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OPTIMIZATION OF PARAMETERS IN MEWMV AND MEWMA CONTROL CHARTS FOR CLEAN WATER QUALITY CONTROL AT PP KRAKATAU TIRTA GRESIK

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ABSTRACT

Water is a vital resource whose quality directly affects public health. In Gresik Regency, water treatment processes must be closely monitored, particularly during production. PT PP Krakatau Tirta, a key provider of clean water in the region, plays a strategic role in treating raw water from the heavily polluted Bengawan Solo River. Ensuring that the treated water consistently meets health standards is crucial, highlighting the need for an effective process. This study aims to evaluate the clean water production process and assess the process capability in maintaining the quality of water produced by PT PP Krakatau Tirta Gresik. Laboratory data on key parameters, including pH, dissolved iron, and total dissolved solids, were collected daily from November 25, 2022, to May 31, 2023. These mandatory indicators were analyzed using Multivariate Exponentially Weighted Moving Variance (MEWMV) and Moving Average (MEWMA) control charts to assess process performance. A key contribution of this research lies in optimizing smoothing parameters to enhance control chart performance. Sixteen combinations of (ω,λ) were tested for MEWMV, with the optimal configuration found at ($\lambda = 0.4$) and ($\omega = 0.4$), indicating that process variability is statistically stable. For MEWMA, nine values of λ were evaluated, and the optimal weight ($\lambda=0.9$) was identified as optimal, yielding a stable process mean after removing two out-of-control points. PT PP Krakatau Tirta, which plays a strategic role in treating raw water from the polluted Bengawan Solo River, was selected as a case study to evaluate the effectiveness of advanced monitoring methods. The results indicate that its clean water production process is well-controlled and capable, with water quality consistently meeting health and safety standards.

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1. INTRODUCTION

Clean water is a basic need for human life because it plays an important role in maintaining health, improving welfare, and supporting the productive activities of the community [1]. A clean lifestyle in the surrounding environment will maintain water quality, while an unclean lifestyle can pollute it [2]. The impact experienced by humans when consuming unclean water is damage to internal organs such as the heart, stomach, and kidneys [3]. However, ensuring access to safe and adequate clean water remains a significant challenge, particularly amid rapid population growth, urbanization, and industrial expansion, as seen in Gresik Regency, East Java.

Gresik Regency, as a strategic industrial area with 6,653 business units in 2020, faces public concerns regarding the quality of clean water, especially in the southern region and around the Benowo Landfill. The turbid and smelly raw water from the Bengawan Solo River was exacerbated by the waste of 63 companies dumped into the river from 2020 to 2021. As a result, around 33% of residents choose to use PDAM services to get safer water [4]. PT PP Krakatau Tirta, a water management company in Gresik, produces clean water with a capacity of 1000 liters per second. The distribution is divided into 650 liters/second for PDAM Gresik and 350 liters/second for the industrial areas JIIPE [5]. Given the condition of the Bengawan Solo River, which is vulnerable to pollution, monitoring the quality of water production is very crucial. Currently, production quality control at PT PP Krakatau Tirta is still limited to matching laboratory test results with the standards set by the Ministry of Health. This approach is not enough to detect irregularities or small changes that may occur systematically in the production process.

However, the challenge faced in water quality monitoring is the multivariate nature of the data, which involves many interrelated parameters. In the context of PT PP Krakatau Tirta, several important parameters monitored include pH**,** Dissolved Iron, and Dissolved Solids. The Dissolved Solids variable will logically be high when the Dissolved Iron concentration is also high, as more acidic water (lower pH) tends to increase the solubility of iron. These three parameters are important indicators in determining water quality because they can affect the taste, color, smell, and safety of water for human health [6]. The use of traditional monitoring methods that focus on only one variable at a time (univariate) is not effective enough in detecting small but significant changes in the combination of these parameters [7]. Therefore, more sophisticated and comprehensive statistical methods are needed for comprehensive and responsive monitoring of water quality.

One of the approaches that can be used is the application of a multivariate control chart [8], including the T^2 Hotelling control chart [9]**-**[12], EWMA (Exponentially Weighted Moving Average) [13], [14], MEWMA (Multivariate EWMA), and MEWMV (*Multivariate Exponentially Weighted Moving Variance*)**.** The T^2 Hotelling control chart is suitable for monitoring process stability, assuming a multivariate normal distribution [15]. Meanwhile, EWMA and MEWMA are more sensitive in detecting small shifts in process averages, with MEWMA capable of handling multivariate data. The MEWMV is useful for monitoring process variations from a diversity perspective, without assuming changes in the average process [16], [17]. MEWMA is effective in monitoring the average shift between intercorrelated parameters, while MEWMV focuses on controlling variations in the production process. Both methods provide a relevant early warning system for the clean water treatment industry [18]. When compared to T^2 Hotelling and Generalized Variance, MEWMA and MEWMV are more sensitive in detecting small shifts, so it is more appropriate to be used to control the quality of water production at PT PP Krakatau Tirta proactively and based on strong statistical data [19].

