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MODELING AND FORECASTING MORTALITY RATES DURING THE COVID-19 PANDEMIC USING THE SECOND ADAPTED NOLFI MODEL AND AUTO ARIMA

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ABSTRACT

Modeling and forecasting mortality rates have been widely performed using various approaches. One such approach is the Second Adapted Nolfi model, which is one of three adaptations derived from the Nolfi and Generalized Nolfi models. Unfortunately, its application remains limited compared to widely used models like Lee-Carter and Cairns-Blake-Dowd. Previous studies on this model have shown satisfactory performance, particularly in residual analysis. However, those studies were conducted before the COVID-19 pandemic, and no study has yet applied it in the pandemic or post-pandemic periods. Although the pandemic may appear less relevant in 2025, the absence of such studies highlights the importance of further investigation into the model's performance under extreme demographic conditions. This study addresses that gap by evaluating the Second Adapted Nolfi model using data from the Human Mortality Database (HMD) for the United States, the United Kingdom, and Italy. The model was applied to data up to 2019, and Auto-ARIMA was used to forecast from 2020 onward. The modeling results indicate that the logarithmic mortality curves align with established patterns, such as high rates at age 0, a decline through childhood, a sharp increase in early adulthood, and a continued rise into old age. The results also show that HMD mortality rates exceed the forecasted values for individuals aged 80 and above, suggesting increased elderly mortality during the pandemic. Three error metrics were used, yielding RMSE values from 0.01 to 0.18, MAE from 0.004 to 0.07, and MAPE from 28 to 286. Although Italy had the highest MAPE, the United States and the United Kingdom also showed notable errors. These findings reveal both the pandemic's demographic impact and limitations of the model in capturing sudden shocks. Future studies may enhance this model through new adaptations, further comparison with other models, or alternative smoothing techniques to develop more robust mortality forecasts.

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1. INTRODUCTION

Modeling mortality rates has been widely studied worldwide, including the analysis of various factors that affect mortality rates, the development of strategies to reduce mortality rates, and other related studies. Several studies on mortality rates in Indonesia, for instance, are regarding the analysis of factors associated with maternal mortality rates [1], strategies to reduce maternal and child mortality rates in East Java [2], Indonesian mortality rates using the Whittaker-Henderson Graduation method [3], and comparison of the modeling of the number of infant mortality in West Java [4]. Meanwhile, modeling and forecasting studies include the application of the Lee-Carter model and the ARIMA method for Indonesia's mortality rate [5], forecasting COVID-19 positive cases in East Java using the Hybrid ARIMA-LSTM method [6], forecasting the number of COVID-19 cases in West Java [7] and Central Java [8] using the ARIMA method, and the use of the PLAT model, ARIMA, and Residual Bootstrap methods for Indonesian male mortality [9].

An approach that may be employed to model the mortality rates is the Nolfi model, first introduced by P. Nolfi in his article in the Bulletin/Association of Swiss Actuaries in 1959 [10]. Furthermore, the model developed into several forms: Generalized, First Adapted, Second Adapted, and Third Adapted Nolfi model. The Generalized Nolfi model is a generalization of the original Nolfi model, achieved by modifying the characteristics of the parameter λ_x. Subsequently, the Adapted Nolfi model generalises the Generalized Nolfi model by incorporating a stochastic process, allowing the first-year mortality rates and/or the parameter $\lambda_x$to vary with time t [11]. Later on, G. Binder compared these models with other established approaches, such as the Lee-Carter model, the Cairns-Blake-Dowd model, and the extrapolation model [12]. In the study, Binder concluded that the Second Adapted Nolfi model gives the best performance because the model best fits the Swiss country population data from 1912 to 2010 in terms of residuals.

Our previous study applied the Nolfi, Generalized Nolfi, and Adapted Nolfi models to Taiwanese mortality data from 1970 to 2014, obtained from the Human Mortality Database (HMD) [13]. Among these models, the Second Adapted Nolfi model again gave the smallest error with RMSE and MAE measures. However, the study was conducted before the COVID-19 pandemic. More recent studies have examined mortality modeling during the pandemic, such as examining mortality shocks induced by the COVID-19 pandemic within the framework of the Lee-Carter model [14] and predicting COVID-19 mortality rates in Indonesia using a Zero-Inflated Negative Binomial (ZINB) model [15]. Unfortunately, to the best of our knowledge, none of these studies have modelled mortality rates using the Adapted Nolfi model.

One of the primary justifications for conducting this research is the limited number of studies employing the Nolfi, Generalized Nolfi, or Adapted Nolfi models for mortality rate modeling. While earlier studies discussing these models have been documented in [10], [11], [12], and [13], they remain significantly outnumbered by studies on more established mortality models like Lee-Carter or Cairns-Blake-Dowd. Furthermore, to our knowledge, no subsequent studies have applied these models during or after the pandemic period. Therefore, this study aims to fill that gap by employing the Second Adapted Nolfi model, one of the three Adapted Nolfi models, to model mortality rates during the COVID-19 pandemic, in combination with the Auto-ARIMA forecasting method.

Beyond its previously demonstrated ability to produce the smallest error, the Second Adapted Nolfi model incorporates a single modification, which is the addition of a stochastic process to the first-year mortality rate. This modification causes its influence on the overall mortality rate to vary from year to year. This is closely related to the smoothing of the first-year mortality rate, which is also one of the key steps in this study. Meanwhile, the Auto-ARIMA method itself has been widely used by researchers as a forecasting method because it offers a suitable model and saves time. Several studies about Auto-ARIMA are [16] about the comparison of ARIMA and Auto-ARIMA methods, [17] on the COVID-19 case in European countries, and [18] on the COVID-19 case in Pakistan. Given the convenience offered by the Auto-ARIMA method, this study employed it to forecast mortality rates during the COVID-19 pandemic from 2020 to 2022.

Although the COVID-19 pandemic may appear less relevant in 2025, the lack of studies exploring the Second Adapted Nolfi model in the context of pandemic-related mortality underscores the importance of further investigation. The findings from this study are expected to provide insights into how this model, in combination with Auto-ARIMA, performs under extreme conditions such as a global health crisis, as well as in future situations that may result in sudden changes or shocks in mortality patterns. This study is expected to further promote the application of the Second Adapted Nolfi model, as well as other Nolfi models, to encourage continued development and evaluation in future studies.

The remainder of this paper is structured as follows. Section 2 presents the data sources and the methodology, including methods and research stages for modeling and forecasting mortality rates using the Second Adapted Nolfi model and the Auto-ARIMA method. Section 3 presents the results of smoothing and parameter estimation, as well as the modeling and forecasting outcomes, along with a discussion that includes error measurements. Section 4 concludes the paper and offers suggestions for future research.

2. RESEARCH METHODS

This research employed quantitative research in which the data originated from secondary sources. The mortality data used in this study were secondary data from the HMD website in the form of mortality and population data for the United States, the United Kingdom, and Italy. Based on the WHO data as of 22 August 2024, the highest number of deaths of COVID-19 patients occurred in the United States of America, with a total of 1.2 million people; the United Kingdom of Great Britain and Northern Ireland occupied the sixth position with a total of 232 thousand people, and Italy occupied the eighth position with a total of 197 thousand people [19]. In this study, the mortality data of the three countries were modelled with the Second Adapted Nolfi model and forecasted with the Auto-ARIMA method.

