, aligning with logistical, economic, and accessibility priorities.

2.2 Solution Method

The proposed mathematical model aims to optimize the placement of RVMs using a multi-objective programming approach with fuzzy parameters. Due to the inherent complexity of solving such models under uncertainty, a structured solution methodology comprising five key steps was developed, as illustrated in **Figure 2**.

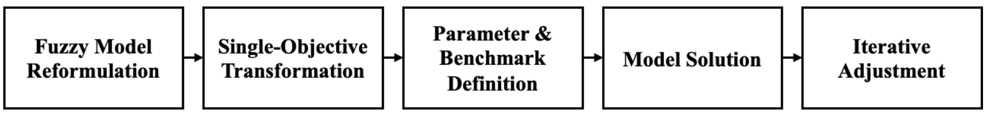


Figure 2. Solution Methodology for Fuzzy Multi-Objective Optimization Model

Step 1: Reformulating the Model with Fuzzy Parameters

These parameters were modeled as fuzzy variables to address the uncertainty in parameters such as waste generation rates, transportation costs, and machine capacities. A triangular fuzzy membership function was employed (o: optimistic, m = most likely, p: pessimistic), following the methodology of **[14]**, **[15]**. For further details, refer to **[16]**. This transformation enables the model to account for variations and uncertainties, producing more robust and realistic outputs. The reformulated equations incorporate fuzzy arithmetic, converting the problem into an equivalent auxiliary crisp model while accounting for the following considerations. An additional parameter, γ, represents the feasibility degree, with values ranging from 0 to 1.

Objective functions

$$Max\ Z1 = \sum_i \sum_k W_{ik} \tag{11}$$

$$Min\ Z2 = \sum_i \sum_j \left(\frac{D_{ij}^p + 2D_{ij}^m + D_{ij}^o}{4}\right) Y_{ij} + \sum_j \sum_m \left(\frac{F_{jm}^p + 2F_{jm}^m + F_{jm}^o}{4}\right) X_j \tag{12}$$

Constraints

(3), (7) - (10)

$$W_{ik} \geq \left[\gamma\left(\frac{V_k^m + V_k^p}{2}\right) + (1-\gamma)\left(\frac{V_k^m + V_k^o}{2}\right)\right]\left[\gamma\left(\frac{R^m + R^p}{2}\right) + (1-\gamma)\left(\frac{R^m + R^o}{2}\right)\right]\left[\gamma\left(\frac{H_i^m + H_i^p}{2}\right) + (1-\gamma)\left(\frac{H_i^m + H_i^o}{2}\right)\right] \quad \forall i,k \tag{13}$$

$$\sum_j \left[\gamma\left(\frac{C_j^m + C_j^p}{2}\right) + (1-\gamma)\left(\frac{C_j^m + C_j^o}{2}\right)\right] X_j \leq B \tag{14}$$

$$\left[\gamma\left(\frac{D_{ij}^m + D_{ij}^p}{2}\right) + (1-\gamma)\left(\frac{D_{ij}^m + D_{ij}^o}{2}\right)\right] Y_{ij} \leq M \quad \forall i,j \tag{15}$$

Step 2: Reformulating Multi-Objective Programming into a Single Objective

Given the dual objectives—maximizing waste collection (**Equation (11)**) and minimizing transportation distances (**Equation (12)**)—the Weighted Sum Method was applied to reformulate the model into a single-objective problem [17], [18]. This approach balances the two through a weighting parameter (α), enabling decision-makers to prioritize environmental sustainability and operational efficiency. The reformulated single-objective function is expressed as a weighted sum of normalized objectives. This reformulation ensures computational feasibility and aligns the model with decision-makers' preferences. The model also includes additional parameters to support the weighted sum approach in multi-objective optimization. The parameter α represents the weighting factor, with values ranging from 0 to 1, allowing decision-makers to adjust the relative importance of the two objectives. $Z1^U$ and $Z1^N$ denote the Utopia and Nadir points of the first objective (Z1), respectively, while $Z2^U$ and $Z2^N$ represent the Utopia and Nadir points of the second objective (Z2). These points define each objective's best and worst achievable values and are used for normalization. Additionally, $O1$ and $O2$ indicate the objective values corresponding to Z1 and Z2, respectively, in the weighted formulation.

Objective functions

$$Min\ Z = \alpha O1 + (1-\alpha) O2 \tag{16}$$

Constraints

(3), (7) - (10), (13) – (15)

$$O1 = \frac{Z1^U - Z1}{Z1^U - Z1^N} \tag{17}$$

$$O2 = \frac{Z2 - Z2^U}{Z2^N - Z2^U} \tag{18}$$

$$Z1 = \sum_i \sum_k W_{ik} \tag{19}$$

$$Z2=\sum_{i}\sum_{j}\left(\frac{D_{ij}^{p}+2D_{ij}^{m}+D_{ij}^{o}}{4}\right)Y_{ij}+\sum_{j}\sum_{m}\left(\frac{F_{jm}^{p}+2F_{jm}^{m}+F_{jm}^{o}}{4}\right)X_{j} \tag{20}$$

Step 3: Defining Feasibility Degrees, Weight Parameter, and Utopia and Nadir Points

To effectively address trade-offs in multi-objective optimization under fuzzy constraints, the proposed model integrates three key elements: feasibility degree (γ), weight parameter (α), and Utopia and Nadir points. The feasibility degree (γ) balances optimistic and pessimistic scenarios, accounting for uncertainties while ensuring practical solutions under varying conditions. By adjusting γ, decision-makers can refine the model's sensitivity to uncertainties, allowing for tailored responses to real-world challenges.

