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ENHANCING VOLATILITY MODELING WITH LOG-LINEAR REALIZED GARCH-CJ: EVIDENCE FROM THE TOKYO STOCK PRICE INDEX

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ABSTRACT

This study compares the Log-linear Realized GARCH (LRG) and its extension with Continuous and Jump components (LRG-CJ) in modeling the volatility of financial assets, using daily data from the Tokyo Stock Price Index (TOPIX) over 2004–2011. The urgency arises from the need for more accurate volatility models during turbulent periods such as the 2008 Global Financial Crisis and the 2011 Great East Japan Earthquake, where markets exhibit both smooth fluctuations and abrupt jumps. Methodologically, the LRG-CJ framework introduces a novel integration of continuous and jump decomposition into the LRG structure, offering an applied innovation to high-frequency volatility modeling. Realized Volatility (RV) was calculated from 1-, 5-, and 10-minute intraday data and decomposed into continuous and jump components. Parameter estimation employed the Adaptive Random Walk Metropolis (ARWM) within a Markov Chain Monte Carlo algorithm, while model performance was assessed using multiple information criteria and out-of-sample forecast evaluations. The empirical results reveal that incorporating continuous and jump components improves volatility modeling accuracy, forecasting, and Value-at-Risk estimation. However, these benefits are frequency-dependent: the LRG-CJ model shows superior in-sample fit for 1-minute RV but provides the strongest out-of-sample forecasting and risk prediction at lower frequencies (5- and 10-minute intervals). This highlights that while jumps are best identified at ultra-high frequencies, their predictive value is most effectively captured in slightly aggregated data. The originality of this study lies in being the first empirical application of LRG-CJ, demonstrating how continuous–jump decomposition interacts with the dual-equation structure of LRG, which has not been examined in TGARCH or APARCH contexts. Limitations include sensitivity to microstructure noise in very high-frequency data and computational challenges in parameter convergence. Overall, the findings underscore the novelty and practical importance of the LRG-CJ framework for risk management, offering actionable guidance for aligning volatility models with data frequency

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1. INTRODUCTION

In the challenging and rapidly changing world of financial markets, a deep understanding of asset price fluctuations is essential for market participants and analysts. Volatility, which refers to asset price fluctuations, plays an important role in decision-making in financial markets [1]. The higher the volatility, the higher the risk of loss and instability in financial markets. Therefore, it is important to develop a model that is able to describe the financial asset volatility.

One of the methods to estimate and measure the financial market asset volatility is by using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model (see [2] for the formula). However, to improve the prediction accuracy, the GARCH model has been extended to the GARCH-X model in [3] by incorporating high-frequency data, such as the Realized Volatility (RV) measure, as exogenous variables, into the volatility dynamics equation. Furthermore, the study by [4] introduced the GARCH-CJ model that decomposes exogenous variables into continuous component *C* and discontinuous jump *J*. The results show that the GARCH-CJ model has a more accurate and better prediction ability to measure the financial market asset volatility.

In stock market analysis, continuous variables refer to data related to gradual or scaled changes, while jump variables indicate sudden or discontinuous changes. The relationship between absolute returns, continuous variables, and jump variables indicates a high level of volatility in the stock market at certain points. For investment decision-making and risk management, a deep understanding of the complex relationship between returns, continuous variables, and jump variables is extremely helpful for market participants and analysts [5]. This study focuses on one of the asymmetric GARCH models, namely the Exponential GARCH (EGARCH) model (see [6] for the formula). In a number of studies, the EGARCH model has been proven to be superior to the GARCH model, such as in international cotton price forecasting [7] and volatility forecasting in the foreign exchange markets of five Eastern and Central European countries [8]. Furthermore, the study by [9] developed the Realized EGARCH (REG) model that utilizes multiple realized measures. Most recently, the study by [10] modified REG to Log-linear Realized GARCH (LRG) and empirically proven that the LRG model is superior to the EGARCH-X model.

The application of these advanced volatility models in the context of Asian markets, and specifically the TOPIX, has yielded significant insights. For instance, the study by [11] examined volatility in the Tokyo Stock Exchange using NIKKEI 225 and TOPIX indices with asymmetric GARCH models (EGARCH, TARCH, PARCH), finding that the EGARCH model performed best, underscoring the prevalence of leverage effects in this market. More recently, the study by [12] explored hybrid models for forecasting TOPIX volatility, demonstrating the superior performance of combined models that leverage both linear stochastic properties and nonlinear machine learning patterns, particularly during turbulent periods. These studies collectively highlight the TOPIX as a rich empirical setting for testing advanced volatility models due to its distinct reactions to both global financial shocks and local structural events.

Furthermore, recent empirical studies on TOPIX using decomposed RV within different GARCH frameworks provide a crucial comparative backdrop for this research. A study on APARCH-type models in **[13]** on the same TOPIX dataset revealed a contrasting result: while a GARCH-CJ model improved fit, the APARCH-CJ model did not outperform the simpler APARCH-X, suggesting that the benefits of jump decomposition might be sensitive to the specific asymmetric structure of the volatility model. These divergent findings underscore a critical nuance in volatility modeling—the interaction between model asymmetry and jump dynamics—and highlight a gap in understanding how these components function within the LRG framework. This study directly addresses this gap by investigating whether the exponential form and dual-equation structure of the LRG model provide a more suitable foundation for utilizing continuous-jump decomposition compared to threshold or power ARCH specifications.

