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GENERALIZED NESTED COPULA REGRESSION TO UNVEIL THE IMPACT OF EXCHANGE RATES AND NIKKEI 225 ON BANK MANDIRI STOCK PRICE

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ABSTRACT

Fluctuations in exchange rates and foreign stock indices strongly influence domestic stock performance, particularly in the banking sector, which is highly sensitive to global economic dynamics. Traditional financial models often fail to capture the complex, non-linear dependencies between these variables, underscoring the need for more advanced approaches. This study examines the effectiveness of copula-based regression models in predicting Bank Mandiri's (BMRI) stock price using exchange rates and the Nikkei 225 Index as predictors. Conventional regression methods, such as Linear Regression, cannot adequately capture nonlinear relationships and tail dependencies in financial time series. To address this, we compare Elliptical Copula, Symmetric Archimedean Copula, Asymmetric Archimedean Copula, and Generalized Nested Copula models. Results show that the Generalized Nested Copula Regression achieves the lowest Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), and Weighted MAPE (wMAPE), effectively modeling asymmetric and tail dependencies that are crucial in financial forecasting. While Elliptical Copula (t-Copula) also provides strong predictive accuracy, Archimedean copulas perform poorly, failing to improve upon linear regression. These findings highlight the importance of flexible statistical models in financial prediction, demonstrating that copula-based regression offers a superior alternative to traditional methods. Unlike prior research that often relied on simpler copula families or linear models, this study introduces a Generalized Nested Copula Regression in the context of the Indonesian banking sector, addressing a gap in emerging market literature. The study assumes correctly specified marginal distributions and a stable dependency structure, which may limit applicability under rapidly changing market conditions. Future work should consider dynamic copula structures and additional economic indicators to further enhance predictive accuracy.

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1. INTRODUCTION

Analyzing the influence of exchange rates and foreign stock indices on domestic stocks is essential, particularly in sectors such as banking that are highly sensitive to international economic fluctuations **[1]**. Traditional financial models have often struggled to capture the complex, non-linear dependencies between these variables, as financial markets are characterized by intricate interactions influenced by both regional and global dynamics [2]. To address these challenges, this study employs a nested copula regression approach, a sophisticated statistical technique designed to model multi-layered, dynamic relationships among variables [3].

The nested copula approach is especially relevant for financial analysis because it allows for the investigation of non-linear dependencies that might otherwise be missed by traditional linear models [4], [5]. In this study, the approach is used to explore how the movements of exchange rates and the Nikkei 225 index influence Bank Mandiri's stock price. By capturing the complex interdependence between these variables, nested copula regression offers a more comprehensive framework for understanding the underlying forces affecting Bank Mandiri’s stock performance.

The concept of copula modeling itself has undergone significant development, beginning with its origins in finance and insurance risk management, where it was used to study dependencies between two variables [6]-[8]. As copula methods have advanced, their applications have extended to fields beyond finance, including environmental science [9], [10] biostatistics [11], [12], engineering [4], [13], and social sciences [14]. This diversity of applications underscores copulas’ flexibility in handling complex data and dependence patterns. Early copula models were limited in scope, often focusing on straightforward, linear relationships and unable to address the complexity of financial markets as they exist today. However, as financial systems grew more interconnected and interdependent, the limitations of simple copula models became apparent. This need for more robust analytical methods led to the development of more advanced copula models capable of handling high-dimensional data and capturing complex dependency structures.

Nested copulas, a more advanced evolution of the copula framework, were introduced to address precisely these challenges [15]-[17]. They allow for the modeling of hierarchical dependencies, enabling researchers to capture multi-dimensional relationships across different economic indicators, such as currency exchange rates and foreign stock indices. By applying a nested copula model, this study delves into the specific interdependencies that affect Bank Mandiri’s stock, offering a more nuanced perspective on how investors might anticipate the impacts of global economic factors on their portfolios in the Indonesian market. This deeper understanding is essential for creating strategies to navigate the volatility and risk associated with international market influences.

Although previous studies have successfully applied copula models to capture dependencies in various financial [18], environmental [19], and engineering [20] contexts, most have been limited to simple copula families or pairwise constructions that fail to represent complex multi-dimensional structures. These approaches often overlook asymmetric and tail dependencies that are crucial in financial markets, particularly for highly volatile assets such as banking stocks. Moreover, existing research on the Indonesian financial sector rarely employs advanced copula frameworks, leaving a gap in understanding how global factors, such as exchange rates and foreign stock indices, jointly influence domestic stock prices. This study addresses this gap by introducing a Generalized Nested Copula Regression model, which provides greater flexibility in modeling hierarchical and asymmetric dependencies, thereby offering a novel contribution to the literature on financial forecasting in emerging markets.

This research also holds significance for regulators and policymakers in the financial sector. Understanding the patterns of volatility and risk that stem from the interconnections between global and domestic markets can inform better decision-making and policy formulation, helping to stabilize markets in times of economic turbulence. Insights derived from this nested copula analysis can be particularly valuable for crafting regulations that mitigate systemic risks, thereby enhancing the resilience of the Indonesian banking sector against global shocks. For academics, this study contributes to the expanding literature on global economic factors influencing Indonesia’s financial sector. It also provides a flexible analytical tool that can be adapted to similar research, potentially inspiring new approaches to understanding global and domestic market interactions. Ultimately, the results of this research aim to guide more strategic investment decisions in Indonesia’s stock market, offering practical implications for investors in sectors with substantial global linkages, such as banking. By applying the nested copula model, this study highlights the importance of sophisticated statistical approaches in navigating today’s interconnected financial environment.

2. RESEARCH METHODS

This section outlines the research methods used to analyze the relationship between exchange rates, the Nikkei 225 index, and the stock price of Bank Mandiri, employing a nested copula regression approach to capture the complex, non-linear dependencies among the variables. However, before delving into the methodology, the basic concept of copula will first be discussed.

2.1. Copula

Definition 1 (Copula Function)**.** A two-dimensional copula is a function C that maps I^2 to I, where $I \in [0,1]$, and satisfies the following properties [21]:

1. C is grounded: $C(u,0) = C(0,v) = 0$,
2. $C(1,v) = u_j$; $\forall u_j \in [0,1]$, and
3. C is n-increasing.

Theorem 1 (Sklar). *Let H be a two-dimensional distribution function with marginal distribution functions F_1 and F_2. Then, there exists a two-dimensional copula C such that for every $x \in R^2$, the following holds:*

$$H(x_1, x_2) = C\big(F_1(x_1), F_2(x_2)\big). \tag{1}$$

If the marginal functions F_1 and F_2 are continuous, then the copula C is unique [21]. Theorem 1 can be extended for n-dimensional case as follows.

