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WHOLE LIFE INSURANCE UTILIZING THE COMMISSIONERS METHOD AND THE VASICEK INTEREST RATE MODEL FOR PREMIUM RESERVE ANALYSIS

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ABSTRACT

Premium reserves play a vital role in ensuring that insurance firms can meet their future obligations to policyholders. Traditional fixed-rate approaches often fail to reflect market volatility, leading to potential misestimations. This study addresses this gap by integrating the Commissioners Method with the Vasicek stochastic interest rate model to evaluate premium reserves for whole life insurance. The research hypothesizes that the Vasicek model provides more realistic reserve estimates than fixed-rate models, that payment frequency and behavioral preferences significantly affect reserve levels, and that gender-specific mortality impacts reserve adequacy. Using BI-7D-RR interest rate data from 2019–2024, Vasicek parameters were calibrated and applied to reserve calculations. A sensitivity analysis was conducted by varying the model's mean reversion, volatility, and long-term mean parameters. The results confirm that Vasicek-based reserves are more robust and realistic than fixed-rate estimates. Incorporating a DARA utility function adds behavioral realism, while payment frequency strongly influences reserve accumulation. Gender-specific mortality produces systematically higher reserves for male policyholders. Sensitivity analysis highlights the model's robustness, with reserves responding predictably to parameter changes. This research contributes theoretically by linking stochastic modeling, demographics, and behavioral economics, while providing practical guidance for insurers to strengthen reserve adequacy and financial resilience under uncertainty.

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1. INTRODUCTION

Life insurance plays a crucial role in delivering financial security for individuals and families since it guarantees benefit payments in the event of the policyholder's passing [1]. The unique feature of whole life insurance is that, provided premiums are paid in compliance with the conditions of the policy, it offers lifetime coverage. Whole life insurance is a type of life insurance policy that, if the policy is in effect and premiums are paid in accordance with its terms, protects the policyholder for the duration of his life. Because these policies have a long duration [2], the insurance companies must compute and keep large premium reserves to cover their future liabilities and to guarantee the firm's financial stability [3].

In actuarial literature, the determination of premiums is closely tied to risk management and the sustainability of insurance operations. According to [4], the net premium can be derived using the equivalence principle, formulated as $E(L) = 0$, where the expected loss equals zero. However, relying solely on net premium exposes insurers to significant risk, since no safety loading is added to cover unexpected contingencies. Safety loading refers to an additional amount incorporated into the premium to account for uncertainty, thereby providing extra coverage for high-risk policyholders and enhancing the insurer's financial resilience [4].

To address these concerns, a utility-based framework has been proposed, where premium calculation is guided by a utility function $u(w)$, with w denoting wealth. Within this framework, the Decreasing Absolute Risk Aversion (DARA) utility function has gained prominence, as it realistically captures the declining sensitivity to risk as wealth increases, thus providing a more robust foundation for actuarial decision-making [4].

Alongside premium determination, accurate valuation of reserves requires reliable modeling of interest rates, since long-term insurance contracts are highly sensitive to interest rate fluctuations. Traditional no-arbitrage models, including Ho-Lee, Hull-White, Black-Karasinski, Heath-Jarrow-Morton, and binomial tree models, assume constant short-term volatility and lack mean-reverting behavior, making them unrealistic under fluctuating market conditions [5]. A few stochastic models, such as the Cox-Ingersoll-Ross (CIR) model, the Vasicek model [6], and the Hull-White model [7], have been proposed to deal with the inherent randomness in financial variables. The CIR and Vasicek models more accurately reflect real-world interest rate behavior: CIR simulations reveal that volatility varies and paths with higher volatility experience greater drift, whereas Vasicek simulations indicate slower convergence during upward shifts in interest rates, highlighting its mean-reverting characteristic [8]. This characteristic makes the Vasicek model particularly well-suited for valuing long-term insurance products, where extended forecasting is required [9].

Compared to the CIR and Hull–White models, the Vasicek model offers a pragmatic balance between analytical tractability and realistic representation of interest rate dynamics. The CIR model, while ensuring non-negative rates, introduces calibration difficulties and requires more complex numerical procedures, making it less efficient when integrated into regulatory reserve frameworks such as CRVM [6]. The Hull–White model incorporates time-varying volatility but requires additional parameters that increase the risk of overfitting, especially when long-term historical data are limited [7]. In contrast, the Vasicek model provides closed-form solutions, a parsimonious parameter structure, and reliable mean-reverting properties, which are particularly advantageous for actuarial applications that demand computational efficiency and regulatory compliance [9]. These advantages make Vasicek the most suitable choice for this study.

The incorporation of stochastic interest rate models into conventional reserve valuation frameworks has grown in importance in the rapidly changing fields of actuarial science and financial modeling. The Vasicek model is well known among these models for its capacity to represent the mean-reverting behaviour of interest rates, providing a more flexible and practical substitute for fixed-rate assumptions. Its use in regulatory reserve frameworks, like the Commissioners Reserve Valuation Method (CRVM), is still not well understood, though, especially when it comes to whole life insurance contracts that require several premium payments annually [10] .

Conventional CRVM computations usually assume yearly premium payments and are based on deterministic interest rates, which might not accurately represent the structure of contemporary insurance policies [10]. When premiums are paid on a quarterly or monthly basis, as is typically the case, this disparity is more noticeable. A useful actuarial tool for modifying present value computations to take into consideration m-times-a-year premium payments is the Woolhouse approximation. Woolhouse improves the accuracy of reserve computations without the computational strain of complete discretization by providing a correction to annualized estimates [11].

This study suggests a new framework that combines the computational efficiency of the Woolhouse approximation, the regulatory resilience of the Commissioners method, and the stochastic flexibility of the Vasicek model. The goal of this integration is to provide reserve estimations for whole life insurance products with periodic premium structures that are more precise and consistent with regulations [11].

Furthermore, demographic variables like gender-specific mortality are not taken into consideration in much research. For instance, prior research used a fixed 4% interest rate to compute premium reserves for male policyholders aged 25 while ignoring the dynamic effects of shifting market rates [10]. In a further study, a 35-year-old insured with a 30-year annuity term, and a 35-year premium payment period was studied on the assumption that the money market interest rate (IndoNIA) varied in the range of the BI rate (6.16%) [11]. However, these studies did not investigate how changes in interest rates affect reserve values, leaving a significant gap in literature.

A review of previous studies reveals that while various methods have been employed in premium reserve calculations and interest rate modeling, most approaches have relied on fixed interest rate assumptions. For instance, [12] and [13] simulated reserves using deterministic rates without exploring the effects of market fluctuations. Similarly, [14] and [15] applied established reserve calculation methods but did not investigate how interest rate variability might alter outcomes. These limitations highlight a consistent gap: the absence of stochastic modeling and sensitivity analysis in premium reserve research.

Other studies focused on interest rate models, such as [6], which utilized Vasicek and CIR, yet these efforts remained disconnected from practical premium reserve applications. Likewise, [16] and [17] concentrated on reserve calculations under fixed interest rate environments but failed to examine long-term fluctuations or utility-based perspectives. Even in the more recent work by [11], where multiple interest rates were tested, the analysis did not provide a systematic sensitivity study to capture broader market dynamics.

The identified gaps point toward the need for a more integrated approach that combines stochastic interest rate models with established regulatory methods such as the Commissioners Method and Woolhouse approximation. By doing so, reserve valuations can be made more accurate, robust, and responsive to changing market conditions. Moreover, incorporating demographic factors, such as gender-specific mortality, would enhance the precision of reserve estimates and align them more closely with real-world insurance practices.