2.1 Methods for Modeling and Forecasting the Mortality Rates

Mortality data for three countries up to 2019 are used as the basis for modeling in this study [19]. Furthermore, stochastic parameter forecasting is carried out for 2020 to 2021 or 2022 to obtain the forecasted mortality rate. The results of mortality rate forecasting are compared with HMD mortality rates to see changes in mortality rates during the COVID-19 pandemic. This mortality rate modeling and forecasting can be extended to subsequent years or extended to other countries according to user needs and applied to various fields that make use of mortality rates, such as mortality tables, insurance premium calculations, and others.

The Second Adapted Nolfi model is one of three Adapted Nolfi models developed by Luthy et. al. by adding stochastic process parameters to the Generalized Nolfi model [11]. Eq. (1) presents the mathematical formulation of the Second Adapted Nolfi Model, while Eq. (2) presents its logarithmic mortality rates formula.

$$q_{x,t} = \{\alpha_{t,1} q_{x.t_0} \exp(-\lambda_x (t - t_0))\}, \tag{1}$$

$$\ln q_{x,t} = \ln\{\alpha_{t,1} q_{x.t_0} \exp(-\lambda_x (t - t_0))\}. \tag{2}$$

Where x: age, t: year, t_0: first year of observation, q_{x,t_0}: mortality rate at age x in year t_0, $q_{x,t}$: mortality rate at age x in year t for $x \geq 0, t \geq t_0$, and there is a condition $\lambda_x > 0$.

The parameters λ_x in the Second Adapted Nolfi model are the same as the parameters λ_x in the Generalized Nolfi model. The estimated value of the parameter λ_x can be calculated using Eq. (3), while the estimated value of the parameter $\alpha_{t,1}$ can be obtained from Eq. (4) below.

$$\hat{\lambda}_x = \left\{ -\frac{\sum_t log\left(\frac{d_{x,t}}{l_{x,t} q_{x,t_0}}\right)(t - t_0)}{\sum_t (t - t_0)^2} \right\}, \tag{3}$$

$$\hat{\alpha}_{t,1} = \frac{\sum_x \left(\frac{d_{x,t}}{l_{x,t} q_{x,t_0}}\right) \exp\left(-\hat{\lambda}_x (t - t_0)\right)}{\sum_x \left(\exp\left(-\hat{\lambda}_x (t - t_0)\right)\right)^2}, \tag{4}$$

$\alpha_{t,1} \in \mathbb{R}_+$ is a stochastic process that makes the effect of $q_{x.t_0}$ on $q_{x,t}$ different each year according to the value $\alpha_{t,1}$ in that year.

Mortality data for three countries up to 2019 are used as the basis for modeling in this study. Furthermore, stochastic parameter forecasting is carried out for 2020 to 2021 or 2022 to obtain the forecasted mortality rate. The results of mortality rate forecasting are compared with HMD mortality rates to observe changes in mortality rates during the COVID-19 pandemic. The forecasting method in the present study was performed by using the Auto-ARIMA method.

2.2 Stages of Modeling and Forecasting using the Second Adapted Nolfi Model and Auto-ARIMA

The study was initiated with a literature review to collect information on the COVID-19 pandemic, mortality data sources, characteristics of the available data, methods for processing mortality data using Python, and other relevant aspects. This information formed the basis for conducting the research, which followed several methodological steps. The stages of this study are as follows:

1. Collecting mortality and population data for the United States, the United Kingdom, and Italy from the HMD website.
2. Smoothing the first-year mortality rate ($q_{x.t_0}$) using the Whittaker-Henderson method in Python. This smoothing method has been previously used to smooth data such as the 2019 World Health Organization (WHO) mortality data for Indonesia [3], series data in the health sector [20], and data from the social security system of the Republic of the Philippines [21].
3. Estimating parameters λ_x and $\alpha_{t,1}$. by employing the functions given in [22] and [23]. In the third stage, the parameters λ_x and $\alpha_{t,1}$ are estimated by first deriving the estimation formula for $\hat{\alpha}_{t,1}$ using the least squares linear regression method. The first parameter to be estimated is the parameter λ_x. The estimation results are then used to estimate the parameter $\alpha_{t,1}$.
4. Calculating the modelled mortality rate. In the fourth stage, the parameter estimation results are substituted into Eq. (1) to obtain the modelled mortality rate ($\hat{q}_{x,t}$) for all ages from the first year to 2019. The results of the modeling were subsequently illustrated by employing several Python library functions, as presented in [24] and [25].
5. Forecasting the mortality rate. In the fifth stage, stochastic parameter forecasting is carried out using the Auto-ARIMA method in Python, as presented in [26].
6. Comparing the forecasted mortality rate with the mortality rate in HMD. In the sixth stage, the forecasted mortality rate is compared with the mortality rate in HMD. A mortality rate plot for the three countries was produced for both female and male genders in 2020-2021 for the United Kingdom and Italy, and 2020-2022 for the United States.
7. Calculating the forecasting error using Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE).
8. Drawing conclusion(s) and suggestion(s).

To provide a clearer overview of the research steps, they are illustrated in the methodological flowchart presented in Fig. 1.