This study compared two models, namely LRG and LRG-CJ, to determine which model is more appropriate for describing financial asset price volatility. To the best of the authors' knowledge, the LRG-CJ model represents an applied modeling innovation that adapts existing theoretical frameworks, specifically by integrating the continuous and jump components from the EGARCH-CJ model introduced in **[4]** into the LRG framework developed by [9]. While the theoretical foundations of these components are well-established, their combined application in the LRG-CJ model offers a novel empirical approach to volatility modeling. The model has been shown to have the potential to better predict future market asset volatility, making it practical for applications such as capital asset pricing, risk measurement, and financial derivative pricing (e.g., options). Therefore, this study contributes to the field by providing an applied alternative for modeling financial asset volatility, utilizing high-frequency (intraday) data with continuous and jump components.

To estimate the model parameters, this study uses the Adaptive Random Walk Metropolis (ARWM) method [14] in the Markov Chain Monte Carlo (MCMC) algorithm. Most recently, this method was utilized in [3], [10] to estimate the parameters of GARCH-X and LRG models, and was shown to be efficient in estimating the model parameters. This study provides additional evidence of the ARWM method ability to estimate the extended models of LRG model.

Therefore, this study contributes to the literature by introducing the first empirical application of the LRG-CJ model, which explicitly decomposes RV into continuous and jump components within the LRG framework. This addresses a critical research gap, as previous studies have not examined how such decomposition impacts volatility forecasting and Value-at-Risk estimation in high-frequency settings. By demonstrating that jump components improve accuracy—particularly at lower frequencies (5- and 10-minute intervals)—we provide novel insights into the frequency-dependent efficacy of volatility models, offering practical advancements for financial risk management.

2. RESEARCH METHODS

2.1 Data

The Tokyo Stock Price Index (TOPIX) data from 2004–2011 was selected for this study due to its unique representation of volatility dynamics during periods of significant financial turbulence, including the 2008 Global Financial Crisis and the 2011 Great East Japan Earthquake. The high-frequency intraday data (1-minute, 5-minute, and 10-minute intervals) was obtained through a licensed purchase from a reputable third-party financial data provider. These events caused significant volatility jumps and structural breaks in TOPIX, providing an ideal empirical setting to test the LRG-CJ model's ability to distinguish between continuous and discontinuous market movements. Although more recent datasets may provide additional insights, high-frequency intraday data with 1, 5, and 10-minute intervals remains scarce and prohibitively expensive for public academic use. Crucially, the purposive sampling of this datasets—limited to actively traded periods—ensures its suitability for benchmarking the proposed model against its predecessors (LRG and EGARCH-CJ), as our primary objective is methodological innovation rather than real-time market analysis. The observed improvements in forecasting accuracy (Section 3.3) confirm the model's robustness even with this older but well-documented dataset.

The selection of 1-, 5-, and 10-minute intervals for RV calculation was carefully considered based on recent literature and empirical needs. Studies have shown that while higher-frequency data (e.g., 1-minute) can better capture intraday jumps and market microstructure effects, they are also more susceptible to noise and biases like bid-ask bounce [15]. Conversely, lower frequencies (e.g., 10-minute) provide smoother volatility estimates but may miss short-term market dynamics. The 5-minute interval has emerged as a practical compromise, often regarded as the "gold standard" in RV research due to its optimal balance between precision and noise reduction [16]. Therefore, by including these three intervals, our study can systematically evaluate how the presence of market microstructure noise and the aggregation of data over different time horizons impact the performance of the LRG and LRG-CJ models, providing valuable insights for both high-frequency and lower-frequency trading strategies.

Fig. 1 displays the relationship pattern between the plot of absolute returns, continuous variables, and jump variables for the daily period of the TOPIX data from the intra-day data between 2004 and 2011 with intervals of 1 minute. The actual stock index data (original price series at time t, P_t) was transformed into three fundamental variables for volatility analysis, each serving distinct purposes in modeling market dynamics. The primary transformation begins with the calculation of daily logarithmic returns:

$$R_t = 100 \times (\log P_t - \log P_{t-1}). \quad (1)$$

This logarithmic transformation corresponds exactly to the solution of geometric Brownian motion (where log returns are normally distributed), while simultaneously achieving stationarity in the time series and preserving the relative scale of price movements. Building upon these returns, we decompose market volatility into its continuous and discontinuous components through intraday realized measures, as detailed in the next subsection.

The graph in Fig. 1 shows that any changes in the absolute returns, either in the form of an increase or decrease in return, directly affect the changes in continuous and jump variables. In other words, when there are significant fluctuations in the market in terms of gains or losses, other variables also respond sharply.

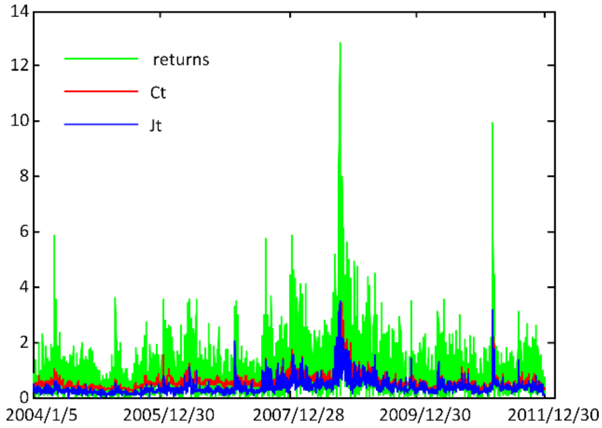


Figure 1. Plot of Absolute Return $|R_t|$, Continuous Variable C_t as well as Jumps Variable J_t over Daily Period of TOPIX

Table 1 presents a number of descriptive statistics values of the TOPIX data between 2004 and 2011. It can be seen that the mean of the returns is close to 0, indicating that overall, there is no general trend towards significant gains or losses during the period. The distribution skewness of the returns is slightly skewed to the left (negative value). It indicates that the distribution of the TOPIX returns in the 8 years observed is characterized by many small gains and few extreme losses [17]. Meanwhile, high kurtosis (more than 3) describes a wider distribution shape with a thicker tail than a normal distribution. Many value fluctuations are far from the mean of the returns [18]. This results in a greater chance of extreme positive or negative events. In line with these results, the Jarque–Bera test statistic strongly rejects the normality hypothesis. However, since this study focuses on the continuous and jump components in the RV, it assumes that the returns follow a normal distribution as the simplest framework.

