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MARKETING GAMES AND SENSITIVITY ANALYSIS FOR ANY NUMBER OF COMPANIES

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ABSTRACT

In the business world, advertising plays a crucial role in shaping market dynamics. This study analyzes advertising competition among companies by calculating the optimal profits for two to four competitors and postulating a general formula for a larger number of companies. Sensitivity analysis is employed to observe how changes in advertising parameters affect profits. The simulation results indicate that in the case of four companies, the optimal spending (Xopt) ranges from 137246 to 191288, while the optimal profit (Popt) ranges from 38693.1 to 92702. The sensitivity analysis shows that companies with higher advertising effectiveness achieve greater profits, whereas those with lower effectiveness tend to experience reductions in both allocation and profit. With the aid of Maple and Excel, this research extends previous advertising competition models by providing a comprehensive framework for optimizing advertising budgets in oligopolistic markets.

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1. INTRODUCTION

In the business context, a product can be supplied by a single provider or producer (monopoly)**[1]**, two (duopoly) [2], or multiple suppliers (oligopoly) [3]. In a monopoly situation, there is no competition, and consumers will purchase the product because there are no other alternative options available. In a duopoly environment, only two major players dominate the market and often engage in fierce competition with each other. While there may be other competitors, their impact is relatively minor. Notable examples of duopolies include Boeing and Airbus, Visa and MasterCard, and Pepsi and Coca Cola. Competition in such markets is ongoing, with each competitor striving to capture a larger market share.

In cases where there are multiple suppliers, consumers have a broader range of options and can opt for a supplier that provides a more affordable cost, provided that the product quality remains comparable [4]. In such cases, advertising plays a vital role [5]. Companies allocate substantial funds to promote their products. For instance, food companies aggressively promote their products on television, particularly targeting children [6]. Other examples include mobile phone [7] and auto motive companies [8], which frequently engage in intensive advertising across various media, including television, newspapers, and magazines. Nowadays, nearly everything can be purchased through online platforms. In Indonesia, online markets are largely dominated by BukaLapak, Shopee, Blibli.com, and Tokopedia. In Malaysia, the majority of the broadband telecommunication market share belongs to Celcom, Maxis, and Digi.

In an oligopolistic environment, there are typically no more than four major players who dominate the market, although it could be more, and there is no precise definition for the number of players that define an oligopoly [9]. However, other competitors in the market usually hold a relatively small market share. Barriers exist for new companies attempting to enter the same market. The monopolistic control and profit potential in oligopolistic sectors are, to some extent, dependent on the level of collaboration among companies in determining product prices, with the aim of achieving stable supplies and consistent profits [10]. However, such collaborative actions may not always conform to legal regulations and can result in penalties imposed by authorities. Additionally, these actions can fail if other non-colluding companies decide to lower their prices, triggering a reduction in prices among the oligopolistic players. An example of collusion in the U.S. airline industry is presented in this context [11].

Setting the pricing strategy is a critical decision for a company, but for many influential companies, there's another significant choice to take into account: how much to allocate for advertising. Since advertising a product can incur significant costs, companies need to carefully decide how much to invest in advertising and where to advertise to maximize their profits. A company that lacks adequate advertising may face challenges in a competitive environment. Even if a product is of excellent quality, effective advertising is crucial for gaining a market share, unless the company enjoys a monopoly. For insights into the significance of advertising in markets where multiple competitors are present, you can consult reference [12].

In today's landscape, digital advertising has become prevalent and its importance is growing, even surpassing traditional print media in the form of periodical print publications for the public. According to a Nielsen Media report, advertising expenditures in Indonesia for the first seven months of 2020 totaled around Rp 122 trillion. This amount was allocated as follows: radio approximately Rp 0.6 trillion, print media about Rp 9.6 trillion, websites about Rp 24.2 trillion, and television about Rp 88 trillion. Therefore, it's clear that product advertising plays a crucial role in a company's sustainability. However, it's important to note that not every product necessarily requires advertising. For instance, products like corn or soybeans already have a well-established market share.

The purpose of this paper is to perform a sensitivity analysis of the optimum profit for two to four companies competing for the same product. The sensitivity analysis is then extended to any number of companies. Our approach is new and unique. From preliminary studies, there is no general formula to determine the optimum profit for any number of companies. This formula will add fresh ideas to the existing knowledge in the field (applied mathematics in microeconomic theory)[13]. The examples for computing the optimum profit for up to four companies [14]. The examples given are fictitious but can be readily applied to real situations. There are many examples of oligopoly in Indonesia, such as instant noodles (brands like Indo Mie, Super Mie, and Mie Sedap) and internet providers (such as IndiHome, MNC Playmedia, Indosat Ooredoo, Biznet Network, and MyRepublic). Competitions among the companies are very tight and often fierce; they spend a lot of money on advertising their products. For instant noodles, the biggest player is Indofood, which produces Indo Mie, Super Mie, and Sarimi. These brands seize the largest share of the instant noodle market in Indonesia.

