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CHAOS CONTROL IN PERMANENT MAGNET SYNCHRONOUS MOTOR BY SLIDING MODEL CONTROLLER WITH LYAPUNOV OBSERVER UNDER UNKNOWN INPUTS

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ABSTRACT

The control of chaotic and hyper-chaotic systems represents a crucial area of research in the field of nonlinear dynamic systems. In this study, we focus on applying chaos control techniques to a permanent magnet synchronous motor (PMSM), a system known to exhibit chaotic behavior under certain conditions. To achieve this, a sliding mode control (SMC) strategy integrated with a Lyapunov-based observer is proposed. The core concept involves designing a candidate Lyapunov function that governs the application of the control law, ensuring system stability while effectively suppressing chaotic dynamics. Through numerical simulations, the proposed sliding mode controller demonstrates its effectiveness in rapidly eliminating chaotic behavior and stabilizing the PMSM system toward a predefined reference trajectory. Notably, the system achieves error convergence within approximately 0.7 seconds under full control (four channels). When control channels are reduced to two, the system still maintains stability, showing flexibility and cost efficiency. In a further simulation, the chaotic PMSM is subjected to two unknown external disturbances, and the proposed controller continues to maintain stability with only a slight increase in convergence time. These quantitative results affirm the robustness, accuracy, and practicality of the proposed control method. This research confirms that integrating sliding mode control with a Lyapunov observer is an effective approach for chaos suppression in PMSMs, offering promising insights for the development of advanced control strategies in nonlinear electromechanical systems.

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1. INTRODUCTION

One of the important fields of nonlinear science that has attracted the attention of researchers in recent decades is the study of chaotic dynamical systems. Chaos has a quasi-random behavior on the surface, but on the in-side, it has a deterministic nature that originates from a class of simple differential equations. Chaotic dynamic equations were first introduced by the famous meteorologist Edward Lorenz [1]. Chaotic dynamical systems exhibit several key characteristics: (1) extreme sensitivity to initial conditions—meaning that even small variations in starting values can lead to significantly different future behaviors; (2) bounded behavior—the system's trajectories remain confined within a finite region and do not diverge to infinity; and (3) recursive properties in phase space, where system states evolve in a deterministic yet non-repeating manner.

Mathematically, if a dynamical system (or first-order differential equations) has at least one positive Lyapunov exponent, then those equations are chaotic. Also, if a dynamical system has at least two positive Lyapunov exponents, then that system will be hyper-chaotic [2][3]. According to the expansion of dynamic systems in engineering sciences, the most important applications of chaotic dynamic systems are guidance and navigation [4][5], aerospace [6][7], secure communications [8][9], image encryption [10][11], power systems [12][13], industrial engineering [14][15] and even economics [16][17].

Two interesting areas of study in chaotic dynamical systems are synchronization and the elimination of chaotic behavior in the model. The synchronization of two chaotic dynamic systems means that by adding a controller to the slave chaotic system, the output can be controlled so that it matches the output of the master system. Therefore, after applying the controller, the chaotic dynamic system of the follower will be synchronized with the master dynamic system. The first synchronization of two chaotic dynamical systems was done in [18]. Sometimes in dynamic systems, the chaotic behavior will be very dangerous and unacceptable, so by designing a control system, the chaos in the dynamic system will be removed or it will be stabilized to the desired point. The first chaos control of dynamic systems was presented in [19].

Other areas of chaotic dynamic systems studies are: bifurcation control [20], anti-chaotic control [21]. Several methods have been introduced to control and synchronize chaos in dynamic systems, including: sliding mode control [22], adaptive control [23][24], impulse control [25][26], backtracking control [27][28], fuzzy control [29][30], and robust control [31][32]. Some methods also combine the mentioned control methods, such as adaptive sliding mode control [33][34], robust adaptive control [35], sliding fuzzy control [36], and adaptive fuzzy [37].

Our main goal in this paper is to control and stabilize a chaotic permanent magnet synchronous motor. The chaos model in PMSM was first introduced in [38]. They have shown that PMSM nonlinear equations can go to a chaotic state under certain conditions. The model under investigation was a PMSM with a uniform air gap. The model of a chaotic permanent magnet synchronous motor with a non-uniform air gap has been investigated in [39].

Then, several methods were introduced to control chaos in PMSM. These methods include passive control [40]. In this method, by separating the linear part from the nonlinear part in the model and applying the passive controller, the chaos in PMSM is controlled. The time to reach zero error in this method is about 1.5 seconds. In [41], the optimal placement method of the Lyapunov face was presented by Ataei et al. for a chaotic PMSM model with uniform air spacing in 2010. They applied the controller to two lines of chaotic PMSM equations. This can be significant in the controller's overall costs. The duration of stability and removal of chaos in PMSM was about 1.5 seconds.

The adaptive control method to eliminate the chaotic behavior in PMSM and reach the desired value was introduced in [42]. In this method, two controllers are used. The time to reach stability and zero error in each control channel is about 0.2 seconds. The most important advantage of this method is to depict the controller signal. Li et al. [43] have applied the adaptive robust control method with uncertainty to eliminate chaos in PMSM. In this method, three controllers are used for the PMSM chaotic equations. The time to reach zero error is about 2.5 seconds. Also, in this method, the controller signal is depicted. Of course, it can be seen in the simulated results that the amplitude of the control signal is relatively high.

In [44], by presenting a nonlinear controller and applying it to a set of PMSM equations, chaos is eliminated in PMSM. The duration of stability in this method is about 7 seconds. Also, the depicted controller signal has low amplitude and fluctuations. In [45], first the chaos removal in PMSM and then the synchronization for secure communication have been done by a finite time control method. In the first part of the simulation, to remove the chaos, the time to reach zero error is about 4.5 seconds. In the second part, where the synchronization is done, the time to reach zero error is about 2 seconds. Chaos control with nonlinear controller design is reviewed in [46]. The duration of reaching stability and reaching zero error in the best design case is 1.5 seconds. The controller signal is not depicted in this method. The backstepping control method based on an adaptive fuzzy system for chaos removal in PMSM is introduced in [47]. By using two controllers and applying them to chaotic PMSM, they have increased the stability time and reached zero error in about 1 second. In this method, the controller signal is also depicted. The control signal in the best scenario of this method is about 1 second. Also, this signal has fluctuations and low amplitude.

In the proposed method of this article, first, the chaotic PMSM equations are described. It will be shown that these equations have at least one positive Lyapunov function. For the stability of the chaotic permanent magnet synchronous motor, a sliding model control method based on a Lyapunov observer will be designed. Numerical simulation shows that the proposed method can remove the chaos in the permanent magnet synchronous motor in an acceptable time. Also, in a part of the simulation, the stability of the chaotic permanent magnet synchronous motor is proved by using two control channels. Finally, the proposed controller is subjected to two unknown inputs, and in this condition, stability and time to reach zero error can be achieved.

The main contribution and novelty of this work are as follows: (1) This paper introduces a novel integration of sliding mode control with a Lyapunov observer to stabilize chaotic behavior in a 4D PMSM system. (2) The controller's effectiveness is validated under reduced control channels and unknown inputs, demonstrating robustness and cost-efficiency. (3) The method achieves fast error convergence and smooth control signals, enhancing its suitability for real-world applications.

This paper is organized as follows: Section 1 provides the introduction and background on chaotic systems and the relevance of chaos control in PMSMs. Section 2 presents the mathematical modeling of the PMSM and verifies its chaotic behavior using Lyapunov exponents. Also, we present the design of the sliding mode controller integrated with a Lyapunov observer to suppress chaos in the PMSM. Section 3 explains the numerical simulation setup and evaluates the controller's performance under various conditions, including reduced control channels and unknown disturbances. Also, we discuss the simulation results, including error convergence and control signal behavior, and compare the proposed method with existing approaches. Finally, Section 4 concludes the paper by summarizing the key findings, contributions, and recommendations for future work in nonlinear electromechanical system control.