Research on the comparison between MEWMA and T^2 Hotelling control charts in polyester fabric production show that in the MEWMA control chart, with a value of $\lambda = 0.7$, the upper control limit (UCL) is 14.56021, and the process average is statistically controlled. On the other hand, in the T^2 Hotelling control chart with a UCL of 10.10928 is statistically controlled after eliminating uncontrolled data four times. Thus, it can be concluded that the MEWMA control chart is more sensitive than the T^2 Hotelling's control chart in detecting shifts in the process average [20]. Previous research [16] applied MEWMV and MEWMA to cardboard quality control at PT Y Kediri, showing the effectiveness of both diagrams in detecting out-of-control process points and helping identify the causes of multivariate product defects. Furthermore, Pratama and team [21] applied the MEWMA control diagram combined with VARIMA (1,1,2) time series modeling to the quality of white granulated sugar at Madukismo Sugar Factory, and proved that this method can accommodate data autocorrelation and remains accurate in detecting process shifts even though the observations are not independent.

The novelty of this research lies in the application of MEWMV and MEWMA control charts for monitoring clean water quality from polluted river sources, such as the Bengawan Solo, enhanced by the case-based optimization of Weighting parameters (ω, λ) in MEWMV, and λ in MEWMA [22]. These parameters play a critical role in determining the sensitivity and responsiveness of the control charts, as they regulate the influence of historical data in detecting small shifts in the process, thereby improving early warning capabilities in water quality monitoring. MEWMV and MEWMA are more sensitive than univariate control charts, T^2-Hotelling, and Generalized Variance (GV), particularly in detecting small shifts in multivariate processes [23], [24]. Their use of exponential weighting on historical data allows them to capture subtle, consistent changes over time and remain robust even when normality assumptions are violated [25]. Univariate charts ignore the correlation between variables, and traditional multivariate methods that rely on multinormality [18]. This approach supports more accurate and adaptive monitoring of complex systems, enabling early detection of variability and small mean shifts to promote proactive quality control in the clean water treatment industry [26]. This study aims to evaluate the clean water production process and assess the capability of PT PP Krakatau Tirta Gresik in maintaining water quality by focusing on key variables such as pH, dissolved iron, and total dissolved solids obtained through laboratory testing.

2. RESEARCH METHODS

2.1. Data Collection

The research data used were obtained from the water company PT PP Krakatau Tirta Gresik. Several clean water quality variables directly related to human health include pH, Dissolved Iron (mg/liter), and Dissolved Solids (mg/liter), which will be presented in Table 1. The analysis of MEWMA and MEWMV is carried out through the stages of forming a weight matrix, calculating monitoring statistics, and evaluating the control limit, which can be seen in the image flow chart, Fig.1.