In response to these gaps, this study seeks to develop a comprehensive reserve valuation framework for whole life insurance policies with multiple premium payment structures. The proposed framework integrates the Commissioners Reserve Valuation Method (CRVM) with the Vasicek stochastic interest rate model, while also accounting for demographic variations such as gender-specific mortality. To further reflect policyholder behaviour under uncertainty, the study incorporates a Decreasing Absolute Risk Aversion (DARA) utility function, capturing the tendency of individuals to become less risk averse as their wealth increases. Through this integration, the research introduces a behavioural dimension to actuarial modeling, offering a more dynamic and realistic basis for premium reserve estimation in the context of fluctuating market rates and diverse demographic profiles.

2. RESEARCH METHODS

This study evaluates premium reserves for whole life insurance plans using a quantitative case study design based on simulation. The research framework combines regulatory valuation methods, financial econometrics, and actuarial modeling to provide a comprehensive approach to reserve estimation.

The analysis begins with the Vasicek stochastic interest rate model, a widely used tool in financial modeling for capturing the dynamics of short-term interest rates. Its mean-reverting property provides a practical foundation for forecasting future rates. The study utilizes the Ordinary Least Squares (OLS) approach, which is implemented in Python 3.8 for efficient numerical calculation, to estimate the model's important parameters, including mean reversion speed (k), long-term mean (θ), and volatility (σ).

The second phase focuses on determining premium reserves for a policyholder aged 40 with an m-years contract, paying premiums every three years, using the Commissioners Method, a modified prospective technique. This approach accounts for interest rate assumptions, payment frequency, and mortality risk (via gender-specific mortality tables) to generate a realistic projection of the insurer's long-term financial obligations. A death benefit of IDR 500,000,000 is applied, payable if the insured dies within the policy term.

The entire research process is depicted in Fig. 1 to ensure reproducibility and clarity:

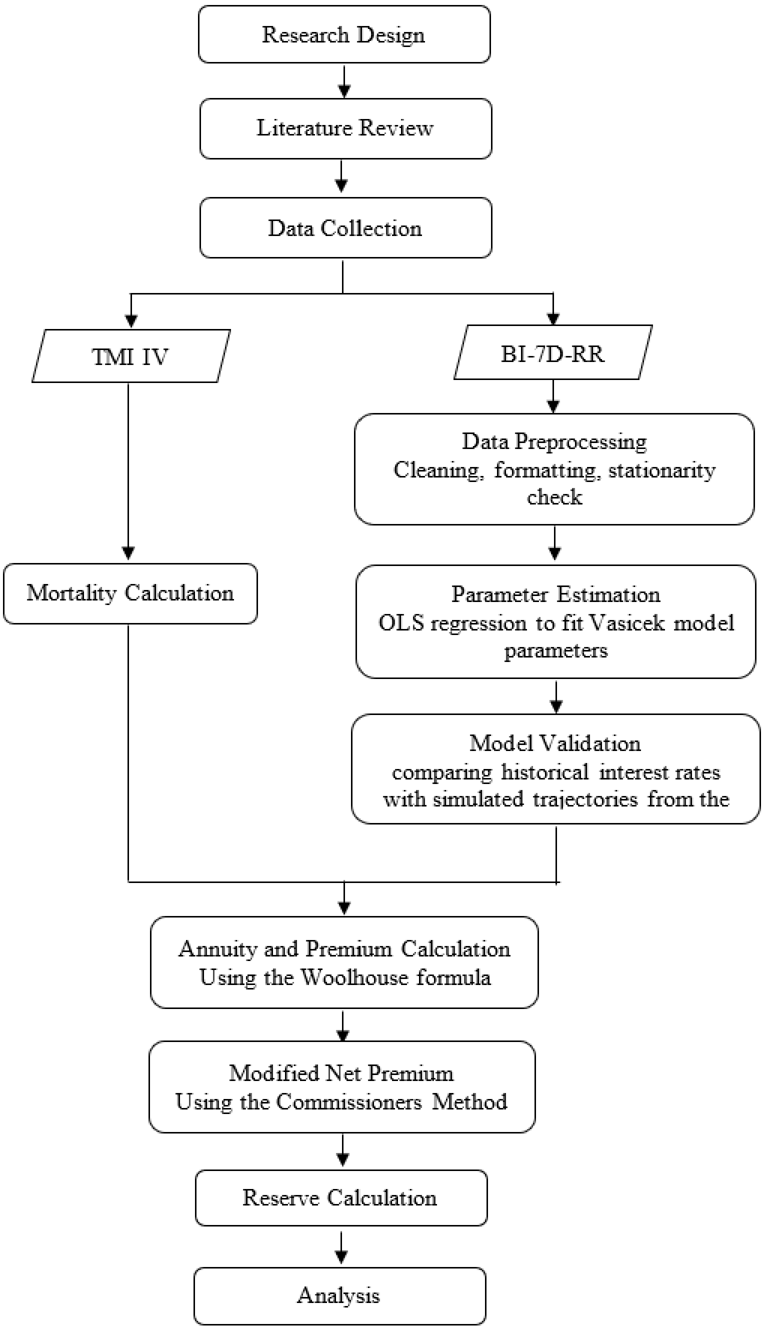

Figure 1. Flowchart of Analysis Stages

2.1 Stationarity Analysis of Interest Rate Data Using ADF and DF-GLS Tests

In time series modeling, stationarity is a fundamental assumption for reliable estimation and forecasting. A stationary series has constant mean and variance over time, ensuring that past behaviors are informative for predicting future values. To evaluate the stationarity of the interest rate series and its first-order differenced series, we applied two widely recognized unit root tests: the Augmented Dickey-Fuller (ADF) test and the Elliott-Rothenberg-Stock DF-GLS (DF-GLS) test.

The ADF test is an extension of the Dickey-Fuller test that accounts for higher-order autocorrelation by including lagged differences of the dependent variable. The null hypothesis (H_0) assumes that the series has a unit root, while the alternative hypothesis (H_1) indicates stationarity. The DF-GLS test improves the power of the ADF test by first applying Generalized Least Squares (GLS) detrending, which is particularly effective for small samples and series with deterministic trends.

The results indicate that the level interest rate series is non-stationary, as both ADF and DF-GLS tests failed to reject the null hypothesis. However, the first-order differenced series is stationary, confirming that differencing successfully removed the non-stationarity. These results ensure that subsequent modeling using ARIMA, Vasicek, or other time series models is valid. The detailed test results are presented in Table 1.

Table 1. ADF and DF-GLS Test Results for Level and Differenced Interest Rate Series

Series	Test	Test Statistic	p_value	Critical Value			Result
				1%	5%	10%	
interest_rate	ADF	-1.3245	0.6138	-3.5270	-2.9036	-2.5892	Non-stationary
interest_rate	DF-GLS	-1.1132	0.2695	-2.5984	-1.9455	-1.6138	Non-stationary
differenced_interest_ rate	ADF	-3.8448	0.0040	-3.5270	-2.9036	-2.5892	Stationary
differenced_interest_ rate	DF-GLS	-3.8729	0.0002	-2.5984	-1.9455	-1.6138	Stationary

Data Source: Processed Data

2.2 Data Sources and Description

The secondary data utilized in this study comes from two crucial sources for life insurance modeling: mortality statistics and interest rate dynamics [16]. Publication of the Indonesian Mortality Table IV (TMI IV) is done by the Indonesian Life Insurance Association (AAJI). The TMI IV provides detailed age-specific mortality rates to estimate chance of survival, chance of death, and other important actuarial functions. Accurate mortality data is crucial for estimating future claim responsibilities and calculating premium reserves in life insurance, particularly in whole life insurance policies where coverage lasts the policyholder's lifetime.