The weight parameter (α) determines the relative importance of the two objectives: maximizing waste collection efficiency (Z1) and minimizing transportation distances (Z2). With values ranging from 0 to 1, α = 0 prioritizes minimizing transportation distances, while α = 1 focuses on maximizing waste collection efficiency. Intermediate values allow for a balanced approach, enabling alignment of model outcomes with strategic goals such as sustainability or cost-effectiveness.

Utopia and Nadir points serve as benchmarks, representing each objective's best and worst achievable values. Each objective is optimized individually to determine these benchmarks, identifying the ideal (Utopia) and worst-case (Nadir) values. These benchmarks enable normalization and practical trade-off analysis. These elements establish a robust framework for navigating multi-objective decision-making, ensuring practical, balanced, and strategic solutions.

Step 4: Solving the Model

The reformulated single-objective model was solved using optimization techniques suitable for fuzzy environments. The outputs include optimal RVM placements, associated waste collection volumes, and minimized transportation costs, ensuring that the solutions meet environmental and operational objectives.

Step 5: Iterative Refinement

If decision-makers are unsatisfied with the initial results, adjustments to the feasibility degree (γ) and weighting parameter (α) can be made. This iterative process enables the model to be fine-tuned to align with evolving priorities, such as increased emphasis on cost reduction or enhanced waste collection efficiency.

3. RESULTS AND DISCUSSION

3.1 Case Study

The case study centers on optimizing 15 communities within Lampang Municipality, situated in the heart of Lampang Province, Thailand. This municipality, comprising 15 communities divided into administrative zones, is an ideal case for addressing waste management challenges. The area was chosen due to its significant waste generation rates and strategic proximity to Bangorn Recycling Group Co., Ltd., a major regional recycling facility. The study's primary objective is to determine optimal RVM placements to maximize waste collection efficiency and minimize transportation distances, while adhering to principles of environmental sustainability and social equity. All relevant data used in the study, including household numbers, waste generation rates, and transportation distances, were collected and verified during 2023.

Seventeen potential locations for RVM installation were identified through centroid analysis and surveys targeting key areas within the municipality. These locations, including public spaces, markets, and community centers, were selected based on their accessibility and high pedestrian traffic volumes, critical for enhancing public participation in recycling efforts.

Several constraints were established to ensure the feasibility and practicality of the proposed solution. RVMs are capped at five, reflecting logistical and financial considerations. The total installation budget is set at 5,000,000 THB, based on discussions and brainstorming sessions with the head of Bangorn Recycling Group. Installation costs for each machine, ranging from 800,000 to 1,200,000 THB, were obtained from a developer currently working on RVM prototypes in collaboration with the Science and Technology Park of

Chiang Mai University. Additionally, the maximum allowable distance between community areas and their assigned RVMs is restricted to 20 kilometers, ensuring ease of access for residents and minimizing transportation-related challenges. These constraints aim to balance operational efficiency, cost-effectiveness, and community convenience.

The study also evaluates seven distinct types of waste: transparent PET bottles, glass bottles, aluminum cans, dark-colored plastic cups, containers, trays, and glasses, paper cups or glasses, drinking straws, and the reflective "glittery bags." By incorporating these diverse waste types, the study seeks to develop a comprehensive recycling approach that addresses the complexities of real-world waste disposal. Disposal rates for these waste types are detailed in **Table 1**, and the assumed household waste generation rates—0.025, 0.05, and 0.075 kg per household—are derived from official municipal statistics in Lampang. The number of households within each community is provided in **Table 2**. The geographic distribution of potential RVM locations is illustrated in **Figure 3**, which also highlights the location of the Bangorn Recycling Group within the case study area of Lampang Municipality. These candidate sites in **Figure 3** are considered potential locations for installing RVMs based on accessibility and community coverage.