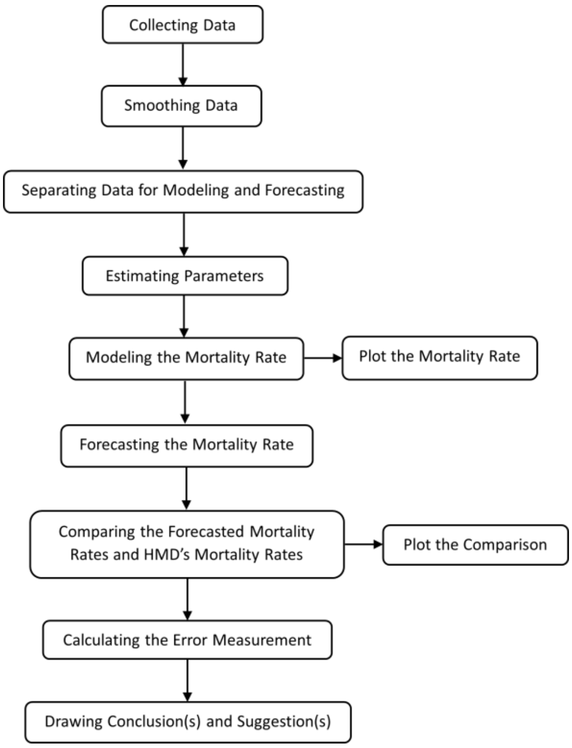


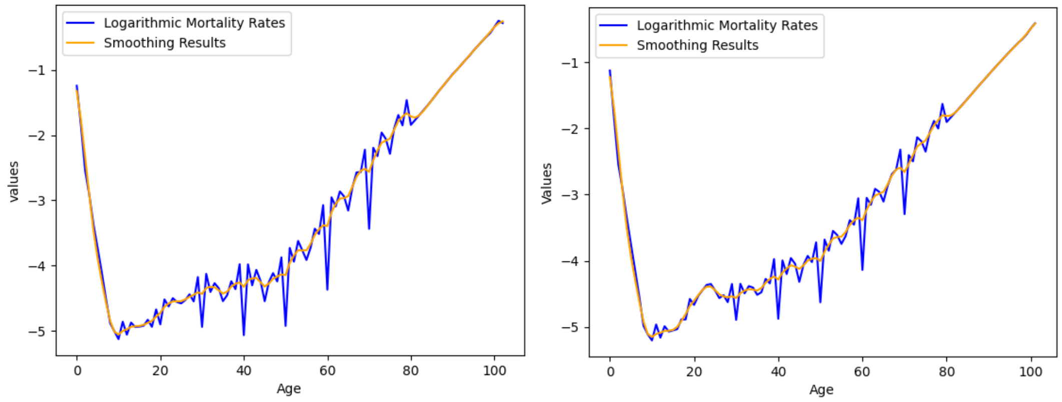
Figure 1. Method Flow

3. RESULTS AND DISCUSSION

The results of this study are presented in four sections. The first section presents the parameter estimation of $\hat{\lambda}_x$ and $\hat{\alpha}_{t,1}$, conducted using Microsoft Excel. The second section presents the mortality rate modeling results using the Second Adapted Nolfi model with HMD data from the first year up to 2019. The third section presents the mortality rate forecasting using the Auto-ARIMA method, conducted using Python. The final section provides further discussion.

3.1 Smoothing and Estimating Parameters Results

By following the research stages above for stages (1) and (2), the smoothed first-year mortality rates for Italy (female and male) and the United Kingdom (female) were obtained. A comparison between these smoothing results and the original data is presented in Figs. 2 and 3.



(a) **(b)**

Figure 2. Italy First-Year Mortality Rates (a) Female, (b) Male

Figure 3. United Kingdom Female First-Year Mortality Rates

The remaining three datasets of first-year mortality rates, which are the United Kingdom male, United States female, and United States male, were not smoothed, as such a procedure would alter the trend of the original data. Comparisons between the smoothed results and original data for these datasets are presented in Figs. 4 and 5.

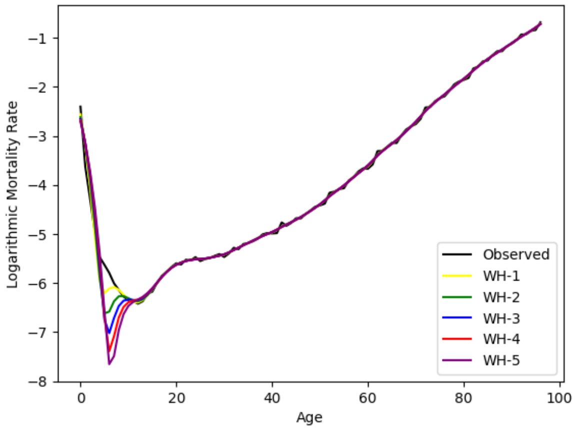
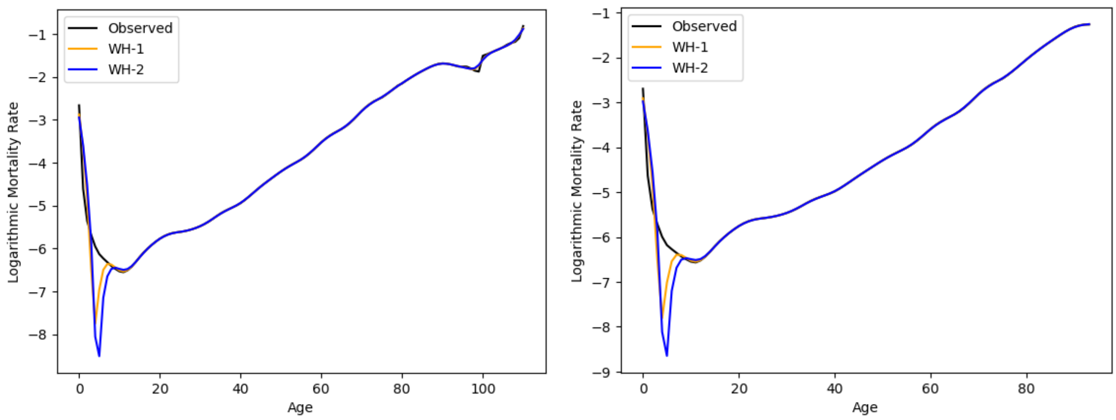


Figure 4. United Kingdom Male First-Year Mortality Rates

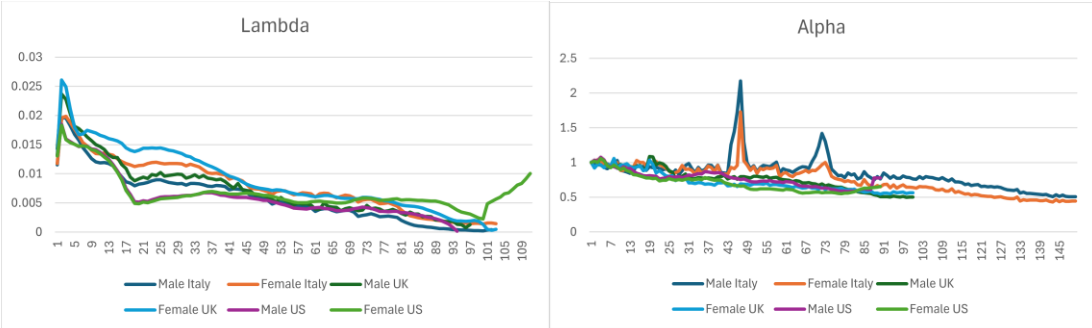


(a) **(b)**

Figure 5. United States First-Year Mortality Rates (a) Female, (b) Male

As illustrated in Figs. 4 and 5, the smoothed first-year mortality rates for United Kingdom males, United States females, and United States males exhibit an increasing trend at age 7 that is not present in the original data. Therefore, the original first-year mortality data, rather than the smoothed data, were used in the modeling process.

The obtained values of $q_{x.t_0}$, along with the data $d_{x,t}$ and $l_{x,t}$, were substituted into Eq. (3) to estimate the parameter $\hat{\lambda}_x$. The value of $\hat{\lambda}_x$ may vary for each age x, subject to the condition $\hat{\lambda}_x > 0$. Subsequently, the estimated $\hat{\lambda}_x$ values were substituted into Eq. (4) to estimate the parameter $\hat{\alpha}_{t,1}$. Fig. 6 (a) and (b) present the estimated parameters $\hat{\lambda}_x$ and $\hat{\alpha}_{t,1}$ for all three countries, as obtained using the Second Adapted Nolfi Model.



(a) **(b)**

Figure 6. Estimated Parameter Values (a) $\hat{\lambda}_x$, (b) $\hat{\alpha}_{t,1}$

The estimated parameters were then used to calculate the modeled mortality rates and logarithmic mortality rates using Eqs. (1) and (2).

3.2 Mortality Rate Modeling Results

Modeling in this study was carried out on mortality data for the United States for 1933-2019, the United Kingdom for 1922-2019, and Italy for 1872-2019. The smoothed values of $q_{x.t_0}$ and the estimated parameters $\hat{\lambda}_x$ and $\hat{\alpha}_{t,1}$ obtained in steps (2) nd (3) are substituted into the mortality rate and logarithmic mortality rate formulas, for the corresponding age x and year t, to obtain the modeled values of $q_{x,t}$ and $\ln q_{x,t}$. Fig. 7 presents the modeling results of mortality rates for United States females, as obtained using the Second Adapted Nolfi model.