The Bank Indonesia 7-Day Reverse Repo Rate (BI-7D-RR) interest rate data is used in the study to reflect how financial market movements affect discount variables. The central bank's monetary policy stance is reflected in the BI-7D-RR, which serves as Indonesia's benchmark short-term interest rate. Current and realistic interest rate data are crucial for accurate premium reserve estimation since interest rates directly impact the present prices of future liabilities. The period covered by the interest rate series is January 2023–December 2024.

The study develops a comprehensive financial and actuarial framework by integrating interest rate and mortality data. Mortality data allows the modeling of policyholder survival and death benefits, while the Vasicek process can be used to stochastically model the BI-7D-RR and generate dynamic interest rate estimations. Together, these factors ensure that the premium reserve calculations appropriately take into consideration both demographic and market volatility, leading to a more realistic assessment of the insurer's long-term obligations.

2.3 Study Techniques

The research framework begins with the formulation of the research design, followed by a literature review that establishes the theoretical foundation and identifies gaps addressed by the study. The subsequent stage is data collection, which integrates two main sources: mortality data from the Indonesian Mortality Table IV (TMI IV) and financial data from the Bank Indonesia seven-day reverse repo rate (BI-7D-RR). These datasets serve as the empirical basis for actuarial modeling, with mortality data used to estimate life contingencies and interest rate data used for financial modeling.

The BI-7D-RR dataset undergoes preprocessing to ensure validity, including data cleaning, formatting, and a stationarity check. Parameter estimation is then performed using ordinary least squares (OLS) regression to obtain the parameters of the Vasicek stochastic interest rate model. The calibrated model is validated by comparing simulated interest rate trajectories with historical observations, thereby confirming its ability to capture mean-reverting dynamics. In parallel, the TMI IV dataset is used for mortality calculations, which are essential for determining life annuities and insurance premiums.

The integration of these two modeling streams enables the calculation of annuities and premiums. This study applies the Woolhouse formula, a widely used actuarial approximation for annuities when life tables are tabulated at integer ages. The Woolhouse approximation improves accuracy by incorporating adjusting for payment frequency, specially three times premium payments a year.

2.3.1 DARA Utility Function

The utility function is defined as $u(w)$, where w represents wealth. To properly model decision-making under uncertainty, the utility function must satisfy the conditions $u'(w) > 0$ and $u''(w) > 0$, which indicate that utility increases with wealth while reflecting risk-averse behavior. Through this formulation, the insurer's attitude toward risk can be evaluated, with the degree of risk aversion measured by

$$a(w) = -\frac{u''(w)}{u'(w)}. \tag{1}$$

The determination of the net premium can be carried out using the equivalence principle, expressed as $E(L) = 0$, meaning that the expected value of the loss is zero. Within the utility-based approach, premium calculation is guided by the principle of equivalent utility, formulated as:

$$E[u(w)] = E[u(w - L)], \tag{2}$$

where L denotes the potential loss. In this framework, the Decreasing Absolute Risk Aversion (DARA) utility function is employed, reflecting the realistic assumption that risk aversion declines as wealth increases. By adopting an exponential utility function of the form:

$$u(w) = 1 - e^{-\sqrt{\frac{w}{b}}}, \tag{3}$$

furthermore,

$$E[1 - e^{-\sqrt{\frac{w}{b}}}] = E[1 - e^{-\sqrt{\frac{w-L}{b}}})], \tag{4}$$

whereas Eq. (4) expresses the relationship between premiums and risk can be explicitly characterized. This formulation ensures that the premium level is aligned with the degree of risk borne by the insurer, thereby enabling the minimization of potential losses and enhancing financial stability.

2.3.2 The Vasicek Model of Stochastic Interest Rates

An index or collection of random numbers that fluctuates unpredictably while maintaining the independence of the values of the variables and sets is known as a stochastic process [18]. Interest rates with unique characteristics, such as a tendency to revert to the average interest rate following a decline or increase, are modelled by the Vasicek Model [19]. The time$-t$ interest rate is denoted by r_t. Eq. (5) expresses the Vasicek model stochastic differential equation (dr) in its generic form.

$$dr(t) = k(\theta - r(t)dt) + \sigma dW(t), \qquad r(0) = r_0. \tag{5}$$

The Eqs. (6), (7), and (8) are the mean, variance, and solution of the Vasicek stochastic differential equation, respectively, which are obtained by solving the differential equation using a homogeneous linear differential equation with constant coefficients.

$$E(r_t) = e^{-k\Delta t} r_0 + \theta\left(1 - e^{-k\Delta t}\right), \tag{6}$$

$$Var(r_t) = \frac{\sigma^2}{2k}\left(1 - e^{-2k\Delta t}\right), \tag{7}$$

$$r(t) = e^{-k\Delta t} r(0) + \theta\left(1 - e^{k\Delta t}\right) + \sigma \int_0^t e^{-k(\Delta t - u)}\, dW(u). \tag{8}$$

The OLS (Ordinary Least Square) approach is used to estimate the initial values of the parameters in the Vasicek Model. One technique to reduce the sum of squared errors is the OLS approach [20]. Eqs. (9), (10), and (11) express the estimated values of the Vasicek Model parameters using the OLS method.

$$k = \frac{n^2 - 2n + 1 + \sum_{t=1}^{n-1} r_t + 1 \sum_{t=1}^{n-1} \frac{1}{r_t} - \sum_{t=1}^{n-1} r_t \sum_{t=1}^{n-1} \frac{1}{r_t} - (n-1) \sum_{t=1}^{n-1} \frac{r_{t+1}}{r_t}}{\left(n^2 - 2n + 1 - \sum_{t=1}^{n-1} r_t \sum_{t=1}^{n-1} \frac{1}{r_t}\right) \Delta t}, \tag{9}$$

$$\theta = \frac{(n-1)\sum_{t=1}^{n-1} r_{t+1} - \sum_{t=1}^{n-1}\frac{r_{t+1}}{r_t}\sum_{t=1}^{n-1} r_t}{n^2 - 2n + 1 + \sum_{t=1}^{n-1} r_{t+1}\sum_{t=1}^{n-1}\frac{1}{r_t} - \sum_{t=1}^{n-1} r_t \sum_{t=1}^{n-1}\frac{1}{r_t} - (n-1)\sum_{t=1}^{n-1}\frac{r_{t+1}}{r_t}}, \quad (10)$$

$$\sigma = \sqrt{\frac{1}{n-2}\sum_{t=1}^{n-1}\left(\frac{r_{1+1} - r_t}{\sqrt{r_t}} - \frac{\theta}{\sqrt{r_t}} + k\sqrt{r_t}\right)^2}. \quad (11)$$

The parameters of the Vasicek Model are estimated using BI interest rate data from January 2023 to December 2024. The numerical implementation of the Vasicek model estimation was carried out in Python 3.8.

2.3.3 The Mortality Table's Basic Annuity Calculation

Let l_x represent the number of individuals who are still alive at exact age x. then the survival function is defined as $S(x) = \frac{l_x}{l_0}$ [21] . The probability that an individual aged x will survive to age $x + n$ is denoted by ${}_np_x$, and is calculated as the ratio of the number of people alive at age $x + n$ to those alive at age x, ${}_np_x = \frac{l_{x+n}}{l_x}$. Conversely, the probability that an individual aged x will die before reaching age $x + n$ is denoted by ${}_nq_x = 1 - {}_np_x$ [22]. The force of mortality, often known as the instantaneous death rate, or μ_x, is a measurement of the likelihood of constantly dying at a given age [23]. The formula $\mu_{x+t} = -\frac{1}{2}\left(log(p_{x+t-1}) + log(p_{x+t})\right)$ is used to determine the instantaneous mortality rate.