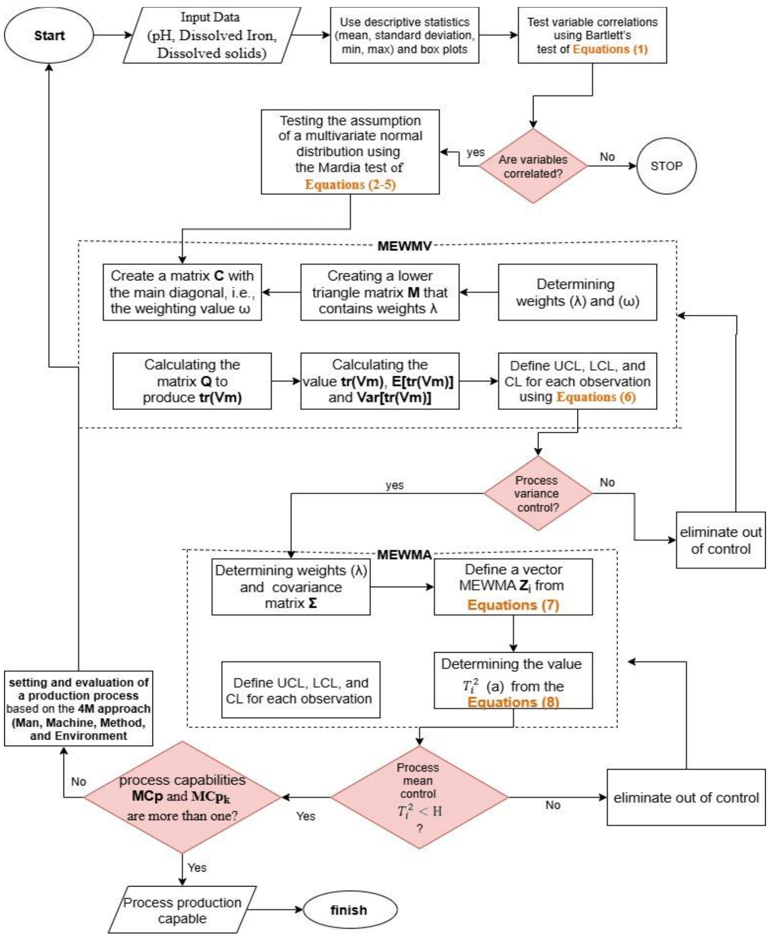


Figure 1. Research Flow Charts

Table 1. **Research Data**

Observation to-	pH	Dissolved iron (mg/L)	Dissolved solids (mg/L)
1	7.50	0.0002	249.78
2	7.59	0.0002	249.72
3	7.42	0.0002	249.75
⋮	⋮	⋮	⋮
189	7.67	0.0002	250.80

Data source*: PT PP Krakatau Tirta Gresik*

2.2. Variable Dependency Test

The Bartlett test is a statistical method that can test several data groups from populations with the same variance. Bartlett's test of sphericity is a form of variable dependency test that is used to test whether there is a significant dependency or correlation between variables in a dataset. Bartlett's test is a crucial first step in identifying variable dependencies before implementing multivariate-based quality control [27]. Bartlett's Test of Sphericity is considered significant if the p-value is less than 0.05, indicating correlation among variables. This indicates that the data are suitable for multivariate analysis because there is a dependence between variables [28]. The formula for Bartlett's test of sphericity is presented in Eq. (1), which is used to determine whether the correlation matrix significantly differs from the identity matrix, indicating the presence of relationships among variables.

$$\chi^2(\mathbf{B}) = -\left[(N-1) - \frac{(2p+5)}{6}\right] ln|\boldsymbol{R}| \tag{1}$$

Where:

N : Number of Observations.
p : Number of Variables.
$|\boldsymbol{R}|$: Determinant of Correlation Matrix.

Hypothesis:

H_0 : Identity correlation matrix.
H_1 : Not an identity correlation matrix.

The null hypothesis (H_0) is rejected if the calculated chi-square statistic in Eq. (1) exceeds the critical chi-square value $\chi^2_{\left(\frac{(p+1)(p-2)}{2};\alpha\right)}$ at the specified α level.

2.2. Multivariate Normal Distribution Test

In testing the assumption of multivariate normal distribution using the Mardia test [29]. Multivariate Skewness and Kurtosis will be used [30]. Multivariate skewness is defined as b_1 and Multivariate kurtosis is defined as b_2, with the following Eqs. (2)-(5):

$$b_1 = \frac{1}{n^2}\sum_{i,j=1}^{n}\left\{(\boldsymbol{X_i} - \bar{\boldsymbol{X}}_i)^T\boldsymbol{S^{-1}}(\boldsymbol{X_j} - \bar{\boldsymbol{X}}_j)\right\}^3, \tag{2}$$

$$b_2 = \frac{1}{n}\sum_{i=1}^{n}\left\{(\boldsymbol{X_i} - \bar{\boldsymbol{X}}_i)^T\boldsymbol{S^{-1}}(\boldsymbol{X_i} - \bar{\boldsymbol{X}}_i)\right\}^2. \tag{3}$$

Where:

b_1 : Skewness Value.
b_2 : Kurtosis Value.
$\boldsymbol{S^{-1}}$: Variance-Covariance Matrix.
$\boldsymbol{X_i}$: Observation of row i.
$\bar{\boldsymbol{X}}_i$: Row average.
$\bar{\boldsymbol{X}}_j$: Column average.

Hypothesis:

H_0 : Data is distributed normally, multivariate.
H_1 : Data is not distributed normally, multivariate.

