BAREKENG: Journal of Mathematics and Its Applications
March 2026 Volume 20 Issue 1 Page 0509–0522
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol20iss1pp0509-0522

HYBRID METHOD IMPLEMENTATION: FUZZY DECISION TREE IN THE CLASSIFICATION OF GENDER INEQUALITY

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Article Info

Article History:
Received: 17th April 2025
Revised: 1st July 2025
Accepted: 4th August 2025
Available online: 24th November 2025

Keywords:
C4.5 decision tree;
Fuzzy decision tree;
Gender inequality;
Hybrid method.

ABSTRACT

The classification of continuous data using the C4.5 decision tree algorithm requires prior discretization based on the calculation of cut points, a process that can be time-consuming and potentially introduce bias. These limitations may negatively impact both the computational efficiency and the classification accuracy of the decision tree model. This study proposes a hybrid method that integrates fuzzy logic with decision tree techniques in the classification process of continuous data types. Fuzzy logic is utilized to manage uncertainty in data variables and enhance flexibility in processing continuous values, while the decision tree plays a role in providing a structured and rule-based framework for decision-making. This proposed method is applied to gender inequality data, encompassing aspects of reproductive health, education and empowerment, and employment across 166 countries worldwide. The results demonstrate that the fuzzy decision tree method, which was constructed using the C4.5 algorithm, achieved a classification accuracy of 91%, while the C4.5 decision tree method without fuzzy only achieved a classification accuracy of 77%. The fuzzy decision tree method successfully improved the classification accuracy by 14%. Additionally, the fuzzy decision tree exhibited more stable and balanced performance in classifying data into four target categories. Therefore, this approach offers an effective and comprehensive alternative for classifying gender inequality, with the potential to support data-driven and targeted policy-making.

How to cite this article:

S. F. M. Irawan, D. Anggraini, and S. Annisa, "HYBRID METHOD IMPLEMENTATION: FUZZY DECISION TREE IN THE CLASSIFICATION OF GENDER INEQUALITY", *BAREKENG: J. Math. & App.*, vol. 20, iss. 1, pp. 0509-0522, Mar, 2026.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · **Open Access**

1. INTRODUCTION

A decision tree is a classification method that divides a large dataset into smaller sets based on specific decision rules. A decision tree connects attributes with possible outcomes based on specific measurements to produce homogeneous groups of data. One of the most popular algorithms for building decision trees is C4.5, which uses gain ratio measurement in selecting the best attribute. The C4.5 algorithm is commonly used due to its ability to handle discrete and continuous types of attributes and data containing missing values. However, the C4.5 algorithm assumes attributes in nominal form, so continuous attributes need to be discretized first. Discretization is performed to divide the range of continuous values using cut points determined by predefined thresholds, which establish clear boundaries within the attribute domain. The optimal cut point is selected based on the threshold that yields the lowest entropy, often achieved by balancing the number of samples and considering the similarity of the target variable categories. This process can be time-consuming, may not always accurately represent the original values, and has the potential to introduce bias, especially in large datasets [1], [2]. These limitations may adversely impact the computational efficiency and classification performance of the decision tree.

To overcome the limitations of discretization, fuzzy logic is applied as an alternative due to its ability to interpret continuous values more accurately and representatively based on fuzzy membership degree calculations [3], [4]. In contrast, fuzzy logic offers a robust framework for managing ambiguity, uncertainty, and incomplete information by emulating human reasoning through the use of linguistic variables. The fuzzy logic concept is implemented through the fuzzification of continuous data and a fuzzy inference system to provide a better classification result. The fuzzification process, by converting crisp data into fuzzy data, is able to represent the uncertainty and variation present in the data. The fuzzy inference system using the General Fuzzy Reasoning Method (GFRM) is capable of performing a comprehensive estimation of the formed decision tree model. Several previous studies have shown that the application of fuzzy decision trees was able to improve accuracy and efficiency in various cases, such as in disease diagnosis, predicting student graduation times, and fraud detection. Previous studies conducted by [5], [6], implemented a fuzzy decision tree using the ID3 algorithm for the purpose of diagnosing diseases. Both of the studies demonstrated that the proposed method successfully outperformed the ID3 algorithm without fuzzy in terms of accuracy, recall, precision, and F1-score. A study conducted by [7], aims to predict students' graduation time using a fuzzy decision tree with the ID3 algorithm and the Mamdani fuzzy inference system. The results show that the proposed method achieves an accuracy of 95.85%, while the method using the ID3 algorithm without fuzzy yields an accuracy of only 93.41%. Another study conducted by [8], developed a fuzzy decision tree using the C4.5 algorithm to detect transactional frauds, resulting in better performance with a higher detection rate and improved computational efficiency compared to traditional method. In recent years, fuzzy reasoning has seen advancements through the integration with ensemble learning, deep learning, and type-2 fuzzy systems, which offer better handling of uncertainties and noise in real-world data including socio-demographic data modeling.