Theorem 2. *For n-dimensional cases, let G be an n-dimensional distribution function with marginal distribution functions $F_1, F_2, \dots F_n$. Then, there exists an n-dimensional copula C_n such that for every $x \in R^n$, the following holds:*

$$G(x_1, x_2, \dots, x_n) = C_n\big(F_1(x_1), F_2(x_2), \dots, F_n(x_n)\big). \tag{2}$$

If all the marginal functions are continuous, then the copula C_n is unique [21].

2.2. Consequence of Sklar's Theorem

Let C is a unique copula, then C can be expressed as:

$$C(u, v) = \int_0^v \int_0^u c(s,t)\, ds\, dt, \tag{3}$$

where $u = F_X(x)$ and $v = F_Y(y)$ and c is the corresponding copula density function. The important consequence of Sklar's theorem [22] (Theorem 1) then stated that every joint probability density h is also writable by the product of its marginal probability densities f_X and f_Y and the copula density c.

Theorem 3. *Let h is a joint density with marginal densities f_X and f_Y, then there exists a copula density c such that*

$$h(x, y) = c\big(F_X(x), F_Y(y)\big) \cdot f_X(x) \cdot f_Y(y). \tag{4}$$

Proof. By deriving the right and left sides of Sklar's Theorem (Theorem 1) with respect to x and y,

$$\frac{\partial}{\partial x \partial y} H(x_1, x_2) = \frac{\partial}{\partial x \partial y} C\big(F_1(x_1), F_2(x_2)\big),$$
$$h(x, y) = c\big(F_X(x), F_Y(y)\big) \cdot f_X(x) \cdot f_Y(y).$$

then Eq. (4) is proven. ■

2.3. Nested Copula

In the case of a 3-dimensional copula, a nested copula structure allows modeling the dependencies among three variables by nesting two-dimensional copulas within one another. The general form of a three-dimensional nested copula can be expressed as follows:

Definition 2. Let C_1 and C_2 be bivariate copulas. A three-dimensional nested copula C can be constructed as:

$$C\big(F_1(x_1), F_2(x_2), F_3(x_3)\big) = C_2\big(C_1[F_1(x_1), F_2(x_2)], F_3(x_3)\big), \tag{5}$$

where F_1, F_2, F_3 are the marginal distribution functions of the three variables. Here, the copula C_1 models the dependency between x_1 and x_2, and C_2 models the dependency between the result of C_1 and x_3. In this case, the nested structure allows the modeling of a hierarchical dependence structure, where the relationship between x_1 and x_2 is first captured, and then the dependence between this pair and the third variable x_3 is modeled. Suppose that $u_i = F(x_i)$ and $U_i = F(X_i)$ for $i = 1,2,3$, then Fig. 1 provides the structural construction of nested copulas in the tri-variate cases.

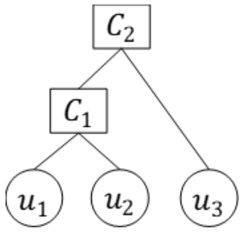

Figure 1. Nested Structure of 3-Dimensional Copulas

Intuitively, the nested copula structure in Fig. 1 can be understood as a two-step process of building dependencies. First, C_1 links two variables, u_1 and u_2, capturing how they move together. Then, C_2 takes the combined result of C_1 and connects it with the third variable, u_3. This hierarchical construction allows us to model complex relationships step by step: we first describe the dependence between two variables and then extend it to include the third. Such a nested approach provides greater flexibility than a single copula, especially when the strength or type of dependence differs across subsets of variables.

The joint probability density function derived from the nested 3-copula follows directly from Sklar's theorem and the theorem of the nested 3-copula. Sklar's theorem ensures that any multivariate distribution can be represented using its marginal distributions and a copula function that captures dependence. Consequently, the joint density function of the nested 3-copula is obtained by differentiating its copula function, providing a precise representation of the dependence structure among the variables.

Theorem 4. *Let X_1, X_2, X_3 be random variables with marginal cumulative distribution functions (CDFs) F_1, F_2, F_3, respectively. Then, the joint distribution $F_{1,2,3}$ can be constructed from a nested structure of two bivariate copula (2-copulas) as follows:*

$$F_{1,2,3}(x_1, x_2, x_3) = C_2\left(C_1\big(F_1(x_1), F_2(x_2)\big), F_3(x_3)\right). \tag{6}$$

As a result, the joint probability density function is written by:

$$f_{1,2,3}(x_1, x_2, x_3) = c_2\left(C_1\big(F_1(x_1), F_2(x_2)\big), F_3(x_3)\right) \cdot c_1\big(F_1(x_1), F_2(x_2)\big) \cdot f_1(x_1) \cdot f_2(x_2) \cdot f_3(x_3). \tag{7}$$

Proof. Using Theorem 2 and Definition 2, we get:

$$F_{1,2,3}(x_1, x_2, x_3) = C_n\big(F_1(x_1), F_2(x_2), \dots, F_n(x_n)\big) = C_2\left(C_1\big(F_1(x_1), F_2(x_2)\big), F_3(x_3)\right),$$

then Eq. (6) proven. By deriving the right and left sides of Eq. (6) with respect to x_1, x_2 and x_3,

$$\frac{\partial}{\partial x_1 \partial x_2 \partial x_3} F_{1,2,3}(x_1, x_2, x_3) = \frac{\partial}{\partial x_1 \partial x_2 \partial x_3} C_2\left(C_1\big(F_1(x_1), F_2(x_2)\big), F_3(x_3)\right),$$

$$f_{1,2,3}(x_1, x_2, x_3) = c_2\left(C_1\big(F_1(x_1), F_2(x_2)\big), F_3(x_3)\right) \cdot c_1\big(F_1(x_1), F_2(x_2)\big) \cdot f_1(x_1) \cdot f_2(x_2) \cdot f_3(x_3),$$

then the joint probability density function in Eq. (7) is proven. ■

Generally, the construction steps for the nested 3-copula are as follows [23].

1. Estimate F_1, F_2, and F_3 (marginal distribution).
2. Select the two variables with the highest degree of dependence, for example X_1 and X_2, then transform the two variables using their respective marginal distributions (notate the results as U_1 and U_2).
3. Estimate C_1 using U_1 and U_2 (first bivariate copula).
4. Transform the remaining variable X_3 using its marginal distribution (notate the result as U_3).
5. Estimate C_2 using U_3 and $C_2(U_1, U_2)$ (second bivariate copula).

However, since the goal of this research is to predict BMRI, the calculation process is simplified by selecting x_1 and x_2 as predictor variables, while x_3 serves as the response variable (BMRI), which will be denoted as y.

2.4. Conditional probability density function and copula regression

Let X_3 be the response variable while X_1 and X_2 are the explanatory variables, then the conditional probability density function of x_3 given x_1 and x_2 is defined by

$$f(x_3|x_1,x_2) = \frac{f(x_1,x_2,x_3)}{f(x_1,x_2)} = c_2(C_1(u_1,u_2),u_3) \cdot f_3(x_3), \tag{8}$$

due to $f(x_1,x_2) = c_1(u_1,u_2) \cdot f_1(x_1) \cdot f_2(x_1)$ [9].