With the following statistical test formula:

$$z_1 = \frac{N}{6} b_1, \tag{4}$$

$$z_2 = \frac{b_2, k - k(k+2)}{\sqrt{\frac{8k(k+2)}{N}}}. \tag{5}$$

The null hypothesis (H_0) is rejected if the value of the test statistic z_1 exceeds the chi-square critical value with degrees of freedom $\frac{(k+1)(k+2)}{6}$ and the value z_2 of exceeds the critical value Z_α

2.3. Multivariate Exponentially Weighted Moving Variance (MEWMV) Control Chart

The MEWMV control chart is one of the control charts that is useful for viewing a process from a variability perspective and without the assumption that there is no change in the average [16]. The MEWMV control chart is different from other control charts. The difference is that there is no assumption that the average must be controlled during the analysis process. The MEWMV control chart without any changes in the average process can detect changes in variability with very high sensitivity.

The control limit formula used in this control chart is as follows, Eq. (6):

$$E[tr(\boldsymbol{V_m})] \pm L\sqrt{Var[tr(\boldsymbol{V_m})]} = p \times tr(\boldsymbol{Q}) \pm L\sqrt{2p\sum_{i=1}^{m}\sum_{j=1}^{m} q_{ij}^2}, \tag{6}$$

where:

$$E[tr(\boldsymbol{V_m})] = E\left[tr\left(\mathbf{XQX^T}\right)\right] = p\sum_{i=1}^{m} q_{ii} = p \cdot tr(\mathbf{Q}).$$

With $\boldsymbol{V_m}$ is the covariance matrix of the data **X,** size $m \times p$. **X** is a data matrix, and **Q** is a weighted rectangular matrix with an $m \times m$ at the time $p = 1$ so $tr(\boldsymbol{V_m})$ will be the MEWMV equation. The value of L is a constant that depends on the specified p, ω, and λ. The value of L is significant for finding the Average Run Length (ARL) value [18].

2.4. Multivariate Exponentially Weighted Moving Average (MEWMA) Control Chart

The MEWMA control chart is one of the control charts that helps find tiny shifts in the mean of multivariate data with individual samples [31]. This control chart will collect information from the past, so it has a very high sensitivity in analyzing mean shifts. This control chart will also be robust to normality in individual observations. The robust nature of the normal assumptions possessed by the MEWMA control chart makes this control chart still usable for analysis, even though the data used is not normally distributed [23]. The MEWMA control chart is based on the following Eqs. (7)-(9):

$$\boldsymbol{Z_i} = \lambda \boldsymbol{X_i} + (1-\lambda)\boldsymbol{Z_{i-1}}, \tag{7}$$

where the value of $Z_0 = 0$ and $0 \leq \lambda \leq 1$. The process of plotting data on a control chart can use the following Eq. (8):

$$T_i^2 = \boldsymbol{Z_i^T} \sum\nolimits_{Z_i}^{-1} \boldsymbol{Z_i}. \tag{8}$$

The data are considered out of control if the value of T_i^2 exceeds $h_4 > 0$. This value is used to obtain the Average Run Length (ARL) value, which will be controlled with the variance-covariance matrix. The covariance matrix $\boldsymbol{\Sigma}$ is analogous to the variance in univariate data [32], as follows Eq. (9):

$$\Sigma_{\boldsymbol{Zi}} = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\boldsymbol{\Sigma} \tag{9}$$

2.5. Capability Process

The capability analysis process functions to determine the capability, using the multivariate process capability analysis calculation [33], [34]. The capability values of MC_p and MC_{pk} are obtained by first calculating the C_p and C_{pk} values for each variable [35]. These are calculated using the following formulas in Eq. (10).

$$C_p(X_i) = \frac{UCL - LCL}{6\sigma},$$
$$C_{pk}(X_i) = min\left\{\frac{UCL-\mu}{3\sigma}, \frac{\mu - LCL}{3\sigma}\right\}. \tag{10}$$

Description:

UCL : Upper Control Limit.
LCL : Lower Control Limit.
σ : Standard Deviation.
μ : Average.

After the C_p and C_{pk} values of each variable have been found, to find the value of the capabilities MC_p and MC_{pk} [35], can use the following formula, Eq. (11).

$$MC_p = \sum_{i=1}^{P} W_i C_p(X_i),$$
$$MC_{pk} = \sum_{i=1}^{P} W_i C_{pk}(X_i)\,. \tag{11}$$

Description:

MC_p : Multivariate data precision level.
MC_{pk} : Multivariate data accuracy level.