Some previous studies have primarily focused on the use of the ID3 or C4.5 algorithms. However, they exhibit limitations in fuzzy integration, as fuzzy concept was applied only in the data fuzzification process. In contrast, this study explores the application of the C4.5 algorithm as an extension of the ID3, alongside a distinct fuzzy inference system known as the General Fuzzy Reasoning Method (GFRM), offering a novel approach to enhance classification performance. In this study, the hybrid method fuzzy decision tree will be tested on the Gender Inequality Index (GII) dataset covering 193 countries. The GII will be grouped into four categories based on the dimensions of reproductive health, education and empowerment, and employment. The indicator variables in this dataset are continuous in type, thereby qualifying them for the application and evaluation of the proposed method. The United Nations Development Programme (UNDP) originally set a goal to achieve global gender equality by 2030. However, with the current level of gender equality at 68.1%, it is projected that it will take approximately 132 years to fully eliminate gender disparities worldwide [9], [10]. Recent study in Indonesia highlighted significant disparities across provinces using GII clustering, addressing that socio-economic conditions, cultural barriers, and governance frameworks significantly influence gender dynamics at the regional level [11]. While previous research has predominantly focused on analyzing gender inequality at the regional or national level, this study addresses the issue by utilizing global data from 193 countries to provide a more comprehensive and comparative perspective on gender disparities worldwide. Thus, this research will identify the accuracy of the fuzzy decision tree method in classifying the Gender Inequality Index (GII) of countries worldwide compared to the C4.5 decision tree method without fuzzy.

2. RESEARCH METHODS

2.1 Materials and Data

This research uses secondary data obtained from the Human Development Reports 2022 and provided by the United Nations Development Programme (UNDP) [9]. This study uses the Gender Inequality Index (GII) data from 193 countries worldwide, categorized into 4 categories which are low value of GII or category 0, moderate value of GII or category 1, high value of GII or category 2, and very high value of GII or category 3. The variables used in this study are presented in Table 1.

Table 1. Data Variables

Variable	Description	Data Type
GII Category (Y)	4 categories that reflected the value of GII.	Ordinal
Maternal Mortality Ratio (X_1)	Pregnancy-related deaths as a percentage of 100,000 live births	Continuous
Adolescent Birth Rate (X_2)	Birth rate among females aged 15 to 19 per 1,000 individuals	Continuous
Female Percentage in Parliament (X_3)	Percentage of seats in the national parliament that are occupied by women	Continuous
Female Population with At Least Some Secondary Education (X_4)	Percentage of the population ages 25 and older that has reached a secondary level of education	Continuous
Female Labor Force Participation Rate (X_5)	Proportion of the working-age population (ages 15 and older) that engages in the labor market	Continuous

***Data source**: United Nations Development Programme (UNDP)*

Several software tools were utilized to support data analysis in this research. Python programming was executed via Google Colaboratory to perform data visualization, data preparation, data fuzzification, data partitioning, and the construction of a decision tree model. Microsoft Excel 2021 was used to perform GFRM calculations and to construct the confusion matrix for evaluating the classification performance. The steps undertaken for the proposed method in this study are summarized in the flowchart below.

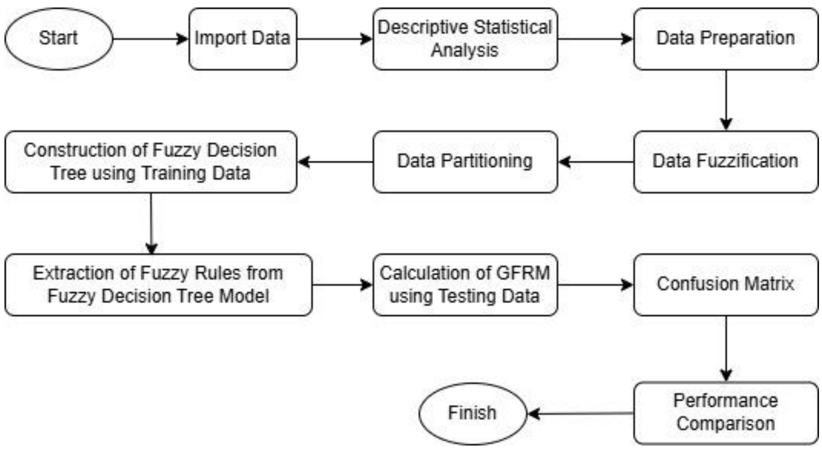


Figure 1. Flowchart of the Proposed Fuzzy Decision Tree Method

Based on Fig. 1, the process includes descriptive statistical analysis through data visualization, data preparation, data fuzzification, model training, and rule extraction, followed by the GFRM using the testing dataset. The model's performance is then evaluated using a confusion matrix to be compared with the traditional C4.5 decision tree without fuzzy integration.