If we wish to predict the value of x_3, then we might take the expected value of the conditional density, which is so-called conditional expectation. The conditional expectation provides the prediction of x_3 with the smallest possible mean square error, which is why it is often referred to as the minimum-mean-square-error predictor. Formally, the conditional expectation of x_3 given x_1 and x_2 is defined as

$$E(x_3|x_1,x_2) = \int_{-\infty}^{\infty} x_3 \cdot f(x_3|x_1,x_2)\, dx_3 = \int_{-\infty}^{\infty} c_2(C_1(u_1,u_2),u_3) \cdot f_3(x_3) \cdot x_3\, dx_3. \tag{9}$$

Since nested copulas are used to construct the conditional density, we call this formula a nested copula regression. Copula regression is often more robust to outliers and non-normality in the data compared to traditional regression techniques. It can handle data with heavy tails and non-standard distributions more effectively.

For computational convenience, we use the Riemann sum approach to estimate the value of the integral:

$$E(x_3|x_1,x_2) \approx \sum_{i=1}^{p} c_2(C_1(u_1,u_2),u_3^{(i)} \cdot f_3\left(x_3^{(i)}\right) \cdot x_3^{(i)} \cdot \Delta x_3^{(i)}\,. \tag{10}$$

where p represents the number of partitions used [24].

2.5. Types of Copula Family

In this study, we explore several families of copulas to model the dependence structure between variables. Copulas offer a flexible framework for separating marginal distributions from their dependence structure, allowing the joint behavior of variables to be modeled without assuming identical distributional forms. This property makes copulas particularly useful for financial and economic data, where relationships are often nonlinear and characterized by asymmetric tail behavior. The families of copulas considered can be broadly classified as follows:

2.5.1. Elliptical Copulas (Gaussian and Student-t)

Elliptical copulas are derived from multivariate elliptical distributions. The Gaussian copula captures symmetric dependence but lacks tail dependence, making it less suitable for modeling extreme events. In contrast, the Student-t copula accommodates tail dependence, thereby providing a better fit for financial data that often exhibits co-movement during extreme market conditions. For further details on the theoretical foundations and applications of elliptical copulas, readers may refer to recent works such as [25], [26], [27], [28] and subsequent references.

2.5.2. Archimedean Copulas (Clayton, Gumbel, Frank, Joe)

Archimedean copulas are widely used due to their simple closed-form expressions and ability to capture asymmetric dependence. For example, the Clayton copula emphasizes lower-tail dependence, making it suitable when joint extreme losses are of interest. The Gumbel copula, on the other hand, captures upper-tail dependence, reflecting simultaneous extreme gains. The Frank copula provides symmetric dependence without tail emphasis, while the Joe copula focuses on strong upper-tail associations. For a more comprehensive discussion of Archimedean copulas and their extensions, readers may consult [29], [30], [31], [32], [33] and subsequent references.

2.5.3. Extreme Value Copulas

Extreme value copulas are designed to capture dependence structures in the tails of distributions, which are critical in risk management and financial forecasting. They provide theoretical consistency with extreme value theory, ensuring that joint tail behavior is properly represented. Further insights into the construction and applications of extreme value copulas can be found in [34], [35], [36], [37] and related references.

2.5.4. Two-Parameter Copulas (BB1, BB6, BB7, BB8)

These copulas extend the flexibility of the Archimedean family by combining features of two different copulas. For instance, BB1 merges Clayton and Gumbel properties, allowing simultaneous modeling of both lower- and upper-tail dependence. Such flexibility is valuable when empirical data shows asymmetric dependence patterns that cannot be captured by single-parameter copulas. Readers interested in detailed theoretical formulations and broader applications of two-parameter copulas are referred to [38], [39], [40], [41], [42] and related works.

To ensure comprehensive coverage of possible dependence structures, we employed eleven copula functions across these families. This diversity allows the analysis to account for a wide range of dependence behaviors—symmetric versus asymmetric, weak versus strong, and central versus tail dependence. Table 1 presents the formulas and parameter domains of the selected bivariate copula functions, which serve as the foundation for constructing the multivariate nested copula models used in this study.

Table 1. Formulas and Parameter Domains of Some Bivariate Copula Functions

No	Copula	$C_X(u_1,u_2)$	Parameter
1	Normal	$F_{N(0,\Sigma)}\left(F^{-1}_{N(0,1)}(u_1), F^{-1}_{N(0,1)}(u_2)\right)$	
2	Student-t	$F_{t(v,\Sigma)}\left(F^{-1}_{t(v)}(u_1), F^{-1}_{t(v)}(u_2)\right)$	
3	Clayton	$\left(u_1^{-\theta}+u_2^{-\theta}-1\right)^{-1/\theta}$	$\theta>0$
4	Gumbel	$\exp\left[-\left(w_1^{\theta}+w_2^{\theta}\right)^{1/\theta}\right]$, where $w_i=-\ln u_i$	$\theta\geq 1$
5	Frank	$-\frac{1}{\theta}\ln\left[1+\frac{w_1w_2}{e^{-1}-1}\right]$, where $w_i=e^{-\theta u_i}-1$	$\theta\neq 0$
6	FGM	$u_1u_2+\theta u_1u_2(1-u_1)(1-u_2)$	$-1\leq\theta\leq 1$
7	Galambos	$u_1u_2\exp\left[\left(w_1^{-\theta}+w_2^{-\theta}\right)^{-1/\theta}\right]$, where $w_i=-\ln u_i$	$\theta\geq 0$
8	BB1	$\left[1+\left(w_1^{\delta}+w_2^{\delta}\right)^{1/\delta}\right]^{-1/\theta}$, where $w_i=u_i^{-\theta}-1$	$\theta>0,\delta\geq 1$
9	BB6	$1-\left\{1-\exp\left[-\left(w_1^{\delta}+w_2^{\delta}\right)^{1/\delta}\right]\right\}^{1/\theta}$, where $w_i=-\ln[1-(1-u_i)^{\theta}]$	$\theta\geq 1,\delta\geq 1$
10	BB7	$1-\left[1-\left(w_1^{-\delta}+w_2^{-\delta}-1\right)^{-1/\delta}\right]^{1/\theta}$, where $w_i=1-(1-u_i)^{\theta}$	$\theta\geq 1,\delta>0$
11	BB8	$\frac{1}{\theta}\left\{1-\left[1-\frac{w_1w_2}{1-(1-\delta)^{\theta}}\right]^{1/\delta}\right\}$, where $w_i=1-(1-\delta u_i)^{\theta}$	$\theta\geq 1$, $0<\delta\leq 1$