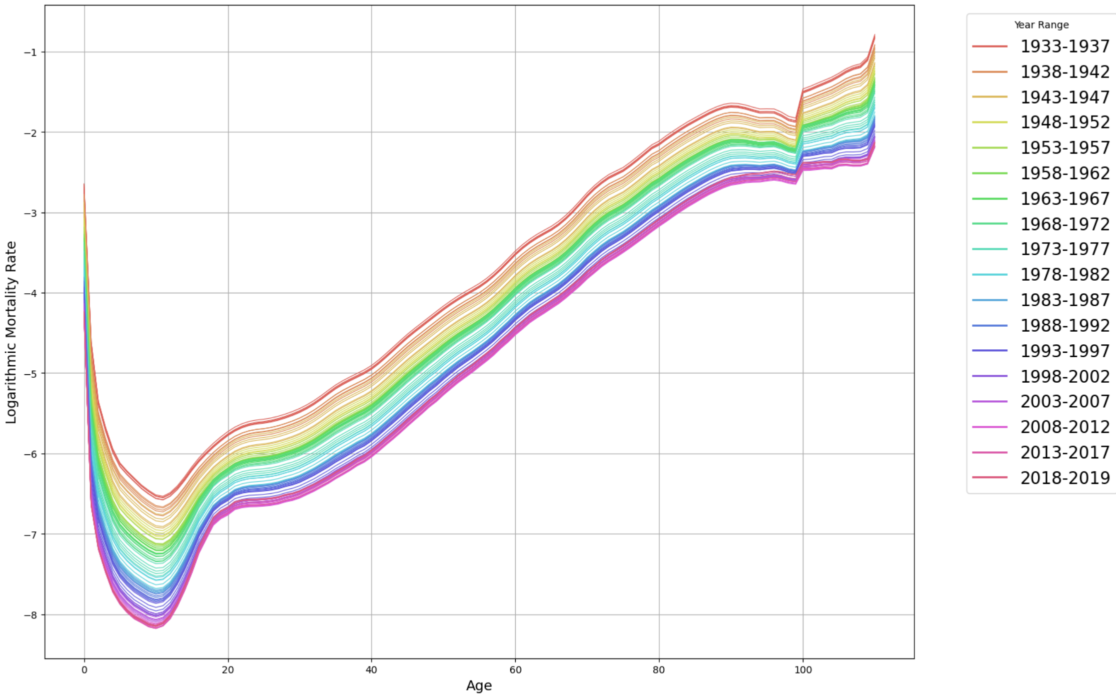


Figure 7. Female Logarithmic Mortality Rates for the United States

The modeling results of the female logarithmic mortality rates in the United States using the Second Adapted Nolfi model from 1933 to 2019 are shown in Fig. 7. In general, the logarithmic values show a declining trend over the years, which aligns with the increase in life expectancy due to advances in health science and technology. However, life expectancy is not examined in this study and may serve as a topic for future research. Subsequently, Fig. 8 presents the modeling results of male logarithmic mortality rates in the United States.

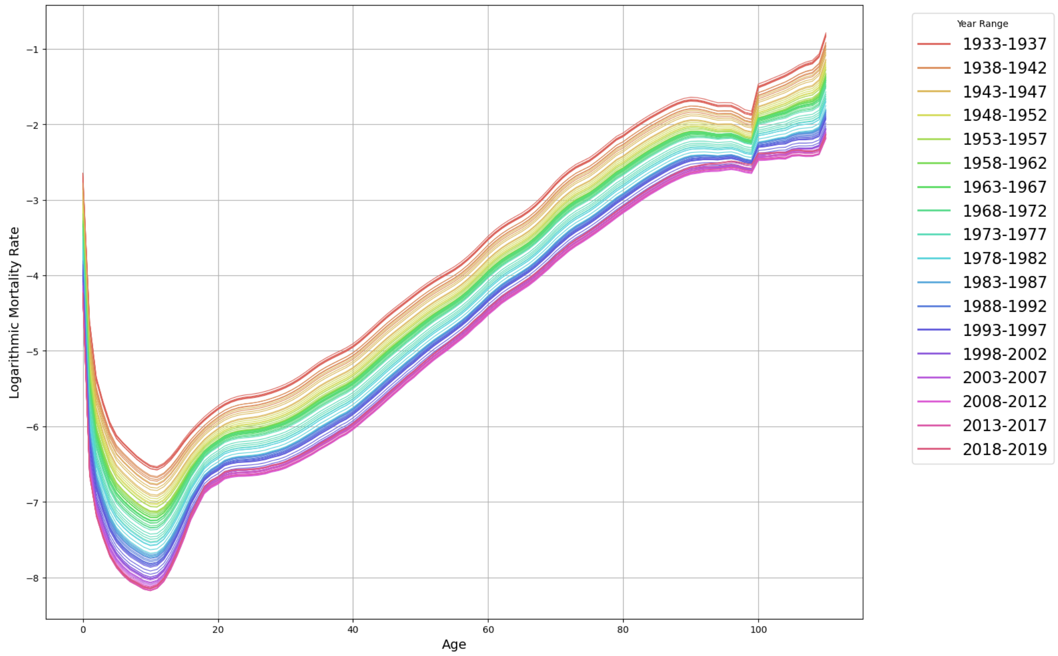


Figure 8. Male Logarithmic Mortality Rates for the United States

The modeled mortality rates for males in the United States show an overall declining trend over time, much like Fig. 7. Fig. 9 shows the outcomes of predicting mortality rates for females in the United Kingdom.