2. RESEARCH METHODS

This study adopts a structured methodology combining theoretical modeling, controller design, and numerical simulation to achieve chaos suppression in a permanent magnet synchronous motor (PMSM). The Flowchart of the research methodology can be seen in Fig. 1. The following stages describe the research workflow:

2.1 Mathematical Modeling of PMSM

The mathematical modeling of the PMSM begins with the derivation of its nonlinear differential equations, which capture the interaction between electrical and mechanical subsystems. The electrical part is typically expressed in the $d-q$ (direct-quadrature) reference frame, involving the d-axis and q-axis stator currents, voltages, and inductances, along with back electromotive force (EMF) terms. The mechanical dynamics are incorporated through the rotor angular velocity and position, which are influenced by electromagnetic torque and load disturbances. Together, these equations form a four-dimensional dynamic system that can display a wide range of behaviors, including periodic, quasi-periodic, and chaotic responses depending on system parameters such as load torque and voltage inputs.

2.2 Chaos Verification Using Lyapunov Exponents

To confirm the chaotic behavior of the PMSM model, Lyapunov exponents are calculated based on the system's nonlinear differential equations. These exponents measure the average rates of divergence or convergence of nearby trajectories in the system's phase space. A positive Lyapunov exponent is a hallmark of chaos, indicating that even infinitesimally close initial conditions will diverge exponentially over time, leading to unpredictable long-term behavior. In this research, numerical methods are employed to compute the spectrum of Lyapunov exponents, and the presence of at least one positive value provides definitive evidence of chaotic dynamics within the PMSM. This verification step is critical in justifying the need for robust control strategies to stabilize the system.

2.3 SMC Design

The controller design in this study is centered on the SMC technique integrated with a Lyapunov-based observer to manage the chaotic dynamics of the PMSM. The first step in the design process involves defining the tracking error between the actual state variables of the PMSM and the desired reference values. These error functions are then used to construct sliding surfaces, which represent the conditions under which the system's dynamics will be forced to evolve in a stable manner. By ensuring that the system states are driven to and maintained on these surfaces, the chaotic behavior can be effectively suppressed.

Following the construction of the sliding surfaces, a candidate Lyapunov function is formulated to assess the stability of the closed-loop system. This function serves as a mathematical tool to guarantee that the energy of the system decreases over time, ensuring convergence to the desired equilibrium point. By taking the time derivative of the Lyapunov function and applying stability theorems, control laws are derived to ensure the function remains decreasing. These laws incorporate discontinuous control actions based on the sign of the sliding surface, which provides robustness against system uncertainties and disturbances. The integration of the Lyapunov observer enhances the accuracy of state estimation, contributing to the reliability and effectiveness of the SMC in stabilizing the PMSM under chaotic conditions.

2.4 Numerical Simulation Using the RK4 Method

To analyze the behavior of the PMSM model under both uncontrolled and controlled scenarios, the Runge-Kutta 4th Order (RK4) method is employed for numerical simulation. This method is chosen due to its high accuracy and stability in solving nonlinear ordinary differential equations. By discretizing the time domain and iteratively computing the system's state variables, the RK4 algorithm provides detailed insights into the dynamic evolution of the PMSM. The simulations are conducted to evaluate how effectively the proposed controller reduces chaotic oscillations and stabilizes the system. Key performance indicators such as error convergence toward zero and the smoothness of the control signal are examined, allowing for a comprehensive assessment of the controller's efficiency and practical applicability.

Specifically, at each time step t_n, the RK4 method calculates four intermediate slope values:

$$k_1 = f(t_n, y_n)$$
$$k_2 = f\left(t_n + \frac{h}{2}, y_n + k_1\frac{h}{2}\right)$$
$$k_3 = f\left(t_n + \frac{h}{2}, y_n + k_2\frac{h}{2}\right)$$
$$k_4 = f\left(t_n + \frac{h}{2}, y_n + k_3\frac{h}{2}\right) \tag{1}$$

These intermediate slopes represent estimations of the system's behavior over the interval and are then combined to compute the next value y_{n+1}.

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{2}$$

This weighted average ensures a high level of accuracy compared to lower-order methods such as Euler's method.

2.5 Performance Evaluation

The performance of the proposed Sliding Mode Controller with Lyapunov observer is evaluated through three distinct simulation scenarios. In the first scenario, full control is applied to all channels of the PMSM model. The results show rapid error convergence and elimination of chaotic behavior within approximately 0.7 seconds, demonstrating high stability and precision. The control signals in this case exhibit minimal fluctuation and smooth transitions, indicating the controller's efficiency and low implementation cost. This full control scenario serves as a benchmark for evaluating the robustness and effectiveness of the system can be seen in Fig. 1.

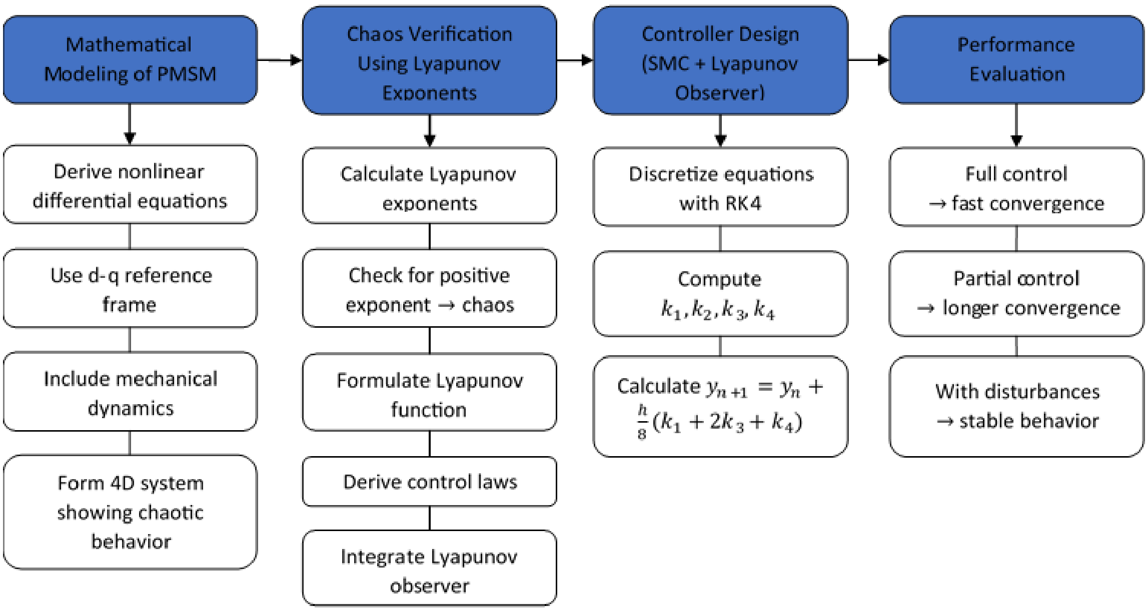


Figure 1. Flowchart of Research Methodology

In the second and third scenarios, the complexity of the control scheme is reduced to assess its resilience and cost-effectiveness. In the partial control setup, only two of the four control channels are activated. Despite this reduction, the system still achieves stabilization, albeit with a slightly longer convergence time for certain variables. In the final scenario, unknown external inputs are introduced to the system to simulate real-world disturbances. The controller continues to maintain system stability, showing strong fault tolerance and adaptability. Across all scenarios, key performance metrics—stability time, error minimization, and the smoothness of the control signal—confirm the robustness and reliability of the proposed method for chaos suppression in PMSMs.