W_i is a weighting with $\sum_{i=1}^{P} W_i = 1$, which has been set by the company regarding the weighting of each variable, if not specified, then the value of $W_i = \frac{1}{m}$ where m is the number of variables used. Decision making on the Bartlett correlation is whether the capability value of MC_p and MC_{pk} produced is > 1, then the data is capable of explaining the truth of the analysis [36].

3. RESULTS AND DISCUSSION

3.1. Descriptive Statistics

To find out the characteristics of each variable, descriptive statistics will be carried out, which will be presented in Table 2.

Table 2. Descriptive Statistics of Research Data

Variable	Minimum	Maximum	Mean	Standard Deviation
pH	6.5	8.21	7.3956	0.34193
Dissolved Iron	0.0001	0.0021	0.0013	0.00088
Dissolved Solid	248.90	249.9961	249.75	0.43343

Based on the statistical analysis results in Table 2, the pH variable has an average value of 7.3956 with a standard deviation of 0.34193, indicating relatively low variation in the data. The pH values range from 6.5 to 8.21, which falls within the acceptable specification limit of 6.5–8.5 as set by the Ministry of Health Regulation. This suggests that the water quality in terms of pH meets the applicable health standards. For the Dissolved Iron variable, the average concentration is 0.0013 mg/L with a relatively small standard deviation, and the observed values are still well below the regulatory limit of < 0.2 mg/L. Similarly, the dissolved solids variable has an average of 249.9961 mg/L, which remains under the maximum allowable limit of < 300 mg/L. Therefore, it can be concluded that all the analyzed water quality parameters are pH (no unit), Dissolved Iron (mg/L), and Total Dissolved Solids (mg/L), comply with the health standards set by the government.

Laboratory results that have been entered into the standards of the Ministry of Health do not necessarily mean that the quality of production from PT PP Krakatau Tirta Gresik is statistically controlled. To determine whether the laboratory result data is no longer statistically out, an analysis will be carried out using a Box Plot, presented in Fig. 2.

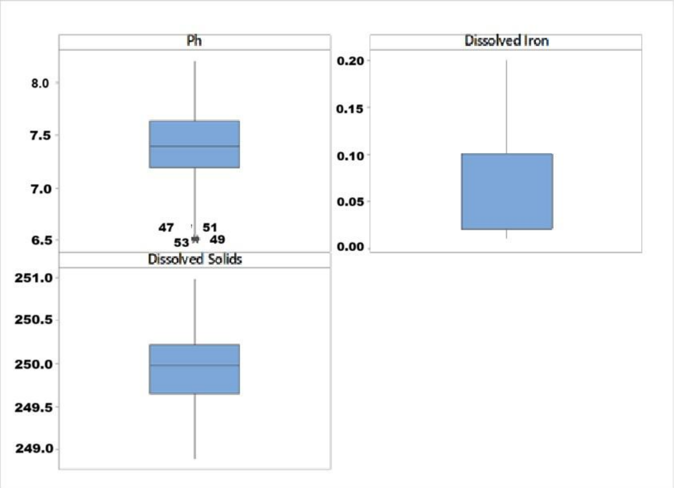


Figure 2. Box Plot of pH, Dissolved Iron, and Dissolved Solids
(Source: Minitab Processing Results)

Based on Fig. 2, in the pH variable, there are four outlier observations, indicating that the quality of pH production at PT PP Krakatau Tirta Gresik is still not controlled. Therefore, in order to monitor and improve these quality issues, control charts based on the MEWMV and MEWMA methods will be applied, as both are effective in detecting small shifts and multivariate deviations in the production process.

3.2. Variable Dependency Test

The requirement for using the MEWMV and MEWMA methods is that the data used must correlate with variables. The correlation test results using the Bartlett's Test method can be presented in Table 3.

Table 3. Bartlett's Test

Chi-Square Test	df	p-value
53.237	3	0.000

It can be seen from Table 3 that the p-value of 0.000 means that the p-value $< \alpha$ (with $\alpha = 0.05$), which means rejecting H_0 so that the data is not included in the identity matrix and can be said to be correlated, which means there is a relationship between the three variables (pH, Dissolved Iron, and Dissolved Solids).

3.3. Multivariate Normal Distribution Test

After conducting the variable dependency test, a multivariate regular data normality test will be performed using Mardia's Test method. The results of the multivariate regular test will be presented in Table 4.