To address the proposed problem in this article, a mathematical model [15] involving sensitivity analysis within the context of advertising competition. The construct an optimization model to calculate the optimal profits for two to four companies competing in a competitive advertising environment. This model employs linear and nonlinear programming approaches to maximize profits, taking into account variables such as advertising costs, advertising effectiveness, and market sensitivity to price and advertising changes. Maple is used for symbolic and numerical calculations, while Excel is utilized for simulation and parameter adjustments. Sensitivity analysis is conducted to observe how changes in advertising parameters affect optimal profits, enabling companies to determine the most effective advertising strategies.

Despite extensive studies on competition models in monopoly and duopoly markets, as well as the influence of advertising in oligopolistic environments, a significant gap remains in analyzing optimal profit strategies for more than two companies under varying advertising conditions. Existing research often focuses on static or simplified scenarios, neglecting the complexities introduced by multi-company competition and sensitivity to advertising parameters. This study addresses this gap by introducing a comprehensive sensitivity analysis framework to assess the optimal profits for two to four companies and postulating a general formula for any number of companies. The inclusion of digitalization in advertising further distinguishes this research from prior studies, as it considers the evolving dynamics of modern marketing. By leveraging advanced computational tools such as Maple and Excel for symbolic and numerical analyses, this paper not only expands the theoretical understanding of advertising competition but also provides a practical framework for real-world applications, offering fresh insights into profit optimization in competitive markets.

2. RESEARCH METHODS

To conduct the research described in this paper, a comprehensive methodology was employed, which integrates theoretical frameworks and practical applications to analyze marketing dynamics and sensitivity analysis for companies involved in advertising competition. A thorough literature review was conducted to build a fundamental understanding of marketing dynamics [3][4], sensitivity analysis [16][17], and the role of advertising in competitive markets. This review encompassed seminal works on sensitivity analysis in practice, the utilization of Maple for problem-solving in scientific computing [18][19], and the applications of linear and nonlinear programming with Maple [14][20].

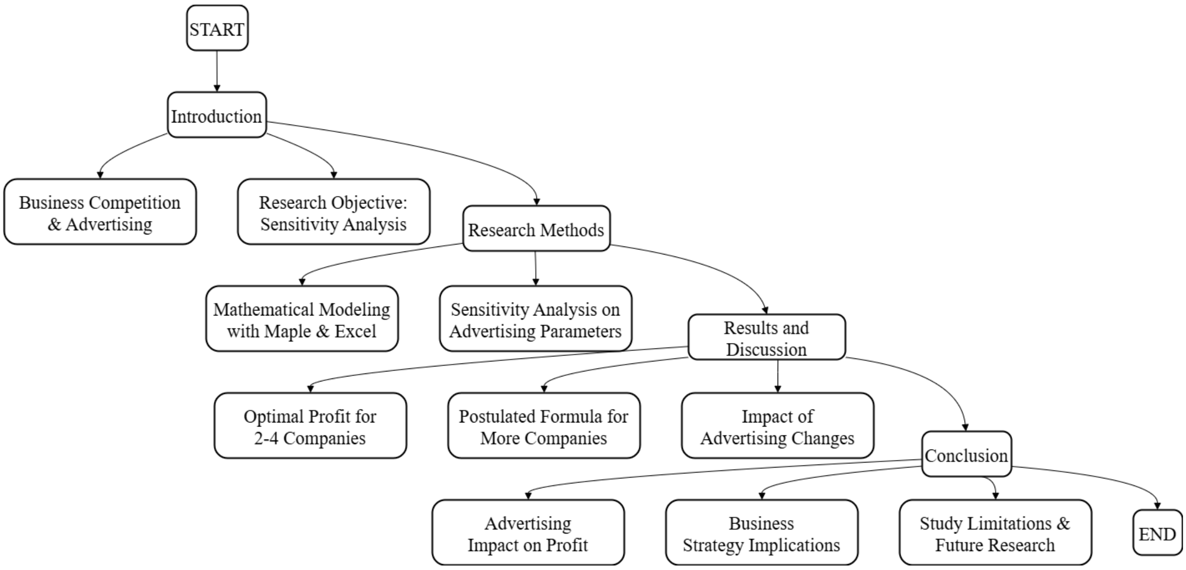

Figure 1. Research Process Diagram of Advertising and Profit Optimization

In Fig. 1, by using Maple, mathematical models were developed to compute optimal profits for up to four companies within a competitive advertising environment. For scenarios involving more than four companies, profits were postulated, and their numerical values were computed using Maple and Excel to facilitate parameter adjustments. Sensitivity analysis was conducted to examine how changes in parameters (such as advertising effectiveness and market sensitivity) influence the optimal profits of participating companies. This analysis leveraged Maple's capabilities for symbolic problem-solving and numerical computation.