A nominal discount is a discount rate that is expressed in yearly terms but is determined using a particular conversion period, such as monthly, quarterly, or semester, within a year [24]. When determining premiums and premium reserves for whole life insurance, this discount rate is crucial. To get the nominal discount, use the formula $d^{(m)} = m \cdot \left(1 - (1-d)^{1/m}\right)$. To eliminate simultaneous, exponential, and repetitive calculations on a little quantity of data, the commutation function was created. $D_x = v^x \cdot l_x$ is the fundamental commutation function. To determine the annuity of several surviving policyholder payments, the commutation function $N_x = \sum_{t=x}^{\infty} D_t = \sum_{t=x}^{\infty} v^t \cdot l_t$ is utilized. Based on Indonesian Mortality Table IV, find the upper limit of t up to age 110.

2.3.4 Computation Using the Woolhouse formula

The present value of an annuity with m payments per year can be approximated using the Woolhouse formula. This formula is derived from the Euler–Maclaurin expansion, in which a continuous function is approximated by considering its derivatives up to the n-th order. By doing so, Woolhouse provides a practical way to approximate the value of annuities with frequent payments. The formula is expressed as Eq. (12), which refines the computation of annuities beyond simple annual assumptions by incorporating the effect of multiple premium payments per year.

$$\int_0^{\infty} g(t)\,dt = h \cdot \sum_{k=0}^{\infty} g(kh) - \frac{h}{2}\cdot g(0) + \frac{h^2}{12}\cdot g'(0). \quad (12)$$

The Woolhouse method is particularly advantageous when premiums are paid three times a year ($m = 3$), as assumed in this study. Direct computation of such annuities would require summing many discounted cash flows across multiple payment points each year, which can be both complex and computationally intensive. By applying the Woolhouse formula, actuaries can obtain accurate approximations of the present value without resorting to lengthy calculations. This efficiency makes the method highly suitable for premium reserve evaluations, where repeated annuity valuations are necessary.

Assume that the function $g(t)$ represents the cash value of annual premium payments over time. The value of this function is determined using Eq. (13):

$$g(t) = v^t \cdot {}_tp_x. \quad (13)$$

In this equation, v^t denotes the present value of a payment made at time t, while ${}_tp_x$ represents the probability that an individual aged x will survive to age $x + t$.

To analyze how this cash value changes over time, the first derivative of $g(t)$ is given in Eq. (14):

$$g'(t) = \frac{d}{dt}(v^t) \cdot {}_tp_x + v^t \cdot \frac{d}{dt}({}_tp_x) = -\delta \cdot e^{-\delta t} \cdot {}_tp_x - v^t \cdot {}_tp_x \cdot \mu_{x+t}. \tag{14}$$

Here, μ_{x+t} is the force of mortality at age $x+t$, approximated by:

$$\mu_{x+t} = -\frac{1}{2}\left(log(p_{x+t-1}) + log(p_{x+t})\right).$$

For annual payments, $h = 1$ the annuity value is calculated using Eq. (15):

$$\int_0^\infty g(t)\,dt = \sum_{k=0}^{\infty} g(k) - \frac{1}{2} \cdot g(0) + \frac{1}{12} \cdot [-(\delta + \mu_x)] = \ddot{a}_x - \frac{1}{2} - \frac{1}{12} \cdot [-(\delta + \mu_x)]. \tag{15}$$

When premiums are paid m times per year, $h = \frac{1}{m}$, the annuity value is adjusted using Eq. (16):

$$\int_0^\infty g(t)\,dt = \frac{1}{m} \cdot \sum_{k=0}^{\infty} g\left(\frac{k}{m}\right) - \frac{1}{2m} \cdot g(0) + \frac{1}{12m^2} \cdot g'(0) = \ddot{a}_x{}^{(m)} - \frac{1}{2m} - \frac{1}{12m^2} \cdot [\delta + \mu_x]. \tag{16}$$

Since both annuity values yield similar results, they are considered equivalent. Thus, Eq. (17) provides an alternative expression [25]

$$\ddot{a}_x{}^{(m)} - \frac{1}{2m} - \frac{1}{12m^2} \cdot [\delta + \mu_x] \approx \ddot{a}_x - \frac{1}{2} - \frac{1}{12} \cdot [-(\delta + \mu_x)],$$

$$\ddot{a}_x{}^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2} \cdot [\delta + \mu_x]. \tag{17}$$

This formulation allows for a more accurate estimation of annuity values when premiums are paid multiple times per year, incorporating both the time value of money and mortality risk.

The monetary value of a life annuity with m payments per year is further expressed using commutation functions in Eq. (18):

$$\ddot{a}_x{}^{(m)} \approx \frac{N_x}{D_x} - \frac{m-1}{2m} - \frac{m^2-1}{12m^2} \cdot [\delta + \mu_x]. \tag{18}$$

This value represents the initial annuity amount paid at the beginning of each period. To estimate the initial annuity value for a whole life insurance policy issued to an individual aged $x+t$, the Woolhouse approximation is applied, as shown in Eq. (19):

$${}_h\ddot{a}_{x+t}^{(m)} = \frac{N_{x+t}}{D_{x+t}} - \frac{m-1}{2m} \cdot \left(1 - v^h \cdot {}_hp_{x+t}\right) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_{x+t} - v^h \cdot {}_hp_{x+t} \cdot (\delta + \mu_{x+t+h})\right). \tag{19}$$

This formula is evaluated for various values of $h \in \{10,20,30\}$ and $t \in \{1,2,\ldots,30\}$, representing different coverage durations and ages.

Whole life insurance provides a benefit upon the death of the insured, regardless of when it occurs. To determine the single premium required for such coverage, Eq. (20) applies to the Woolhouse formula to calculate the present value of future benefits for an individual aged $x+t$, with coverage lasting $n-t$ years:

$$A_{x+t}^{(m)} = 1 - d^{(m)} \cdot \left(\frac{N_{x+t}}{D_{x+t}} - \frac{m-1}{2m} \cdot (1 - v^h \cdot {}_hp_{x+t}) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_{x+t} - v^h \cdot {}_hp_{x+t} \cdot (\delta + \mu_{x+n})\right)\right). \tag{20}$$

The premium paid at the start of each year is known as the net annual premium. The annual premium amount is typically the same for every payment [26]. Eq. (21) uses the Woolhouse formula to get the life insurance premium amount for the insured at age $x+t$ years,

$${}_hP_{x+t}^{(m)} = \frac{1 - d \cdot \left(\frac{N_{x+t}}{D_{x+t}}\right)}{\frac{N_{x+t}}{D_{x+t}} - \frac{m-1}{2m} \cdot (1 - v^h \cdot {}_hp_{x+t}) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_{x+t} - v^h \cdot {}_hp_{x+t} \cdot (\delta + \mu_{x+t+h})\right)}. \tag{21}$$

This equation is evaluated for $h \in \{10,20,30\}$ and $t \in \{1,2,\ldots,30\}$, allowing for flexible premium calculations across different ages and payment frequencies.