2.6. Parameter Estimation using the Inference of Function for Margin

The Inference of Function for Margin (IFM) method is a parametric method consisting of two steps, with the basis of each step containing the log likelihood approach. This method is usually used to estimate the parameters of a multidimensional copula. The first step in this method is to construct a log likelihood function to estimate the marginal parameter vector $\hat{\alpha}_i$, i.e.

$$\hat{\alpha}_i=\arg\max\ln L_i=\arg\max\ln\prod_{t=1}^{N} f_i\left(x_i^t;\alpha_i\right), \tag{11}$$

where f_i is the probability density function of the random variable X_i. The second step of the IFM method is to estimate the copula parameters by maximizing the log value of the copula likelihood function L. For bivariate cases, it is written as follows.

$$\hat{\theta}=\arg\max\ln L=\arg\max\ln\prod_{t=1}^{N} c(F_1(x_1^t;\hat{\alpha}_1),F_2(x_2^t;\hat{\alpha}_2);\theta), \tag{12}$$

where $\hat{\theta}$ is the estimate of the copula parameter θ and c is the copula probability density function [43], [44]. For the trivariate case, copulas are formed through a nested structure of bivariate copulas as in Algorithm 1.

2.7. Goodness-of-fits

In this study, we use the Kolmogorov–Smirnov (K–S) test [45], Root Mean Square Error (RMSE), and Akaike's information criterion (AIC) [46] to measure the goodness of fit of the joint distributions as follows:

$$KSE = \max_{i=1,2,..,n} \left|P_{E_i} - P_{T_i}\right|, \tag{13}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(P_{E_i} - P_{T_i}\right)^2}, \tag{14}$$

$$AIC = 2k - 2\,ln\,L, \tag{15}$$

where n is the sample size, k is the number of parameters of different distributions, L is the maximum likelihood function value of distributions, P_{Ei} and P_{Ti} are the empirical and theoretical frequency, respectively. We use Gringorten's formula [47] to estimate the empirical frequency of X and Y as follow

$$P_{E_i} = P(X \le x_i, Y \le y_i) = \frac{\#(X \le x_i, Y \le y_i) - 0.44}{n + 0.12}, \tag{16}$$

where $\#(X, Y)$ is the combination of the i-th values of the increased order in the X and Y series. Meanwhile, theoretical frequency is the models from marginal and copula distributions. In this study, the Kolmogorov–Smirnov (KS) test, Anderson–Darling (AD) test, RMSE, and AIC were selected as the primary model comparison tools because they provide complementary insights into both distributional fit and predictive performance. Other criteria, such as the Bayesian Information Criterion (BIC) or likelihood ratio tests, were not considered for two reasons. First, BIC tends to penalize model complexity more strongly than AIC, which may be less suitable in the context of copula models where flexibility is required to capture complex dependencies. Second, likelihood ratio tests are not always straightforward to apply in copula-based frameworks, particularly when comparing non-nested models, making them less practical for the objectives of this study. Thus, the chosen criteria strike a balance between statistical rigor and applicability to the copula modeling framework.

2.8 Datasets

In this study, three datasets are utilized to develop a predictive model for BMRI. The response variable is BMRI, while the predictor variables include Exchange Rates and the Nikkei 225 index. The datasets consist of monthly observations of BMRI stock prices, exchange rates (USD/IDR), and the Nikkei 225 index over the study period. The details of each dataset are described as follows.

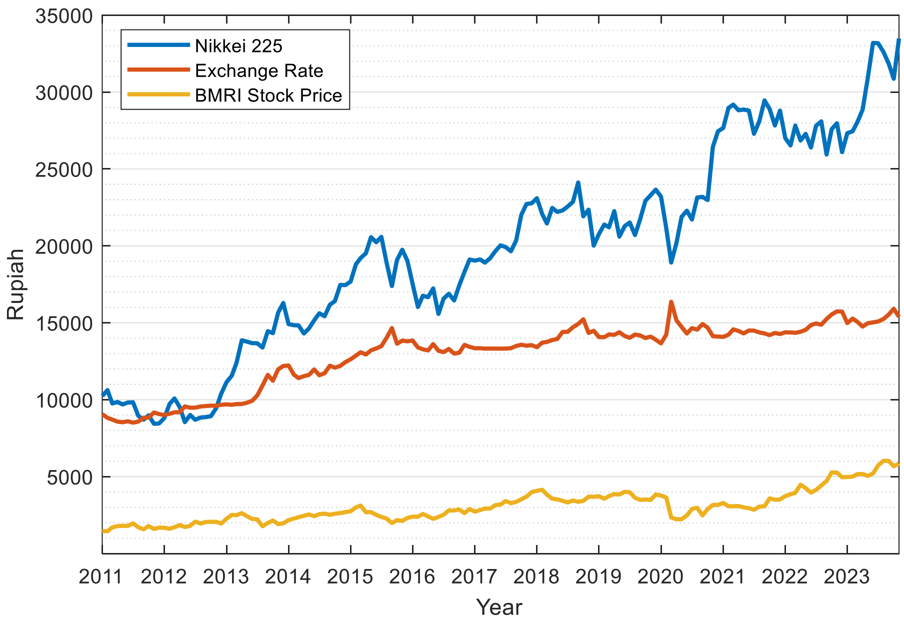

Figure 2. Time Series of the BMRI Stock Price, USD to IDR Exchange Rate and Nikkei 225 Index.

The above Fig. 2 provides an overview of the historical trends and fluctuations in BMRI stock prices, exchange rates, and the Nikkei 225 index over the observed period. By analyzing these time series, patterns and potential correlations among the variables can be identified, which is essential for building an accurate predictive model. The integration of these three datasets enables a comprehensive analysis of the factors affecting BMRI stock prices. By leveraging historical data, the model aims to enhance prediction accuracy and support data-driven decision-making in financial markets.

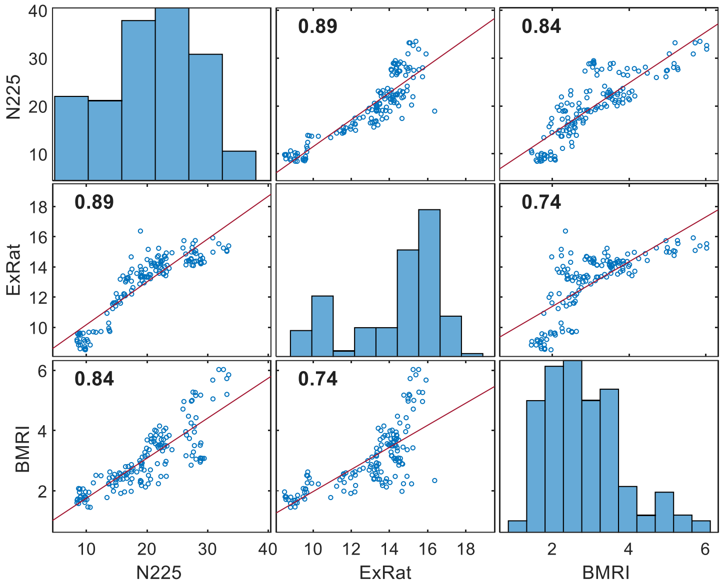

Figure 3. Correlation Matrix among the Three Variables: Bank Mandiri Stock Price (BMRI), Exchange Rates (Exrat), and the Nikkei 225 Index (N225).