The results obtained from the mathematical models and sensitivity analysis were carefully analyzed to draw conclusions about the impact of advertising competition on company profits and the effectiveness of different advertising strategies. This methodological approach, rooted in a combination of theoretical analysis and practical application, enables a comprehensive examination of marketing dynamics and sensitivity analysis in the context of competitive advertising. The use of advanced computational tools such as Maple and Excel facilitates detailed exploration of complex mathematical models and the effects of parameter variations on company profits

In the following we discuss various cases for obtaining optimum that is for two companies, three companies, four companies and any number of companies. It should be noted that the findings of this research are based on hypothetical data. While the models and sensitivity analysis provide a theoretical foundation, the lack of comparison with real-world advertising budget data is acknowledged as a limitation. Future studies could incorporate actual data to further validate the models and conclusions presented in this paper.

3. RESULTS AND DISCUSSION

In this section the formulas for dollars allocated and profit gained for each company. The discuss cases for two, three and four companies. The result for five and six companies since the derivations are very long and complicated. For any number of companies, the postulate of the formulas. All formulas are derived by using Maple, a symbolic programming language.

3.1 Case for Two Companies (Duopoly)

In a duopoly scenario, two companies allocate X_i dollars ($i = 1,2$) for advertising the same product in order to gain a share of the market. The variables a_i and β_i represent the advertising effectiveness and the sensitivity of each company's appeal to the market, respectively. Here, $0 < a_i, \beta_i \leq 1$. Market share for each company is given by companies (competitors)' are given by [16].

$$s_i = \frac{a_i X_i \beta^i}{a_1 X_1 \beta^1 + a_2 X_2 \beta^2}. \tag{1}$$

Let S be the total market size of the product. Assume that the market will be proportional to the money poured for advertisement of the product. In the real world, this assumption is not always true. An old player which has established itself in the market may not necessarily spend more money to capture more shares in the market. However, new players tend to spend much money to lure new customers. The sale for each company is then given by s_i:

$$\frac{S a_i {X_i}^{\beta_i}}{a_1 {X_1}^{\beta_1} + a_2 {X_2}^{\beta_2}}. \tag{2}$$

For a special case, let β be 1. The sale for each company is then given by

$$\frac{S a_i X_i}{a_1 X_1 + a_2 X_2}. \tag{3}$$

If each company's products generate a gross profit of g_i, then the profit P_i for company i is subsequently calculated as follows:

$$P_i = \frac{S g_i a_i X_i}{a_1 X_1 + a_2 X_2} - X_i. \tag{4}$$

Let $f_i = a_i g_i$, We obtain

$$P_i = \frac{S f_i X_i}{a_1 X_1 + a_2 X_2} - X_i. \tag{5}$$

The optimum profit is found by differentiating P_i with respect to X_i and make the result zero or

$$\frac{\partial P_i}{\partial X_i} = 0.$$

After simplification, from $\frac{\partial P_1}{\partial X_1} = 0$ and $\frac{\partial P_2}{\partial X_2} = 0$ then:

$$S f_1 a_2 X_2 - (a_1 X_1 + a_2 X_2)^2 = 0, \quad (6)$$

$$S f_2 a_1 X_1 - (a_1 X_1 + a_2 X_2)^2 = 0. \quad (7)$$

Solving Eqs. (6) and (7) respectively gives the values of X_1 and X_2, optimum spending:

$$X_{opt,1} = \frac{S g_1 f_1 f_2}{(f_1 + f_2)^2}; \quad X_{opt,2} = \frac{S g_2 f_1 f_2}{(f_1 + f_2)^2}; \quad (8)$$

Substituting to Eq. (4) yields the optimum profit:

$$P_{opt,1} = \frac{S g_1 {f_1}^2}{(f_1 + f_2)^2}; \quad P_{opt,2} = \frac{S g_2 {f_2}^2}{(f_1 + f_2)^2}. \quad (9)$$

The problem can be readily addressed with the assistance of Maple, as illustrated in an example presented in [17]. Maple is a programming language designed for symbolic computation, enabling us to address the problem symbolically, rather than just numerically, which is different from many other programming languages. In addition to Maple, Mathematica is another robust symbolic computation language that can be utilized. A significant benefit shared by both Maple and Mathematica is their capability to generate the required formulas, as exemplified in [19].