Table 1. Waste Disposal Rates for Different Waste Types under Fuzzy Scenarios (Unit: Kilogram)

Waste type	1	2	3	4	5	6	7
Optimistic	0.072	0.024	0.024	0.048	0.024	0.008	0.024
Most Likely	0.090	0.030	0.030	0.060	0.030	0.010	0.030
Pessimistic	0.108	0.036	0.036	0.072	0.036	0.012	0.036

Table 2. Household Population Data under Fuzzy Scenarios (Unit: Families)

Municipality	Optimistic	Most Likely	Pessimistic	Municipality	Optimistic	Most Likely	Pessimistic
1	29559	29659	29759	9	2115	2255	2395
2	25010	25160	25310	10	2342	2522	2702
3	6690	6890	7090	11	2340	2540	2740
4	2092	2242	2392	12	4262	4402	4542
5	2671	2871	3071	13	1795	1945	2095
6	2804	2904	3004	14	1464	1624	1784
7	2257	2337	2417	15	3651	3751	3851
8	4128	4228	4328				

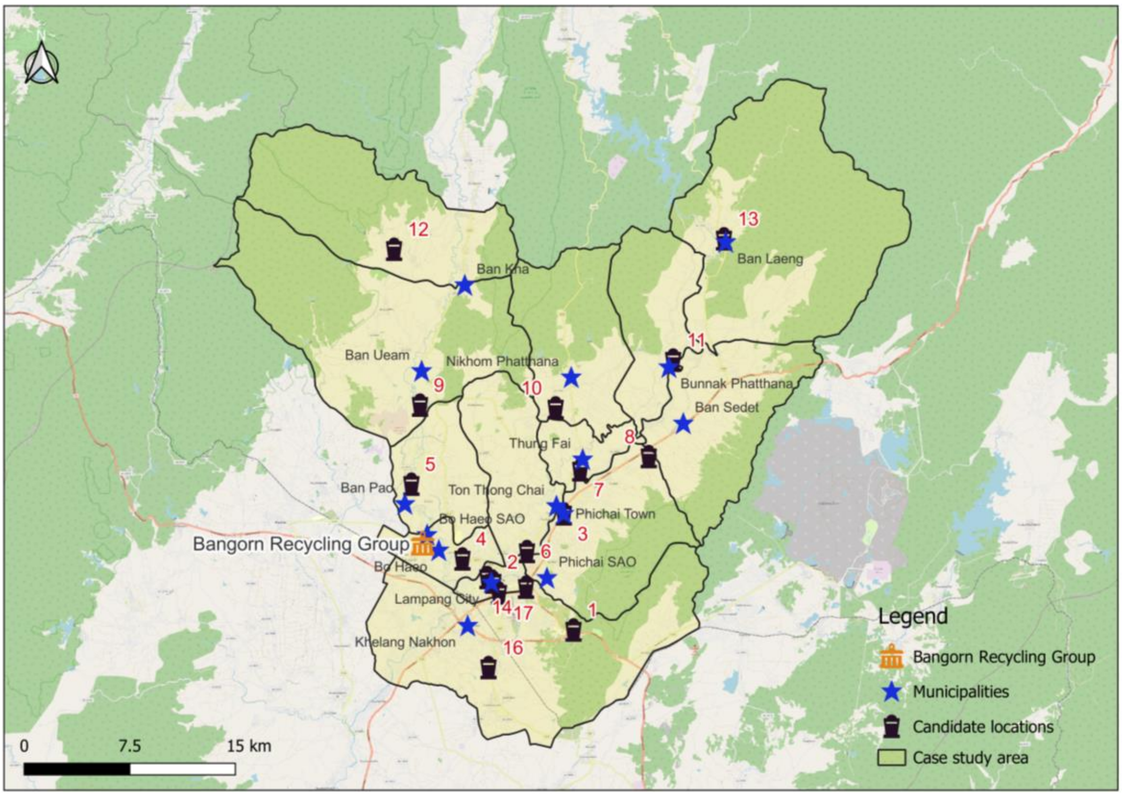


Figure 3. Geographic Map of Locations for Reverse Vending Machines in Lampang Municipality

3.2 Results from the Mathematical Model

The proposed mathematical model was coded and solved using LINGO 14.0 optimization software on a PC with an Intel Core i7-6700 CPU (3.40GHz) and 16 GB RAM to address the case study. The feasibility degree (γ) was set to 0.5 to balance the dual objectives of maximizing waste collection efficiency and minimizing transportation distances, aligning with methodologies used in similar studies. Initially, the model was solved as single-objective programming for each of the two objectives independently to evaluate their respective optimal solutions.

When prioritizing waste collection efficiency, the model identified five optimal RVM locations: Location 1, Location 4, Location 9, Location 11, and Location 15, achieving a total waste collection of 23,911.50 kilograms. These results highlight the model's effectiveness in strategically placing RVMs in high-waste-generation areas to maximize collection. However, this configuration did not minimize transportation distances, leading to higher logistics costs.

In contrast, when the model was focused on minimizing transportation distances, four RVM locations were identified as optimal: Location 4, Location 7, Location 11, and Location 12. This configuration minimized the total transportation distance from communities to RVMs and from RVMs to the recycling facility to 178.50 kilometers, significantly reducing transportation costs and environmental impacts. However, this solution did not maximize waste collection, as fewer RVMs were deployed, limiting overall coverage.

Table 3 highlights the inherent trade-offs in multi-objective optimization. The solution for maximizing waste collection efficiency resulted in greater waste coverage but longer transportation distances, while the solution for minimizing transportation distances achieved cost and environmental benefits at the expense of waste collection efficiency. As both objectives could not be simultaneously optimized in a single solution, the Weighted Sum Method was applied to reformulate the problem into a single-objective model. By assigning equal weights to both objectives, the model achieved a compromise solution that balanced waste collection efficiency and transportation distance, offering a practical and sustainable strategy for the deploying of RVMs in Lampang Municipality.