2.2 C4.5 Algorithm

An extension of the ID3 algorithm, the C4.5 algorithm can handle both discrete and continuous types of attributes, contain missing values, perform pruning, and derive a series of rules. The C4.5 algorithm is estimated based on the gain ratio measurement, where the attribute that achieves the maximum gain ratio will be chosen as the splitting attribute. In constructing a decision tree, C4.5 applies several calculations, namely entropy calculation, information entropy, information gain, split info, and gain ratio. Entropy measures the impurity of the input in a set to be classified. Information gain is calculated for each attribute based on the entropy value. The value of information gain is then compared with the value of split info to obtain the gain ratio [12]. Calculation of the entropy and information entropy is obtained through Eqs. (1) and (2).

$$Info(D) = \sum_{i=1}^{m} -p_i log_2(p_i)\,, \tag{1}$$

$$Info_A(D) = \sum_{j=i}^{n} \frac{|D_j|}{|D|} \times Info(D_j)\,. \tag{2}$$

Then the information gain is calculated through Eq. (3).

$$Gain(A) = \ Info(D) - Info_A(D). \tag{3}$$

The following step is the calculation of the gain ratio, which is obtained by dividing the information gain by the split info.

$$SplitInfo_A(D) = -\sum_{j=1}^{n} \frac{|D_j|}{|D|} \times log_2\left(\frac{|D_j|}{|D|}\right). \tag{4}$$

Thus, the gain ratio is calculated by the equation below.

$$GainRatio(A) = \frac{Gain(A)}{SplitInfo_A(D)}. \tag{5}$$

Where D denotes the set of cases, i is a partition of D, and m is the number of partitions D, $C_{i,D}$ is the number of samples with class C_i in D, p_i is the proportion of $C_{i,D}$ to D. Then A denotes attribute, j is a partition of A and n is the number of partitions of A. Thus, D_j is the number of samples in partition j, $|D_j|$ denotes proportion of D_j to D, and $|D|$ is the number of cases in D.

2.3 Fuzzy Set Theory

A fuzzy set $\tilde{A}$ in X, where X is a series of objects represented by x, defined by Eq. (6) below:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x))|x \in X\} \tag{6}$$

where $\boldsymbol{\mu_{\tilde{A}}(x)}$ denotes the membership degree to where x belongs to the fuzzy set $\boldsymbol{\tilde{A}}$, which maps elements of X to the membership range M. A fuzzy set is a collection of objects with a range of membership levels [13]. Fuzzy set members are interpreted in the form of linguistic variables as words or sentence in natural or synthetic language. The characteristic of a fuzzy set is that its members or objects are grouped based on their degree or level of membership [14]. The membership value encompasses all real numbers within the interval 0.0 – 1.0, where the maximum membership is indicated by a membership value of 1. Meanwhile, an object that has a membership value of 0 does not belong to that membership.

2.4 Triangular Fuzzy Membership Function

The triangular fuzzy membership curve is one of the most common membership functions that is constructed by three parameters, namely a, b, and c [15]. These parameters formed two linear lines, which can be seen in Fig. 2 [16]. The triangular membership function was chosen due to its methodological simplicity and practical ease of assignment. With only three defining points or parameters (a, b, c), it simplifies the process of data assignment and reduces potential ambiguity during the fuzzification stage.

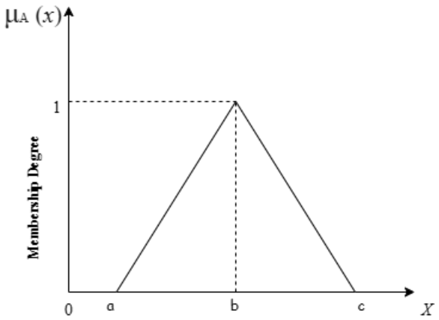


Figure 2. The Curve of Triangular Fuzzy Membership Function

The x coordinates of the three corners of the triangular fuzzy membership function are determined by those parameters with $a < b < c$ [17]. The equation of this membership function is written as follows.

$$\mu_A(x) = \begin{cases} 0, & x \leq a \\ (x-a)/(b-a), & a < x \leq b \\ (c-x)/(c-b), & b < x \leq c \\ 0, & c < x \end{cases} \tag{7}$$

Where a denotes the minimum value of a variable class, b denotes the value between a dan b with a membership degree equal to 1, and c denotes the maximum value of a variable class.

2.5 Fuzzy Decision Tree

The fuzzy concept is integrated into the process of discretizing the values of continuous variables and is carried out based on fuzzy membership function rules, a process known as fuzzification [18]. Fuzzy membership functions convert continuous attribute values into linguistic variables with several categories. Fuzzy rules can enhance the interpretability of algorithms in a decision tree for classifying linguistic variables, compared to computational models [19]. The steps of constructing the fuzzy decision tree that will be carried out are explained as follows.

1. Fuzzification of data using a triangular fuzzy membership function. The purpose of this step is to convert continuous-valued attributes by calculating how much the continuous value represents linguistic categories in the fuzzy set, which is referred to as the degree of membership value. The linguistic category with the highest degree of membership will be chosen to represent the continuous value.
2. Splitting the fuzzified dataset into training and testing data.
3. The C4.5 algorithm is implemented in the formation of a decision tree using fuzzified training data. Splitting attributes in the construction of the decision tree is based on the calculation of the gain ratio.
4. Rule-based extraction is carried out from the decision tree that has been constructed in step 3, which will then be used in decision-making. Fuzzy rules are generated from each path of the decision tree from the root node to the leaf node. These rules are obtained in linguistic form with an if-then format.