3.2 Case for Three Companies

The sale for company i is then given by:

$$\frac{S a_i X_i}{a_1 X_1 + a_2 X_2 + a_3 X_3}. \quad (10)$$

As a result, the company's profit P_i is determined as follows:

$$P_i = \frac{S g_i a_i X_i}{a_1 X_1 + a_2 X_2 + a_3 X_3} - X_i, \quad (11)$$

Thus. Eq. (3) can be written as

$$P_i = \frac{S f_i X_i}{a_1 X_1 + a_2 X_2 + a_3 X_3} - X_i.$$

Taking $\frac{\partial P_1}{\partial X_1} = 0, \frac{\partial P_2}{\partial X_2} = 0$ and $\frac{\partial P_3}{\partial X_3} = 0$ then have after simplification:

$$S f_1 (a_2 X_2 + a_3 X_3) - (a_1 X_1 + a_2 X_2 + a_3 X_3)^2 = 0, \quad (12)$$

$$S f_2 (a_3 X_3 + a_1 X_1) - (a_1 X_1 + a_2 X_2 + a_3 X_3)^2 = 0, \quad (13)$$

$$S f_3 (a_1 X_1 + a_2 X_2) - (a_1 X_1 + a_2 X_2 + a_3 X_3)^2 = 0. \quad (14)$$

Solutions of this set of linear equations are given by

$$X_{opt,1} = 2S \frac{g_1 f_2 f_3 [f_1 (f_2 + f_3) - f_2 f_3]}{(f_1 f_2 + f_2 f_3 + f_3 f_1)^2}, \quad (15)$$

$$X_{opt,2} = 2S \frac{g_2 f_3 f_1 [f_2 (f_3 + f_1) - f_3 f_1]}{(f_1 f_2 + f_2 f_3 + f_3 f_1)^2}, \quad (16)$$

$$X_{opt,3} = 2S \frac{g_3 f_1 f_2 [f_3 (f_1 + f_2) - f_1 f_2]}{(f_1 f_2 + f_2 f_3 + f_3 f_1)^2}. \quad (17)$$

Substituting $X_{opt,1}, X_{opt,2}$ and $X_{opt,3}$ to Eq. (2) yields profit for each company. After simplification, the profits are given as follow:

$$P_{opt,1} = S\frac{g_1[f_1(f_2+f_3)-f_2f_3]^2}{(f_1f_2+f_2f_3+f_3f_1)^2}, \tag{18}$$

$$P_{opt,2} = S\frac{g_2[f_2(f_3+f_1)-f_3f_1]^2}{(f_1f_2+f_2f_3+f_3f_1)^2}, \tag{19}$$

$$P_{opt,3} = S\frac{g_3[f_3(f_1+f_2)-f_1f_2]^2}{(f_1f_2+f_2f_3+f_3f_1)^2}. \tag{20}$$

Note that Eqs. (15) to (20) are circular. X_1 and P_1 can easily find X_2 and P_2 as well as X_3 and P_3.

Write Eqs. (15) to (20) in simpler forms. Let A_1 be f_2+f_3 and B_1 be f_2f_3. We obtain:

$$X_{opt,1} = 2S\frac{g_1B_1[A_1f_1-B_1]}{[A_1f_1+B_1]^2}, \tag{21}$$

$$P_{opt,1} = S\frac{g_1[A_1f_1-B_1]^2}{[A_1f_1+B_1]^2}. \tag{22}$$

$X_{opt,2}$ and $X_{opt,3}$ can be written easily by changing index i from 1 to 2 and 3. For example, $f_1 \to f_3, g_1 \to g_3, A_1 \to A_3$ and $B_1 \to B_3$ where the symbol " $\to$ " means becomes. Here, $A_3 = f_1 + f_2$ and $B_3 = f_1f_2$.

3.3 Case for Four Companies

The sales of company i are subsequently determined by

$$\frac{a_iX_i}{a_1X_1+a_2X_2+a_3X_3+a_4X_4}. \tag{23}$$

The profit for company i can be expressed as follows:

$$P_i = \frac{g_ia_iX_i}{a_1X_1+a_2X_2+a_3X_3+a_4X_4} - X_i. \tag{24}$$

The Eq. (24) can be written as

$$P_i = \frac{f_iX_i}{a_1X_1+a_2X_2+a_3X_3+a_4X_4} - X_i. \tag{25}$$

Taking $\partial P_1/\partial X_1 = 0, \partial P_2/\partial X_2 = 0, \partial P_3/\partial X_3 = 0$ and $\frac{\partial P_4}{\partial X_4} = 0$ then have after the simplification:

$$Sf_1(a_2X_2+a_3X_3+a_4X_4) - (a_1X_1+a_2X_2+a_3X_3+a_4X_4)^2 = 0, \tag{26}$$

$$Sf_2(a_3X_3+a_4X_4+a_1X_1) - (a_1X_1+a_2X_2+a_3X_3+a_4X_4)^2 = 0, \tag{27}$$

$$Sf_3(a_4X_4+a_1X_1+a_2X_2) - (a_1X_1+a_2X_2+a_3X_3+a_4X_4)^2 = 0, \tag{28}$$

$$Sf_4(a_1X_1+a_2X_2+a_3X_3) - (a_1X_1+a_2X_2+a_3X_3+a_4X_4)^2 = 0. \tag{29}$$

The solutions of this set of linear equations for $X_{opt,1}$ (optimum spending for company 1) as follow:

$$X_{opt,1} = 3S\frac{g_1f_2f_3f_4[f_1(f_2f_3+f_3f_4+f_4f_2)-2f_2f_3f_4]}{(f_1f_2f_3+f_2f_3f_4+f_3f_4f_1+f_4f_1f_2)^2}. \tag{30}$$

The values of $X_{opt,1}$, $X_{opt,3}$ and $X_{opt,4}$ can be found easily as described in the last part of section 3. Substituting the values of $X_{opt,i}$ to Eq. (19) produces the optimum profit for company 1

$$P_{opt,1} = S\frac{g_1[f_1(f_2f_3+f_3f_4+f_4f_2)-2f_2f_3f_4]^2}{(f_1f_2f_3+f_2f_3f_4+f_3f_4f_1+f_4f_1f_2)^2}. \tag{31}$$

Eqs. (24) and (25) can be written in simpler notations as

$$X_{opt,1} = 3S\frac{g_1B_1[A_1f_1-2B_1]}{[A_1f_1+B_1]^2}, \tag{32}$$

$$P_{opt,1} = S\frac{g_1[A_1f_1 - 2B_1]^2}{[A_1f_1 + B_1]^2}. \tag{33}$$

where $A_1 = f_2f_3 + f_3f_4 + f_4f_2$ and $B_1 = f_2f_3f_4$.

3.4 Case for Any Number of Companies

Looking at formulas for X_1 and P_1 for three and four companies, the postulate formulas if n is the number of companies where $n \geq 3$. Here the postulated formulas:

$$X_{opt,1} = (n-1)S\frac{g_1B_1[A_1f_1 - (n-2)B_1]}{[A_1f_1 + B_1]^2}, \tag{34}$$

$$P_{op1} = S\frac{g_1[A_1f_1 - (n-2)B_1]^2}{[A_1f_1 + B_1]^2}. \tag{35}$$

Here, A_i is the sum of combinations of $n-1$ elements taken at $n-2$ each where B_i is the combination of $n-1$ elements taken at $n-1$. Elements here refer to f_i. See [20] or [21] for examples of combinations. Many textbooks on probabilities also describe about combinations.

At first, it did not verify them. Later, the prove of the formulas for $n = 5$ and $n = 6$ by using Maple. The derivations of the formulas are quite lengthy and tedious so they are not shown here. Formulas for $n > 6$ are not conducted but the postulation is correct.

For $n = 5$, then

$$X_{opt,1} = 4S\frac{g_1B_1[A_1f_1 - 3B_1]}{[A_1f_1 + B_1]^2}, \tag{36}$$

$$P_{opt,1} = S\frac{g_1[A_1f_1 - 3B_1]^2}{[A_1f_1 + B_1]^2}. \tag{37}$$

where $A_1 = f_2f_3f_4 + f_3f_4f_5 + f_4f_5f_2 + f_5f_2f_3$ and $B_1 = f_2f_3f_4f_5$.

For $n = 6$ then

$$X_{opt,1} = 5S\frac{g_1B_1[A_1f_1 - 4B_1]}{[A_1f_1 + B_1]^2}, \tag{38}$$

$$P_{opt,1} = S\frac{g_1[A_1f_1 - 4B_1]^2}{[A_1f_1 + B_1]^2}. \tag{39}$$

where $A_1 = f_2f_3f_4f_5 + f_3f_4f_5f_6 + f_4f_5f_6f_2 + f_5f_6f_2f_3 + f_6f_2f_3f_4$ and $B1 = f_2f_3f_4f_5f_6$.

Values of other A_i and B_i can be found easily by changing indices appropriately. This is where understanding of combinations is useful.

Formulas given in Eqs. (36) to (39) have been verified using Maple. You can of course use Mathematica or any symbolic language software to verify them. However, the results are not straight forward. You must manipulate them in order to have them simplified as shown in those equations.

A program written in C language has been written to calculate the spending values and optimum profits for up to six companies. Numerical examples by using C have been verified to those of examples by using Maple. See also [14] for a few examples for up to four companies.