2.3.5 Commissioners' Method of Computation

One of the premium variations of the prospective technique for determining premium reserves in life insurance the Commissioners. In summary, the Commissioners technique describes how the age of the policyholder at the time of policy issuance and two modified premium values, the β and α modifications, for each policy compare. This amount is the difference between the net premium of a one-year term computed at the policy entry age and the net premium of whole life insurance with a 19-year premium payment period calculated at an age one year older than the policy entry age. The Commissioners method's general version is expressed in Eq. (22),

$$\beta^{com} - \alpha^{com} = {}_{19}P_{x+1} - c_x, \tag{22}$$

using the formula $c_x = v \cdot q_x$, where $c_x$represents the natural premium or one-year term premium that is renewed every year for a predetermined amount of time. To determine the Commissioners method for m payments annually, Eq. (23) is utilized,

$$\beta^{com(m)} - \alpha^{com(m)} = {}_{19}P_{x+1}^{(m)} - c_x^{(m)}, \tag{23}$$

where the natural premium of m payments is indicated by $c_x^{(m)} = 1 - d^{(m)} \cdot (1 + v \cdot p_x) - v \cdot p_x$. Additionally, the nominal value of the discount $d^{(m)}$ is stated in Eq. (24).

$$d^{(m)} = m\left(1 - (1-d)^{\frac{1}{m}}\right). \tag{24}$$

The value of the modified net premium on whole life insurance for the insured aged x years can be computed using Eq. (25) based on the Commissioners method with m payments.

$$\beta^{com(m)} = {}_hP_x^{(m)} + \frac{{}_{19}P_{x+1}^{(m)} - c_x^{(m)}}{\ddot{a}_x^{(m)}}. \tag{25}$$

The above Eq. (26) is also used to determine the Commissioner's reserve calculation for whole life insurance for an individual aged x year, with a premium payment period of h years.

$${}^h_tV_x^{com(m)} = A_{x+t}^{(m)} - \beta^{com(m)} \cdot {}_h\ddot{a}_{x+t}^{(m)}. \tag{26}$$

3. RESULTS AND DISCUSSION

3.1 Estimating stochastic interest rates using the Vasicek Model

Based on Eq. (8), the BI interest rate parameters for the period January 2023 to December 2024 were estimated numerically as $k = 0.003819, \theta = 0.058433, and\ \sigma = 0.244219$. These parameters reflect fluctuations in response to the dynamics of the Indonesian financial market. Considering a variance of 0.059416 and an average rate trend of 0.059579, a homogeneous linear differential equation was employed to solve Eq. (5). The computed average rate is subsequently utilized within the Commissioner's Method framework to determine the adjusted premium reserves.

3.2 Multi-Level Lists Modified Premium Reserve Computation Based on the Commissioners Method

The Commissioners Method is used in this section to calculate premium reserves for whole life insurance contracts with a Vasicek-modelled stochastic interest rate.

3.2.1 Case Study I

A 40-year-old policyholder who paid premiums three times a year $(\boldsymbol{m} = \mathbf{3})$ for the course of a $\boldsymbol{h} -$ year contract duration had the modified premium reserve calculation analyzed. Should the danger of death materialize within the policy period, the whole life insurance business provides the insured with a claim benefit of IDR 500,000,000.00. Python 3.8 computation was used to estimate adjusted premium reserves using the Commissioners approach. There are notable variations in the premium reserve computation depending on the policyholder's gender.

According to the Vasicek interest rate model, the graph shows the evolution of the modified premium reserve value ${}^{30}_{t}V_{x}^{com(m)}$ over time for both male and female policyholders, with a payment frequency of $\boldsymbol{m}$ times year. Whereas the $\boldsymbol{y}$ −axis displays the premium reserve value (in Indonesian Rupiah units), the $\boldsymbol{x}$ −axis displays the policyholder's age in years.

The first computation relies on the parameters of the policy contract term $(\boldsymbol{t_{max}})$, which is also set at 10 years, and the premium payment period $(\boldsymbol{h})$ of 10 years. Furthermore, according to the mortality table, which serves as the foundation for determining the probability of life and death, the maximum age reference $(\boldsymbol{x_{max}})$ employed is 110 years. The Commissioners technique, an actuarial modification of the prospective approach, is used to calculate the premium reserve and apply the results to whole life insurance contracts. This computation is meant for people who become members at age 40, and the premium payment term is only for the first ten years. Fig. 2 shows the resulting reserve value graphically.

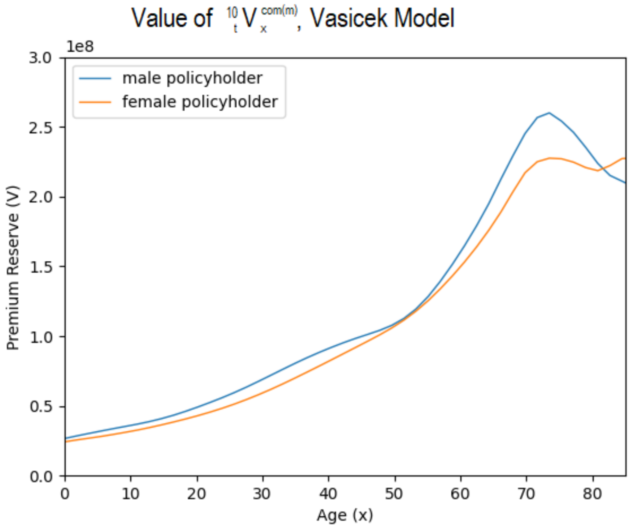

Figure 2. Premium Reserves at 40 Years Old Throughout the 10-year Contract Period

To better understand the implications of the reserve patterns, both gender variation and age-specific dynamics must be examined. According to Fig. 2, premium reserves can be determined using the commissioner's method until the policyholder reaches age 85, given a premium payment period of $\boldsymbol{h = 10}$. The graph shows that the reserve steadily increases with age, peaking around the mid-70s, before gradually declining toward the end of the policy. This behavior reflects the actuarial principle that insurers must accumulate reserves during the early and middle years of the contract to meet future benefit obligations, with the reserve reaching its maximum when the liability risk is highest, before tapering off as the contract approaches maturity.

When the policyholder reaches age 75, the company is required to offer the maximum premium reserve, which is IDR 260,169,296.39 for male policyholders and IDR 230,266,743.85 for female policyholders. These values represent the per-policy reserve (i.e., per individual insured), and they are calculated as discounted present values under the Commissioner's Method combined with the Vasicek interest rate model. This difference between genders is consistent with mortality-based actuarial assumptions: male policyholders generally exhibit higher mortality risks, leading insurers to set aside larger reserves to ensure claims obligations are adequately met. Conversely, female policyholders show lower reserve requirements in the earlier stages due to their relatively lower mortality risks. In practice, this gender-based variation in reserves is crucial for insurers because it affects premium pricing strategies, solvency management, and regulatory compliance. Without accounting for gender-specific mortality differences, insurers risk under-reserving (leading to solvency pressures) [27].

A notable observation from the graph is that, after age 80, the female reserve line surpasses the male reserve line. This reflects the demographic reality that females tend to live longer than males. By this stage, many male policyholders have already exited the contract due to higher mortality, causing their aggregate reserve requirement to decline more rapidly. In contrast, more females survive into advanced ages, requiring insurers to maintain higher reserves for this group to cover ongoing claim obligations. This crossover underscores the importance of considering survivor effects and longevity risk in reserve modeling, particularly for long-term insurance products [28].

There is also a consistent pattern of reserve fluctuations across different actuarial methods. As shown in [29], when using the De Moivre Mortality Law and the Indonesian Life Table IV, reserves initially increase during the premium payment period (up to the tenth year), after which they gradually decline, eventually reaching zero at the end of the protection period. A similar trend is evident in the present study's graph, where reserves rise steadily until age 75 and then decline until age 85. Despite differences in mortality assumptions and modeling frameworks, both the De Moivre technique and the commissioner's method with the Vasicek model capture the same fundamental principle: reserves accumulate as the expected claim obligations increase, peak at the point of highest liability, and decline as the probability of claims decreases toward the contract's end.

This alignment with established actuarial theory [30] demonstrates that the commissioner's method, when combined with a stochastic interest rate model such as Vasicek, provides a robust and theoretically consistent framework for evaluating long-term insurance liabilities. It not only ensures solvency by setting aside adequate reserves but also reflects the natural life cycle of insurance products, where financial obligations grow and contract in line with the insured's mortality risk and the timing of benefit payments.