2.6 General Fuzzy Reasoning Method

General fuzzy reasoning method (GFRM) is a fuzzy inference method used to classify data that has been fuzzified based on the formed fuzzy rules. This method is based on the calculation of the compatibility degree between each sample and each fuzzy rule. GFRM calculates the final result based on the sum of aggregates from all fuzzy rules, rather than selecting only the rule with the highest firing strength, leading to a more inclusive and comprehensive reasoning process. In contrast, other inference method, such as Mamdani, often uses a max-aggregation approach, which only retains the maximum value among the rule outputs. This inclusive nature of GFRM makes it more suitable for complex classification problems, such as those involving socio-economic indicators like the Gender Inequality Index (GII). The steps taken in GFRM are as follows.

1. Calculating the degree of compatibility between each sample and each fuzzy rule, as written in the following equation [20].

$$compat(e_p, R_k) = t[M_i(a_{pi}), \dots, M_m(a_{pm})] \tag{8}$$

Where:

e_p : the p-th sample in the data testing with $p = 1, 2, 3, \dots, n$.
R_k : the k-th fuzzy rules with $k = 1, 2, 3, \dots n$.
t : t-norm (min-max function)
$M_i(a_{pi})$: membership degree of a sample in the data testing with $i = 1, 2, 3, \dots, m$.

2. Calculating the classification value for each category in the target variable. Fuzzy rules with the same category classification are aggregated for each sample, which is written in the equation as follows [20].

$$class_j = f\{compat(e_p, R_k)\}. \tag{9}$$

Where:

$class_j$: the category of R_k.
f : the operator of aggregation.

Steps 1 and 2 are repeated for each sample. The category with the highest aggregate value is determined as the classification result for that sample.

2.7 Confusion Matrix

In this study, multi-class classification was performed and measured using an evaluation matrix known as a confusion matrix. A confusion matrix consists of several measurement values arranged in columns and rows, namely true positive (TP), false positive (FP), true negative (TN), and false negative (FN). Multi-class classification obtains these values by assuming one class to be the positive class, while the rest of the classes become the negative classes [21].

Table 2. The Measurements of the Confusion Matrix

Measurement	Definition
TP	Samples are correctly classified into the positive class
TN	Samples are correctly classified into the negative class
FP	Samples from the negative class are incorrectly classified into the positive class
FN	Samples from the positive class are incorrectly classified into the negative class

The information obtained from the confusion matrix is then used to measure the accuracy for each class and the average accuracy for all classes. Accuracy is measured by the number of samples that are correctly classified out of the total samples. The accuracy measure indicates how closely the analysis results approach the true value, which reflects the precision of the data and is related to systematic errors or biases in the analysis. The equations used to calculate the accuracy value per class and the average accuracy are written in Eqs. (10) and (11).

$$\text{Class Accuracy} = \frac{TP + TN}{TP + FP + FN + TN} \times 100\%, \tag{10}$$

$$\text{Average Accuracy} = \frac{\sum_{u=1}^{v} \frac{TP_u + TN_u}{TP_u + FP_u + FN_u + TN_u}}{v} \times 100\%. \tag{11}$$

Where u denotes the category of the target variable and v denotes the number of categories in the target variable.

2.8 Gender Inequality Index

The Gender Inequality Index (GII) illustrates how gender-based inequality reduces the potential for advancements in human development, making it an important indicator for assessing the success of development and empowerment for women [22]. The GII is formed by a combination of measurements using three dimensions, namely reproductive health, education and empowerment, and labor force participation. These dimensions are further divided into several indicators that are measured separately for women and men [9]. The GII value ranges from 0 to 1, where a high GII value indicates a significant disparity between women and men based on these three dimensions.

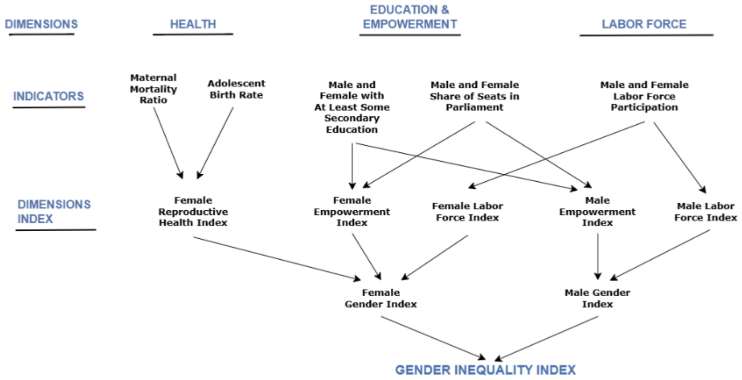


Figure 3. Dimensions of the GII

The indicators used in the model are presented in Fig. 3, namely Maternal Mortality Ratio, Adolescent Birth Rate, Female Share of Seats in Parliament, Female with At Least Some Secondary Education, and Female Labor Force Participation. These five indicators were selected based on their relevance to measuring gender inequality and serve as the input variables for the classification model.