The examples demonstrate in a market characterized by rivalry, those with superior effectiveness factors and increased gross profits will attain higher profits. Conversely, companies with the least advertising effectiveness factor and minimal gross profit will have the lowest optimal advertising investments and profits. Nevertheless, alterations in the effectiveness factor or gross profit can lead to changes in the optimal advertising investments and profits, potentially enhancing their competitive standing. Larger companies typically can expect higher demand from the market when they allocate more funds to advertising. On the other hand, smaller companies often have budget limitations for advertising, making it challenging to capture a larger market share. Without market regulations, larger companies can acquire smaller ones, leading to a scenario where only major corporations dominate the market. Frequently, major companies acquire smaller ones but do not necessarily integrate them in marketing specific products, allowing the smaller companies to maintain a small share of the market for those products.

Nevertheless, some argue that there are situations in which smaller companies can still secure a larger market share. This argument is valid, contingent upon the marketing strategy adopted by the company. In this context, branding assumes great importance. Established customers tend to remain loyal to products they love and may resist switching to other products. In this scenario, lets assume that advertising will aid companies in enhancing the sales of their products.

3.5 Sensitivity Analysis

Sensitivity analysis is used to detect the change of a variable when other variables parameters change. See [22], [23], and [24] for the use of sensitivity analysis in practice.

Formula for sensitivity analysis for four companies:

$$X_1 = F(g_1, f_1, f_2, f_3, f_4). \tag{40}$$

Substituting $f_i = a_i g_i$ yields

$$X_1 = F(a_1, a_2, a_3, a_4, g_1, g_2, g_3, g_4). \tag{41}$$

To make a more general function, change each variable to a single letter $a, b, c, d, p, q, r, s, t$. So, the aforementioned equation becomes

$$X = F(a, b, c, d, p, q, r, s), \tag{42}$$

Then, we obtain:

$$dX = \frac{\partial X}{\partial a}da + \frac{\partial X}{\partial b}db + \frac{\partial X}{\partial c}dc + \frac{\partial X}{\partial d}dd + \frac{\partial X}{\partial p}dp + \frac{\partial X}{\partial q}dq + \frac{\partial X}{\partial r}dr + \frac{\partial X}{\partial s}ds, \tag{43}$$

Assuming $da, db, \dots, ds$ are small, change them with $\Delta a, \Delta b, \dots, \Delta s$. Eq. (43) becomes

$$\Delta X \approx \frac{\partial X}{\partial a}\Delta a + \frac{\partial X}{\partial b}\Delta b + \frac{\partial X}{\partial c}\Delta c + \frac{\partial X}{\partial d}\Delta d + \frac{\partial X}{\partial p}\Delta p + \frac{\partial X}{\partial q}\Delta q + \frac{\partial X}{\partial r}\Delta r + \frac{\partial X}{\partial s}\Delta s. \tag{44}$$

Eq. (43) is an exact formula for the change of X with respect to its parameters while Eq. (44) is an approximation of Eq. (43). However, since X is a function of eight variables, finding $\frac{\partial X}{\partial a}, \frac{\partial X}{\partial b}, \dots, \frac{\partial X}{\partial s}$ is quite difficult. While it can be done symbolically, the result is quite complicated. It is therefore better to do the sensitivity analysis by using numerical values. So, we will have

$$X + \Delta X = F(a + \Delta a, b + \Delta b, c + \Delta c, d + \Delta d, p + \Delta p, q + \Delta q, r + \Delta r, s + \Delta s), \tag{45}$$

or simply

$$X^* = F(a^*, b^*, c^*, d^*, p^*, q^*, r^*, s^*), \tag{46}$$

where

$$X^* = X + \Delta X, a^* = a + \Delta a, b^* = b + \Delta b, \dots, s^* = s + \Delta s. \tag{47}$$

Eq. (46) is easier to use numerically rather than Eq. (44) which uses a lot of partial derivatives. Moreover, since Eq. (44) is only approximation, it will lose accuracy compared to Eq. (46) which is numerically exact.

Next, the explanation of the Maple program we created to calculate the impact of changes in advertising effectiveness and gross profits on the optimal fund allocation and company profits through sensitivity analysis. This program is designed to evaluate how small changes in these variables affect resource allocation decisions and the company's potential profits.

```
> restart; # Sensitivity Analysis. Medan, 31/8/24.
  fA := (f2,f3,f4)   f2*f3 + f3*f4 + f4*f2;
  fB := (f2,f3,f4)   f2*f3*f4; # for a company

  fXopt := (f1,g1,A1,B1,S)   3*S*g1*B1*(A1*f1 - 2*B1) / (A1*f1 + B1)^2;
      # dollars allocated to the company
  fPopt := (f1,g1,A1,B1,S)   S*g1*(A1*f1 - 2*B1)^2 / (A1*f1 + B1)^2;
      # optimum profit gained by the company

  # advertising effectiveness (a[i]) and gross profit (g[i]) for each company
  a[1] := 0.75; g[1] := 83;
  a[2] := 0.82; g[2] := 87;
  a[3] := 0.77; g[3] := 80;
  a[4] := 0.73; g[4] := 85;

  S := 10000; # Total market size

  # Calculate f[i] for each company
  for i from 1 to 4 do
    f[i] := a[i]*g[i];
  od;
```