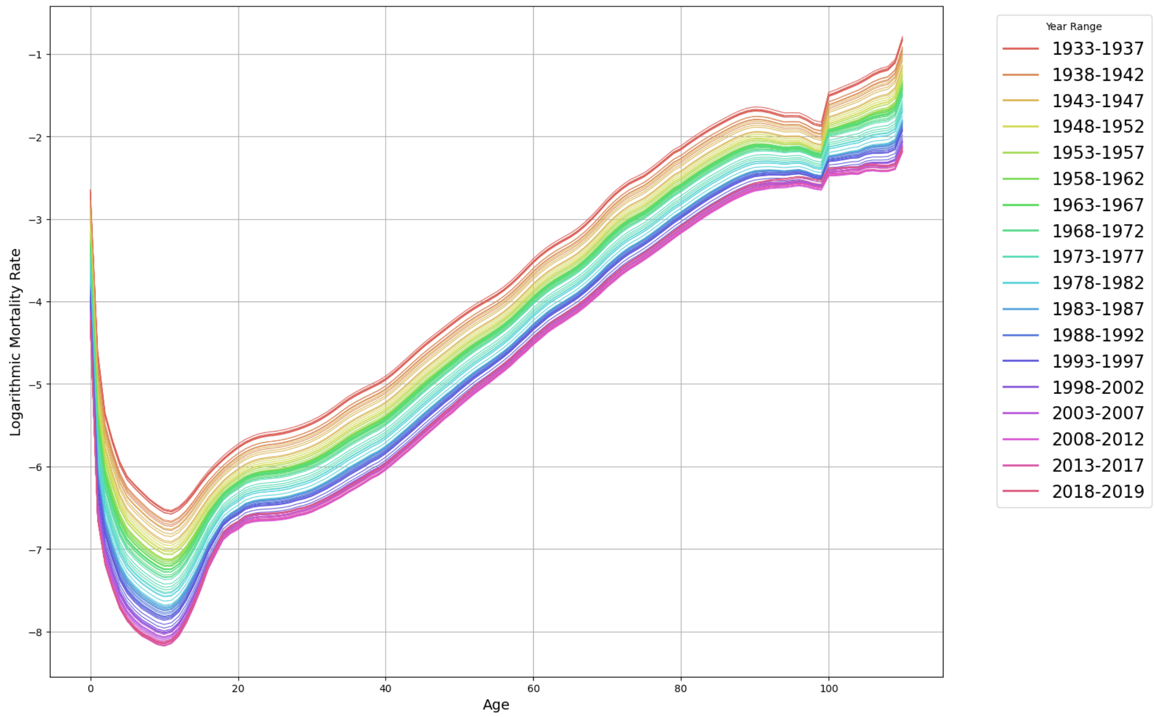


Figure 9. Female Logarithmic Mortality Rates for the United Kingdom

The female logarithmic mortality rates in the United Kingdom likewise exhibit a general declining tendency over time, which is consistent with the findings of mortality rates modeling for the United States. Fig. 10 then displays the findings of the modeling of males' mortality rates in the United Kingdom.

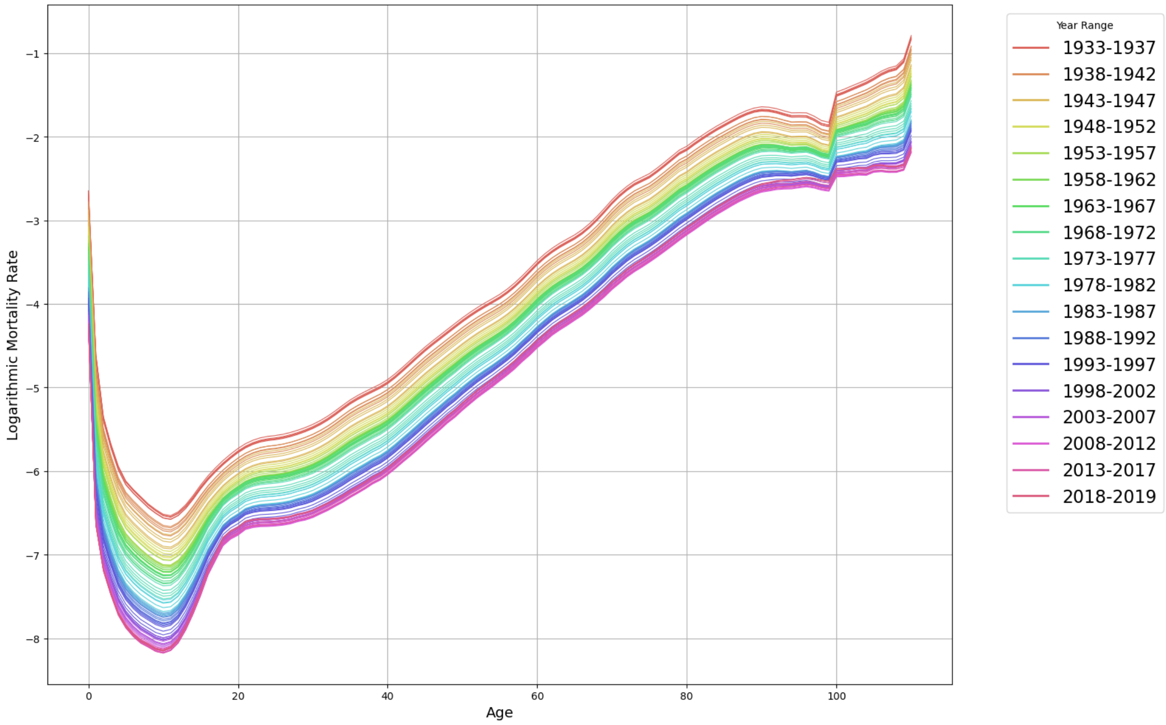


Figure 10. Male Logarithmic Mortality Rates for the United Kingdom

Similar to the modeling results presented in Fig. 7 through Fig. 9, the mortality rates for males in the United Kingdom also show a general decline over time. As for Italy, the results of mortality rate modeling for females are presented in Fig. 11.

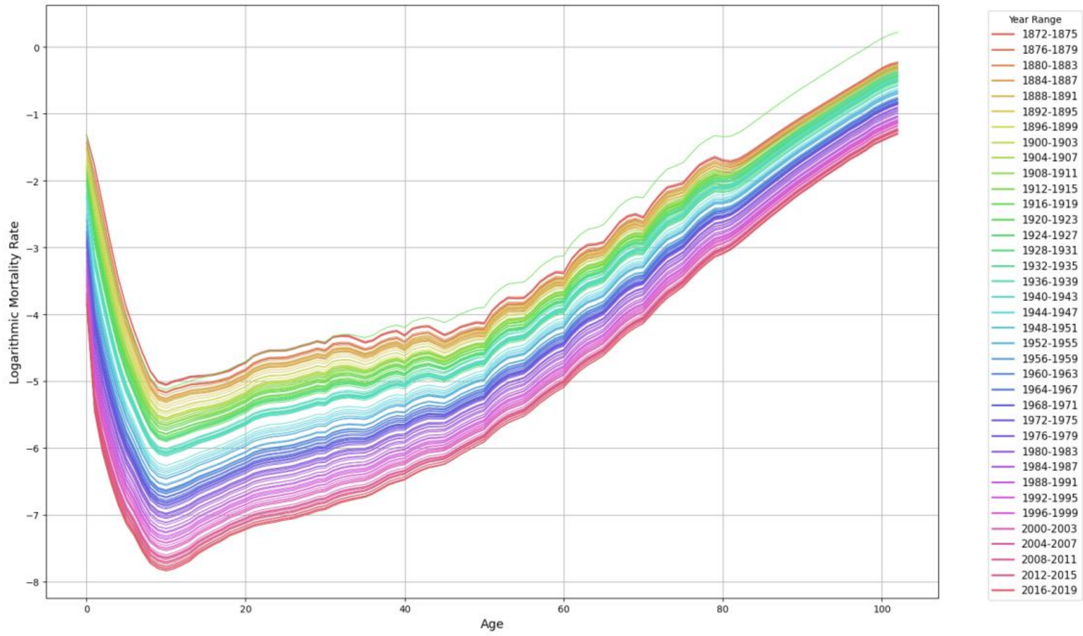


Figure 11. Female Logarithmic Mortality Rates for Italy

The female logarithmic mortality rates in Italy also show a declining trend over the years, like those observed in the United States and the United Kingdom. However, the curve of female logarithmic mortality rates presented in Fig. 11 appears less smooth compared to the other two countries. Higher smoothing levels or alternative smoothing methods may yield different results, but it is important to carefully analyze the associated error values. Subsequently, the results of modeling males' mortality rates in Italy are presented in Fig. 12.