3. RESULTS AND DISCUSSION

The relation of each indicator variable to the target variable is presented in the boxplot diagram, which can be seen in Fig. 4. The boxplots illustrate the relationship between each of the five GII input indicators and the target GII category. These visualizations help reveal how indicator values vary across the four GII categories.

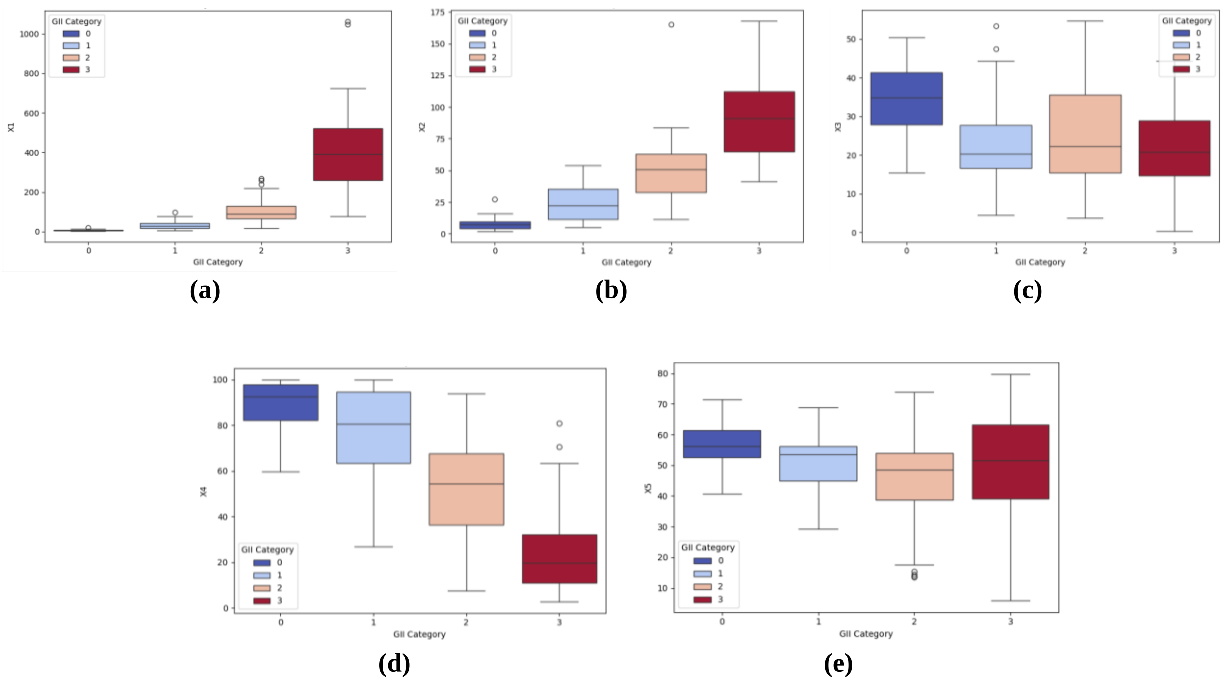
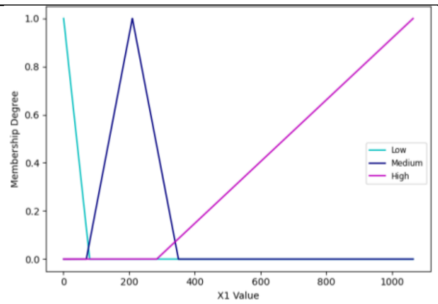
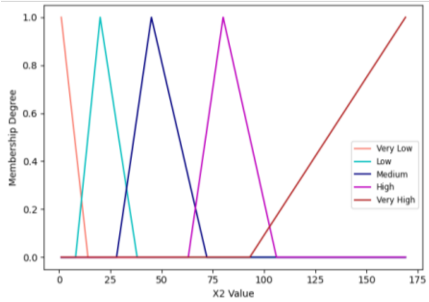
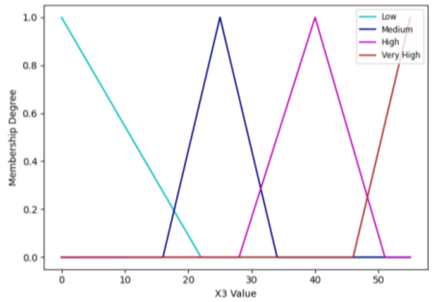
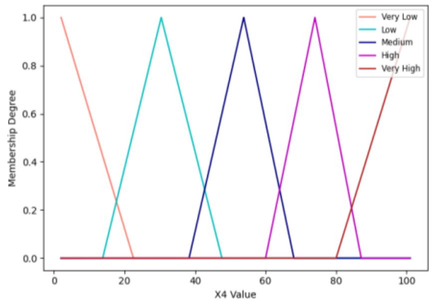
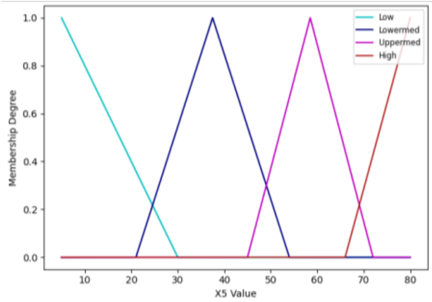