Figure 2. Program Initiation and Calculation Section

This program calculates the optimal allocation of funds and company profits based on advertising effectiveness and gross profits through sensitivity analysis. As shown in Fig. 2, The functions fA and fB are used to compute additional parameters from several company performance variables. Using these formulas, fund allocation ($fXopt$) and optimal profits ($fPopt$) are calculated based on total market size ($S = 10{,}000$), gross profits ($g[i]$), and other parameters. The values for advertising effectiveness ($a[i]$) and gross profits ($g[i]$) are defined for four companies, and the performance values ($f[i]$) are calculated as the product of $a[i]$ and $g[i]$. This process aims to understand how changes in advertising effectiveness and gross profits affect resource allocation and the profits companies can achieve.

```
# Calculate A[i] and B[i] for each company
A[1] := evalf(fA(f[2],f[3],f[4]),6);
B[1] := evalf(fB(f[2],f[3],f[4]),6);
A[2] := evalf(fA(f[3],f[4],f[1]),6);
B[2] := evalf(fB(f[3],f[4],f[1]),6);
A[3] := evalf(fA(f[4],f[1],f[2]),6);
B[3] := evalf(fB(f[4],f[1],f[2]),6);
A[4] := evalf(fA(f[1],f[2],f[3]),6);
B[4] := evalf(fB(f[1],f[2],f[3]),6);

# Calculate optimum allocation (Xopt) and profit (Popt)
for i from 1 to 4 do
  Xopt[i] := evalf(fXopt(f[i],g[i],A[i],B[i],S),6);
  Popt[i] := evalf(fPopt(f[i],g[i],A[i],B[i],S),6);
od;

# Sensitivity analysis: Changes in a[i] and g[i]
da[1] := 0.041; da[2] := 0.038; da[3] := 0.036; da[4] := 0.035;
dg[1] := 3.3; dg[2] := 5.2; dg[3] := 4.0; dg[4] := 2.6;

# Calculate as[i] and gs[i] for sensitivity analysis
for i from 1 to 4 do
  as[i] := a[i] + da[i];
  gs[i] := g[i] + dg[i];
```

Figure 3. Advanced Program Calculation Section

This part of the program continues by calculating the values of $A[i]$ and $B[i]$ for each company using the previously defined functions fA and fB as we can see in Fig. 3. The calculations are performed by multiplying the f values from three other companies for each specific company. Afterward, the program computes the optimal fund allocation ($Xopt[i]$) and optimal profit ($Popt[i]$) for each company using the functions $fXopt$ and $fPopt$, which take into account variables like advertising effectiveness, gross profit, and the $A[i]$ and $B[i]$ parameters that were just calculated. The program then moves on to sensitivity analysis, where small changes in advertising effectiveness ($da[i]$) and gross profit ($dg[i]$) are introduced to measure their impact on fund allocation and profit. The adjusted values for advertising effectiveness ($as[i]$) and gross profit ($gs[i]$) are calculated by adding these changes to the initial values, which will be used in the subsequent sensitivity calculations.

```
od;

# Calculate fs[i] based on as[i] and gs[i]
for i from 1 to 4 do
  fs[i] := as[i] * gs[i];
od;

# Calculate As[i] and Bs[i] for the sensitivity analysis
for i from 1 to 4 do
  As[i] := evalf(fA(fs[2], fs[3], fs[4]), 6);
  Bs[i] := evalf(fB(fs[2], fs[3], fs[4]), 6);
od;

# Calculate optimum allocation (Xsopt) and profit (Psopt) after changes
for i from 1 to 4 do
  Xsopt[i] := evalf(fXopt(fs[i], gs[i], As[i], Bs[i], S), 6);
  Psopt[i] := evalf(fPopt(fs[i], gs[i], As[i], Bs[i], S), 6);
od;

# Calculate changes in allocation (dXopt) and profit (dPopt)
for i from 1 to 4 do
  dXopt[i] := Xsopt[i] − Xopt[i];
  dPopt[i] := Psopt[i] − Popt[i];
od;
```

Figure 4. Final Program Calculation Section

This section continues the sensitivity analysis by calculating the impact of changes in advertising effectiveness and gross profits on the optimal fund allocation and company profits. First, as we can see in Fig. 4, the new values $fs[i]$ are calculated as the product of the new advertising effectiveness ($as[i]$) and the new gross profits ($gs[i]$). After that, additional parameters $As[i]$ and $Bs[i]$ are computed based on the $fs[i]$ values for the sensitivity analysis. Using these new parameters, the optimal fund allocation ($Xsopt[i]$) and optimal profit ($Psopt[i]$) are calculated to assess the impact of the changes. Finally, the program calculates the differences in fund allocation ($dXopt[i]$) and profits ($dPopt[i]$) before and after the changes, providing insights into how small variations in these variables affect the company's performance.