3. RESULTS AND DISCUSSION

3.1 Chaotic Permanent Magnet Synchronous Motor Model

Consider the chaotic synchronous motor model as follows [48].

$$
\begin{aligned}
\frac{d\theta}{dt} &= \omega \\
\frac{d\omega}{dt} &= \sigma(i_q - \omega) - \bar{T}_L \\
\frac{di_q}{dt} &= -i_q - i_d\omega - \gamma\omega + \bar{u}_q \\
\frac{di_d}{dt} &= -i_d - i_q\omega + \bar{u}_d
\end{aligned} \tag{3}
$$

In Eq.(3), θ, ω, i_q, i_d represent the engine position, angular velocity, and currents in the $d-q$ axes, which are system variables. Also, σ, γ are constant parameters and $\bar{T}_L, \bar{u}_q, \bar{u}_d$ are load torque and $d-q$ axes voltages, respectively. As can be seen from Fig. 2, there is at least one positive Lyapunov exponent.

The behavior of chaotic synchronous motor variables for $\bar{T}_L = \bar{u}_q = \bar{u}_d = 0$ and $\sigma = 5.46, \gamma = 14.93$ as well as initial conditions $[\theta, \omega, i_q, i_d]^T = [5\ 5\ 5\ 5]^T$ are shown in Fig. 3. The phase space of the chaotic permanent magnet synchronous motor can be seen in Fig. 4.

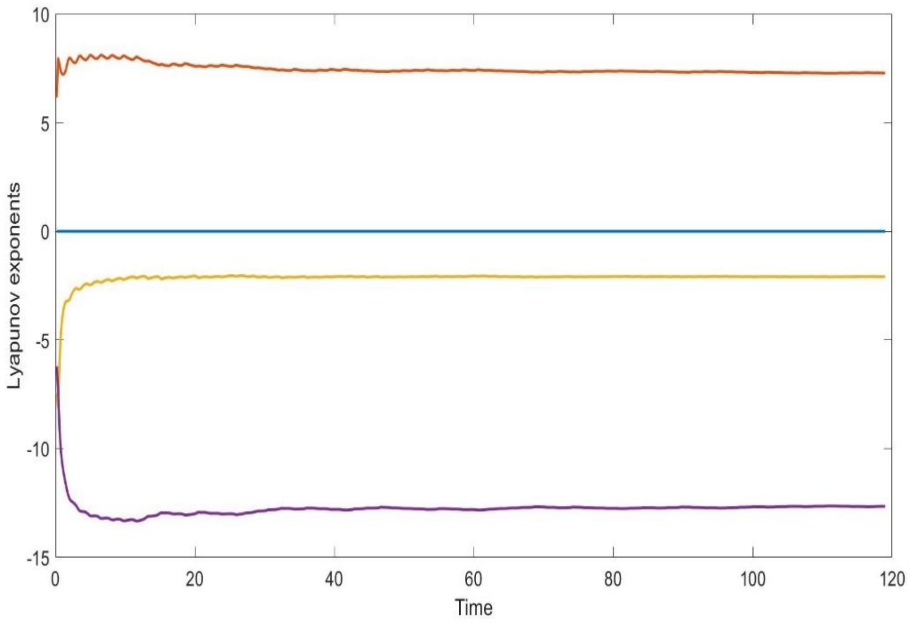


Figure 2. Lyapunov Exponent of Permanent Magnet Synchronous Motor Equations while $\sigma = 5.46, \gamma = 14.93$

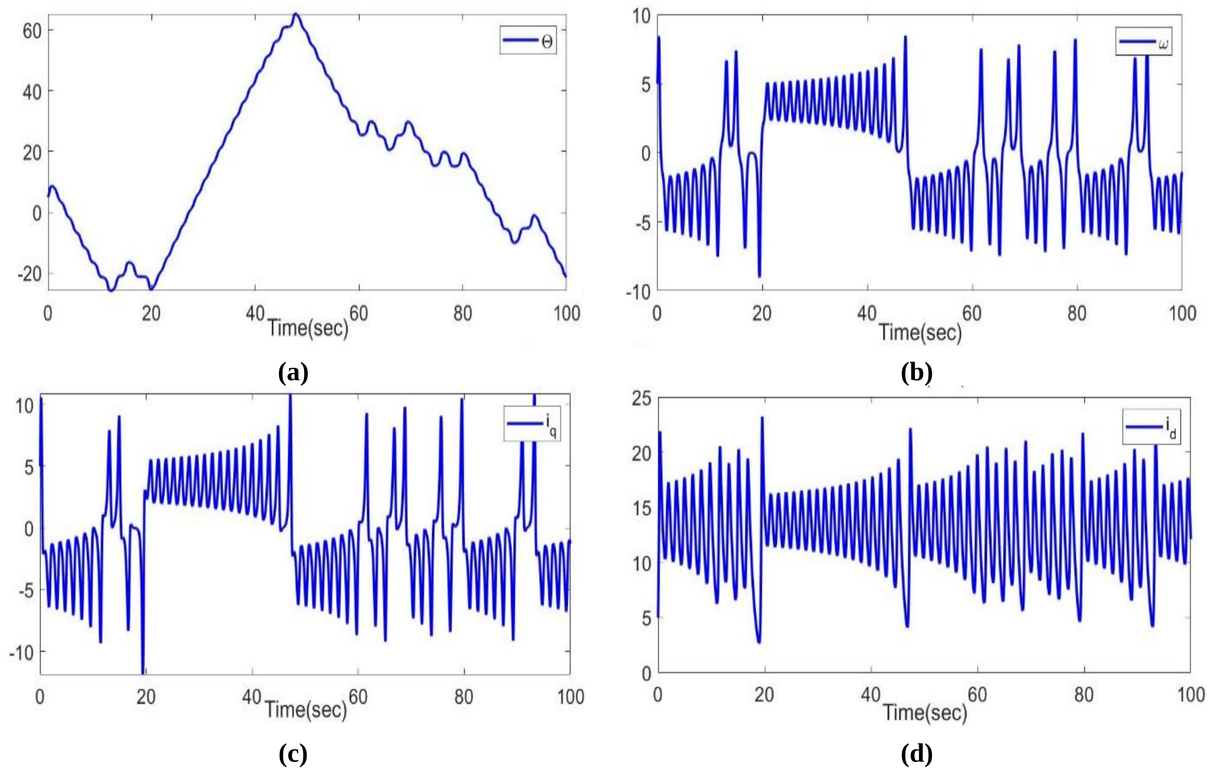
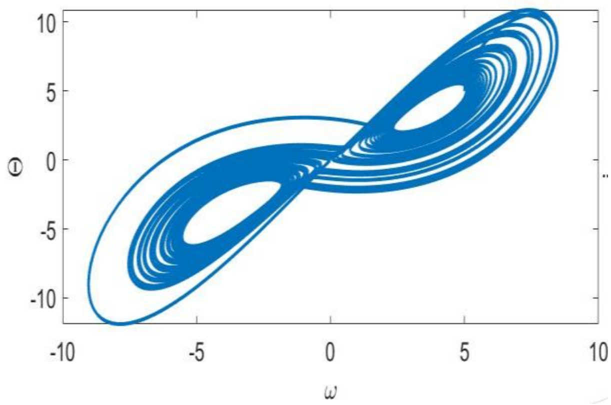
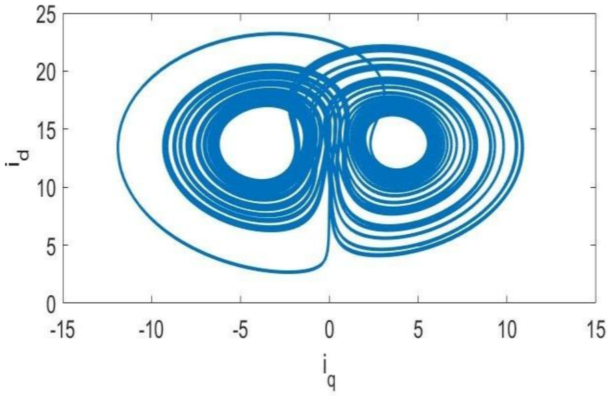