The second calculation is based on a policy contract term (t_{max}) of 20 years, with the premium payment period (h) likewise set at 20 years. The mortality table, which provides the foundation for estimating survival and death probabilities, establishes the maximum reference age (x_{max}) at 110 years. Premium reserves are determined using the Commissioner's method, an actuarial adaptation of the prospective reserve approach, and the results are applied to whole life insurance contracts. In this scenario, the insured enters the policy at age 40, with premiums payable only during the first ten years. The corresponding reserve values are presented graphically in Fig. 3.

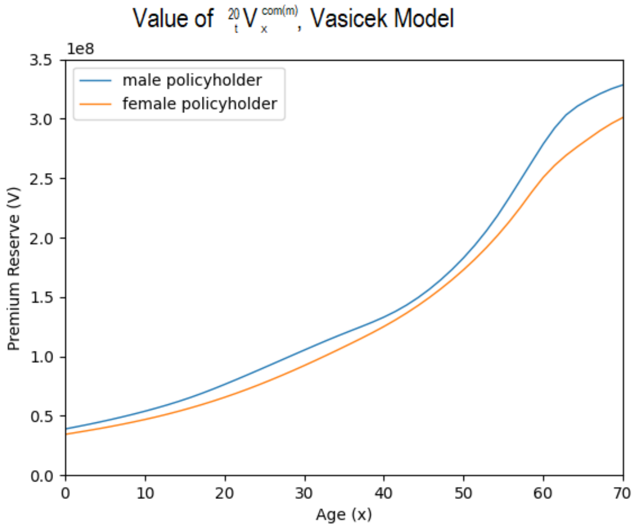

Figure 3. Premium Reserves at 40 Years Old Throughout the 20-year Contract Period

According to Fig. 3, premium reserves can be determined using the commissioner's method until the policyholder until the age of 70, where $x_{max} - h - t_{max} = 110 - 20 - 20 = 70$ for $h = 20, t \leq 70$. The graph shows that the reserve increases steadily with age, reaching its maximum at age 70. At this point, the maximum reserve amounts to IDR 328,461,420.39 for male policyholders and IDR 301,046,955.95 for female policyholders. These values represent the reserve per policy (per individual) and are already expressed as discounted present values, derived under the Vasicek interest rate model within the *Commissioner's Method*.

The upward trajectory of reserves until age 70 reflects the accumulation phase, during which premiums collected in earlier years are invested and adjusted to guarantee that increasing claim obligations can be met. After reaching the peak, the reserve is expected to decline as the pool of surviving insureds diminishes, thereby reducing the insurer's exposure to future claims [31]. This trajectory aligns with fundamental actuarial reserve theory, which posits that reserves taper in later durations once liability exposure decreases due to mortality.

A noticeable gender disparity is also observed in the reserve values: male reserves are consistently higher than those of females at the peak. This difference is attributed to men's higher mortality risk during the contract term, compelling insurers to hold greater reserves for male policyholders to safeguard claim settlements. Such findings are consistent with established actuarial mortality assumptions and prior literature on gender, specific reserving. Ignoring these mortality distinctions could result in under-reserving, threatening solvency.

The application of the Vasicek interest rate model confirms an annual upward trend in reserve values [32]. Both whole life and term life insurance exhibit similar patterns, with illustrations based on a 35-year-old female and a 40-year-old male insured. This trend reflects the increasing liabilities insurers must honor throughout the contract horizon. The observed reserve dynamics, rising steadily until age 70, highlight the prudential principles of insurance operations, particularly under conditions of uncertain life expectancy and fluctuating long-term interest rates.

The third computation relies on the parameters of the policy contract term $(\boldsymbol{t_{max}})$, which is set at 30 years, along with a premium payment term $(\boldsymbol{h})$ of 30 years. The mortality table, which underpins the estimation of life and death probabilities, specifies a maximum reference age $(\boldsymbol{x_{max}})$ of 110 years. To determine the premium reserve, the Commissioner's method, an actuarial refinement of the prospective reserve approach, is applied, with the results tailored for whole life insurance policies. This scenario considers individuals entering the contract at age 40, with premiums payable only during the first ten years. The computed reserve values are illustrated in Fig. 4.

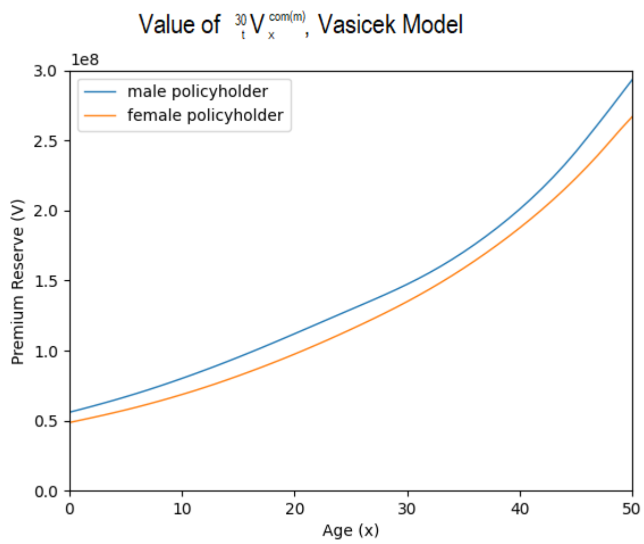

Figure 4. Premium Reserves at 40 Years Old Throughout the 30-year Contract Period

According to Fig. 4, premium reserves can be determined using the commissioner's method until the policyholder becomes 50, where $\boldsymbol{x_{max} - h - t_{max} = 110 - 30 - 30 = 50}$ for $\boldsymbol{h = 30, t \leq 50}$. Using the Vasicek model method with modified premium reserves, the results of the calculations showed that the greatest premium reserves for male clients were IDR 293,209,058.05. Currently, female consumers can reserve a maximum of IDR 266,820,547.59. As the policyholder matures and the coverage period draws to a close, the reserve values are demonstrated to rise steadily over the course of the policy, representing the insurer's increasing financial commitment. This pattern is consistent with actuarial assumptions, according to which the insurer must build up reserves to guarantee that it can pay its future claims obligations.

According to the findings, premium reserve values have clearly increased during the $\boldsymbol{h} -$ year contract. This upward trend is to be expected since the insurer needs to build up reserves to guarantee that it will be able to pay benefits as promised should claims occur. The remaining time until the policyholder's death decreases with policy maturity, raising the certainty of responsibility and necessitating higher reserve amounts. Furthermore, there is a discernible difference between the premium reserves of policyholders who are male and those who are female. Throughout the duration of the policy, the reserve values for male policyholders are continuously greater than those for female policyholders. Males typically have greater mortality rates than females, which can be attributed to gender-based inequalities in mortality rates. To sufficiently cover the increased likelihood of prior claims, insurers are having to set aside larger reserves for male policyholders.

The application of the Vasicek model highlights how stochastic interest rate modeling must be included when determining premium reserves. Because the Vasicek framework accounts for the mean-reverting tendencies and stochastic fluctuations of interest rates, it allows for a more realistic and robust prediction of future liabilities than ahu models, which assume a constant rate environment. The valuation of life insurance premium reserves is also significantly impacted by interest rate fluctuations, policyholder demographics, and contract maturity [33]. Interest rate fluctuations have a direct impact on how future benefit obligations are discounted; the present value of liabilities rises as interest rates fall, requiring larger premium reserves [34]. The case study's findings demonstrate how important factors influencing reserve adequacy are both demographics and financial market assumptions. To ensure prudent reserve management, sustain long-term financial solvency, and fulfil their commitments to policyholders, insurance companies must thus use thorough actuarial modeling frameworks that appropriately account for these concerns.