Table 4. Mardia's Test

Test	Statistic	p-value
Mardia Skewness	246.68	2.70×10^{-47}
Mardia Kurtosis	8.3	0.00

Based on Table 4, it can be concluded that the p-value of Mardia Skewness is 2.70×10^{-47}, and the p-value of Mardia Kurtosis is 0.00. Both p-values are smaller than alpha ($\alpha = 0.05$), which means that they fail to reject H_0. The data is not multivariate normally distributed.

This study uses the MEWMV and MEWMA methods, which do not affect the assumption of multivariate normal distribution. Although the data used is not multivariate normally distributed, the analysis can still be continued using these methods. This shows the advantages of the MEWMV and MEWMA methods, which can analyze data that does not meet the assumption of normal distribution [18].

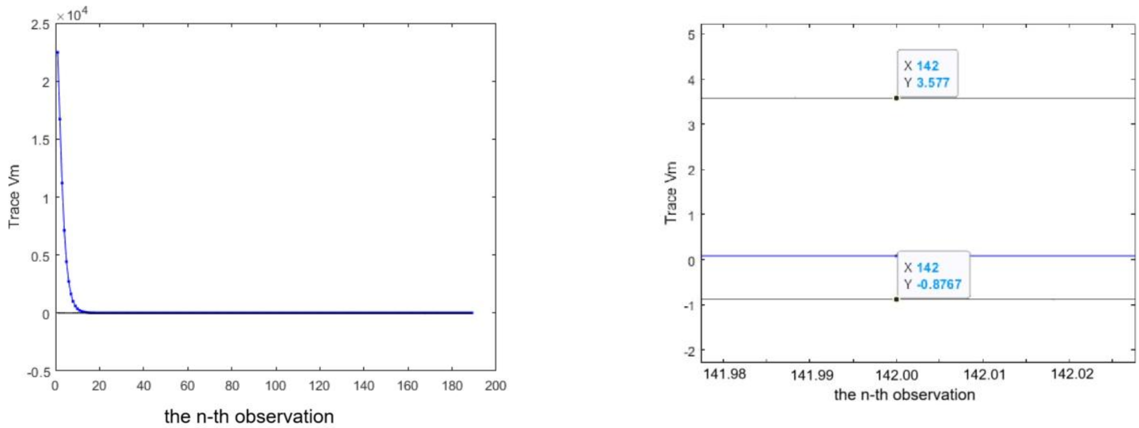
3.4. Quality Control in the Variance Water Treatment Process

Several studies that are the reference sources for this research state that the most optimal weighting is between 0.1 and 0.9. To find the optimal weighting, a combination of weightings from 0.1 to 0.9 will be used for ω and λ as follows.

Table 5. Experiments with Equal Weighting Combinations

ω	λ	L	max $tr(V_m)$	UCL	LCL	Sum Out of control
0.1	0.1	2.7900	50582	3.9057	1.2100	176
0.2	0.2	3.3086	39966	4.0856	0.1811	135
0.3	0.3	3.6602	30599	3.9520	-0.4932	27
0.4	0.4	3.9219	22481	3.5767	-0.8767	19
0.5	0.5	4.1191	15612	3.0099	-1.0099	13
0.6	0.6	4.2715	9991	2.3104	-0.9390	10
0.7	0.7	4.3836	5620	1.5468	-0.7160	8
0.8	0.8	4.4590	2497	0.8134	-0.4134	6
0.9	0.9	4.4984	624	0.2394	-0.1303	4

The effect of weighting parameters on the MEWMV control chart, along with two important findings, is presented in Table 5. First, as the weighting value ω increases, the number of observations that fall outside the control limits decreases, indicating improved process stability. Second, the maximum value of $\boldsymbol{tr(V_m)}$, which reflects the variability within the system, tends to decrease as the λ value increases, suggesting that a higher λ results in smoother monitoring of the process. Furthermore, the experiment also reveals that the observations identified during the burn-in period coincide with those detected as out-of-control when using the weighting combination of $\omega = 0.4$ and $\lambda = 0.4$. This reinforces the results visualized in Fig. 3, confirming that the weighting achieves a balance between sensitivity and stability in detecting shifts.



(a) **(b)**

Figure 3. Experiment with the Exact Weighting of is (a) $\boldsymbol{\omega = 0.4}$; and (b) $\boldsymbol{\lambda = 0.4}$

(Source: Matlab Processing Results)

As shown in Fig. 3, the variance is statistically controlled when the weighting is $\omega = 0.4$ and $\lambda = 0.4$. However, to determine the most optimal weighting combination that ensures better sensitivity in detecting process shifts, further analysis will be conducted by testing various combinations of λ and ω ranging from 0.1 to 0.9. The results of this analysis, which aim to identify the most stable and responsive configuration for the control chart, are summarized in Table 6.