The above Fig. 3 presents a correlation matrix that reveals key relationships between Bank Mandiri Stock Price (BMRI), exchange rates (ExRat), and the Nikkei 225 index (N225). The upper triangle shows Pearson correlation coefficients, indicating strong positive correlations: 0.89 between N225 and ExRat, 0.84 between N225 and BMRI, and 0.74 between ExRat and BMRI. These values suggest that movements in the Nikkei 225 and exchange rates significantly affect BMRI stock prices, with N225 exerting a slightly stronger influence. The lower triangle features scatter plots with upward-sloping red regression lines, visually confirming the positive associations among the variables. Diagonal elements contain histograms showing right-skewed distributions for BMRI and ExRat, while N225 appears more symmetric. Collectively, the matrix highlights that BMRI is influenced by both exchange rates and the Japanese stock market, with implications for predictive modeling that underscore the need to consider both variables when forecasting BMRI performance.

3. RESULTS AND DISCUSSION

This study aimed to predict the stock price of Bank Mandiri (BMRI) using two predictor variables: the Nikkei 225 Index (N225) and Exchange Rates (ExRat). Traditional linear regression and several copula-based models were applied to assess the predictive performance of different approaches. The models tested include Linear Regression, Elliptical Copula, Symmetric Archimedean Copula, Asymmetric Archimedean Copula, and Generalized Nested Copula. Each model was evaluated based on metrics such as Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), and weighted MAPE (wMAPE).

3.1. Performance of Linear Regression Model

The parameter estimation process for the linear regression model was conducted using the Ordinary Least Squares (OLS) method. OLS aims to minimize the sum of squared residuals, ensuring that the estimated regression line best fits the given data. In this study, the dependent variable (BMRI) was modeled as a function of two explanatory variables: Nikkei 225 (N225) and Exchange Rates (ExRat). The general form of the linear regression model is expressed as:

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon,$$

where $\hat{y}$ represents the predicted value of BMRI, β_0 is the intercept, β_1 and β_2 are the estimated regression coefficients for N225 (x_1) and Exchange Rates (x_2), respectively, and ϵ denotes the error term. After performing OLS estimation, resulting in:

$$\hat{y} = 0.1419x_1 - 0.0328x_2 + \epsilon.$$

This indicates that BMRI has a positive relationship with N225, as evidenced by the coefficient 0.1419. This suggests that an increase in the Nikkei 225 index is associated with an increase in BMRI, potentially reflecting the influence of global stock market trends on BMRI stock performance. Conversely, the negative coefficient of -0.0328 for Exchange Rates suggests that a rise in exchange rates tends to decrease BMRI, which may be linked to investor concerns over currency depreciation and its effects on financial markets.

To evaluate the model's predictive performance, three error metrics were computed. The Root Mean Squared Error (RMSE) of 0.56461 quantifies the average magnitude of prediction errors, providing a measure of how well the model fits the data. Additionally, the Mean Absolute Percentage Error (MAPE) of 0.13808 indicates that the model's predictions deviate by approximately 13.81% from actual values on average. The Weighted Mean Absolute Percentage Error (wMAPE) of 0.13894 further confirms the model's consistency across different observations.

While linear regression serves as a useful baseline model due to its simplicity and interpretability, it assumes a linear relationship between variables. However, financial data often exhibits nonlinear dependencies, tail risks, and complex interactions that cannot be effectively captured by traditional regression techniques. As a result, more advanced approaches, such as copula-based regression, may provide better predictive accuracy by accounting for the intricate dependencies and extreme fluctuations observed in financial markets.

3.2. Performance of Copula Regression Model

In this section, copula regression will be performed following the procedures outlined in the methodology. The process begins with estimating the marginal distributions of all variables to ensure that each variable's individual characteristics are well captured. Next, a nested copula structure is constructed, allowing the model to capture the dependence between the predictor variables, Nikkei 225 and Exchange Rates, and the response variable, BMRI. Finally, the expected conditional probability density function (PDF) of BMRI is computed, given known values of the predictor variables. This step is essential in obtaining accurate predictions while incorporating the complex dependency structure among the financial variables.

To evaluate the effectiveness of different copula-based regression models, four 3-nested copula approaches are considered: Elliptical Copula, Symmetric Archimedean Copula, Asymmetric Archimedean Copula, and Generalized Nested Copula. The Elliptical Copula, which includes the Gaussian and t-copulas, provides a flexible dependence structure suitable for capturing symmetric relationships. The Symmetric Archimedean Copula assumes a common dependence pattern among variables and is often used when dealing with exchangeable data structures. The Asymmetric Archimedean Copula allows for asymmetric tail dependencies, making it particularly useful for financial data where extreme movements tend to be more pronounced in one direction. Finally, the Generalized Nested Copula combines different copula families, offering a more advanced and adaptable framework for modeling complex dependencies.

By applying these four approaches, this study aims to determine the most effective nested copula structure for predicting BMRI stock prices. Given the nonlinear and heavy-tailed nature of financial data, incorporating copula-based regression models provides a more flexible alternative to traditional methods, ensuring that both linear and nonlinear dependencies between financial variables are properly accounted for.

3.2.1. Marginal Distributions Estimation

In copula-based modeling, an essential step before constructing the dependence structure is estimating the marginal distributions of each variable. Marginal distributions describe the individual behavior of each variable independently, allowing for the separation of dependency modeling from univariate characteristics. This step ensures that the selected copula function captures only the dependence structure and not the individual variability of the variables.

3.2.2. Selection of Marginal Distributions

To accurately model the behavior of the predictor variables (Nikkei 225 and Exchange Rates) and the response variable (BMRI), different probability distributions were tested. The best-fitting marginal distributions were selected based on statistical criteria such as the Kolmogorov-Smirnov Test (KS) and Anderson-Darling (AD) tests, and Akaike Information Criterion (AIC). The identified marginal distributions for each variable are:

1. Nikkei 225 (X1):
   Generalized Extreme Value (GEV) distribution with parameters $k = -0.33371, \sigma = 6.6589$, and $\mu = 17.5862$ was selected based on the Kolmogorov–Smirnov (KS) and Anderson–Darling (AD) statistics, despite not yielding the lowest Akaike Information Criterion (AIC). This choice was made because goodness-of-fit tests (KS and AD) provide a more direct measure of the model's ability to capture the empirical distribution, particularly in the tails, which is critical for extreme value analysis. In contrast, AIC emphasizes parsimony and overall likelihood, but it may not adequately reflect the fit in the distribution's extremes, which are the primary focus in extreme value modeling.