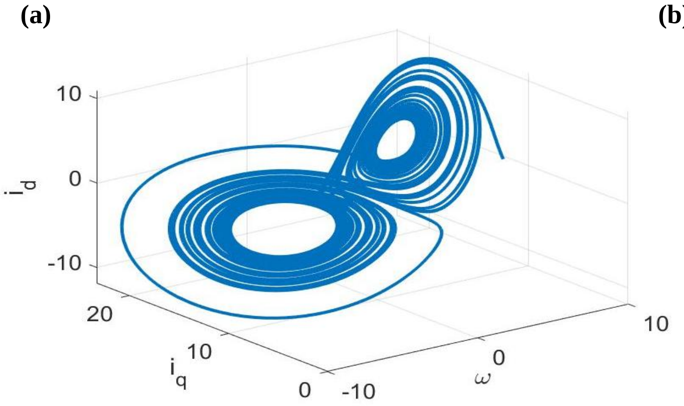
Figure 3. Chaotic Behavior in Permanent Magnet Synchronous Motor Variables: (a) θ Signal, (b) ω Signal, (c) i_q Signal, and (d) i_d Signal with Initial Conditions $[\theta, \omega, i_q, i_d]^T = [5\ 5\ 5\ 5]^T$



(a)



(b)



(c)

Figure 4. Phase Space of Chaotic Permanent Magnet Synchronous Motor: (a) $\theta - \omega$ Plane, (b) $i_d - i_q$ Plane, and (c) $i_d - i_q - \omega$ Plane

The calculate the Lyapunov exponents numerically based on the PMSM's nonlinear differential equations. The process involves evaluating how small perturbations to the system's initial conditions evolve over time in the phase space. If at least one exponent is positive, the system is confirmed to be chaotic. Although the specific algorithm used is not explicitly detailed, such computations are typically done using standard methods like the Wolf algorithm or QR decomposition-based approaches. The result, shown in Figure 2, confirms that the PMSM model has at least one positive Lyapunov exponent, indicating chaos.

3.2 Sliding model control with Lyapunov observer

In order to design the sliding mode controller based on the Lyapunov observer, the model of the chaotic permanent magnet synchronous motor is rewritten as below.

$$\begin{aligned}\frac{d\theta}{dt} &= \omega + u_1\\ \frac{d\omega}{dt} &= \sigma(i_q - \omega) - \bar{T}_L + u_2\\ \frac{di_q}{dt} &= -i_q - i_d\omega - \gamma\omega + \bar{u}_q + u_3\\ \frac{di_d}{dt} &= -i_d - i_q\omega + \bar{u}_d + u_4\end{aligned} \tag{4}$$

In Eq. (4), u_1, u_2, u_3, u_4 are the controllers that must be designed. In other words, these controllers try to eliminate the chaos in the permanent magnet synchronous motor.

The first step is to calculate the error:

$$e_\theta = \theta - \theta^*, \qquad e_\omega = \omega - \omega^*, \qquad e_{i_q} = i_q - {i_q}^*, \qquad e_{i_d} = i_d - {i_d}^* \tag{5}$$

Where $\theta^*, \omega^*, {i_q}^*, {i_d}^*$ are the desired values. Thus, the derivative of the error is equal to:

$$\dot{e}_\theta = \dot{\theta} - \dot{\theta}^*, \qquad \dot{e}_\omega = \dot{\omega} - \dot{\omega}^*, \qquad \dot{e}_{i_q} = i_q - {i_q}^*, \qquad \dot{e}_{i_d} = i_d - {i_d}^* \tag{6}$$

In the design of the proposed controller, the sliding surface is considered as follows:

$$S_1 = C_1 e_\theta, \qquad S_2 = C_2 e_\omega, \qquad S_3 = C_3 e_{i_q}, \qquad S_4 = C_4 e_{i_d} \tag{7}$$

In Eq. (7), C_i are positive constant coefficients. Also, the derivative of the sliding surface will be equal to:

$$\dot{S}_1 = D_1 \dot{e}_\theta, \qquad \dot{S}_2 = D_2 \dot{e_\omega}, \qquad \dot{S}_3 = D_3 \dot{e}_{i_q}, \qquad \dot{S}_4 = D_4 \dot{e}_{i_d} \tag{8}$$

In Eq. (8), D_i are constant coefficients obtained.

The proposed sliding model method is based on a Lyapunov observer. Thus, you should first consider the candidate Lyapunov function as follows:

$$V(s) = \frac{1}{2}(s_1^2 + s_2^2 + s_3^2 + s_4^2) \tag{9}$$

By deriving the Lyapunov candidate function, we have:

$$\dot{V}(s) = (s_1\dot{s}_1 + s_2\dot{s}_2 + s_3\dot{s}_3 + s_4\dot{s}_4) < 0 \tag{10}$$

Now, by breaking Eq. (10) into four separate parts, we have:

$$\dot{S}_1 = D_1 \dot{e}_\theta, \qquad \dot{S}_2 = D_2 \dot{e_\omega}, \qquad \dot{S}_3 = D_3 \dot{e}_{i_q}, \qquad \dot{S}_4 = D_4 \dot{e}_{i_d} \tag{11}$$

To reach $\dot{V}_1(s) < 0$, the sliding mode control design rule with Lyapunov observer is defined as:

$$\begin{aligned} \dot{V}_1(s) &= s_1\dot{s}_1 < 0 \\ &\Rightarrow s_1(D_1\dot{e}_\theta) < 0 \end{aligned} \tag{12}$$

In order for the condition expressed in Eq. (12) to always hold, the Lyapunov candidate function supervises it. So:

$$\Rightarrow \begin{cases} if\ s_1 > 0, then\ \dot{s}_1 < 0 \Rightarrow u_1 = D_1\big(-\theta + \theta^* + k_1 s_1 + \xi_1 sign(s_1)\big) \Rightarrow s_1(D_1\dot{e}_\theta) < 0\ and\ k_1, \xi_1 < 0 \\ if\ s_1 > 0, then\ \dot{s}_1 > 0 \Rightarrow u_1 = D_1\big(-\theta + \theta^* + k_1 s_1 + \xi_1 sign(s_1)\big) \Rightarrow s_1(D_1\dot{e}_\theta) < 0\ and\ k_1, \xi_1 > 0 \\ if\ s_1 > 0, then\ \dot{s}_1 = 0 \Rightarrow u_1 = D_1\big(-\theta + \theta^* + k_1 s_1 + \xi_1 sign(s_1)\big) \Rightarrow s_1(D_1\dot{e}_\theta) < 0\ and\ k_1, \xi_1 = 0 \end{cases} \tag{13}$$

where k_1, ξ_1 are control parameters and have constant values, also the sign function is the same as the sign function. According to the conditions observed for the sliding surface, the sign of the coefficient changes. Now, for the second part of Eq. (13), we have:

$$\begin{aligned} \dot{V}_2(s) &= s_2\dot{s}_2 < 0 \\ &\Rightarrow s_2\left(D_2\dot{e}_{i_q}\right) < 0 \end{aligned} \tag{14}$$

$$\Rightarrow \begin{cases} if\ s_2 > 0, then\ \dot{s}_2 < 0 \Rightarrow u_2 = D_2\left(-\sigma(i_q - \omega) + k_2 s_2 + \xi_2 sign(s_2)\right) \Rightarrow s_2(D_2\dot{e}_\omega) < 0\ and\ k_2, \xi_2 < 0 \\ if\ s_2 > 0, then\ \dot{s}_2 > 0 \Rightarrow u_2 = D_2\left(-\sigma(i_q - \omega) + k_2 s_2 + \xi_2 sign(s_2)\right) \Rightarrow s_2(D_2\dot{e}_\omega) < 0\ and\ k_2, \xi_2 > 0 \\ if\ s_2 > 0, then\ \dot{s}_2 = 0 \Rightarrow u_2 = D_2\left(-\sigma(i_q - \omega) + k_2 s_2 + \xi_2 sign(s_2)\right) \Rightarrow s_2(D_2\dot{e}_\omega) < 0\ and\ k_2, \xi_2 = 0 \end{cases} \tag{15}$$

In the same way, for the third and fourth parts of Eq. (15), we will have:

$$\begin{aligned} \dot{V}_3(s) &= s_3\dot{s}_3 < 0 \\ &\Rightarrow s_3(D_3\dot{e}_\omega) < 0 \end{aligned} \tag{16}$$

$$\begin{cases} if\ s_3 > 0, then\ \dot{s}_3 < 0 \Rightarrow u_3 = D_3\left(i_q + i_d\omega + \gamma\omega + k_3 s_3 + \xi_3 sign(s_3)\right) \Rightarrow s_3\left(D_3\dot{e}_{i_q}\right) < 0\ and\ k_3, \xi_3 < 0 \\ if\ s_3 < 0, then\ \dot{s}_3 > 0 \Rightarrow u_3 = D_3\left(i_q + i_d\omega + \gamma\omega + k_3 s_3 + \xi_3 sign(s_3)\right) \Rightarrow s_3\left(D_3\dot{e}_{i_q}\right) < 0\ and\ k_3, \xi_3 > 0 \\ if\ s_3 = 0, then\ \dot{s}_3 = 0 \Rightarrow u_3 = D_3\left(i_q + i_d\omega + \gamma\omega + k_3 s_3 + \xi_3 sign(s_3)\right) \Rightarrow s_3\left(D_3\dot{e}_{i_q}\right) < 0\ and\ k_3, \xi_3 = 0 \end{cases} \tag{17}$$

$$\begin{aligned} \dot{V}_4(s) &= s_4\dot{s}_4 < 0 \\ &\Rightarrow s_4\big(D_4\dot{e}_{i_d}\big) < 0 \end{aligned} \tag{18}$$

$$\begin{cases} if\ s_4 > 0, then\ \dot{s}_4 < 0 \Rightarrow u_4 = D_4\left(-i_d - i_q\omega + k_4 s_4 + \xi_4 sign(s_4)\right) \Rightarrow s_4\big(D_4\dot{e}_{i_d}\big) < 0\ and\ k_4, \xi_4 < 0 \\ if\ s_4 < 0, then\ \dot{s}_4 > 0 \Rightarrow u_4 = D_4\left(-i_d - i_q\omega + k_4 s_4 + \xi_4 sign(s_4)\right) \Rightarrow s_4\big(D_4\dot{e}_{i_d}\big) < 0\ and\ k_4, \xi_4 > 0 \\ if\ s_4 = 0, then\ \dot{s}_4 = 0 \Rightarrow u_4 = D_4\left(-i_d - i_q\omega + k_4 s_4 + \xi_4 sign(s_4)\right) \Rightarrow s_4\big(D_4\dot{e}_{i_d}\big) < 0\ and\ k_4, \xi_4 = 0 \end{cases} \tag{19}$$

3.3 Numerical simulation

In this section, a simulation has been done using MATLAB software and the ode113 differential equation solution method. Eq. (4) will be unchanged for the initial conditions of $[\theta, \omega, i_q, i_d]^T = [5\ 5\ 5\ 5]^T$. The desired values are $\theta = 0,\ \omega^* = 0,\ i_q^* = 0,\ i_d^* = 0$. The values of $C_i = 1, (i = 1, 2, 3, 4)$ and the controlling coefficients are equal to $k_i = 5, (i = 1, 2, 3, 4)$ and $\xi_i = 0.01, (i = 1, 2, 3, 4)$. It can be seen in Fig. 5 that the error of the proposed method tends to zero with the passing of a limited time. If chaos occurs in a dynamic system, it can cause a lot of damage to the system and related components. Therefore, in the design of controllers of chaotic systems, the time to reach zero error is very important. As seen in Fig. 5, the time to reach zero error in this design method is approximately equal to 0.7 seconds.