Figure 4. Boxplot between each Indicator Variable and Target Variable, (a) Boxplot of X_1 vs Y, (b) Boxplot of X_2 vs Y, (c) Boxplot of X_3 vs Y, (d) Boxplot of X_4 vs Y, (e) Boxplot of X_5 vs Y

Based on Fig. 4, the variable maternal mortality ratio and adolescent birth rate show an increase in line with the increase in the GII category. On the other hand, the variable female population with at least some secondary education shows a decrease in line with the decrease in the GII category. Meanwhile, for the variable female percentage in parliament and female labor force participation rate, it can be observed that there is no consistent pattern of increase or decrease between these variables and the GII category.

Based on Fig. 2 and using Eq. (7), fuzzification is performed on the indicator variables to obtain the membership degree values of each membership function shown in Table 3.

Table 3. Parameters of Each Membership Function

Variable	Graph of Membership Function	Membership Function	Parameters
X_1		Low	1; 1; 80
		Medium	70; 210; 350
		High	284; 1064; 1064
X_2		Very Low	1; 1; 14
		Low	8; 20; 38
		Medium	28; 45; 72
		High	63; 80; 106
		Very High	93; 169; 169
X_3		Low	0; 0; 22
		Medium	16; 25; 34
		High	28; 40; 51
		Very High	46; 55; 55
X_4		Very Low	2; 2; 22,5
		Low	13,8; 30,4; 47,6
		Medium	38,3; 53,8; 68
		High	60; 74; 87,1
		Very High	80; 101; 101
X_5		Low	5; 5; 30
		Lowermed	21; 37,5; 54
		Uppermed	45; 58,5; 72
		High	66; 80; 80

The fuzzification process is guided by classifications and thresholds established by the United Nations. For example, the fuzzification of variable X_1 or maternal mortality ratio was based on the Sustainable Development Goal (SDG) Target 3.1, which sets the global target for maternal mortality at 70 deaths per 100,000 live births. Values above this threshold were further partitioned into two fuzzy sets due to the broad variability in the data exceeding 70. Therefore, the number of fuzzy membership functions and the parameters assigned to each variable were determined based on the distribution of the data and aligned with relevant global standards, such as the UN or SDG classifications. Then the observations are labeled based on the highest membership degree value obtained. Labeling starts with label 0 for the first membership function, label 1 for the second membership function, and so on until the last membership function.

The base decision tree model is first created using the C4.5 algorithm. Several parameter scenarios were tested to find the optimal configuration. The highest accuracy was produced by the max depth parameter set to 5, which will be used in the formation of the fuzzy decision tree model. The implementation of the C4.5 algorithm was carried out with a training data proportion of 70%. The first calculation uses Eq. (1) to obtain the entropy value. Then, the information entropy is calculated using Eq. (2), information gain is calculated using Eq. (3), split info is calculated using Eq. (4), and gain ratio is calculated using Eq. (5). Thus, the calculation results for determining the root node are obtained in Table 4.

Table 4. Calculation Result for the Root Node Selection using the C4.5 Algorithm

Variable	GII Category				N	Entropy	Info Gain	Split Info	Gain Ratio
	0	1	2	3					
Total	30	29	27	30	116	1.999			
X_1							0.837	1.390	0.602
0	0	13	25	30	68	1.508			
1	8	16	2	0	26	1.239			
2	22	0	0	0	22	0.000			
X_2							0.885	2.210	0.401
0	0	0	7	25	32	0.758			
1	0	7	13	5	25	1.469			
2	10	16	7	0	33	1.503			
3	10	5	0	0	15	0.918			
4	10	1	0	0	11	0.439			
X_3							0.129	1.744	0.074
0	11	9	9	2	31	1.822			
1	14	12	12	10	48	1.990			
2	5	6	5	16	32	1.790			
3	0	2	1	2	5	1.522			
X_4							0.665	2.258	0.294
0	13	0	0	0	13	0.000			
1	11	9	1	0	21	1.222			
2	4	10	6	2	22	1.790			
3	2	8	8	8	26	1.854			
4	0	2	12	20	34	1.221			
X_5							0.181	1.536	0.118
0	4	6	0	0	10	0.971			
1	11	9	9	5	34	1.949			
2	11	12	18	24	65	1.927			
3	4	2	0	1	7	1.379			

Based on Table 4, the GII Category columns represent fuzzy membership groupings assigned during data fuzzification, and N refers to the number of samples in each category. It is known that the highest gain ratio value is obtained from the variable X_1, with the value of 0.602. Therefore, this variable is chosen as the root node in the decision tree. In the variable X_1, those three categories become the branches of the root node. Category 2 with an entropy of 0 indicates maximum data homogeneity, so this branch directly leads to a leaf node and produces a decision. Meanwhile, categories 0 and 1 will become branches leading to internal nodes at the next depth of the decision tree. Recalculations need to be performed in the same manner to determine the internal nodes. The iteration will be stopped if the entropy value in all category branches has reached maximum homogeneity, indicated by an entropy value equal to 0. Although variable X_1 was selected as the root node due to its highest gain ratio, gain ratio calculations for other variables (as shown in Table 4) also offer insights into their relative importance. X_3 and X_5 show lower gain ratio values (0.074 and 0.118, respectively), suggesting a weaker contribution to the model. These variables may not contribute consistently across splits, which may explain their absence in early splits. After the fuzzy decision tree model is formed, visualization of the model is carried out to obtain the fuzzy rules.