Table 2. Results of the Marginal Distribution Fitting for the Nikkei 225 Index.

Dist. Name	KS stat	KS p-val	AD stat	AD p-val	AIC
GEV	**0.068710**	**0.437650**	**1.111500**	**0.303620**	1026.0
NOR	0.071322	0.391240	1.179200	0.275520	1028.1
WB	0.073160	0.360470	1.182100	0.274360	**1023.7**
LOG	0.071058	0.395790	1.245400	0.250830	1037.4
EV	0.100270	0.082661	1.917900	0.102010	1042.1
GAM	0.098653	0.091434	2.310700	0.062447	1033.5
LL	0.094723	0.116100	2.755100	0.036571	1048.6
LN	0.121990	0.018129	3.416500	0.016948	1043.5
ING	0.129160	0.010290	3.707900	0.012162	1043.5
EXP	0.347580	0.000000	30.841000	0.000004	1236.8

2. Exchange Rates (X2):
   Extreme Value distribution with parameters $\mu = 13.8707$ and $\sigma = 1.5136$ was selected, as all goodness-of-fit statistics consistently indicated that this distribution provides the best fit to the observed data. This result suggests that the Extreme Value distribution is more capable of capturing the behavior of the empirical distribution, particularly in representing the central tendency and spread of the data, compared to other candidate distributions. The superiority across multiple statistical tests reinforces its robustness, implying that the Extreme Value distribution is the most reliable choice for further analysis and modeling in this study.

Table 3. Results of the Marginal Distribution Fitting for the Exchange Rates.

Dist. Name	KS stat	KS p-val	AD stat	AD p-val	AIC
EV	**0.122250**	**0.017766**	**3.697100**	**0.012312**	**634.66**
GEV	0.136950	0.005359	3.800900	0.010947	638.38
WB	0.147180	0.002147	5.699500	0.001343	645.47
LOG	0.137480	0.005119	6.778200	0.000425	675.84
NOR	0.192230	0.000017	7.931700	0.000124	671.94
LL	0.157530	0.000795	8.801500	0.000047	696.50
GAM	0.211920	0.000001	9.592900	0.000018	687.31
LN	0.220350	0.000000	10.383000	0.000007	696.33
ING	0.222600	0.000000	10.528000	0.000006	696.36
EXP	0.482050	0.000000	50.883000	0.000004	1105.50

3. BMRI (Y):
   Inverse Gaussian distribution with $\mu = 3.049$ and $\lambda = 26.4443$ was selected, as all goodness-of-fit statistics consistently demonstrated that this distribution provides the best agreement with the observed data. This indicates that the Inverse Gaussian distribution effectively represents both the shape and variability of the dataset, making it the most suitable candidate among the evaluated distributions for subsequent analysis.

Table 4. Results of the Marginal Distribution Fitting for BMRI.

Dist. Name	KS stat	KS p-val	AD stat	AD p-val	AIC
ING	**0.040691**	**0.950330**	**0.390110**	**0.858280**	**428.90**
LN	0.041779	0.939070	0.404770	0.843770	429.77
GEV	0.054525	0.725040	0.437850	0.810300	432.01
LL	0.056915	0.675260	0.587380	0.659730	437.15
GAM	0.059564	0.619610	0.715350	0.545900	433.98
LOG	0.072874	0.365150	1.648700	0.144670	455.39
WB	0.075641	0.321470	2.167100	0.074555	453.65
NOR	0.087769	0.172940	2.320300	0.061722	456.16
EV	0.153160	0.001219	6.993900	0.000338	513.48
EXP	0.390530	0.000000	32.514000	0.000004	657.59

The parameters for each distribution were estimated using the Maximum Likelihood Estimation (MLE) method, which finds the parameter values that maximize the likelihood function given the observed data. These parameters determine the shape, scale, and location of the respective marginal distributions, ensuring a precise representation of the data. To validate the suitability of the selected marginal distributions, goodness-of-fit tests were performed. The Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) test was used to compare the empirical distribution with the fitted theoretical distribution, while the Akaike Information Criterion (AIC) provided a measure of model selection by balancing goodness-of-fit with model complexity. The selected distributions showed a good fit, ensuring that the copula construction accurately captures the dependency structure without distorting the individual behaviors of the variables.

By accurately estimating the marginal distributions, the dependency modeling using copulas becomes more reliable. The marginal transformations allow the variables to be mapped into the uniform (0,1) domain, where the copula function is applied. This transformation ensures that the dependency structure is analyzed independently of marginal behavior, enhancing the flexibility and robustness of the copula-based regression model. Accurate marginal distribution estimation is crucial for the effectiveness of copula models, as incorrect marginal choices can lead to biased dependency structures. Therefore, selecting the appropriate distributions and validating their fit plays a fundamental role in achieving reliable predictions in copula-based modeling.

3.2.3. Elliptical Copula Model Regression

The Elliptical Copula Model Regression is applied to estimate BMRI stock prices using Nikkei 225 and Exchange Rates as predictor variables. This process involves constructing the copula, selecting the best-fitting copula, performing copula-based regression, and evaluating the model's predictive accuracy. In this study, two elliptical copula families were considered: Gaussian Copula, which assumes a normal dependency structure without tail dependence, and t-Copula, which allows for stronger tail dependence, making it more suitable for financial data with extreme fluctuations. The copulas were parameterized using maximum likelihood estimation (MLE) to best fit the observed data. Table 5 shows the results of the 3-nested copula fitting using elliptical copula with their goodness of fits.

Table 5. Results of the 3-Nested Copula Fitting Using Elliptical Copula.

Name	param1*)	param2	CvM	pValue	RMSE	AIC
t	r	6.148	0.27798	0.15429	0.042349	-477.92
Gaussian	r	-	0.27365	0.15838	0.042017	-458.36

*) $r = \begin{pmatrix} 1 & 0.8963 & 0.8602 \\ 0.8963 & 1 & 0.8244 \\ 0.8602 & 0.8244 & 1 \end{pmatrix}$

To determine the most suitable elliptical copula, model selection was performed using the Akaike Information Criterion (AIC) and Cramér–von Mises (CvM) statistic. The copula with the lowest AIC value was chosen as the best fit for modeling the dependencies between BMRI, Nikkei 225, and Exchange Rates. In this case, the t-Copula provided the best performance, indicating that financial data exhibit significant tail dependencies, which are better captured by the t-copula structure. Once the best-fitting copula was selected, copula regression was applied to estimate the conditional probability density function (PDF) of BMRI given observed values of Nikkei 225 and Exchange Rates.