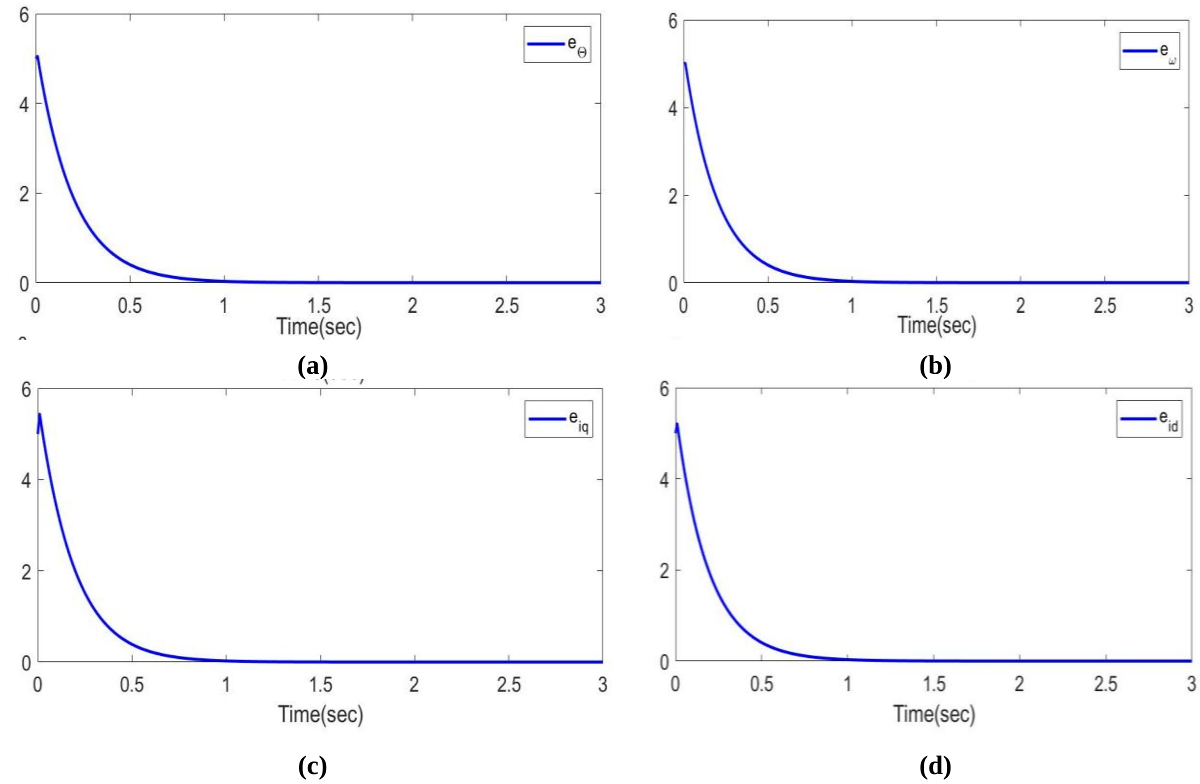
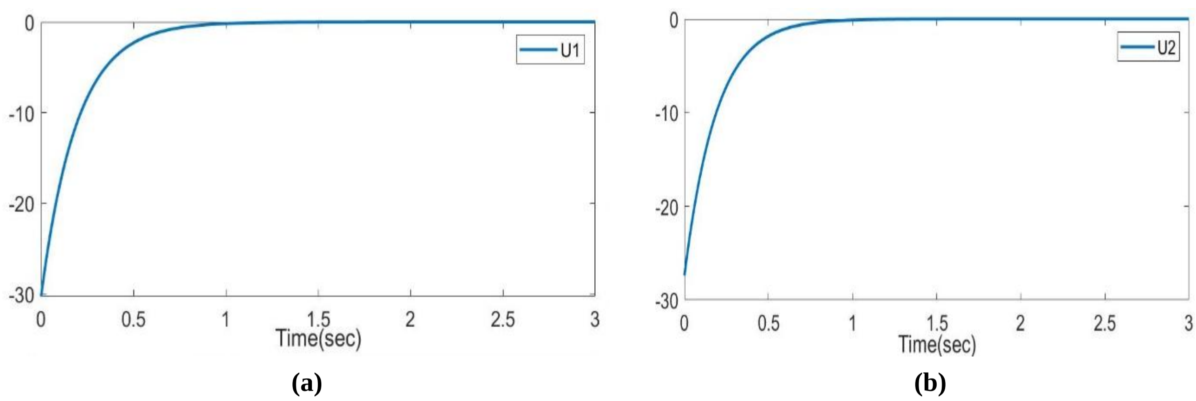
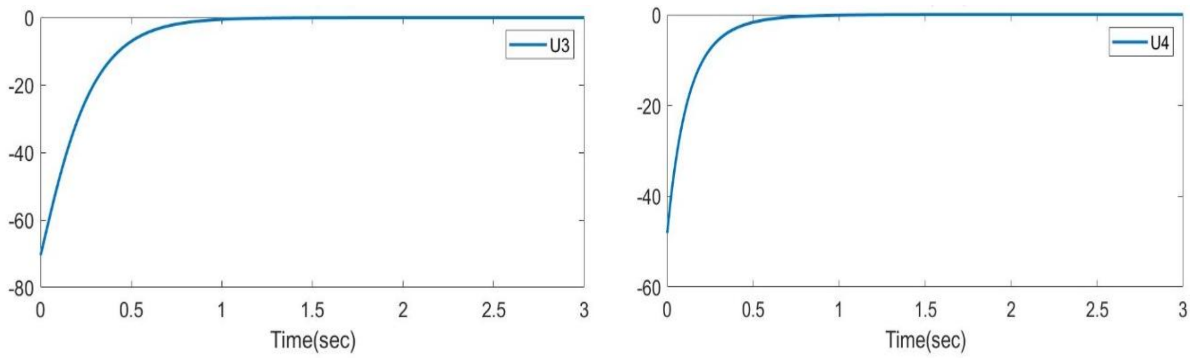


Figure 5. Time History Error of SMC with Lyapunov Observer: (a) θ signal, (b) ω Signal, (c) i_q Signal, and (d) i_d Signal with Initial Conditions $[\theta, \omega, i_q, i_d]^T = [5\ 5\ 5\ 5]^T$.

The signal behavior of the sliding model controller based on the Lyapunov observer is shown in Fig. 6. In the real world, the cost of the controller design can be determined by the behavior of the control signal. If the control signal has strong fluctuations or a large amplitude, it will be expensive to implement in the real world. Therefore, the controller signal of the proposed method is without fluctuation and has a large amplitude, and even after reaching zero error, it tends to zero. This shows that real-world implementation with the proposed control method will be low-cost.





(c) (d)

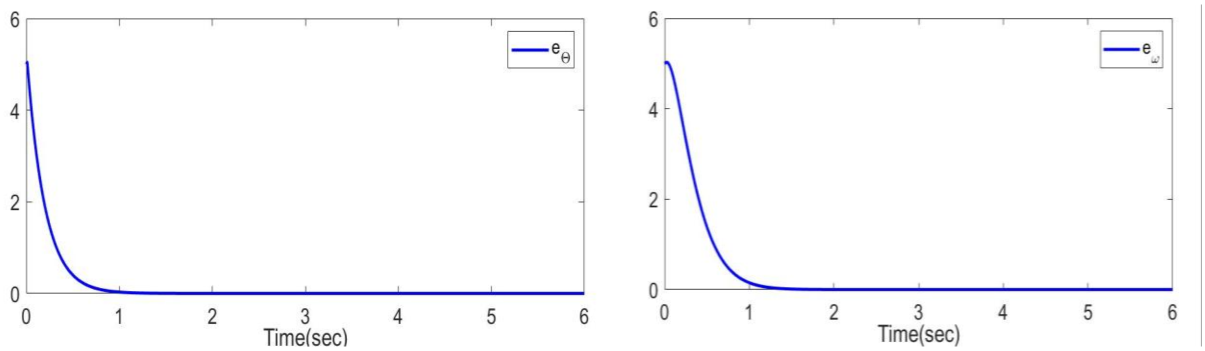
Figure 6. Behavior of SMC based on Lyapunov Observer Without Unknown Input: (a) U_1 Signal, (b) U_2 Signal, (c) U_3 Signal, and (d) U_4 Signal

In the second part of the numerical simulation, we reduce the number of controllers. In other words, by using two sliding model controllers with a Lyapunov observer, the chaos in the permanent magnet synchronous motor will be eliminated. Therefore, Eq. (4) will be rewritten as follows:

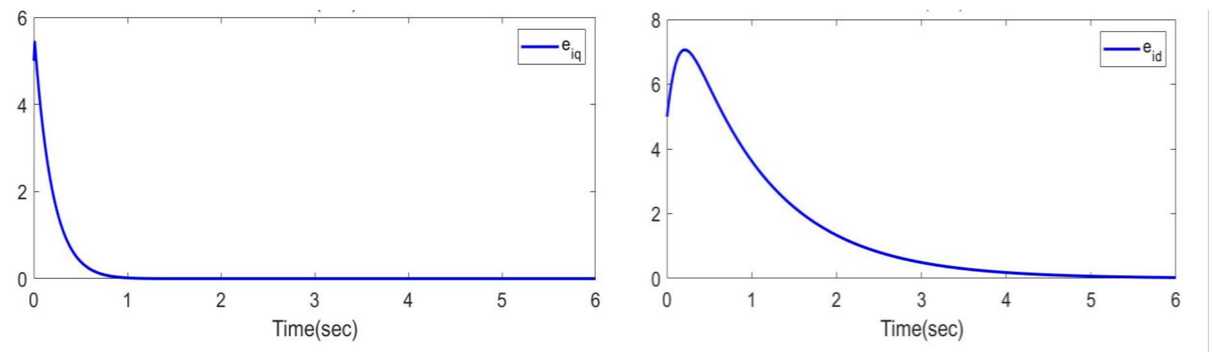
$$\begin{aligned} \frac{d\theta}{dt} &= \omega + u_1 \\ \frac{d\omega}{dt} &= \sigma(i_q - \omega) - \bar{T}_L \\ \frac{di_q}{dt} &= -i_q - i_d\omega - \gamma\omega + \bar{u}_q + u_3 \\ \frac{di_d}{dt} &= -i_d - i_q\omega + \bar{u}_d \end{aligned} \tag{20}$$

As Eq. (20) shows, the sliding model controller based on the Lyapunov observer is only added to the first and third lines. Other simulation parameters are unchanged. Fig. 7 shows the error of the sliding model controller based on the Lyapunov observer. As can be seen from Fig. 7, the behavior of the error e_θ, e_{i_q} tends to zero in a period of about 0.7 seconds, but the behavior of the error e_ω, e_{i_d} has a longer stability time.