Rules are extracted based on each path formed in the fuzzy decision tree, from the root node to the leaf node. A total of 81 paths were obtained, which were then extracted to form 81 fuzzy rules. The formation of fuzzy rules is written based on a fuzzy *if-then* format that consists of 2 parts, namely the antecedent, which is located between *if* and *then*, and the consequent, which is located after *then*. In the antecedent part, the *and* operator is used because the value of the target variable is essentially obtained from the calculations of the five indicator variables, so all five indicator variables have equal importance in forming the target variable.

The GFRM method begins with calculating the compatibility degree of each sample toward each extracted fuzzy rule using Eq. (8). The *min* operator is used at this step, so the degree of compatibility is obtained from the minimum membership degree of each variable that forms the fuzzy rule. Thus, each sample will have a total of 81 compatibility degree values. The process for one of the samples, denoted as sample B, is presented in Table 5. Out of the 81 rules generated, the calculation of the compatibility degree for sample B is demonstrated with respect to two selected fuzzy rules, each having a different consequent.

Table 5. Membership Degree of Sample B

Variable \ Category	0	1	2	3	4
$\mathbf{X_1}$	0.676	0.000	0.000		
$\mathbf{X_2}$	0.000	0.015	0.572	0.000	0.000
$\mathbf{X_3}$	0.000	0.174	0.369	0.000	
$\mathbf{X_4}$	0.000	0.000	0.000	0.082	0.287
$\mathbf{X_5}$	0.000	0.407	0.169	0.000	

1. Rule 1 : *If* X_1=0 *and* X_2=1 *and* X_3=2 *and* X_5=1, *then* the sample is classified in the GII category 0.

$$compat(e_B, R_1) = min[0.676, 0.015, 0.369, 0.407] = 0.015 \tag{12}$$

2. Rule 2 : *If* X_1=0 *and* X_2=2 *and* X_3=1 *and* X_4=4 *and* X_5=1, *then* the sample is classified in the GII category 1.

$$compat(e_B, R_2) = min[0.676, 0.572, 0.174, 0.287, 0.407] = 0.174 \tag{13}$$

Rule 1 reflects the condition under which the sample B most likely belongs to the GII category 0, whereas rule 2 reflects the condition under which the sample B most likely belongs to the GII category 1. Next, the classification value is calculated using Eq. (9), where the classification values for each GII category are obtained by aggregating the compatibility degree values corresponding to the same GII category. Thus, the classification value for sample B is presented in Table 6.

Table 6. Classification Value of Sample B

GII Category	Number of Aggregated Rules	Classification Value
0	13 rules	0.015
1	25 rules	1.140

GII Category	Number of Aggregated Rules	Classification Value
2	26 rules	0.000
3	17 rules	0.000

To determine the classification result for each sample, the *max* operator is used. The GII category with the highest classification value is assigned as the predicted category for the sample. As shown in Table 6, the sample B is classified into GII category 1, which has the highest classification value of 1.140. As the predicted classification is obtained, the evaluation process is conducted using the confusion matrix for the C4.5 decision tree model and fuzzy decision tree model. Model evaluation was conducted on the testing set, and the result was obtained in the form of a confusion matrix, which is shown in a heatmap as follows.

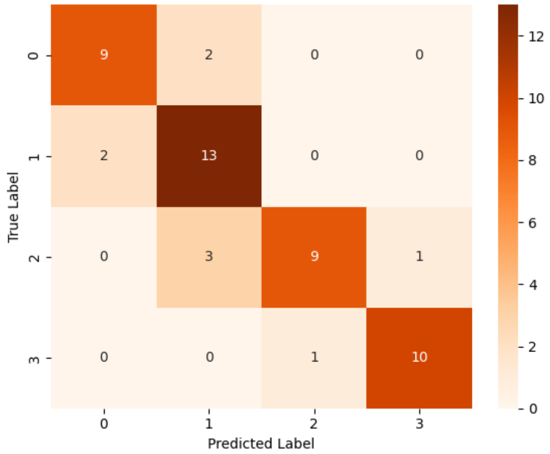


Figure 5. Heatmap of Fuzzy Decision Tree Confusion Matrix

Visualization of the classification performance of the fuzzy decision tree model across four GII categories is shown in Fig. 5, where the diagonal cells from top left to bottom right represent correctly classified samples. Then, the accuracy of each GII category is calculated using Eq. (10) and the average accuracy is calculated using Eq. (11). Thus, a comparison of the accuracy values of the two models is obtained as follows.