Table 1. Descriptive Statistics for Returns of TOPIX

Mean	Median	Standard Deviation	Skewness	Kurtosis
16	0.0476	0.0399	0.0287	0.0456

2.2 EGARCH-type Models

One of the most popular asymmetric GARCH models is the exponential GARCH (EGARCH) model. This model is specifically designed to deal with the asymmetric characteristics between returns and volatility. It is assumed that the time series of the return R_t follows a normal distribution with zero mean and variance σ_t^2. The EGARCH(1,1) model equation for the volatility σ_t of the return R_t can be expressed as follows:

$$R_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0,1), \tag{2}$$

$$h_t = \omega + \alpha_1 |\varepsilon_{t-1}| + \alpha_2 \varepsilon_{t-1} + \beta h_{t-1}, \tag{3}$$

where $h_t = \log \sigma_t^2$ and $\varepsilon_t = \frac{R_t}{\sigma_t}$. The parameter α_2 indicates the asymmetric effect of volatility on returns, meaning that positive or negative values of returns (which have the same absolute value) at time $t-1$ have different effects on volatility at time t. When negative returns have a greater impact on volatility, compared to positive returns, the effect is called the leverage effect [19].

By incorporating exogenous variables into the volatility equation in the EGARCH model, the EGARCH-X model becomes able to capture more complex patterns in market fluctuations that may be difficult to explain by simpler models [20]. Following [21], the EGARCH-X(1,1) volatility equation can be expressed as follows:

$$h_t = \omega + \alpha_1 |\varepsilon_{t-1}| + \alpha_2 \varepsilon_{t-1} + \beta h_{t-1} + \gamma_1 \log X_{t-1}, \tag{4}$$

where X represents the exogenous component and normally uses RV measures calculated from the intra-day data.

Furthermore, the study by [21] developed the EGARCH-X model into the EGARCH-CJ model by decomposing the exogenous component X into two components, namely the continuous component C and the jump component J. The volatility equation of EGARCH-CJ(1,1) can be expressed as follows:

$$h_t = \omega + \tau(\varepsilon_{t-1}) + \beta h_{t-1} + \gamma_1 \log C_{t-1} + \gamma_2 \log(J_{t-1} + 1). \tag{5}$$

The components C and J are calculated based on the RV ($\equiv X$), Median RV ($\equiv D$), and Median Realized Tri-power Quarticity ($\equiv Q$) measures as follows:

$$C_t = I_{Z_t > \emptyset_\alpha}(X_t - D_t), \tag{6}$$

$$J_t = I_{Z_t \le \emptyset_\alpha} X_t + I_{Z_t > \emptyset_\alpha} D_t, \tag{7}$$

where $\emptyset_\alpha$ is the α-quantile of the standard normal distribution function, I is the indicator function, and the Z_t statistic can be expressed as follows:

$$Z_t = \frac{\frac{X_t - D_t}{X_t}}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right)\left(\frac{1}{M}\right)\max\left(1, \frac{Q_t}{D_t}\right)}} \tag{8}$$

with

$$X_t^{\,2} = \sum_{i=2}^{M-1} R_{t,i}^2, \tag{9}$$

$$D_t = \frac{\pi}{6 - 4\sqrt{3} + \pi}\left(\frac{M}{M-2}\right) \times \sum_{i=2}^{M-1} Median\left(|r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}|\right)^2, \tag{10}$$

$$Q_t = \frac{3\pi M}{9\pi + 72 - 52\sqrt{3}}\left(\frac{M}{M-2}\right) \times \sum_{i=2}^{M-1} Median\left(|r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}|\right)^4, \tag{11}$$

where $M = N + 1$ represents the number of partitions plus one, and $R_{t,i}$ represents the intraday return at specific time intervals.

With a different method, the study by [9] extended the EGARCH-X model by adding a measure equation to characterize a more flexible modeling of the dependence between returns and volatility. This model is called the LRG(1,1) model and is expressed as follows:

$$h_t = \omega + \tau(\varepsilon_{t-1}) + \beta h_{t-1} + \gamma_1 \ln X_{t-1}, \tag{12}$$

$$\ln X_t = \xi + \varphi h_t + \delta(\varepsilon_t) + \eta u_t, \tag{13}$$

where $u_t \sim N(0,1)$ and the leverage functions are expressed by:

$$\tau(\varepsilon_t) = \tau_1 \varepsilon_t + \tau_2(\varepsilon_t^2 - 1) \text{ and } \delta(\varepsilon_t) = \delta_1 \varepsilon_t + \delta_2(\varepsilon_t^2 - 1). \tag{14}$$

This study specifically proposes a development of the EGARCH-CJ(1,1) and LRG(1,1) models into the LRG-CJ(1,1) model which is expressed as follows:

$$h_t = \omega + \tau(\varepsilon_{t-1}) + \beta h_{t-1} + \gamma_1 \log C_{t-1} + \gamma_2 \log(J_{t-1} + 1), \tag{15}$$

$$\log C_t = \xi + \varphi h_t + \delta(\varepsilon_t) + \eta u_t. \tag{16}$$

To provide a clear conceptual overview of the modeling framework, particularly for the novel LRG-CJ model, Fig. 2 presents a flowchart illustrating the data flow and key components of the volatility forecasting process. The diagram summarizes how intraday high-frequency data is transformed into realized measures, decomposed, and then integrated into the dual-equation structure of the LRG-CJ model to produce volatility forecasts and Value-at-Risk (VaR) estimates.