Table 6. Experiment Using Weighting Combination 0.1 - 0.4

ω (a)	λ (b)	L (c)	max $tr(V_m)$ (d)	UCL (e)	LCL (f)	UCL – LCL (f)	max $tr(V_m)$- UCL (g)	Sum Out of control (h)
0.1	0.1	2.7900	50582	3.9057	1.2100	2.6957	50578	176
0.1	0.2	2.7939	39966	3.2733	0.9934	2.2799	39963	176
0.1	0.3	2.7949	30599	2.6644	0.7918	1.8726	30596	175
0.1	0.4	2.7988	22481	2.0945	0.6055	1.4890	22479	175
0.2	0.1	3.3105	50582	4.8756	0.2402	4.6354	50577	138
0.2	0.2	3.3086	39966	4.0856	0.1811	3.9045	39962	135
0.2	0.3	3.3164	30599	3.3359	0.1229	3.2130	30596	124
0.2	0.4	3.3213	22481	2.6240	0.0760	2.5480	22478	97
0.3	0.1	3.6484	50582	5.7717	-0.6559	5.7717	50576	47
0.3	0.2	3.6523	39966	4.8395	-0.5728	4.8395	39961	30
0.3	0.3	3.6602	30599	3.9520	-0.4932	3.9520	30595	27
0.3	0.4	3.6699	22481	3.1118	-0.4118	3.1118	22478	26
0.4	0.1	3.8984	50582	6.6407	-1.5249	6.6407	50575	45
0.4	0.2	3.9063	39966	5.5684	-1.3018	5.5684	39960	24
0.4	0.3	3.9121	30599	4.5438	-1.0850	4.5438	30594	20
0.4	0.4	3.9219	22481	3.5767	-0.8767	3.5767	**22477**	**19**

From Table 6, it can be concluded that based on two evaluation criteria, namely the difference value of (max $tr(V_m)$ – UCL) and the number of out-of-control observations, the weighting with a value of $\omega = 0.4$ and $\lambda = 0.4$ is considered the most optimal weighting, because it has the smallest difference of (max $tr(V_m)$ – UCL) Column (g), which is 22477. This difference reflects how far the maximum value of $tr(V_m)$ is from the upper control limit (UCL), and a smaller value indicates less uncontrolled variation. In addition, the number of out-of-control observations was found to be the least in this weighting, which was 19 observations. This shows that this weighting setting produces a better level of change detection. Considering both criteria, the weighting with a value of $\omega = 0.4$ and $\lambda = 0.4$ is regarded as the most optimal for creating an MEWMV control chart in this study.

3.5. Quality Control in the Mean Water Treatment Process

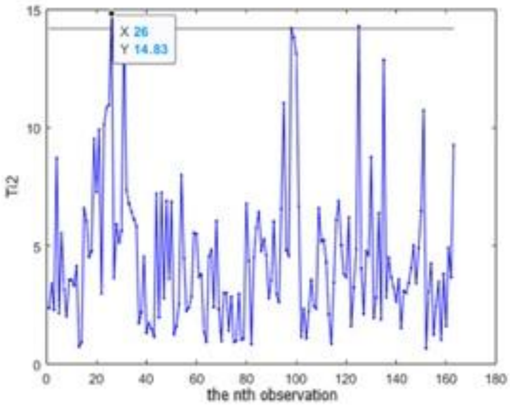
Process variability has been controlled using optimum weighting, namely, $\omega = 0.4$ and $\lambda = 0.4$. For that, it will be continued for control in terms of the process mean. The diagram that will be used here is the MEWMA control diagram. The initial step of a series of control processes is determining the weighting λ of 0.1 to 0.9, with a difference between the weightings of 0.1. The weighting here helps determine the value of T_i^2 as the point of each observation. The results of each experiment will be presented in Table 7.