The expected value of BMRI is computed using Eqs. (9) and (10). It resulted in RMSE = 0.547, MAPE = 0.13688, and wMAPE = 0.13438. The results indicate that the t-Copula model outperformed linear regression, achieving lower RMSE and MAPE values. This suggests that incorporating tail dependence in financial modeling significantly enhances predictive performance.

3.2.4. Symmetric Archimedean Copula Model

The Symmetric Archimedean Copula Model Regression was applied to estimate BMRI stock prices using the Nikkei 225 Index and Exchange Rates as predictors. Unlike elliptical copulas, which allow for greater flexibility in capturing dependence structures, symmetric Archimedean copulas rely on a single-parameter dependency structure. This simplicity makes them computationally efficient and relatively easy to interpret but also limits their ability to represent asymmetric or complex tail behavior commonly observed in financial data.

In this study, four Archimedean families—Clayton, Frank, Gumbel, and Joe—were tested. Each family has distinct properties: the Clayton copula emphasizes lower-tail dependence, the Gumbel and Joe copulas focus on upper-tail dependence, and the Frank copula provides a more symmetric dependence without tail dominance. Parameter estimation was performed using Maximum Likelihood Estimation (MLE), and model adequacy was evaluated using the Cramér–von Mises (CvM) statistic, Akaike Information Criterion (AIC), and error measures. Table 6 summarizes the results. Among the four families, the Clayton copula achieved the most favorable AIC (–432.72) and the lowest RMSE (0.03593), suggesting that lower-tail dependence plays a more prominent role in the joint movements of BMRI with its predictors. In contrast, the Joe copula performed poorly, with the weakest fit indices and the highest RMSE (0.07308).

Table 6. Results of the 3-Nested Copula Fitting Using Symmetric Archimedean Copula.

Name	param1	param2	CvM	pValue	RMSE	AIC
Clayton	2.6913	-	0.20004	0.24702	0.03593	-432.72
Frank	9.4214	-	0.33061	0.11228	0.04618	-431.60
Gumbel	2.5048	-	0.43180	0.06094	0.05278	-390.22
Joe	2.8102	-	0.82775	0.00558	0.07308	-290.00

Model selection was guided by AIC and CvM, leading to the Clayton copula as the best candidate within this family. However, further validation using conditional expectation showed that its predictive performance remained limited, with RMSE = 1.3217, MAPE = 0.35305, and wMAPE = 0.31786. These results indicate that although the Clayton copula captures dependency patterns better than other Archimedean copulas, its predictive accuracy is inferior to that of elliptical copula models. Overall, symmetric Archimedean copulas provide a straightforward means of modeling dependence, particularly when interpretability is prioritized. Nevertheless, their restrictive single-parameter form makes them less suitable for highly dynamic and asymmetric financial relationships, underscoring the need for more flexible copula structures in practice.

3.2.5. Asymmetric Archimedean Copula Model Regression

Given the limitations of symmetric Archimedean copulas in capturing complex dependencies, the Asymmetric Archimedean Copula Model Regression was employed as an alternative. Unlike the symmetric version, which assumes a uniform dependency strength across all variables, the asymmetric structure introduces greater flexibility by nesting multiple copulas. This design allows for varying dependence strengths between the lower and upper tails, which is particularly important in financial applications where joint extreme losses and gains do not necessarily occur with equal probability.

In this study, a nested Clayton copula was selected because of its ability to capture lower-tail dependence more accurately. Such dependence is relevant in financial markets, where extreme negative shocks in exchange rates and foreign indices may disproportionately affect stock prices. Table 7 summarizes the results of the 3-nested copula fitting using asymmetric Archimedean families.

Table 7. Results of the 3-Nested Copula Fitting Using Asymmetric Archimedean Copula.

Family	Parameter	Goodness of Fits
Clayton	Nested Layers: No. 1: Copula Name = Clayton param1 = 4.2896 No. 2: Copula Name = Clayton param1 = 2.2128	AIC (Joint PDF) = 1632.8 CvM = 0.171 RMSE = 0.033 pVal = 0.294

Family	Parameter	Goodness of Fits
Gumbel	Nested Layers: No. 1: Copula Name = Gumbel param1 = 2.7335 No. 2: Copula Name = Gumbel param1 = 2.3913	AIC (Joint PDF) = 1720.7 CvM = 0.422 RMSE = 0.052 pVal = 0.065
Frank	Nested Layers: No. 1: Copula Name = Frank param1 = 12.713 No. 2: Copula Name = Frank param1 = 8.9316	AIC (Joint PDF) = 1633.0 CvM = 0.329 RMSE = 0.046 pVal = 0.114
Joe	Nested Layers: No. 1: Copula Name = Joe param1 = 2.8583 No. 2: Copula Name = Joe param1 = 2.81	AIC (Joint PDF) = 1833.4 CvM = 0.821 RMSE = 0.073 pVal = 0.006

The model construction followed a two-step process: first, the marginal distributions of BMRI, Nikkei 225, and Exchange Rates were estimated, and then the copula structure was determined using Akaike Information Criterion (AIC). The nested Clayton copula was identified as the best fit, highlighting strong lower-tail dependence while allowing for varying dependency strengths between predictors and the response variable.

Using the selected copula, the conditional probability density function (PDF) of BMRI stock prices was estimated given the observed values of Nikkei 225 and Exchange Rates. The expected value was computed numerically following Eq. (5) from the methodology, approximated using Riemann sums. This model achieved RMSE = 0.64323, MAPE = 0.14261, and wMAPE = 0.1519, demonstrated that the Asymmetric Archimedean Copula Model significantly outperformed its symmetric counterpart. Lower error values suggest that accounting for asymmetric dependencies leads to a more precise stock price prediction. This confirms that allowing for different strengths of dependency in different market conditions is essential for financial modeling, making the asymmetric approach a more suitable choice than the symmetric Archimedean copula model.

3.2.6. Generalized Nested Copula Model

Since the Asymmetric Archimedean Copula Model failed to outperform the Elliptical Copula Regression and even showed lower accuracy than traditional linear regression, a more advanced approach was considered: the Generalized Nested Copula Model. This framework extends the flexibility of copula constructions by allowing different copula families to be combined hierarchically. Through such nesting, the model can simultaneously capture multiple forms of dependence, including asymmetric relationships, nonlinear interactions, and tail dependencies, which are often present in financial time series data.

The modeling process began with the estimation of marginal distributions for BMRI stock prices, Nikkei 225, and Exchange Rates. Afterward, various candidate nesting structures were evaluated using the Akaike Information Criterion (AIC), Cramér–von Mises (CvM) statistic, and root-mean-square error (RMSE). The best-fitting specification was obtained by combining the BB8-180 copula in the first layer (linking Nikkei 225 and Exchange Rates) with a t-copula in the second layer (incorporating BMRI). This hybrid structure enabled the model to represent lower- and upper-tail dependence as well as symmetric tail co-movement, yielding a more accurate and realistic representation of joint dynamics.