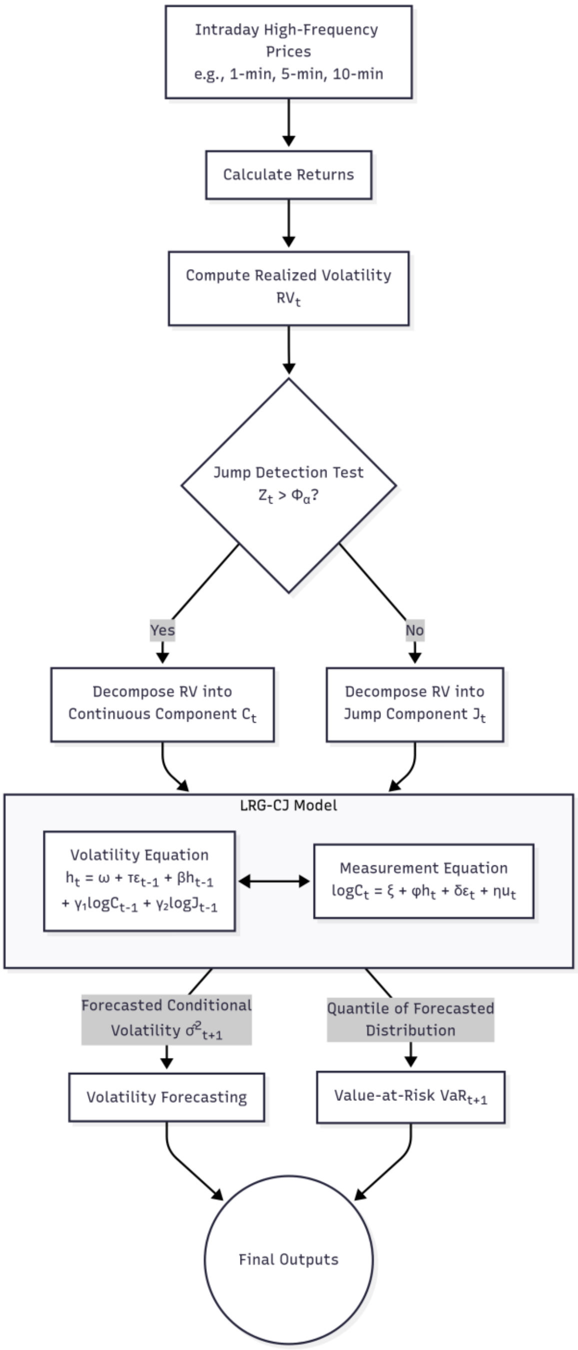


Figure 2. Conceptual Flowchart of the LRG-CJ Volatility Modeling and Forecasting Process

2.3 Estimation Method

According to [22], MCMC is an increasingly popular method in Bayesian inference to obtain approximate information about the posterior distribution, especially when the posterior distribution is difficult to obtain analytically. This method consists of two components, namely the Markov chain and the Monte Carlo method. The Monte Carlo part refers to random samples from a distribution to obtain numerical results that describe the distribution (such as mean, standard deviation, and Bayesian interval).

One of the methods for generating Markov chains is the ARWM method. This method is an improvement on the RWM method by adaptively updating the parameter estimations. The Markov chains are constructed by generating samples from the posterior distribution. In the Bayesian context, the posterior distribution of the parameter θ, given the data D, is expressed by Bayes' theorem as follows:

$$f(\theta|D) \propto L(D|\theta) \times p(\theta), \tag{17}$$

where f represents the posterior distribution, L represents the likelihood function, and p represents the prior distribution.

To measure the efficiency of the ARWM method in obtaining statistically independent samples, this study used Integrated Autocorrelation Time (IACT). A smaller IACT value indicates that the estimation method is more efficient, so the estimation is more accurate [23]. The study by [24] recommended Effective Sample Size (ESS), calculated as the length of the Markov chains divided by the IACT, to be greater than 400 to state that the estimation method is statistically efficient.

2.4 Model Selection Criteria

In determining which model has a better data fit, this study applied four information criteria, namely the Akaike Information Criterion (AIC), the Consistent Akaike Information Criterion (CAIC), the Bayesian Information Criterion (BIC), and the Adjusted Bayesian Information Criterion (ABIC), which can be expressed as follows [25]:

$$\text{AIC} = -2\log L + 2k, \tag{18}$$

$$\text{CAIC} = -2\log L + k(1+\log n), \tag{19}$$

$$\text{BIC} = -2\log L + k\log n, \tag{20}$$

$$\text{ABIC} = -2\log L + k\log\left(\frac{n+2}{24}\right), \tag{21}$$

where k represents the number of parameters, L represents the likelihood value, and n represents the number of data samples. If the given value is smaller, then the model shows a better fit.