Furthermore, this point should not be hidden that in this situation, two control signals have been completely removed, and by using only two controllers, the chaos in the permanent magnet synchronous motor has been removed. In Fig. 8, the behavior of the controller signal is depicted. As you can see, u_2, u_4 the controller has been removed. The controller signal u_1, u_3 remains unoscillated and has a low amplitude.

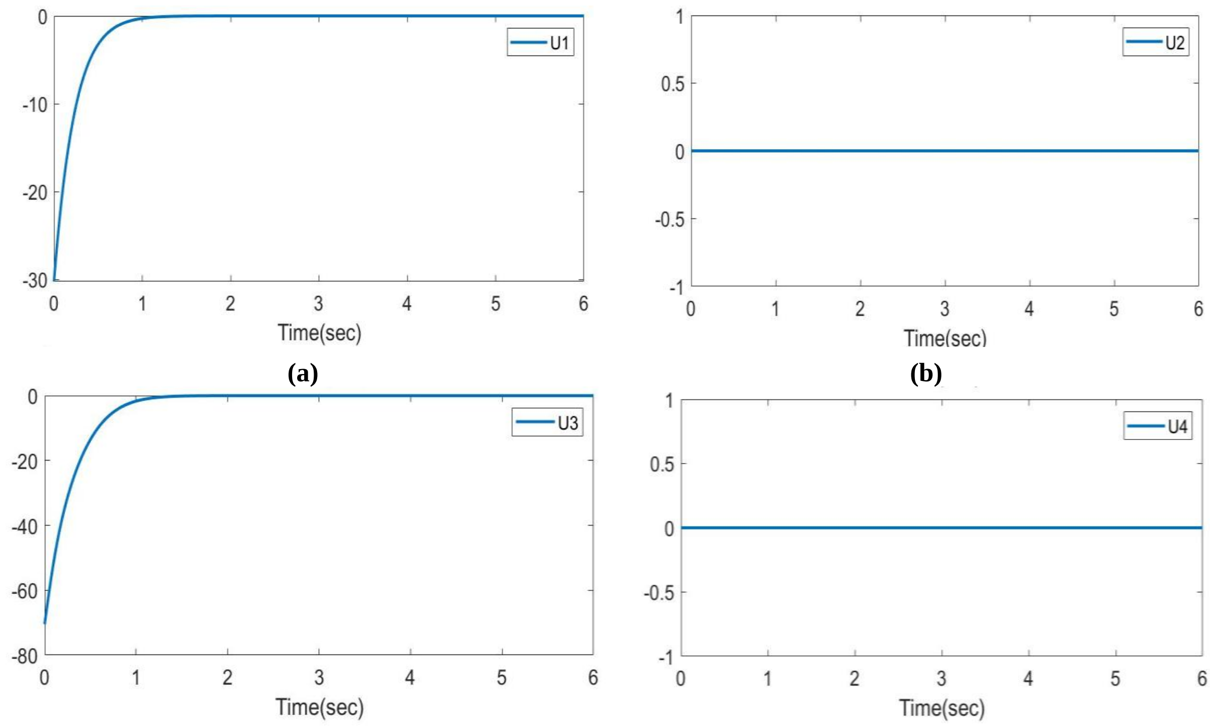


(a) (b)



(c) (d)

Figure 7. Behavior of Error for Chaotic PMSM in the Presence of Two SMC Based on Lyapunov Observer Without Unknown Input: (a) θ Signal, (b) ω Signal, (c) i_q Signal, and (d) i_d Signal



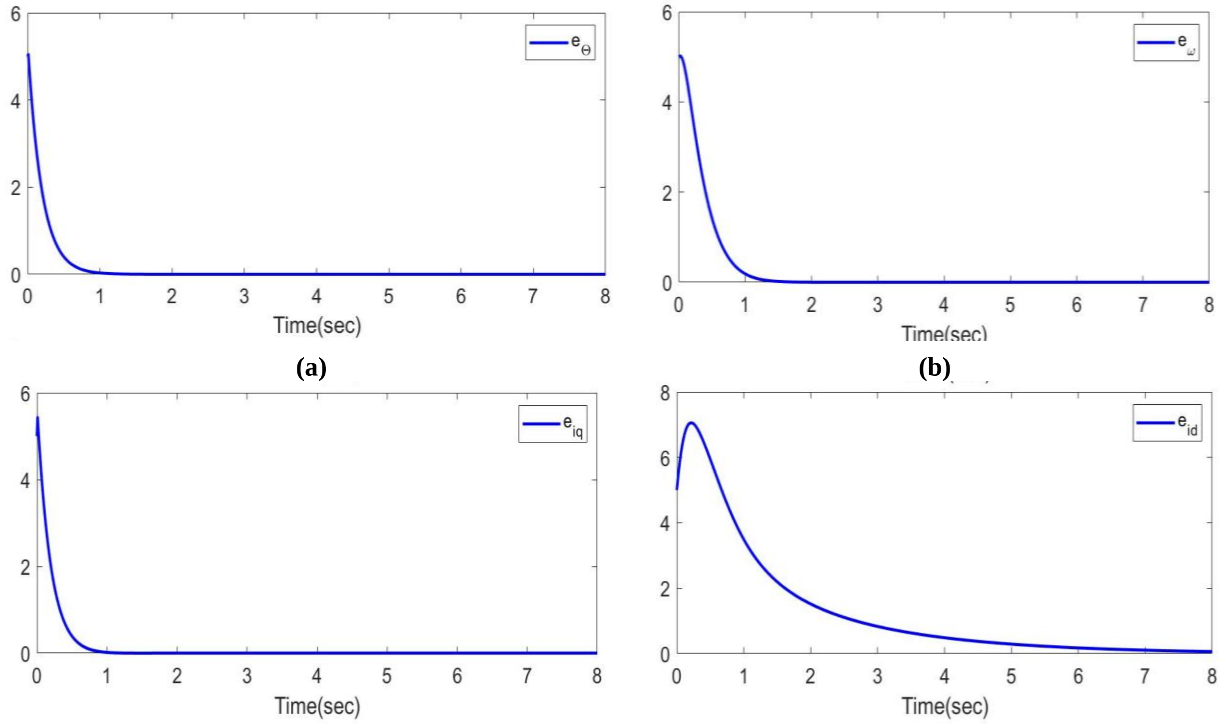
(c) (d)

Figure 8. Behavior of SMC Based on Lyapunov Observer in the Condition of Controller Elimination u_2, u_4 and without Unknown Input: (a) U_1 Signal, (b) U_2 Signal, (c) U_3 Signal, and (d) U_4 Signal

In the third part of the numerical simulation, an unknown input will be added to the chaotic permanent magnet synchronous motor model. It is reminded that the engine model will still have two controllers. Thus, Eq. (3) is rewritten as follows:

$$\frac{d\theta}{dt} = \omega + u_1$$
$$\frac{d\omega}{dt} = \sigma(i_q - \omega) - \bar{T}_L + P(\omega)$$
$$\frac{di_q}{dt} = -i_q - i_d\omega - \gamma\omega + \bar{u}_q + u_3$$
$$\frac{di_d}{dt} = -i_d - i_q\omega + \bar{u}_d + P(i_q) \quad (21)$$

which in Eq. (21), $P(\omega), P(i_q)$ are unknown inputs with a limited range, and in the form of $P(\omega) = 0.5 \times \sin(\omega)$, $P(i_q) = 0.5 \times sin\ (i_q)$. Other simulation parameters and initial conditions are unchanged. Fig. 9 depicts the error of the sliding model controller based on the Lyapunov observer in the presence of two unknown inputs.