Table 3. Utopia and Nadir Points for Objectives

Objectives	Results		Utopia point	Nadir point
	Z1	Z2		
Z1	23911.50	5386.14	23911.50	5386.14
Z2	265.20	178.50	178.50	265.20

After determining both objectives' Utopia and Nadir points, the study applied the Weighted Sum Method to achieve a balanced solution. The weight parameter (α) was set to 0.5, ensuring equal prioritization of waste collection efficiency and transportation distance minimization.

Based on the results, the optimal solution identified five locations for RVMs: Location 2, Location 4, Location 7, Location 11, and Location 12. Each RVM was strategically positioned to optimize waste collection, achieving a total coverage of 23,911.50 kilograms across all sites. Additionally, the total transportation distance, including community-to-RVM and RVM-to-recycling facility distances, was minimized to 179.90 kilometers.

This configuration represents a balanced trade-off between the two objectives, achieving maximum waste collection efficiency while minimizing transportation costs and environmental impacts. The results demonstrate the effectiveness of the Weighted Sum Method in addressing the inherent trade-offs in multi-objective optimization, providing a practical and sustainable strategy for waste management in Lampang Municipality. The selected locations ensure accessibility for residents and support the region's waste recycling goals, aligning with environmental sustainability and social equity priorities.

3.3 Sensitivity Analysis

The sensitivity analysis of distance limitations (**Figure 4** (a)) reveals its impact on waste coverage (Z1) and total transportation distance (Z2). The results indicate that waste coverage remains constant at 23,911.5 kg across all tested distance limits (11–20 km), demonstrating the model's robustness in maintaining maximum collection efficiency. This suggests that expanding the allowable distance between communities and RVM locations does not affect the total waste collected, as the optimized placements effectively cover high-waste-generation areas.

In contrast, transportation distance (Z2) is highly sensitive to changes in distance limitations. At 11 km, Z2 is 214 km, reflecting routing inefficiencies. As the distance limit increases, Z2 gradually decreases, reaching 179.9 km at 18 km, beyond which it stabilizes. This indicates that further increases beyond 18 km provide no additional transportation efficiency gains, identifying 18 km as the optimal threshold for balancing waste collection and transportation costs.

These findings highlight a trade-off between distance limitations and transportation efficiency. While strict limits (≤11 km) increase transportation costs without improving waste collection, expanding the limit to 18 km significantly enhances efficiency. However, beyond this threshold, further increases offer no additional benefits. This insight is crucial for urban planners and policymakers, as setting a practical distance limit of 18 km ensures maximum waste collection efficiency while minimizing transportation costs and environmental impacts, supporting a sustainable and cost-effective waste management strategy.

The sensitivity analysis of budget limitations (**Figure 4** (b)) reveals a trade-off between financial investment, waste collection efficiency (Z1), and transportation costs (Z2). As the budget increases from 4 to 15 million Baht, Z1 steadily rises, reflecting the direct relationship between higher investments and improved waste collection efficiency. At 4 million Baht, the model collects 19,129.2 kg of waste, increasing to 71,734.5 kg at 15 million Baht, demonstrating that a larger budget allows for more RVM installations, enhancing waste collection capacity.