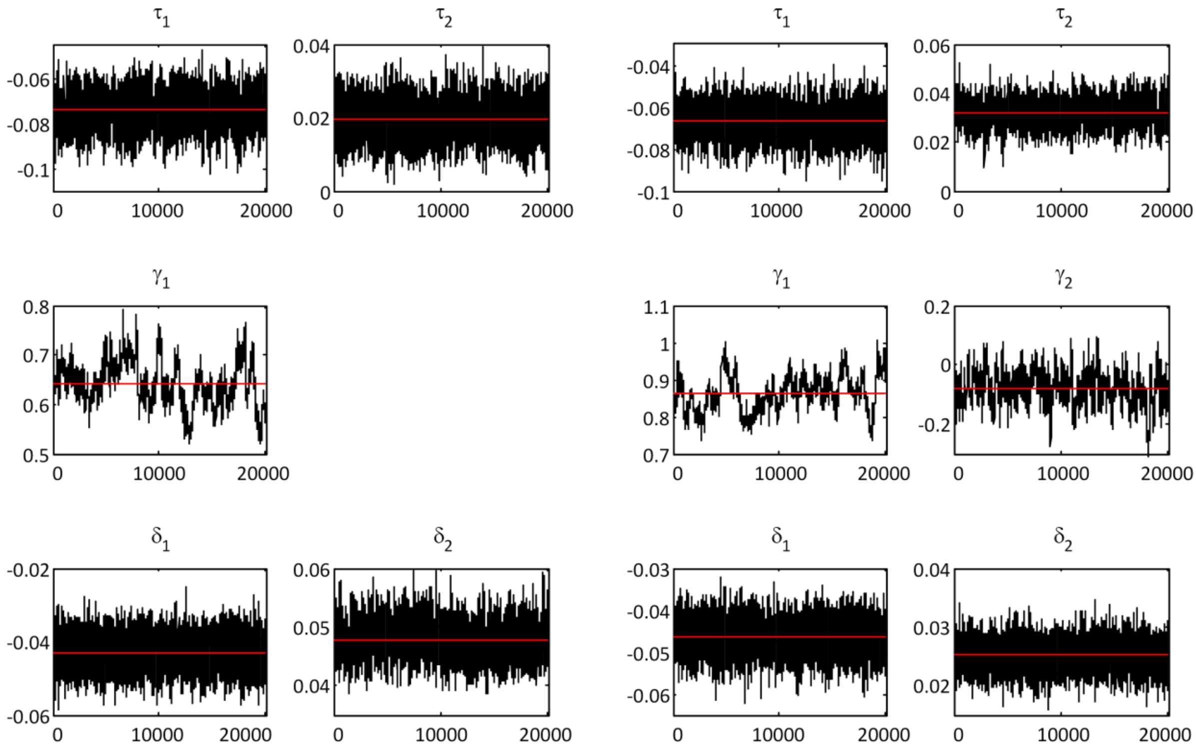
3. RESULTS AND DISCUSSION

In this section, research results related to existing theories are presented. Furthermore, a careful discussion is elaborated to explore the significance of the analysis results to see the extent to which the results match the relevant research topic.

3.1 Parameter Estimation Results

For example, Fig. 3 displays the trace plots of the estimated key parameters for the LRG and LRG-CJ models with 1-minute RV, obtained by running the MCMC for 25,000 iterations (with the first 5,000 iterations discarded as a burn-in period). Visually, the parameter values fluctuate around their means (indicated by the red lines), suggesting that the Markov chains generated by the ARWM method have been reached a stationary distribution. This indicates convergence of the chains to their target distributions. However, slow convergence is observed for the parameters γ_1 and γ_2. Statistically, the ESS values (see Table 2) further confirm that the ARWM method is less efficient in estimating γ_1. These findings align with the results of [10] in the context of the LRG model. With this convergence established, the analysis of the parameter estimates can now proceed.

From the posterior samples of key parameters, we compute posterior mean, Standard Deviation (SD), and 95% HPD (Highest Posterior Density) interval (LB: Lower Bound, UB: Upper Bound), with additional diagnostics (IACT and ESS) presented in Table 2. For all leverage function parameters (τ_1, τ_2, δ_1, δ_2) in both models, the 95% HPD intervals exclude zero regardless of intra-day data frequency, demonstrating their statistical significance. Notably, the estimated values of τ_1 and δ_1 are negative, suggesting that past negative returns have a stronger impact on current conditional volatility than positive returns of equal magnitude. These findings collectively demonstrate that leverage effects in both conditional volatility and RV processes play a crucial role in LRG-type modeling. This evidence reinforces existing literature on the significance of leverage effects in intra-day data-based volatility modeling and forecasting (e.g., [10], [26]), while extending its application to more sophisticated modeling frameworks.



(a) (b)

Figure 3. Trace Plot of the Posterior Samples of the Key Parameters for: (a) the LRG Model and (b) the LRG-CJ Model; with 1-minute RV

The estimated parameters of both LRG and LRG-CJ models provide significant insights into volatility dynamics and their implications for financial risk management. The consistently negative values of the leverage effect parameters (τ_1 in the volatility equation and δ_1 in the RV equation) across all frequency bands confirm the presence of asymmetric volatility responses to negative returns in the TOPIX market. The estimation results reveal that in both models (LRG and LRG-CJ), the absolute values of leverage parameters tend to increase as intraday frequency decreases (from 1-minute to 10-minute intervals), indicating a stronger leverage effect over longer time windows. From a practical standpoint, the estimated τ_1 value of –0.0748 in the LRG model with 1-minute data indicates that a 1% negative return exerts a 7.48% greater impact on conditional volatility than an equivalent positive return, while the corresponding δ_1 of –0.0426 shows RV measures increase about 4.26% more for negative returns. This dual effect intensifies at lower frequencies, with τ_1 reaching –0.0836 and δ_1 reaching –0.0533 for 10-minute intervals, suggesting that prolonged negative trends captured by lower-frequency data significantly exacerbate both conditional volatility and RV asymmetry—a crucial finding for risk managers monitoring multi-day market stress conditions. This reflects how market participants process and react to negative information over different time horizons, with more sustained selling pressure emerging in longer timeframes.