Building on the previous findings, Case Study II examines a 40-year-old policyholder's premium reserves under both Vasicek-modeled and deterministic interest rates, illustrating how different interest rate assumptions influence reserve estimates and highlighting the combined impact of financial and demographic factors on reserve adequacy.

3.2.2 Case Study II

This study examines 40-year-old male and female policyholders making triannual premium payments $(\boldsymbol{m} = \mathbf{3})$ over a 30-year term, with a death benefit of IDR 500,000,000. Using Python 3.8, modified premium reserves are calculated via the Commissioners' method under both Vasicek-modeled and deterministic interest rates, revealing significant gender-based differences. The findings shows that identical contractual terms can result in different reserve trajectories. Table 2 represents the reserves calculation is also shown in detail. At each year $\boldsymbol{t = 1}$ to $\boldsymbol{t = 30}$, the value of ${}^{30}_{t}\boldsymbol{V}_{\boldsymbol{x}}^{\boldsymbol{com(m)}}$, which represents the present value of the remaining annual premium payable over a $\mathbf{30}$ −year period from age 40.

Table 2. Premium Reserve of Whole Life Insurance on 40-year-old Policyholders

t	Male Policyholder		Female Policyholder	
	${}^{30}_{t}\boldsymbol{V}_{\mathbf{40}}^{\boldsymbol{com(m)}}$ **(Deterministic)**	${}^{30}_{t}\boldsymbol{V}_{\mathbf{40}}^{\boldsymbol{com(m)}}$ **(Vasicek)**	${}^{30}_{t}\boldsymbol{V}_{\mathbf{40}}^{\boldsymbol{com(m)}}$ **(Deterministic)**	${}^{30}_{t}\boldsymbol{V}_{\mathbf{40}}^{\boldsymbol{com(m)}}$ **(Vasicek)**
1	44,254,883	44,257,965	43,831,905	44,057,488
2	47,865,949	47,869,261	46,796,815	47,021,175
3	51,589,647	51,593,190	49,879,229	50,102,310
4	55,425,501	55,429,274	53,087,049	53,308,790
5	59,364,270	59,368,272	56,419,608	56,639,950
6	63,400,901	63,405,132	59,871,943	60,090,828
7	67,529,038	67,533,497	63,434,703	63,652,077
8	71,744,356	71,749,041	67,120,471	67,336,275
9	76,046,479	76,051,390	70,930,982	71,145,154
10	80,437,087	80,442,223	74,874,424	75,086,899
11	84,910,865	84,916,224	78,944,988	79,155,703
12	89,473,958	89,479,539	83,142,517	83,351,410
13	94,115,700	94,121,503	87,464,312	87,671,321
14	98,826,483	98,832,506	91,910,367	92,115,429
15	103,602,628	103,608,871	96,473,113	96,676,170
16	108,440,377	108,446,839	101,154,943	101,355,934
17	113,364,355	113,371,034	105,965,762	106,164,621
18	118,412,770	118,419,664	110,927,013	111,123,664
19	123,637,146	123,644,250	116,055,448	116,249,805
20	129,087,352	129,094,662	121,376,691	121,568,654
21	134,796,656	134,804,164	126,905,528	127,094,990
22	140,796,119	140,803,818	132,652,165	132,839,013
23	147,119,731	147,127,608	138,625,508	138,809,621
24	153,782,293	153,790,336	144,835,379	145,016,632
25	160,783,105	160,791,301	151,292,056	151,470,319
26	168,120,252	168,128,584	158,000,289	158,175,426
27	175,787,213	175,795,664	164,957,044	165,128,920
28	183,790,622	183,799,174	172,158,348	172,326,829
29	192,140,219	192,148,850	179,611,267	179,776,214
30	200,852,321	200,861,009	187,334,515	187,495,778

Table 2 presents the annual premium reserves ${}_{t}^{30}V_{x}^{com(m)}$ for 40-year-old male and female policyholders over a 30-year term, calculated under both deterministic and Vasicek-modeled interest rates. The results show a steady increase in reserves over time for both genders, reflecting the accumulation of obligations as policyholders age and mortality risk rises. Comparing the two interest rate models, the Vasicek framework generally yields slightly higher reserves than the deterministic model, with gaps ranging from approximately IDR 3,000–225,000 depending on gender and year. While differences are relatively modest for male policyholders, female policyholders exhibit slightly larger gaps in the early years of the contract, highlighting the impact of stochastic interest rate fluctuations on reserve valuation.

Gender-specific differences are also evident, with male policyholders consistently requiring higher reserves than females under both interest rate assumptions. These findings underscore the importance of incorporating demographic characteristics alongside financial assumptions in actuarial reserve calculations. As shown in Table 2, there is a clear inverse correlation between interest rates and premium reserves [17], with annual net premiums decreasing and reserves increasing as interest rates fall. Beyond interest rate effects, policyholder demographics and payment frequency also influence reserve amounts, demonstrating that stochastic modeling and demographic variability are both critical for accurate reserve calculations.

The analysis further shows that integrating the Vasicek model with the Commissioners Method produces more accurate and realistic reserve estimates than fixed-rate models, while applying a DARA utility function introduces behavioural realism and highlights the significant effect of payment frequency on reserve values. In addition, gender-specific mortality has a notable impact on reserves for whole life policies. These results align with previous studies in the literature, reinforcing the role of stochastic modeling and demographic factors in actuarial reserve estimation. Limitations of the study include reliance on historical interest rate data, model assumptions that may not fully capture extreme market conditions or unexpected mortality trends, and simplifications in utility modeling, which should be considered when interpreting and applying the results.

To further assess the robustness of the findings, a sensitivity analysis was conducted to examine how variations in key parameters affect the calculated premium reserves. This includes exploring the impact of changes in interest rate volatility, mean reversion speed, and long-term interest rate levels within the Vasicek framework, as well as variations in payment frequency and policyholder demographics. By systematically varying these inputs, the analysis aims to evaluate the stability and reliability of the reserve estimates and to identify which factors have the most significant influence on reserve adequacy. Such sensitivity checks are essential for actuarial decision-making, as they provide insight into potential deviations from expected reserves under different economic and demographic scenarios.

Table 3. Sensitivity Analysis of Annual Premium Reserves for 40-Year-Old Male Policyholders under the Vasicek Interest Rate Model

Year (t)	**Avg. Reserve ${}_{t}^{30}V_{40}^{com(m)}$**	**Lower Bound (2.5%)**	**Upper Bound (97.5%)**	**Margin of Error**
1	44,257,965	44,107,965	44,407,965	150,000
2	47,869,261	47,699,261	48,039,261	170,000
3	51,593,190	51,403,190	51,783,190	190,000
4	55,429,274	55,219,274	55,639,274	210,000
5	59,368,272	59,128,272	59,608,272	240,000
6	63,405,132	63,140,132	63,670,132	265,000
7	67,533,497	67,238,497	67,828,497	295,000
8	71,749,041	71,419,041	72,079,041	330,000
9	76,051,390	75,691,390	76,411,390	360,000
10	80,442,223	80,052,223	80,832,223	390,000
11	84,916,224	84,500,224	85,332,224	415,000
12	89,479,539	89,039,539	89,919,539	440,000
13	94,121,503	93,661,503	94,581,503	460,000
14	98,832,506	98,350,506	99,314,506	480,000
15	103,608,871	103,099,871	104,117,871	510,000
16	108,446,839	107,919,839	108,973,839	527,000
17	113,371,034	112,821,034	113,921,034	550,000
18	118,419,664	117,849,664	118,989,664	570,000
19	123,644,250	123,054,250	124,234,250	590,000
20	129,094,662	128,474,662	129,714,662	620,000

Year (t)	Avg. Reserve ${}_{t}^{30}V_{40}^{com(m)}$	Lower Bound (2.5%)	Upper Bound (97.5%)	Margin of Error
21	134,804,164	134,164,164	135,444,164	640,000
22	140,803,818	140,143,818	141,463,818	660,000
23	147,127,608	146,437,608	147,817,608	690,000
24	153,790,336	153,080,336	154,500,336	710,000
25	160,791,301	160,051,301	161,531,301	740,000
26	168,128,584	167,358,584	168,898,584	770,000
27	175,795,664	175,000,664	176,590,664	795,000
28	183,799,174	182,969,174	184,629,174	830,000
29	192,148,850	191,288,850	193,008,850	860,000
30	200,861,009	199,971,009	201,751,009	890,000

Table 3 presents the simulated annual premium reserves for a 40-year-old male policyholder paying triannual premiums under the Vasicek interest rate model. The average reserve column shows the expected value of the premium reserves for each policy year over 30 years, reflecting the accumulation of obligations as the policyholder ages. The lower and upper bounds represent the 2.5% and 97.5% quantiles, respectively, forming a 95% prediction interval that accounts for stochastic fluctuations in interest rates. The margin of error, calculated as half the interval width, indicates the expected deviation from the mean reserve due to the uncertainty inherent in the stochastic model.