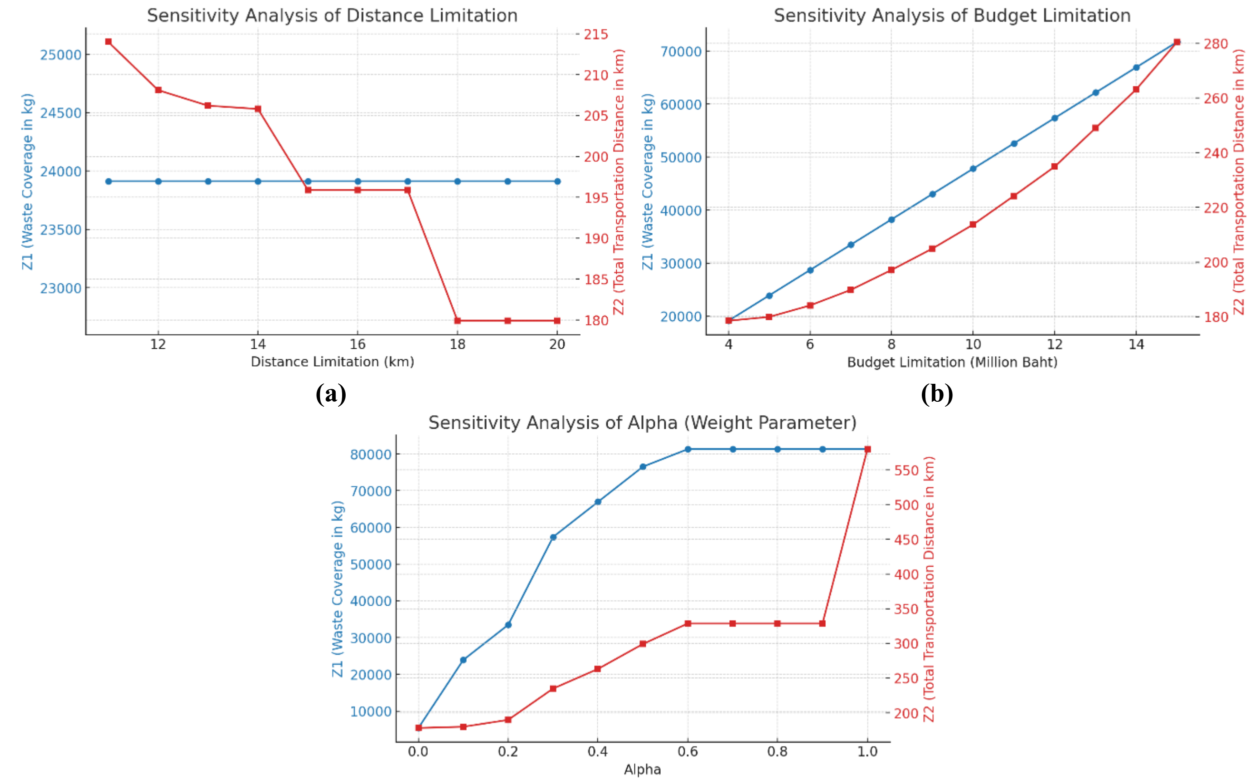
However, Z2 follows a non-linear trend. Initially, increasing the budget slightly reduces transportation efficiency, as Z2 increases from 178.5 km (at 4 million Baht) to 179.9 km (at 5 million Baht). Beyond this point, Z2 rises significantly, reaching 280.5 km at 15 million Baht. Excessive RVM installations can lead to more complex and inefficient transportation routes, suggesting diminishing returns beyond the optimal threshold.

These findings emphasize the importance of strategic budget allocation. A budget range of 4–6 million Baht ensures optimal waste collection while maintaining minimal transportation inefficiencies. Beyond this range, additional investments may increase costs without proportionate benefits. Decision-makers should balance RVM deployment and transportation logistics to achieve a cost-effective and sustainable waste management strategy. In addition to previous sensitivity analyses, we also examined the impact of the weight parameter (α) while removing the constraint on the maximum number of machines and limiting the budget in the model. This approach allows us to better understand how prioritizing waste collection efficiency affects transportation costs when machine placement is not restricted.

Figure 4 (c) illustrates the trade-off between waste coverage (Z1) and total transportation distance (Z2) as α increases. Initially, Z1 rises sharply, reaching 81,299.1 kg at α = 0.6, and remains constant until α = 1. This indicates that placing more weight on waste collection enhances efficiency; however, beyond α = 0.6, further prioritization does not improve waste collection, suggesting that an upper limit has been reached.

Conversely, Z2 (transportation distance) follows a non-linear trend. At lower α values (0–0.5), Z2 gradually increases from 178.1 km to 299.1 km, reflecting moderate inefficiencies in transportation as waste collection is emphasized. In the range of α = 0.6 to 0.9, Z2 stabilizes at 328.7 km, indicating that additional prioritization of waste collection does not significantly alter transportation distances. However, at α = 1, Z2 rises sharply to 579.9 km, demonstrating that excessive focus on waste collection leads to inefficient routing and significant transportation costs.

These findings emphasize balancing waste collection efficiency and transportation costs when selecting an appropriate α value. A moderate α $(0.5-0.6)$ achieves high waste coverage while keeping transportation costs manageable, whereas higher α values result in diminishing returns and increased logistical inefficiencies. This insight benefits decision-makers in waste management planning, ensuring that Reverse Vending Machines (RVMs) are optimally placed to support economic and environmental sustainability.



(a) (b)

(c)

Figure 4. Sensitivity Analysis of (a) Distance Limitation, (b) Budget Limitation, and (c) Weight Parameter

3.4 Discussion

This study demonstrates the efficacy of a multi-objective mathematical model for optimal RVM placement, aiming to maximize waste collection efficiency while minimizing transportation distances. This research advances prior models that relied on deterministic assumptions by incorporating fuzzy parameters to handle uncertainties and conducting sensitivity analyses. The integration of triangular fuzzy membership functions, as proposed in [14], enables the model to capture real-world variability in waste generation, transportation costs, and installation budgets.

Unlike previous studies, such as [6], which focused on location-routing problems for infectious waste collection, this study incorporates advanced waste management technologies like RVMs. Similarly, while [3] and [4] explored multi-objective facility location models, their application was limited to hazardous waste management, without addressing urban recyclable waste or modern technological systems. The sensitivity analyses further enhance practical applicability, demonstrating how budget levels and distance constraints influence waste collection efficiency and transportation costs [19]. Notably, higher budgets significantly improve waste coverage (Z1) in alignment with the United Nations Sustainable Development Goals (SDGs). However, overly restrictive distance constraints may hinder effectiveness, underscoring the need for a balanced approach.