The inclusion of jump components in LRG-CJ systematically reduces the leverage effect in the volatility equation (τ_1 becomes less negative by 0.0074 to 0.0108 across frequencies), suggesting continuous jumps partially offset asymmetric volatility responses. Conversely, the RV equation's leverage parameter (δ_1) intensifies in LRG-CJ (more negative by 0.0039 to 0.0114), indicating discontinuous jumps amplify RV's sensitivity to negative returns. This divergence is most pronounced at lower frequencies (10-minute), where τ_1 values nearly converge between models while δ_1 differences peak, implying jumps dominate RV dynamics in lower-frequency data. The results align with theoretical expectations: jumps redistribute leverage effects, weakening them in conditional volatility while strengthening them in RV, with frequency-dependent magnitudes reflecting competing discontinuous jump influences. These findings support the results of [27] in their analysis of the Heterogeneous Autoregressive (HAR) model.

The differential behavior between τ_1 and δ_1 parameters reveal important temporal dynamics in market responses: while τ_1 captures the immediate overreaction of conditional volatility to negative returns, δ_1 reflects the more gradual incorporation of these effects into RV measures. This lagged transmission mechanism enables algorithmic trading systems to optimize strategies by initially responding to τ_1 signals during market shocks, and then progressively adjusting positions as δ_1 confirms the sustained volatility impact, particularly valuable when the 10-minute δ_1 shows more persistent effects than 1-minute readings. For instance, a 2% market drop triggers instant volatility alerts via τ_1, but prudent algorithms would await δ_1 stabilization in subsequent 5 to 10 minutes windows before re-entering positions, thereby avoiding overtrading during transient spikes while capitalizing on more reliable RV signals. This dual-parameter approach effectively bridges short-term market overreactions with medium-term volatility reality, enhancing both risk management and profitability in high-frequency environments.

In the LRG-CJ model, the negative estimates of the jump component parameter (γ_2), such as the value of –0.0899 for 1-minute RV, reveal that discontinuous jumps contribute disproportionately to volatility persistence. Economically, each 1% increase in the jump component reduces conditional volatility by approximately 9%, reflecting the characteristic of market jumps as short-lived shocks that decay more rapidly than continuous volatility components. This finding substantiates the importance of decomposing RV into continuous and jump components for derivative pricing applications, where jump risks require distinct hedging strategies.

Table 2. Estimation Results of the Key Parameters for the LRG and LRG-CJ Models on the TOPIX Data

	Parameter					
	τ_1	τ_2	γ_1	γ_2	δ_1	δ_2
LRG with RV 1-min						
Mean	–0.0748	0.0199	0.6125	-	–0.0426	0.0479
SD	0.0074	0.0045	0.0501	-	0.0043	0.0029
LB	–0.0901	0.0124	0.5153	-	–0.0509	0.0425
UB	–0.0617	0.0296	0.6991	-	–0.0343	0.0539
ESS	1250.8	2026.3	35.1	-	2617.8	655.5
LRG-CJ with RV 1-min						
Mean	–0.0674	0.0327	0.8070	–0.0899	–0.0465	0.0254
SD	0.0074	0.0050	0.0459	0.0527	0.0039	0.0024
LB	–0.0816	0.0226	0.7211	–0.1950	–0.0542	0.0208
UB	–0.0536	0.0420	0.8968	0.0105	–0.0390	0.0300
ESS	1555.2	847.1	33.7	165.8	2681.0	1826.5
LRG with RV 5-min						
Mean	–0.0780	0.0261	0.6231	-	–0.0569	0.0484
SD	0.0085	0.0055	0.0409	-	0.0050	0.0033
LB	–0.0952	0.0158	0.5405	-	–0.0660	0.0421
UB	–0.0622	0.0370	0.7010	-	–0.0464	0.0552
ESS	1554.0	2506.4	39.4	-	2462.5	1252.6
LRG-CJ with RV 5-min						
Mean	–0.06719	0.0354	0.6443	0.0279	–0.0626	0.0394
SD	0.0083	0.0058	0.0516	0.0683	0.0051	0.0033
LB	–0.0830	0.0245	0.5499	–0.1054	–0.0725	0.0328
UB	–0.0506	0.0467	0.7412	0.1567	–0.0526	0.0457
ESS	1889.9	1910.0	37.2	114.5	1927.9	1234.4
LRG with RV 10-min						
Mean	–0.0836	0.0269	0.4840	-	–0.0533	0.0615
SD	0.0087	0.0062	0.0411	-	0.0059	0.0041
LB	–0.1010	0.0148	0.4095	-	–0.0647	0.0535
UB	–0.0671	0.0387	0.5689	-	–0.0419	0.0698
ESS	1417.3	2404.1	35.7	-	2660.4	872.4
LRG-CJ with RV 10-min						
Mean	–0.0821	0.0361	0.5155	0.0482	–0.0647	0.0396
SD	0.0092	0.0068	0.0458	0.0559	0.0062	0.0039
LB	–0.1004	0.0231	0.4186	–0.0599	–0.0765	0.0318
UB	–0.0644	0.0496	0.5974	0.1624	–0.0525	0.0468
ESS	1737.0	1079.8	44.9	169.7	2846.2	2107.1

3.2 Model Selection

Table 3 presents the log-likelihood values and four information criteria for assessing the fitting performance of the LRG and LRG-CJ models based on three RV measures. Models with better performance can be seen through the criteria dominance that have lower values. By considering each case of the RV measure, the LRG-CJ model is superior only in the use of 1-minute RV data. Moreover, overall, the application of the RV data provides evidence of the superiority of the LRG-CJ model. These findings not only highlight the importance of incorporating jump components in RV modeling, particularly for high-frequency data applications, but also support three key studies: [26] on HAR model jump dynamics, [28] on jump magnitude in total price variance, and [29] on jumps in Heston stochastic volatility model.