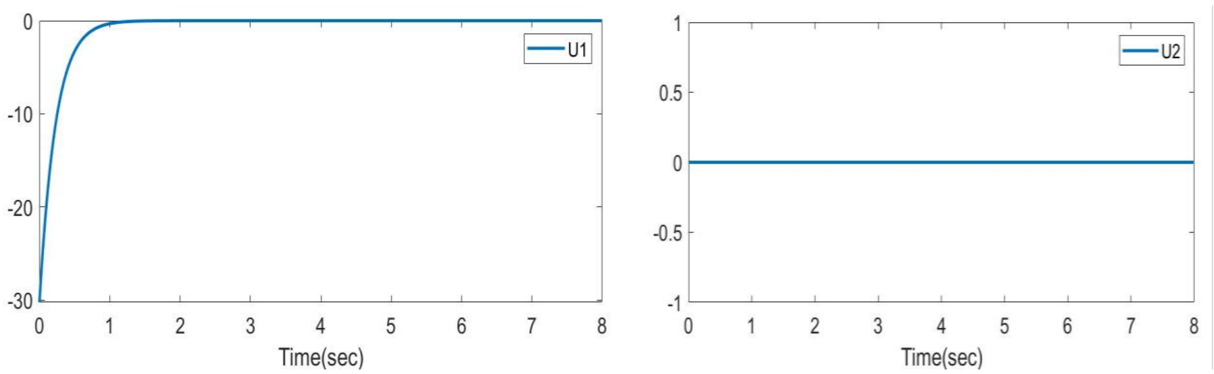


(a) (b)

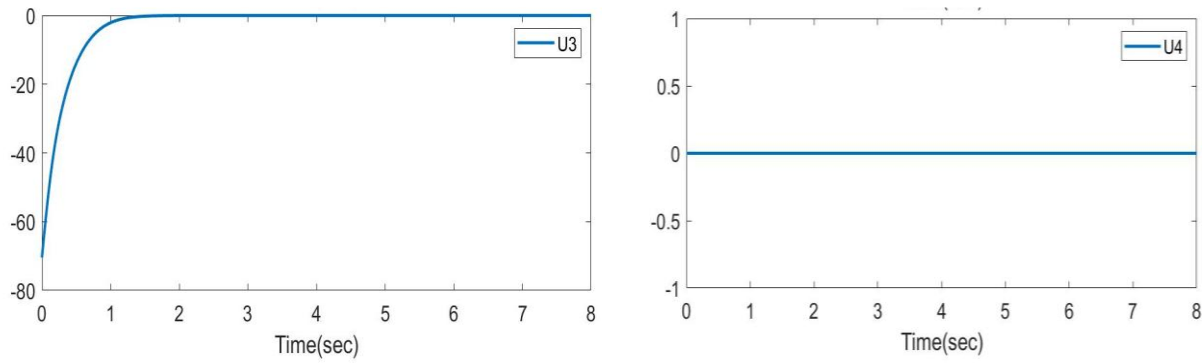
(c) (d)

Figure 9. Error of Behavior for Chaotic PMSM in the Presence of Two SMC Based on Lyapunov Observer in the Presence of Two Unknown Variables $P(\omega) = 0.5 \times \sin(\omega)$, $P(i_q) = 0.5 \times sin\ (i_q)$: (a) θ Signal, (b) ω Signal, (c) i_q Signal, and (d) i_d Signal

As can be seen from Fig. 9, the duration of stability is unchanged in all faults except e_{i_d}. Fig. 10 shows the behavior of the controller signal. In the situation where the two controllers are eliminated and the permanent magnet synchronous motor is controlled by only two controllers, the overall control system cost can potentially be reduced by approximately 40–50%, considering the hardware, complexity, and energy consumption. This significant reduction makes the proposed approach more practical and cost-effective for real-world applications.



(a) (b)



(c) (d)

Figure 10. Behavior of SMC Signal Based on Lyapunov Observer in the Presence of Two Unknown Inputs $P(\omega) = 0.5 \times \sin(\omega)$, $P(i_q) = 0.5 \times sin\ (i_q)$ via Two Controls u_2, u_4: (a) U_1 Signal, (b) U_2 Signal, (c) U_3 Signal, and (d) U_4 Signal

Table 1. Chaos Control Methods in PMSM

No. Ref	Dimension Model	Method	Number of controllers	Unknown input or disturbance	Signal control	Time stability error
[40]	3-D	Passive control	one	-	-	≈ 13 sec
[41]	3-D	Optimal Lyapunov exponents placement	Two	-	Available	≈ 0.1 sec
[42]	3-D	Adaptive Control	Two	-	-	≈ 2 sec
[43]	3-D	Adaptive robust nonlinear control	Three	-	Available	≈ 2 sec
[44]	3-D	Nonlinear Control	One	-	Available	≈ 7 sec
[45]	3-D	Finite Time control	Three	-	-	≈ 0.5-2 sec
[46]	3-D	Nonlinear Simple control	One	-	-	≈ 1-6 sec
[47]	3-D	Backstepping-based adaptive fuzzy	Three	d1=2x1Sin(3πt/2) d2=3x2Cos(4πt/3) d3=5x2Sin(5πt/4)	Available	≈ 0.3 sec
This paper	4-D	Sliding mode control based on Lyapunov	Four and Two	P(ω)=0.5Sin(ω) P(iq)=0.5Sin(iq)	Available	≈ 0.5-2 sec

In Table 1, different methods of disturbance control in a permanent magnet synchronous motor are described. As can be seen, the most important factor in chaos control is the time it takes to achieve zero error. Also, the number of controllers is directly related to the design cost.

4. CONCLUSION

In this study, the chaotic behavior and control of a 4-D PMSM model were investigated. Based on the reviewer's recommendation, the key conclusions are presented as follows:

1. This study examined the 4-D PMSM model, revealing that variations in system parameters can transition the motor's behavior from nonlinear to chaotic. This was substantiated through the calculation of Lyapunov exponents.
2. A SMC based on a Lyapunov observer was developed. The controller dynamically adjusted its parameters using a candidate Lyapunov function, ensuring stability by responding to the conditions of the sliding surface.

3. Numerical simulations demonstrated that the proposed SMC effectively stabilizes the chaotic PMSM rapidly. Notably, chaos was suppressed using both four and two control channels, with the latter approach reducing design complexity and cost without compromising performance. Additionally, the controller-maintained stability even when unknown inputs were introduced to the angular velocity and current i_d , while control was applied to the motor position and current i_q.

Future studies are encouraged to explore alternative control strategies, such as impulsive, adaptive, and intelligent methods (e.g., fuzzy logic and neural networks), to enhance the decision-making capabilities of the Lyapunov observer.

Author Contributions

Seyed Mohamad Hamidzadeh: Writing – Original Draft, Software, Investigation. Amiral Aziz: Writing—Review and Editing, Formal Analysis, Validation. Mohamad Afendee Mohamed: Supervision, Conceptualization, Validation. Sundarapandian Vaidyanathan: Writing – Original Draft, Formal analysis, Software. Muhamad Deni Johansyah: Writing – Original Draft, Visualization, Formal Analysis. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare no competing interests.

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