While this study advances the field, scalability remains a key challenge. The model's application to Lampang Province provides valuable insights, but its adaptability to larger urban areas or regions with complex topographies requires further investigation. Solving large-scale multi-objective fuzzy optimization models can be computationally intensive in densely populated cities with hundreds of potential RVM locations. As the number of candidate sites and decision variables increases, exact optimization methods (e.g., LINGO) may become impractical [18].

To address scalability, future research could explore metaheuristic approaches, such as Genetic Algorithms (GA), Particle Swarm Optimization (PSO), or Hybrid Evolutionary Techniques, to achieve near-optimal solutions within reasonable computation times. Additionally, integrating Geographic Information System (GIS) data could refine site selection criteria, incorporating real-world spatial constraints like terrain complexity, infrastructure availability, and accessibility factors.

Applying the model to megacities with high population densities would also require additional considerations, including multi-zone waste flow dynamics, traffic congestion effects on transportation distances, and variations in public participation levels across different districts. Addressing these factors would enhance the robustness and adaptability of the model, ensuring its effectiveness in diverse urban environments.

This study advances sustainable waste management by integrating fuzzy multi-objective optimization with practical constraints and real-world validation. It provides actionable insights for urban planners and policymakers, supporting environmental sustainability, logistical efficiency, and social equity. This study lays the foundation for future advancements in data-driven, scalable waste management strategies by bridging gaps in prior research.

3.5 Managerial Insights

This research provides valuable managerial insights for optimizing RVM placement in urban areas, offering actionable strategies for sustainable waste management. The results demonstrate that strategic budget allocation significantly enhances waste collection efficiency. Budgets up to a threshold, such as 14,000,000 THB, yield substantial gains in waste coverage (Z1), while higher investments show diminishing returns. Decision-makers can use these findings to prioritize cost-effective RVM deployments.

The study highlights the importance of balancing waste collection efficiency and distances. Managers can align this balance with sustainability goals by locating RVMs strategically in high-waste-generation areas while minimizing transport costs and emissions. Flexibility in distance thresholds ensures accessibility without compromising efficiency, especially in dense or geographically constrained areas.

Additionally, the model's integration of fuzzy parameters ensures adaptability to uncertainties, such as fluctuating waste generation and costs, allowing scalability to different urban or regional contexts. This adaptability aligns with the United Nations Sustainable Development Goals (SDGs), supporting sustainable cities and climate action.

Finally, the research emphasizes equitable service distribution, ensuring diverse communities benefit from accessible recycling solutions. By leveraging these insights, policymakers can implement data-driven, sustainable waste management strategies that enhance environmental outcomes and community engagement.

3.6 Limitations and Future Research Directions

This study provides a valuable optimization framework for RVM placement; however, several limitations should be acknowledged. First, the model operates within a single planning horizon, assuming that waste generation rates, community participation, and budget constraints remain static. These parameters fluctuate due to seasonal tourism, economic shifts, and policy interventions. Future research could develop a multi-period dynamic model that captures temporal variations, allowing for adaptive RVM deployment strategies that adjust to changing waste generation patterns and financial constraints. A rolling optimization approach or stochastic modeling could improve real-world applicability [18].

Second, while the study broadly refers to "community engagement," it does not explicitly model user compliance, public incentives, or behavioral influences that impact RVM utilization rates. Factors such as public awareness campaigns, financial incentives, and cultural acceptance significantly determine actual participation levels. Future research should integrate behavioral modeling techniques, such as agent-based simulations or survey-based empirical analysis, to better understand how different populations respond to RVM implementation.

Third, although the study includes an installation cost constraint, it lacks detailed operational cost modeling, critical for assessing long-term financial feasibility. Future work should incorporate a comprehensive cost-benefit analysis, covering maintenance expenses, energy consumption, and revenue from collected recyclables. Additionally, the integration of economic sensitivity analysis would allow policymakers to evaluate the financial sustainability of RVM systems under fluctuating market conditions. Reviewing existing literature on operational cost structures in automated waste collection systems could further strengthen financial modeling in future studies [20].

Addressing these limitations by incorporating dynamic planning, behavioral insights, and detailed cost modeling would significantly enhance the real-world applicability and policy relevance of RVM optimization strategies.

4. CONCLUSION

This research proposed a multi-objective mathematical model for optimizing the placement of RVMs to maximize waste collection efficiency while minimizing transportation distances. The model was applied to a Lampang Province, Thailand case study, integrating constraints such as budget limitations, RVM capacity, and accessibility. Using the Weighted Sum Method, the study provided a balanced solution that effectively addressed the trade-offs between the two objectives. The findings demonstrated the potential of strategic RVM placement in enhancing waste management efficiency, achieving significant waste collection coverage, and reducing environmental impacts through optimized transportation routes. The results highlight the practicality of the proposed model in supporting sustainable waste management strategies in urban settings. Future studies could expand on this research in several ways. First, incorporating dynamic parameters such as real-time waste generation, seasonal variations, and demographic shifts could enhance the model's accuracy and applicability. Second, integrating behavioral data, such as user preferences and public compliance rates, would provide deeper insights into community engagement and the success of RVM deployment. Third, exploring multi-dimensional constraints, including environmental regulations, socio-economic considerations, and public acceptance, would offer a more holistic perspective. Finally, applying advanced optimization techniques, such as machine learning-based heuristics, could improve computational efficiency and scalability for larger urban areas or regional implementations. These advancements would further enhance the model's robustness and utility in diverse waste management scenarios.