Table 3. Log-likelihood (LL) and Information Criteria Values

Data	Model	LL	AIC	ABIC	BIC	CAIC
RV 1-min	LRG	–2526.6	5073.1	5107.2	5128.9	5138.9
	LRG-CJ	–2337.9	4697.9	4735.3	4759.3	4770.3
RV 5-min	LRG	–2846.9	5713.8	5747.9	5769.6	5779.6
	LRG-CJ	–2885.4	5792.7	5830.1	5845.1	5865.1
RV 10-min	LRG	–3189.8	6399.6	6433.6	6455.4	6465.4
	LRG-CJ	–3233.7	6489.3	6526.8	6550.7	6561.7

3.3 Forecast Evaluation

The primary objective of volatility modeling is not only to find the model that best fits historical data but also to generate accurate forecasts for future periods. Thus, the evaluation process should not stop at analyzing the model's in-sample fit to the entire dataset. Out-of-sample performance assessment becomes an essential step to measure how reliable a model is in predicting volatility outside the data sample used for estimation [30].

Testing the accuracy of out-of-sample volatility forecasting relies on the use of statistical loss functions. These functions quantify the forecasting error, which represents the difference between the actual volatility and the forecasted volatility. Consequently, the accuracy of the out-of-sample forecast can be analyzed, where a smaller value indicates the most accurate forecast.

Since no single accuracy measure is considered superior for comparing various volatility models [31], this study adopts three different goodness-of-fit measures commonly used in the relevant literature [32]: the Mean Squared Error (MSE), mean absolute error (MAE) and Mean Absolute Percentage Error (MAPE). The formula for calculating these three approaches are as follows:

$$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n}(\sigma_t^2 - \hat{\sigma}_t^2)\text{^}2,\ \text{MAE} = \frac{1}{n}\sum_{t=1}^{n}|\sigma_t^2 - \hat{\sigma}_t^2|,\ \text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{\sigma_t^2 - \hat{\sigma}_t^2}{\sigma_t^2}\right|, \quad (22)$$

where σ_t^2 represents the true volatility, $\hat{\sigma}_t^2$ is the forecasted volatility, and n is the total number of forecasts.

Although the calculation of forecast error implicitly assumes that RV is the true volatility value, in reality, RV is merely an approximation. To address this, the study approximates the true (daily) volatility by using five-minute RV. The selection of this measure is supported by evidence from studies in [33], [34], which showed its superior performance over alternative measures within the context of GARCH models.

For each LRG-type model, this study used the observations from 15 November 2011 to 30 December 2011 to perform the one-day-ahead volatility forecasts using the recursive method. Forecasting is estimated through three RV measures. As displayed in Table 4, we have compared the models via three loss function to select the model with the most accurate forecasts.

Table 4. Evaluations of Volatility Forecasts for One-Day Out-of-Sample Using Loss Functions

Loss Function	RV 1-min		RV 5-min		RV 10-min	
	LRG	LRG-CJ	LRG	LRG-CJ	LRG	LRG-CJ
MSE	2.290	2.626	1.965	1.575	1.878	1.463
MAE	1.422	1.541	1.310	1.158	1.263	1.096
MAPE	3.788	4.332	3.239	2.607	3.128	2.450

In the out-of-sample forecasting analysis (Table 4), the LRG-CJ model demonstrated significant superiority when using 5-minute and 10-minute RV data. However, during the same period, the standard LRG model performed slightly better when employing 1-minute RV data. This suggests that, for predictive purposes within this timeframe, the "CJ" components was more effective in capturing volatility dynamics at lower data frequencies.

The observed performance discrepancies—where LRG-CJ excels with 1-minute RV data in full-sample fitting but outperforms in volatility forecasting with 5-minute and 10-minute RV data in out-of-sample tests—lead to a key conclusion: the superiority of the "CJ" decomposition in RV data depends on the performance objective. These inconsistent results likely stem from the distinction between a model's ability to fit historical data (in-sample fit) and its capacity to predict unseen data (out-of-sample forecast), where market conditions in each period may exhibit distinct characteristics.

VaR has emerged as one of the most widely used methods for market risk estimation. As defined in [35], VaR represents the maximum potential loss over a specified t-day holding period, calculated at a (1-α)% confidence interval. Building on this foundation, the current study further examines the application of volatility forecasts in calculating risk measures, with particular focus on VaR estimation. VaR is computed according to the formula:

$$\text{VaR}_\alpha(t) = N_\alpha \hat{\sigma}_t, \tag{23}$$

where N_α represents the quantile of Normal distribution $N(0,1)$.

The evaluation of VaR model performance has led to the development of various testing methodologies. Existing literature classifies backtesting procedures into two distinct categories: statistical hypothesis-based backtesting and loss-function-based evaluation. Statistical backtesting approaches primarily assess the accuracy of VaR estimates, yet this framework lacks the capacity for comparative model ranking. In contrast, loss-function-based evaluation incorporates both the frequency and magnitude of VaR exceptions. This methodology enables financial regulators/supervisors and risk managers/firms to establish a model hierarchy, where preference is given to specifications that minimize aggregate loss metrics.