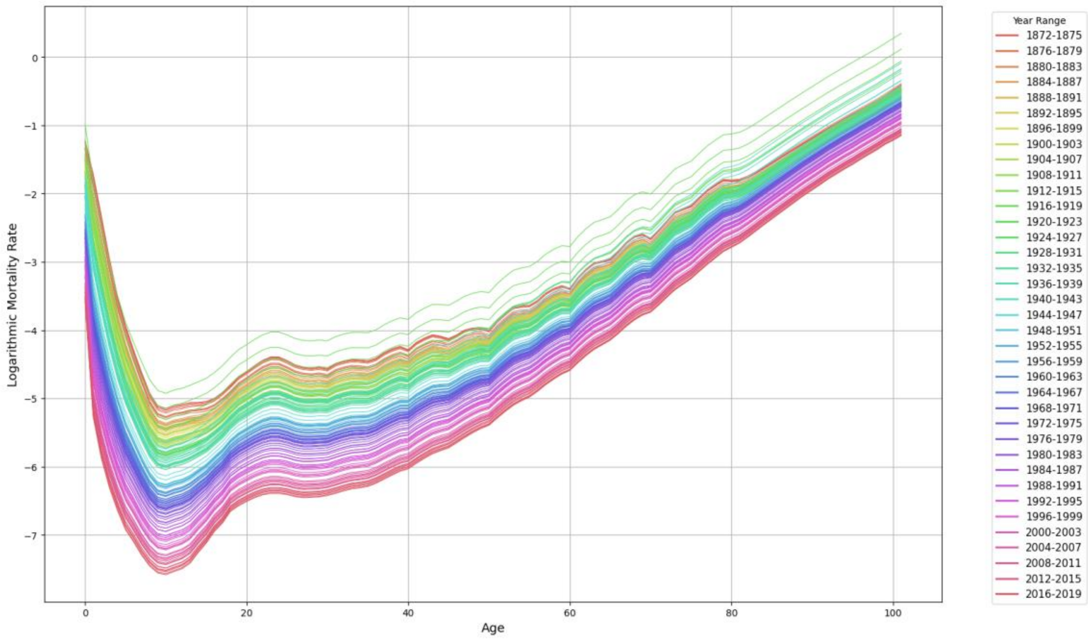
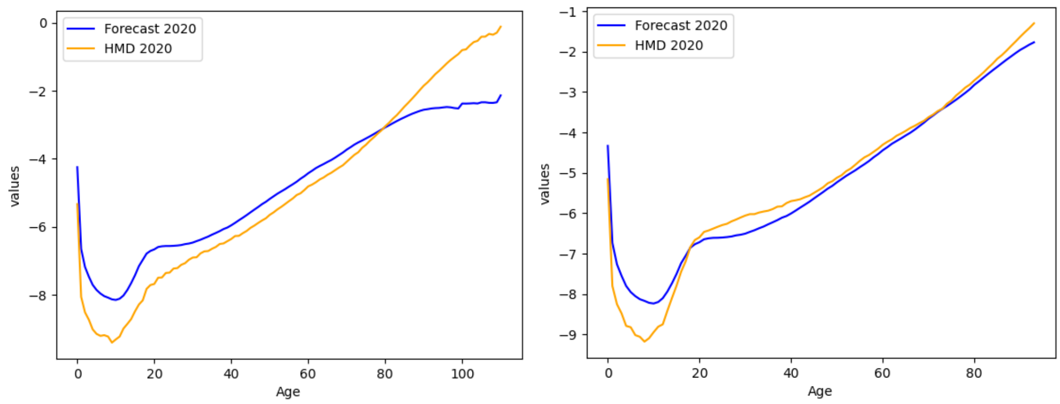


Figure 12. Male Logarithmic Mortality Rates for Italy

Similar to Fig. 11, Fig.12 also presents logarithmic mortality rate curves that appear less smooth compared to those of the other countries. A comparison of various smoothing methods and different smoothing levels applied to Italy's mortality data appears promising for further investigation to improve modeling and forecasting results.

3.3 Mortality Rate Forecasting Results

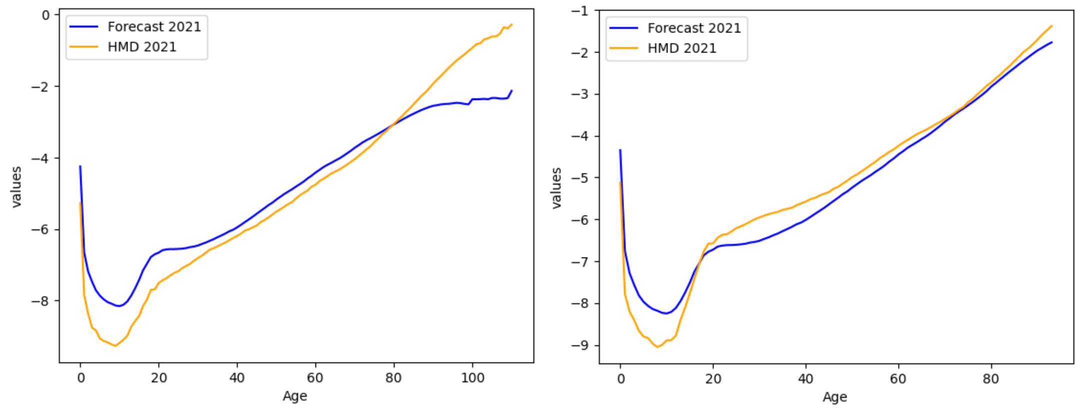
The parameter estimation values $\alpha_{t,1}$, obtained through Microsoft Excel, are then used as the basis for forecasting the parameter $\alpha_{t,1}$ with Python. Auto-ARIMA forecasting in this study is carried out using the 'Auto-ARIMA program' in Python, which produces the forecasting results for $\alpha_{t,1}$ for 2020-2021 (for the United Kingdom and Italy) and 2020-2022 (for the United States). The forecasted values of $\alpha_{t,1}$ is then substituted into the Second Adapted Nolfi model formula to obtain the forecasted mortality rates $q_{x,t}$, as shown in the following figures.



(a) (b)

Figure 13. Logarithmic Mortality Rates for the United States in 2020 (a) Female, (b) Male

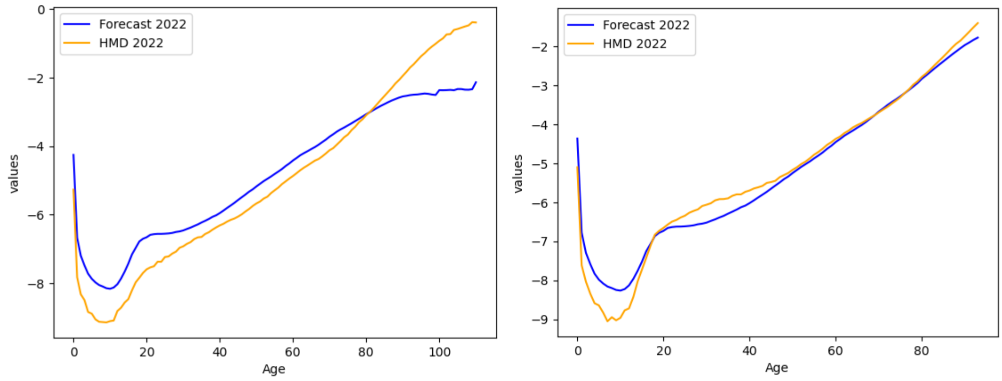
The mortality rate forecasting results for the United States and the HMD in 2020 are shown in Fig. 13. For the age range of 20 to 90 years, the forecasted male mortality rates for the United States in 2020 closely align with the HMD mortality rates. Subsequently, the forecasting results in 2021 are presented in Fig. 14.