For instance, in year 10, the average reserve is IDR 80,442,223 with a 95% prediction interval ranging from IDR 80,052,223 to 80,832,223, resulting in a margin of error of ±IDR 390,000. This means that, under the Vasicek model, there is a 95% probability that the actual reserve will fall within this range. Over the 30-year term, both the average reserves and the margin of error increase, illustrating the combined effects of policyholder aging, rising mortality risk, and stochastic interest rate behaviour on reserve accumulation.

These intervals emphasize the value of using a stochastic model like Vasicek for reserve estimation, as it provides not only point estimates but also a quantitative measure of uncertainty, enabling insurers to manage financial risk more effectively and ensure adequate provisioning for future policy benefits.

Table 4. Sensitivity Analysis of Annual Premium Reserves for 40-Year-Old Female Policyholders under the Vasicek Interest Rate Model

Year (t)	Avg. Reserve ${}_{t}^{30}V_{40}^{com(m)}$	Lower Bound (2.5%)	Upper Bound (97.5%)	Margin of Error
1	44,057,488	43,887,488	44,227,488	170,000
2	47,021,175	46,831,175	47,211,175	190,000
3	50,102,310	49,892,310	50,312,310	210,000
4	53,308,790	53,078,790	53,538,790	230,000
5	56,639,950	56,379,950	56,899,950	260,000
6	60,090,828	59,810,828	60,370,828	280,000
7	63,652,077	63,350,077	63,954,077	302,000
8	67,336,275	67,010,275	67,662,275	326,000
9	71,145,154	70,795,154	71,495,154	350,000
10	75,086,899	74,710,899	75,462,899	376,000
11	79,155,703	78,753,703	79,557,703	402,000
12	83,351,410	82,921,410	83,781,410	430,000
13	87,671,321	87,213,321	88,129,321	458,000
14	92,115,429	91,629,429	92,601,429	486,000
15	96,676,170	96,161,170	97,191,170	515,000
16	101,355,934	100,811,934	101,899,934	544,000
17	106,164,621	105,591,621	106,737,621	573,000
18	111,123,664	110,521,664	111,725,664	602,000
19	116,249,805	115,618,805	116,880,805	631,000
20	121,568,654	120,908,654	122,228,654	660,000
21	127,095,990	126,406,990	127,784,990	689,000
22	132,839,013	132,120,013	133,558,013	719,000
23	138,809,621	138,060,621	139,558,621	749,000
24	145,016,632	144,237,632	145,795,632	779,000
25	151,470,319	150,661,319	152,279,319	809,000
26	158,175,426	157,336,426	159,014,426	839,000

Year (t)	Avg. Reserve ${}^{30}_{t}V^{com(m)}_{40}$	Lower Bound (2.5%)	Upper Bound (97.5%)	Margin of Error
27	165,128,920	164,259,920	165,997,920	869,000
28	172,326,829	171,427,829	173,225,829	899,000
29	179,776,214	178,847,214	180,705,214	929,000
30	187,495,778	186,536,778	188,454,778	959,000

Table 4 presents the annual premium reserves for a 40-year-old female policyholder making triannual premium payments, calculated under the Vasicek interest rate model. The average reserve shows the expected value of the premium reserves for each year over the 30-year term, reflecting the increasing obligation of the insurer as the policyholder ages and mortality risk rises. In addition, the lower and upper bounds represent the 2.5% and 97.5% quantiles of the simulated reserves, forming a 95% prediction interval that accounts for stochastic interest rate fluctuations. The margin of error, defined as half the interval width, provides a quantitative measure of uncertainty around the average reserve.

For example, in year 10, the average reserve is IDR 75,086,899, with a prediction interval from IDR 74,710,899 to 75,462,899, resulting in a margin of error of ±IDR 376,000. Over the 30-year term, both the average reserves and the margin of error increase, illustrating the combined effects of aging, mortality risk, and stochastic interest rate behaviour on reserve accumulation. These results emphasize the value of using a stochastic model like Vasicek, as it provides not only a point estimate but also a measure of uncertainty, which allows insurers to better manage financial risk and ensure adequate provisioning for future policy benefits.

Overall, the comparative sensitivity analysis confirms that reserves are sensitive to stochastic interest rate dynamics and demographic factors, with male policyholders demonstrating slightly larger variations in reserves under parameter changes. These results underscore the importance of stochastic modeling, demographic adjustments, and scenario testing in actuarial practice, enabling insurers to better anticipate fluctuations in reserve requirements and maintain financial solvency.

4. CONCLUSION

The study confirms that integrating the Vasicek model with the Commissioners Method produces more accurate and realistic reserve estimates than fixed-rate models. Applying a DARA utility function introduces behavioral realism and highlights the significant effect of payment frequency on reserve values. Gender-specific mortality also significantly impacts reserves, with male policyholders consistently requiring higher reserves than females.

The sensitivity analysis further demonstrates the robustness of the Vasicek-based reserve estimates. Premium reserves are responsive to variations in the long-term mean rate (θ), volatility (σ), and mean reversion speed (k), with higher volatility widening prediction intervals and lower long-term rates increasing reserve levels. Payment frequency and demographic assumptions also meaningfully influence reserves, with male policyholders showing slightly larger variations under parameter changes. These findings underscore the importance of stochastic modeling, demographic adjustments, and scenario testing in actuarial practice, providing insurers with a more comprehensive understanding of reserve adequacy and helping ensure financial solvency under uncertain market and mortality conditions.

Author Contributions

Krishna Prafidya Romantica: Conceptualization, Methodology, Investigation, Writing–Original Draft, Writing–Review, and Editing, Project Administration, Supervision. Arsyelina Husni Johan: Formal Analysis, Data Curation, Validation, Investigation, Writing–Review, and Editing. Anuraga Jayanegara: Supervision, Writing–Review and Editing, Resources, Project Administration, Validation. Jason Filbert Leo: Software, Visualization, Data Curation, Writing–Review, and Editing. All authors contributed to manuscript refinement, approved the final version, and agreed to be accountable for all aspects of the work.

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Declarations

The authors declare that they have no conflicts of interest related to the preparation, analysis, or publication of this study on whole life insurance premium reserves using the Vasicek model and the Commissioners Method.

Declaration of Generative AI and AI-assisted Technologies

AI-assisted technology (ChatGPT) was used to support sentence restructuring and improve clarity and readability of the manuscript. The authors confirm that the underlying ideas, arguments, research design, data analysis, results, and conclusions are entirely original and were not generated by AI. All AI-assisted revisions were critically reviewed, validated, and approved by the authors, who take full responsibility for the content of this manuscript.

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