Further developments in VaR validation methodologies include the approach introduced by Lopez, which focuses on evaluating the accuracy of VaR estimation. This method was later refined by Sarma, Thomas, and Shah, who proposed modifications to enhance its effectiveness. In their framework, a loss function is calculated for each analyzed period based on the corresponding rate of returns, following the formula specified below [36]:

$$\text{Regulator's loss function: Lopez's Quadratic (RQL)} = \begin{cases} 1 + (\text{VaR}_t - R_t)^2 & \text{if } R_t < \text{VaR}_t \\ 0 & \text{if } R_t \geq \text{VaR}_t \end{cases}, \tag{24}$$

$$\text{Firms's loss function: Sarma, Thomas, and Shah (STS)} = \begin{cases} (\text{VaR}_t - R_t)^2 & \text{if } R_t < \text{VaR}_t \\ -0.6\text{VaR}_t & \text{if } R_t \geq \text{VaR}_t \end{cases}. \tag{25}$$

For the competing models, we compute the loss functions at both the 1% and 5% significance levels. The optimal model is determined by selecting the specification that yields the minimal loss function value. The corresponding results for each RV measure are presented in Table 5.

Table 5. Results of RQL and STS Loss Functions Test

Loss Function	**RV 1-min**				**RV 5-min**				**RV 10-min**			
	LRG		**LRG-CJ**		**LRG**		**LRG-CJ**		**LRG**		**LRG-CJ**	
% VaR	1	5	1	5	1	5	1	5	1	5	1	5
RQL	7.72%	4.41%	8.42%	5.10%	7.12%	4.13%	6.04%	3.42%	6.76%	3.98%	5.71%	3.47%
STS	6.48%	3.21%	7.16%	3.95%	6.12%	3.07%	5.04%	2.32%	5.76%	2.91%	4.71%	2.42%

From a regulatory point of view, the RQL function emphasizes strict adherence to VaR thresholds, especially at the 1% significance level, where underestimation of risk would have severe systemic consequences. The results show that LRG-CJ consistently outperforms LRG for both 5-minute and 10-minute RV measures, with lower RQL values (e.g., 6.04% vs. 7.12% at 5-minute RV, 1% VaR). This suggests that incorporating CJ components improves the accuracy of risk estimation when volatility is sampled at lower frequencies. However, for 1-minute RV, the standard LRG model yields better performance (7.72% vs. 8.42% at 1% VaR), likely due to noise in the very high-frequency data that complicates jump detection. The 5% VaR level universally shows lower RQL values, confirming that the model performs better under less extreme market conditions. Regulators should prioritize LRG-CJ for 5- and 10-minute RV applications, but may prefer the simpler LRG for high-frequency settings.

For financial firms, the Sarma-Thomas-Shah (STS) loss function provides a more balanced view, weighing the frequency and magnitude of VaR violations to optimize capital allocation. Like the RQL results, LRG-CJ dominates for both 5-minute and 10-minute RVs, providing the lowest STS losses (e.g., 4.71% vs.

5.76% at 10-minute RV, 1% VaR). This is in line with the company's need to minimize unexpected losses while avoiding excessive capital buffering. However, at the 1-minute RV level, LRG again proved superior (6.48% vs. 7.16% at 1% VaR), reinforcing that jump adjustments can render noisy high-frequency data inappropriate. The smaller gap between STS and RQL values at the 5% VaR level (e.g., 2.42% vs. 3.47% for LRG-CJ at 10-minute RV) suggests that firms face a milder trade-off for a moderate risk threshold. Firms should adopt LRG-CJ for lower-frequency RVs, but stick with LRG for high-frequency trading desks.

The practical implications of these forecasting and VaR results are very important for financial institutions and traders. The finding that the LRG-CJ model performs better with 5- and 10-minute data for out-of-sample prediction and risk management, but not with 1-minute data, provides a clear, actionable guidance for model selection based on data frequency. For risk managers focusing on daily VaR calculations and intraday strategies that rely on slightly smoothed signals (e.g., 5- or 10-minute intervals), adopting the LRG-CJ model can lead to more accurate risk assessments and better capital allocation, ultimately reducing unexpected losses. Conversely, for ultra-high-frequency trading desks operating at the 1-minute level, the standard LRG model remains the more robust and parsimonious choice, as the noise in the data at this frequency outweighs the benefits of jump decomposition. This frequency-dependent performance underscores the critical importance of aligning model complexity with the specific characteristics of the available data to achieve optimal practical outcomes.

4. CONCLUSION

This study compared the LRG and LRG-CJ models in modeling financial asset volatility using TOPIX data. The results show that integrating continuous and jump components significantly improves volatility modeling accuracy, forecasting, and Value-at-Risk estimation. A key and novel contribution is the first empirical application of the LRG-CJ framework, which demonstrates frequency-dependent performance: while it fits ultra-high-frequency (1-minute) data better in-sample, its forecasting and risk estimation are most effective at 5- and 10-minute intervals.

Practically, these findings provide guidance for implementation in financial systems. The LRG-CJ model is particularly suitable for risk management and trading strategies that rely on lower-frequency data, where noise is reduced and jump dynamics are better captured. For implementation, model calibration should prioritize out-of-sample forecasting accuracy and ensure convergence diagnostics in parameter estimation.

Limitations remain, including the model’s sensitivity to high-frequency market noise and computational challenges in estimation. Future work could refine jump-detection techniques, extend the framework to multivariate settings, and test robustness across different asset classes. In sum, this study contributes a novel empirical framework that enhances volatility modeling by explicitly incorporating continuous and jump components, offering both theoretical advancement and actionable value for financial risk management.

Author Contributions

Didit Budi Nugroho: Conceptualization, Methodology, Formal Analysis, Writing-Original Draft, Validation. Zefania Sasongko Putri: Draft Preparation, Software. Bambang Susanto: Validation, Writing-Review and Editing. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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