(a) **(b)**

Figure 14. Logarithmic Mortality Rate for the United States in 2021 (a) Female, (b) Male

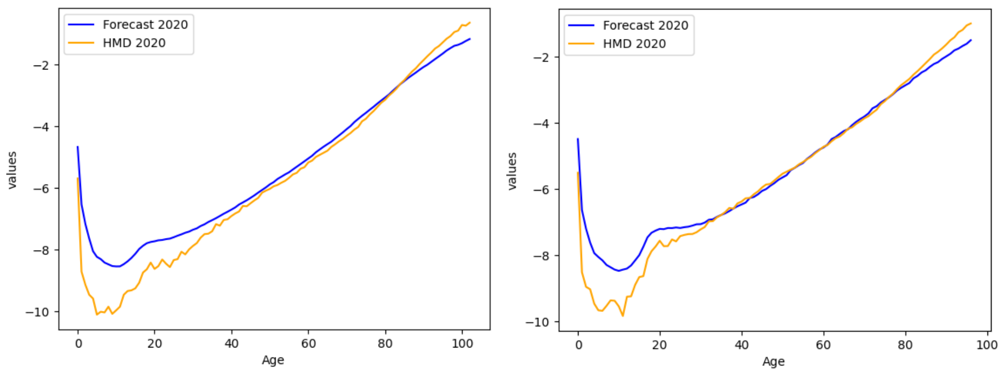
As seen in Fig. 14, the 2021 male mortality rate forecasting results nearly match the 20–90 age group HMD mortality rates. The HMD mortality rates are continuously greater than the forecasted mortality rates for ages beyond 80, according to all comparisons of the preceding curves. Following that, Fig. 15 displays the forecasting findings for 2022.



(a) **(b)**

Figure 15. Logarithmic Mortality Rate for the United States in 2022 (a) Female, (b) Male

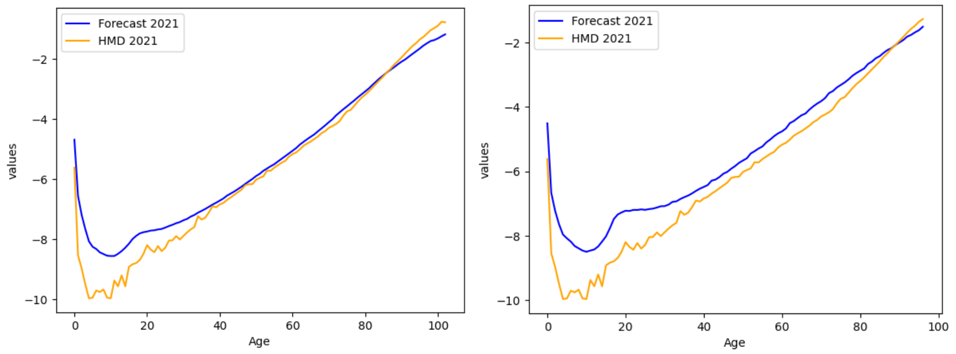
Similar to the previous comparisons, Fig. 15 also shows that the HMD mortality rates are higher than the forecasted mortality rates for ages above 80. Following this, Fig. 16 displays the forecasted findings for the United Kingdom in 2020.



(a) **(b)**

Figure 16. Logarithmic Mortality Rate for the United Kingdom in 2020 (a) Female, (b) Male

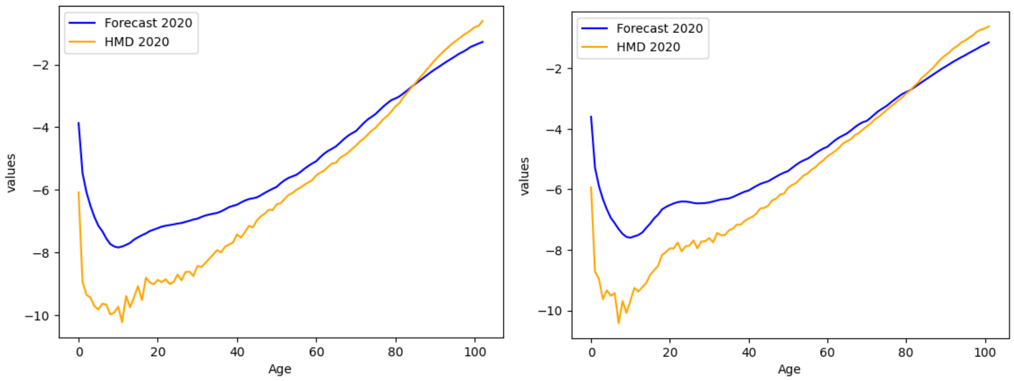
The forecasted findings for the United Kingdom in 2020, in Fig. 16, demonstrate that the HMD mortality rates are greater than the forecasted death rates for ages beyond 80, in line with the earlier comparisons. Subsequently, Fig. 17 below displays the forecasted findings for the United Kingdom in 2021.



(a) (b)

Figure 17. Logarithmic Mortality Rate for the United Kingdom in 2021 (a) Female, (b) Male

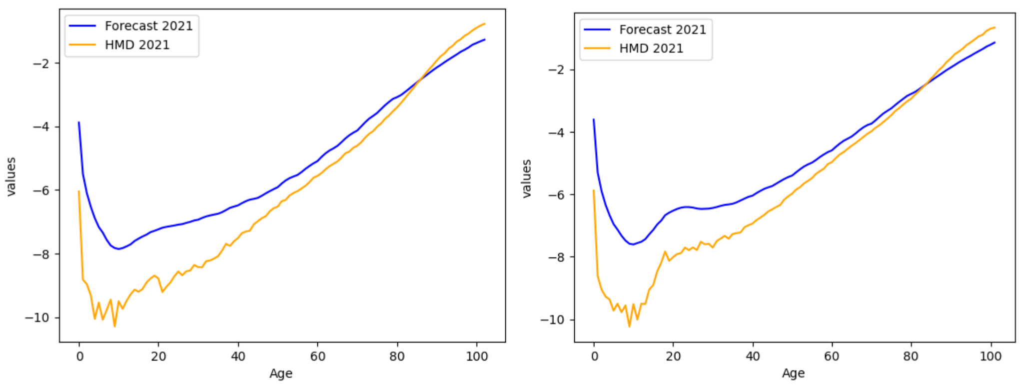
Similar to the previous forecasted findings, Fig. 17 shows that the HMD mortality rates rise again at older ages, eventually exceeding the forecasted mortality rates, consistent with the earlier comparisons. Fig. 18 then displays the forecasted findings for Italy in 2020.



(a) (b)

Figure 18. Logarithmic Mortality Rate for Italy in 2020 (a) Female, (b) Male

Consistent with the previous comparisons, Fig. 18 also indicates that the HMD mortality rates are higher than the forecasted mortality rates for ages above 80. Subsequently, the forecasting results for Italy in 2021 are presented in Fig. 19.



(a) (b)

Figure 19. Logarithmic Mortality Rate for Italy in 2021 (a) Female, (b) Male

The HMD mortality rates are greater than the predicted death rates for ages beyond 80, as shown in Fig. 19, as was the case with the earlier comparisons.