AUTHOR CONTRIBUTIONS

Chawis Boonmee: Conceptualization, Formal Analysis, Funding Acquisition, Investigation, Methodology, Project Administration, Resources, Software, Supervision, Validation, Visualization, Writing - Original Draft. Nopphamart Khuankaew: Conceptualization, Data Curation, Investigation, Methodology, Resources, Software, Writing - Review and Editing. Phavika Mongkolkittaveepol: Formal Analysis, Investigation, Supervision, Validation, Writing - Review and Editing. All authors discussed the results and contributed to the final manuscript.

FUNDING STATEMENT

This research is part of the project "A Strategic Roadmap Toward the Next Level of Intelligent, Sustainable, and Human-Centered SME: SME 5.0," funded by the European Union's Horizon 2021 research and innovation program under the Marie Skłodowska-Curie Grant Agreement No. 101086487. Moreover, this research work was supported by Chiang Mai University.

ACKNOWLEDGEMENT

The authors thank Bangorn Recycling Group Co., Ltd. for providing valuable data support.

CONFLICT OF INTEREST

The authors declare no competing interests.

REFERENCES

[1] T. Susilawati, I. Darmawan, and M. S. Alfaresa, "OPTIMIZATION OF WASTE MANAGEMENT IN SUMBAWA DISTRICT USING A DYNAMIC SYSTEM MODEL," *BAREKENG: J. Math. & App.*, vol. 18, no. 3, pp. 1403–1410, Jul. 2024, doi: ttps://doi.org/10.30598/barekengvol18iss3pp1403-1410

[2] S. Octarina *et al.*, "DETERMINING THE OPTIMAL TEMPORARY WASTE DISPOSAL SITES IN THE ALANG-ALANG LEBAR SUB-DISTRICT PALEMBANG USING THE P-CENTRE LOCATION PROBLEM AND P-MEDIAN PROBLEM

MODELS," *BAREKENG: J. Math. & App.*, vol. 18, no. 4, pp. 2685–2702, Oct. 2024, doi: https://doi.org/10.30598/barekengvol18iss4pp2685-2702

[3] M. Wan *et al.*, "A BI-OBJECTIVE MODEL FOR THE LOCATION AND OPTIMIZATION CONFIGURATION OF KITCHEN WASTE TRANSFER STATIONS," *Systems*, vol. 12, no. 12, p. 571, Dec. 2024, doi: https://doi.org/10.3390/systems12120571

[4] H. Yu and W. D. Solvang, "A MULTI-OBJECTIVE LOCATION-ALLOCATION OPTIMIZATION FOR SUSTAINABLE MANAGEMENT OF MUNICIPAL SOLID WASTE," *Environ Syst Decis*, vol. 37, no. 3, pp. 289–308, Sep. 2017, doi: https://doi.org/10.1007/s10669-017-9632-y

[5] B. Cicekalan, A. Pekpazar, and F. Çebi, "A MULTI-OBJECTIVE OPTIMIZATION MODEL FOR WASTE-TO-ENERGY FACILITY LOCATION IN SUSTAINABLE MUNICIPAL SOLID WASTE MANAGEMENT," in *Engineering and Technology Management in Challenging Times*, vol. 1137, F. Çebi, A. T. Tekin, A. Çorum, G. D. Cömert, and G. I. S. Bolatan, Eds., in Lecture Notes in Networks and Systems, vol. 1137. , Cham: Springer Nature Switzerland, 2024, pp. 55–67. doi: https://doi.org/10.1007/978-3-031-70935-7_5

[6] S. Suksee and S. Sindhuchao, "GRASP WITH ALNS FOR SOLVING THE LOCATION ROUTING PROBLEM OF INFECTIOUS WASTE COLLECTION IN THE NORTHEAST OF THAILAND," *10.5267/j.ijiec*, vol. 12, no. 3, pp. 305–320, 2021, doi: https://doi.org/10.5267/j.ijiec.2021.2.001

[7] N. Wichapa and P. Khokhajaikiat, "A HYBRID MULTI-CRITERIA ANALYSIS MODEL FOR SOLVING THE FACILITY LOCATION–ALLOCATION PROBLEM: A CASE STUDY OF INFECTIOUS WASTE DISPOSAL," *J. Eng. Technol. Sci.*, vol. 50, no. 5, pp. 698–718, Nov. 2018, doi: https://doi.org/10.5614/j.eng.technol.sci.2018.50.5.8