Table 7. Comparison of Accuracy Value

Evaluation Measurement	Model	
	C4.5 Decision Tree	Fuzzy Decision Tree
GII Category 0 Accuracy	66%	92%
GII Category 1 Accuracy	70%	86%
GII Category 2 Accuracy	80%	90%
GII Category 3 Accuracy	92%	96%
Average Accuracy	77%	91%

Based on Table 7, the fuzzy decision tree model is able to improve the accuracy values. The accuracy of the GII category 0 increased by 26%, the accuracy of the GII category 1 increased by 16%, the accuracy of the GII category 2 increased by 10%, and the accuracy of the GII category 3 increased by 4%. In terms of average accuracy, the C4.5 decision tree model achieved an accuracy of 77%, while the fuzzy decision tree model achieved an accuracy of 91%. Thus, an accuracy improvement of 14% was obtained in the fuzzy decision tree model compared to the C4.5 decision tree model. In the C4.5 decision tree, there is quite a contrasting difference between each GII category, where the highest accuracy is at 92%, while the lowest accuracy is at 66%. Meanwhile, in the fuzzy decision tree, the highest accuracy is at 96%, while the lowest accuracy is at 86%. Thus, the difference in accuracy between GII categories remains in a relatively stable range.

Table 8. **Precision, Recall, and F1-Score of the Proposed Method**

GII Category	Precision	Recall	F1-Score
0 (Low)	82%	82%	82%
1 (Medium)	72%	87%	79%
2 (High)	90%	69%	78%
3 (Very High)	91%	91%	91%
Macro Average	84%	82%	82%

In addition to accuracy, Table 8 presents a more detailed evaluation of model performance. Macro precision, recall, and F1-Score values above 80% indicate that the model performs consistently well across all GII categories, reflecting both accurate and balanced classification performance without bias toward any specific class. To evaluate overfitting due to the relatively small dataset size and the large number of fuzzy rules, we conducted a performance comparison between the training and testing datasets. The fuzzy decision tree achieved an average accuracy of 92.2% on the training data and 91% on the testing data. This small difference of 1.2% indicates that the model generalizes well to unseen data and does not exhibit significant overfitting. The results demonstrate that, despite the high number of fuzzy rules, the model maintains consistent predictive performance across both datasets.

The quantitative results show that the model performs best in identifying regions with Very High GII, indicated by a 96% accuracy value. This suggests that countries with severe gender inequality tend to have more distinct and recognizable patterns in the input variables, making them easier to classify. Conversely, the Medium and High categories yield slightly lower accuracy (86% and 90%), which may reflect the nuanced and overlapping socio-economic conditions within these groups. In practical terms, this means that while the model is highly reliable in detecting areas of critical concern, it may require more refined features or policy-based context to distinguish between intermediate levels of inequality. These insights can help inform targeted policy interventions, especially in regions where the classification results are less clear.

The main limitation of the proposed method lies in the GFRM calculation process, which was conducted manually without full computational automation. As a result, the procedure is highly susceptible to human error, especially when applied to datasets with a large number of records or when the data has numerous GII categories. The lack of automation increases the potential for inconsistencies and inefficiencies in the rule derivation and inference stages. This limitation is acknowledged, and future work will aim to develop or adopt computational tools to automate the GFRM process, thereby improving the accuracy of classification performance.

4. CONCLUSION

The conclusion obtained from this research is written as follows.

1. The fuzzy decision tree model achieved an improvement in the classification result compared to the C4.5 decision tree model without fuzzy. The improvement can be seen in the measurement of average accuracy and variation in accuracy between different GII categories, which is relatively more consistent.
2. The application of the fuzzy concept through data fuzzification and fuzzy inference systems has proven to provide more accurate results in the case of classification. The fuzzification process, which was conducted by converting continuous data into fuzzy linguistic data, has proven to be able to represent the uncertainty in the data and address clear boundaries in the continuous value. The fuzzy inference system GFRM has also proven capable of providing more accurate decisions by performing a comprehensive estimation of the formed decision tree model. This study has successfully offered a clearer understanding of how the fuzzy inference system differentiates outcomes based on varying rule structures and the membership values associated with the input samples. Classification using the fuzzy decision tree model has proven to be particularly relevant for continuous data types, as in this research, because continuous data tends to have ambiguous value boundaries and diverse variations.

Author Contributions

Siti Fatimah Marliany Irawan: Conceptualization, methodology, software, visualization, formal analysis, validation, writing-original draft. Dewi Anggraini: Conceptualization, draft preparation, validation, funding acquisition, supervision. Selvi Annisa: Methodology, visualization, validation, funding acquisition, supervision. All authors discussed the results and contributed to the final manuscript.

Funding Statement

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Acknowledgment

The authors would like to express their sincere gratitude to the Department of Statistics, Faculty of Mathematics and Natural Sciences, Universitas Lambung Mangkurat, for the continuous support and academic resources provided throughout this research. The authors also gratefully acknowledge the reviewers for the insightful feedback and recommendations, which have significantly improved the quality of this manuscript.

Declarations

The authors declare that they have no conflicts of interest to report related to this study.

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