Table 8 summarizes the fitting results. The nested BB8-180 and t-copula combination outperformed all competing models, achieving RMSE = 0.58557, MAPE = 0.13632, and wMAPE = 0.14709. These error metrics were substantially lower than those of the symmetric and asymmetric Archimedean copulas and also improved upon the elliptical copula regression. The relatively strong p-value (0.197) further indicated that the dependence structure was statistically consistent with the data.

Table 8. Results of the 3-Nested Copula Fitting Using Generalized Nested Copula

Family	Parameter	Goodness of Fits
Clayton	Nested Layers: No. 1: Copula Name = BB8 180 Param 1 = 7.8361, param 2 = 0.91401	AIC (Joint PDF) = 1585.3 CvM = 0.238 RMSE = 0.039

Family	Parameter	Goodness of Fits
	No. 2: Copula Name = t Param 1 = 0.85455, param 2 = 11831819.2288	pVal = 0.197

Overall, the results demonstrate that the Generalized Nested Copula Model provides the most robust framework for capturing the complex dependencies inherent in financial markets. By flexibly combining copulas with different strengths, the model successfully reconciles tail dependence, asymmetry, and nonlinearity, i.e., features that simpler models fail to capture simultaneously. This superior performance underscores the importance of hybrid copula constructions in financial econometrics and positions the Generalized Nested Copula Regression as the most effective predictive tool among all the models tested.

3.3. Discussion

This section presents a comparative analysis of the predictive performance of various models, including linear regression, elliptical copula regression, symmetric Archimedean copula, asymmetric Archimedean copula, and generalized nested copula regression. The discussion focuses on evaluating the strengths and weaknesses of each approach based on key performance metrics such as Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), and Weighted MAPE (wMAPE).

Table 9. Comparison of Performance Metrics from Each Model Structure

Model Structure	RMSE	MAPE	wMAPE
Generalized Nested Copula	**0.58557**	**0.13632**	**0.14709**
Elliptical Copula (t)	0.54700	0.13688	0.13438
Linear Regression	0.56461	0.13808	0.13894
Asymmetric Archimedean Copula (Clayton)	0.64323	0.14261	0.15190
Symmetric Archimedean Copula (Clayton)	1.32170	0.35305	0.31786

From Table 9, the Generalized Nested Copula Model demonstrates the best overall performance, achieving the lowest MAPE and wMAPE, confirming its superior ability to model financial dependencies. The Elliptical Copula (t-Copula) Regression follows closely behind, showing strong predictive accuracy, particularly in capturing tail dependencies. Linear Regression, while performing relatively well, lacks the flexibility to handle complex dependency structures, making it less effective than copula-based approaches. The Asymmetric Archimedean Copula Model, despite allowing for different dependency strengths in the lower and upper tails, does not outperform elliptical copulas or linear regression. Lastly, the Symmetric Archimedean Copula Model performs the worst, highlighting its limitations in modeling real-world financial relationships due to its rigid structure.

The results indicate that nested copula models outperform traditional linear regression and simpler copula structures by better capturing nonlinear and asymmetric dependencies. The Generalized Nested Copula Model, which combines BB8-180 and t-Copula layers, successfully adapts to varying dependency structures, making it the most effective model for stock price prediction. The Elliptical Copula (t-Copula) Regression also exhibits strong predictive power, particularly due to its ability to model tail dependencies, which are crucial in financial markets. This confirms that models capable of handling extreme market conditions tend to perform better in stock price forecasting.

The underperformance of the Symmetric and Asymmetric Archimedean Copulas suggests that a single-parameter dependency structure is insufficient for capturing the complexity of stock market data. While the Asymmetric Archimedean Copula improves upon its symmetric counterpart, it still fails to match the accuracy of elliptical and nested copula models. This reinforces the notion that financial time series data require greater modeling flexibility, which can be achieved through hybrid or hierarchical copula structures.

Overall, the findings emphasize the importance of selecting an appropriate copula model for financial forecasting. While linear regression remains a simple and interpretable baseline model, it is outperformed by copula-based approaches that incorporate nonlinear, asymmetric, and tail-dependent relationships. The Generalized Nested Copula Model emerges as the most effective, proving that combining multiple dependency structures enhances predictive accuracy in financial markets. However, it should be noted that these findings are specific to the dataset employed in this study and may not necessarily generalize to other time periods or market conditions. Future research could explore further refinements in copula selection and parameter estimation to optimize predictive performance even further.

4. CONCLUSION

This study has demonstrated the effectiveness of Generalized Nested Copula Regression in predicting Bank Mandiri's (BMRI) stock price based on Exchange Rates and the Nikkei 225 Index. By comparing multiple models, we found that traditional linear regression methods fail to capture the complex, nonlinear dependencies inherent in financial data. Copula-based models, particularly the Generalized Nested Copula, emerged as the most effective approach for modeling stock price movements by allowing for both asymmetric and tail-dependent relationships.

The findings reveal that BMRI stock prices are significantly influenced by both exchange rates and the Nikkei 225, with the latter showing a stronger correlation. While Elliptical Copulas, especially t-Copula Regression, provided a strong baseline for capturing tail dependencies, the Generalized Nested Copula Model outperformed all alternatives by adapting to varied dependency structures, leading to the lowest RMSE, MAPE, and wMAPE values. This result highlights the importance of hybrid copula models in accurately predicting financial trends, particularly in highly volatile markets.

From a broader perspective, this research underscores the need for more sophisticated statistical techniques in financial forecasting. The superiority of copula-based models suggests that market analysts, investors, and policymakers should move beyond traditional regression techniques and adopt more flexible, dependencies-aware approaches. Future research could further refine copula selection strategies, incorporate additional economic indicators, and explore dynamic copula structures to enhance forecasting accuracy. By leveraging these advanced methodologies, financial market predictions can become more precise, resilient, and adaptable to changing economic conditions.

Author Contributions

Alfi Khairiati: Data Curation, Software, Investigation, Writing—Original Draft Preparation, Validation, Funding Acquisition. Retno Budiarti: Resources, Formal Analysis, Validation, Writing—Review and Editing. Mohamad Khoirun Najib: Conceptualization, Methodology, Software, Supervision, Project Administration, Formal Analysis, Visualization, Writing, Critical Revision. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare that he/she has no conflicts of interest to report study.

Declaration of Generative AI and AI-assisted Technologies

Generative AI tools (e.g., ChatGPT) were used solely for language refinement (grammar, spelling, and clarity). The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. The authors reviewed and approved all final text.

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