[8] N. Boonpermpoon, "SELECTION OF SUITABLE METHOD AND LOCATION FOR MUNICIPAL WASTE DISPOSAL IN CHIANG MAI AND LAMPHUN PROVINCES," Chiang Mai University, 2008. [Online]. Available: https://cmudc.library.cmu.ac.th/frontend/Info/item/dc:110056

[9] A. Singh, "MANAGING THE UNCERTAINTY PROBLEMS OF MUNICIPAL SOLID WASTE DISPOSAL," *Journal of Environmental Management*, vol. 240, pp. 259–265, Jun. 2019, doi: https://doi.org/10.1016/j.jenvman.2019.03.025

[10] N. Wichapa and P. Khokhajaikiat, "SOLVING MULTI-OBJECTIVE FACILITY LOCATION PROBLEM USING THE FUZZY ANALYTICAL HIERARCHY PROCESS AND GOAL PROGRAMMING: A CASE STUDY ON INFECTIOUS WASTE DISPOSAL CENTERS," *Operations Research Perspectives*, vol. 4, pp. 39–48, 2017, doi: https://doi.org/10.1016/j.orp.2017.03.002.

[11] Y. P. Li, G. H. Huang, and S. L. Nie, "A MATHEMATICAL MODEL FOR IDENTIFYING AN OPTIMAL WASTE MANAGEMENT POLICY UNDER UNCERTAINTY," *Applied Mathematical Modelling*, vol. 36, no. 6, pp. 2658–2673, Jun. 2012, doi: https://doi.org/10.1016/j.apm.2011.09.049

[12] D. Kim, S. Lee, M. Park, K. Lee, and D.-Y. Kim, "DESIGNING OF REVERSE VENDING MACHINE TO IMPROVE ITS SORTING EFFICIENCY FOR RECYCLABLE MATERIALS FOR ITS APPLICATION IN CONVENIENCE STORES," *Journal of the Air & Waste Management Association*, vol. 71, no. 10, pp. 1312–1318, Oct. 2021, doi: https://doi.org/10.1080/10962247.2021.1939811

[13] H. Zia *et al.*, "PLASTIC WASTE MANAGEMENT THROUGH THE DEVELOPMENT OF A LOW COST AND LIGHT WEIGHT DEEP LEARNING BASED REVERSE VENDING MACHINE," *Recycling*, vol. 7, no. 5, p. 70, Sep. 2022, doi: https://doi.org/10.3390/recycling7050070

[14] M. Jiménez, M. Arenas, A. Bilbao, and M. V. Rodrı´guez, "LINEAR PROGRAMMING WITH FUZZY PARAMETERS: AN INTERACTIVE METHOD RESOLUTION," *European Journal of Operational Research*, vol. 177, no. 3, pp. 1599–1609, Mar. 2007, doi: https://doi.org/10.1016/j.ejor.2005.10.002.

[15] M. Arenas Parra, A. Bilbao Terol, B. Pérez Gladish, and M. V. Rodríguez Uría, "SOLVING A MULTIOBJECTIVE POSSIBILISTIC PROBLEM THROUGH COMPROMISE PROGRAMMING," *European Journal of Operational Research*, vol. 164, no. 3, pp. 748–759, Aug. 2005, doi: https://doi.org/10.1016/j.ejor.2003.11.028

[16] C. Boonmee and C. Kasemset, "THE MULTI-OBJECTIVE FUZZY MATHEMATICAL PROGRAMMING MODEL FOR HUMANITARIAN RELIEF LOGISTICS," *iems*, vol. 19, no. 1, pp. 197–210, Mar. 2020, doi: https://doi.org/10.7232/iems.2020.19.1.197.

[17] C. Boonmee and P. Thoenburin, "TEMPORARY SAFETY ZONE SITE SELECTION DURING HAZE POLLUTION: AN INTEGRATED APPROACH WITH FAHP and FTOPSIS," *Expert Systems with Applications*, vol. 245, p. 123002, Jul. 2024, doi: https://doi.org/10.1016/j.eswa.2023.123002

[18] C. Boonmee, K. Legsakul, and M. Arimura, "MULTI-OBJECTIVE TWO-STAGE STOCHASTIC OPTIMIZATION MODEL FOR POST-DISASTER WASTE MANAGEMENT," *Production Engineering Archives*, vol. 29, no. 1, pp. 58–68, Mar. 2023, doi: https://doi.org/10.30657/pea.2023.29.8

[19] C. Boonmee, M. Arimura, and C. Kasemset, "POST-DISASTER WASTE MANAGEMENT WITH CARBON TAX POLICY CONSIDERATION," *Energy Reports*, vol. 7, pp. 89–97, Sep. 2021, doi: https://doi.org/10.1016/j.egyr.2021.05.077

[20] T. Yankova and I. Grigorova, "AUTOMATED WASTE COLLECTION SYSTEMS - POSSIBILITIES, TRENDS AND APPLICATION," presented at the 20th International Multidisciplinary Scientific GeoConference Proceedings SGEM 2020, Sep. 2020, pp. 219–226. doi: https://doi.org/10.5593/sgem2020/4.1/s18.028