3.4 Discussion

In general, the shape of the logarithmic mortality curves for each country is consistent with the patterns observed in our previous study [13] as well as [3] which presents Indonesian mortality rates based on WHO data; [9] which displays the logarithmic values of estimated mortality and data from the World Population Prospects; and [12], which illustrates the logit-rate patterns produced by the Generalized Nolfi Model, the Lee-Carter model, and the Adapted Nolfi model. This similarity in shape is indicated by relatively high logarithmic mortality rates at age 0, followed by a decline approaching adolescence. During adolescence, the logarithmic mortality rate curves exhibit a sharp increase, peaking around age 20 and the early twenties. This rise suggests an increased risk of mortality during this age period and is reflected not only in the curves generated by the Second Adapted Nolfi Model but also in the curves derived from the HMD data. After this period, the logarithmic mortality rate continues to show an upward trend, increasing into old age.

In addition to the graphical comparisons above, error measurements using RMSE, MAE, and MAPE were also conducted to quantitatively assess the differences between the forecasted mortality rates and those from the HMD. The error measurement results for the three countries are presented in Table 1.

Table 1. RMSE, MAE, and MAPE Values for Forecasted Mortality Rates

Country	Year	Sex	RMSE	MAE	MAPE
United States	2020	Female	0.184004648	0.073782445	85.31661933
		Male	0.017904343	0.006332479	35.46963059
	2021	Female	0.157128267	0.063178657	69.97607619
		Male	0.014512721	0.005697734	36.77197821
	2022	Female	0.155363739	0.063225963	75.60102928
		Male	0.012886374	0.004323143	28.34253109
United Kingdom	2020	Female	0.04517895	0.015474572	90.53402631
		Male	0.030735854	0.010334874	59.05325757
	2021	Female	0.0350076	0.012085115	77.19254811
		Male	0.022833809	0.007624545	62.13244917
Italy	2020	Female	0.052628122	0.019639248	286.5172872
		Male	0.053251688	0.020777432	283.4422284
	2021	Female	0.040744759	0.015706425	263.1976437
		Male	0.045884028	0.01785296	263.7672034

In this study, pre-COVID-19 data were used as the basis for modeling, which subsequently served as the foundation for forecasting. As a result, the forecasted outcomes tend to follow pre-pandemic patterns. A similar situation was observed in mortality modeling using the Lee-Carter model, as noted in [22], where it is assumed that mortality rates evolve smoothly over time without sudden changes or shocks. Another study, as noted in the reference [23], also applied other mortality models without jump effects, such as the Renshaw and Haberman model and the Cairns-Blake-Dowd model, by fitting the models to pre-pandemic data and forecasting mortality during and after the pandemic. Therefore, considering that the comparison between the forecasted values and the actual data in Table 1 was conducted during the COVID-19 pandemic, a period that claimed many lives and potentially caused sudden changes or shocks, high errors may have occurred due to significant deviations in the actual values from the expected forecast patterns.

Regarding the error results, this study found that error values for United States males and United Kingdom males were lower than those for females across all observed years. This aligns with the fact that the age intervals for males in both countries were narrower than those for females. This difference is attributed to limitations in the HMD database, the requirement to meet the condition $\lambda_x > 0$, and the absence of interpolation and extrapolation processes, which resulted in some age intervals being unobservable. Consequently, the contribution of error differences at older ages was more prominently captured in the female data than in the male data.

In the case of Italy, the MAPE value was significantly higher compared to the other two countries. This can be attributed to the contrast between the fluctuating HMD mortality rates for Italy and the smooth forecasted mortality rates, which had undergone a smoothing process, as illustrated in Fig. 2. Furthermore, Figs. 18 and 19 also reveal that the discrepancies between the forecasted and HMD mortality rates at younger ages in Italy were greater than those observed in the other countries.

Although Italy's MAPE values were notably high, the MAPE values for the United States and the United Kingdom were also not negligible. This indicates a significant difference between the forecasted mortality rates, which were generated based on pre-pandemic modeling data, and the actual mortality rates observed during the COVID-19 pandemic. However, it is also possible that these high errors reflect limitations in the model and forecasting method in capturing the dynamics of the data. Therefore, further comparisons between the Second Adapted Nolfi model and other mortality models, or adjustments to the Second Adapted Nolfi model to accommodate sudden changes or shocks, may be useful for assessing its effectiveness. Additionally, the RMSE, MAE, and MAPE values are influenced by the data smoothing process and the level of smoothing applied. Since smoothing was applied only to three datasets (Italy females, Italy males, and the United Kingdom females), while the other three used raw data, the resulting error values may vary if alternative smoothing techniques and levels were implemented.

4. CONCLUSION

The Nolfi model, the Generalized Nolfi model, and the Adapted Nolfi models are some of the approaches that can be used to model mortality rates. Unfortunately, discussions surrounding these models in the context of mortality modeling remain limited, and, to our knowledge, no studies have applied these models to the pandemic or post-pandemic period. Although the pandemic may appear less relevant in 2025, the absence of such studies highlights the importance of further investigation into the model's performance under extreme demographic conditions. Accordingly, this study aimed to fill that gap by employing the Second Adapted Nolfi model, one of the three Adapted Nolfi models, to model mortality rates during the COVID-19 pandemic. The findings from this study are expected to provide insights into how this model, in combination with Auto-ARIMA, performs under extreme conditions such as a global health crisis, as well as in future situations that may result in sudden changes or shocks in mortality patterns. By applying the Second Adapted Nolfi model to data from the United States, the United Kingdom, and Italy, we found that the modeling mortality rates results remains consistent with our previous findings, such as high logarithmic mortality rates at age 0, followed by a decline approaching adolescence, then a sharp increase around age 20 and the early twenties, and a continued rise into old age. The mortality rate forecasts using the Auto-ARIMA method also revealed that the HMD mortality rates were higher than the forecasted rates for individuals aged above 80 years. This suggests that mortality rates for the elderly population increased during the COVID-19 pandemic in the three countries examined. It also exhibited relatively high forecasting errors that may reflect deviations caused by the pandemic's impact, but also highlight potential limitations in the model and forecasting method in capturing sudden changes or shocks. This is supported by the error values obtained in this study, with RMSE ranging from 0.01 to 0.18, MAE from 0.004 to 0.07, and MAPE from 28 to 286, varying across countries and genders. Furthermore, differences in smoothing techniques across datasets affected model performance, underscoring the importance of consistent preprocessing in mortality modeling.

Despite its limitations, the Second Adapted Nolfi model shows potential and warrants further exploration. Future studies may enhance this model by introducing new adaptations, applying it to broader datasets, or experimenting with alternative smoothing techniques. These steps will support the development of more resilient mortality models capable of handling both normal and exceptional demographic conditions.

Author Contributions

Made Diyah Putri Martinasari: Conceptualization, Data Curation, Formal Analysis, Funding Acquisition, Investigation, Methodology, Project Administration, Resources, Software, Supervision, Validation, Visualization, Writing-Original Draft, and Writing – Review and Editing. Krishna Prafidya Romantica: Conceptualization, Funding Acquisition, Methodology, Resources, Software, Validation, Writing-Original Draft, and Writing – Review and Editing. Putu Tika Dinda Gentari: Conceptualization, Funding Acquisition, Resources, Software, And Writing – Review and Editing. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare that there is no conflict of interest in the report study.

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