3.6 Maple as a Tool for Solving Operations Research Problems

Maple is highly effective for tackling problems in a symbolic manner. This is particularly advantageous when it comes to deriving formulas and solving problems across various inputs. Numerous well-regarded books are available on Maple and its practical applications for problem-solving, including references like [25], [19], [26], and [27]. Maple's effectiveness in solving mathematical problems is also remarkable. It provides tools for solving operations research issues, including finding the best solution from a number of possible solutions, as demonstrated in [28]. Nonetheless, the situations outlined in this document require the creation of specialized software solutions for resolution. With the growing quantity of participating companies, achieving precise solutions becomes progressively intricate, even though Maple is still capable of finding them. The solutions provided by Maple often require manipulation and simplification to enhance their practical utility. In such scenarios, resorting to numerical methods is frequently more practical.

In addition to Maple, MATLAB can also be employed for symbolic problem-solving as discussed in this paper, as illustrated in [26]. However, MATLAB is relatively slower, and the displayed outcomes may lack a similar level of user-friendliness. Furthermore, the symbolic programs must be imported from Maple. Therefore, it is advisable to initially establish generalized forms using Maple and subsequently transition to MATLAB.

An example of calculating sensitivity analysis (Equations 40 and 41) for four companies is done by using Maple. Necessary comments regarding parameters of the program are shown for each parameter shown either as a text (inside the square bracket or at the end of the statements where the parameters are used using the # symbol. Since the program is rather long, it is put in the Appendix. Summary of the results from the Maple program is shown in Table 1.

Table 1. Results from the Maple Program

Company	A	g	X_{opt}	P_{opt}	Δa	Δg	ΔX_{opt}	ΔP_{opt}
1	0.75	83	168347	66382.7	0.041	3.3	−16092	−21223.5
2	0.82	87	191288	92702.0	0.038	5.2	13952	10449.0
3	0.77	80	137246	38693.1	0.036	4.0	8013	2848.4
4	0.73	85	148490	43254.0	0.035	2.6	−952	−2993.7

***Data source**: the data was processed using Maple*

From Table 1, we can see that dollars allocated for companies 2 and 4 decrease. Optimum profits for companies 2 and 4 are also reduced.

This study assumes that advertising is the primary factor influencing market share, as reflected in the mathematical models and sensitivity analyses developed. However, it is important to acknowledge that in real-world scenarios, other critical factors such as brand loyalty, product innovation, and external economic conditions significantly influence market dynamics. For instance, well-established brands with strong customer loyalty may require less advertising to maintain their market position, while economic downturns could impact consumer purchasing power and alter market competition.

4. CONCLUSION

Based on the findings of this study, several key points can be highlighted:

First, this research successfully formulates a general equation to calculate the optimal profit for n companies in an oligopolistic market. The formula extends previous advertising competition models that were limited to duopoly or simple oligopoly settings. Second, sensitivity analysis demonstrates that advertising effectiveness and gross profit are the main determinants of advertising budget allocation strategies. Companies with higher advertising effectiveness achieve greater profits, whereas those with lower effectiveness tend to experience reductions in both allocation and profit.Third, model validation through symbolic computation using Maple and numerical simulation with Excel for cases of two to four companies confirms the consistency of the results, strengthening the reliability of the model for application to larger numbers of companies.

This study has certain limitations, including the use of simulated data and the assumption that market share is directly proportional to advertising expenditure. These assumptions simplify the complexity of real-world market conditions, and therefore the results should be regarded as normative and requiring further empirical validation. For future studies, the model can be extended by applying it to real-world oligopolistic industries, particularly in the context of digital advertising, which has become increasingly dominant. In addition, testing the model under market conditions influenced by external factors such as economic crises or regulatory changes will provide a more realistic understanding of the effectiveness of advertising strategies.

Author Contributions

Almira Amir: Conceptualization, Methodology, and Writing – Original Draft. Ruth Mayasari Simanjuntak: Data Curation, Visualization, and Writing – Review and Editing. Yenny Suzana: Investigation and Formal Analysis. Riri Syafitri Lubis: Formal Analysis and Validation. Fatmah Syarah: Software Development and Data Curation. Fajriana: Visualization and Writing – Review and Editing. Zahedi: Supervised the Research, Project Administration, and Writing – Review and Editing. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare no conflict of interest.

Declaration of Generative AI and AI-assisted Technologies

Generative AI tools were used solely for language refinement (grammar, spelling, and clarity). The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. The authors reviewed and approved all final text.

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