Table 7. Experiment Using Weighting 0.1 - 0.9

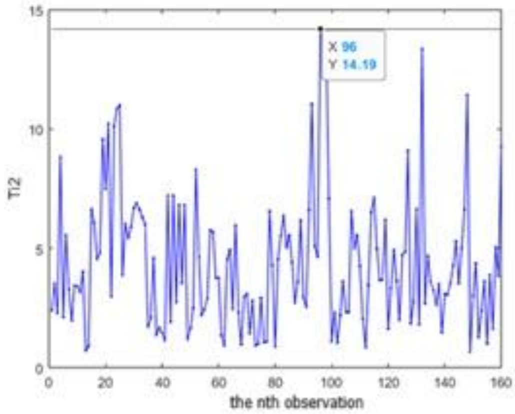
λ	max T_i^2	UCL	difference	Sum Out of control
0.1	278.9	12.4	266.5	98
0.2	294.2	13.39	280.81	69
0.3	242.5	13.79	228.71	53
0.4	195.4	13.99	181.41	48
0.5	158	14.1	143.9	45
0.6	130.1	14.16	115.94	40
0.7	112.6	14.19	98.41	34
0.8	95.38	14.21	81.17	31
0.9	79.73	14.21	65.52	26

Based on Table 7 containing the evaluation results of the MEWMA control chart, it can be seen that the weighting with a value of 0.9 is the most optimum weighting value to detect the mean process of PT PP Krakatau Tirta Gresik because it has the smallest out-of-control observation and the difference between the maximum value of T_i^2 and the Control Limit is also the smallest.

To control the mean production process of PT PP Krakatau Tirta Gresik, cause detection will be carried out by searching for out-of-control data, and data removal will be carried out in the observation. Data cleaned from out-of-control observations will be analyzed again using the most optimal weighting, namely 0.9, as seen in Fig. 4.



(a)



(b)

Figure 4. Detection of (a) First Cause, (b) Second Cause
(Source: Matlab Processing Results)

Based on Fig. 4, it can be seen that for image (a), there are still three observations out of control, so that detection is still carried out in image (b). In image (b), no more observations are out of control, meaning the mean process is statistically controlled. To see whether the production process of PT PP Krakatau Tirta Gresik is classified as good production or not, it can be seen from how many times the cause detection process and elimination of out-of-control data are carried out. If the maximum cause detection process is two times, then the production process of PT PP Krakatau Tirta Gresik is classified as good production. From the analysis that has been carried out, the cause detection process was only carried out two times, and the mean process was statistically controlled, meaning that the production process of PT PP Krakatau Tirta Gresik is classified as good production.

3.6 Process Capability Analysis of Production Water Quality

To see whether the process that has been carried out is good or not, a process capability analysis will be carried out, and previously, the C_p and C_{pk} values of each variable will be sought, which will be presented in Table 8.

Table 8. Univariate Process Capability

Variable	C_p	C_{pk}
pH	0.97	0.88
Dissolved Iron	37.88	0.49
Dissolved Solids	115.36	38.91

From Table 8, the C_p and C_{pk} values of each variable are obtained, which are used to find the value of the capabilities MC_p and MC_{pk} , and the results of the MC_p and MC_{pk} values can be seen in Table 9.

Table 9. Multivariate Process Capability

MC_p	MC_{pk}
50.89	13.29

To determine whether the data is capable, the capability values of MC_p and MC_{pk} must be more than one (>1). From Table 9**,** it is known that the capability values obtained from the analysis that has been carried out are MC_p of 50.89 and MC_{pk} of 13.29, both of which are >1. As a result, it can be concluded that the clean water production process at PT PP Krakatau Tirta Gresik meets the criteria of a capable production system.

When the data is not normally distributed or contains outliers, a control map based on Median Absolute Deviation (MAD) becomes a more robust alternative. MAD effectively monitors process averages and variability due to its extreme value-resistant nature [37]. However, in the context of multivariate processes, control maps such as the Multivariate Exponentially Weighted Moving Average (MEWMA) and the Multivariate Exponentially Weighted Moving Median Variance (MEWMV) were developed to increase the sensitivity of cumulatively small changes on multiple variables at once. MEWMA gives exponential weight to the latest data to detect changes in multivariate process averages, while MEWMV combines median strength and exponential weights to monitor variability in a more resilient manner to outliers, making them a powerful complement to the monitoring of abnormal multivariate quality and containing outliers.

To address the limitations of MEWMA and MEWMV in detecting small variations and dealing with outliers, multivariate control maps based on bivariate ranked set schemes [35] were developed. This method quickly detects changes in the average process and measures the capabilities of the process effectively. In addition, a new multivariate process monitoring scheme that is more adaptive and robust can be developed, namely, the two robust MEMWA [25]. This method uses rank-based Lepage and Cucconi testing based on marginal distribution and pseudo-copula to detect shifts in product quality features and dependency structures.

MEWMA and MEWMV control maps have been widely used in multivariate process control to monitor averages and variability simultaneously. However, both have limitations in detecting small changes in the process when the data is distributed and complex or involves high dimensions. Therefore, for further research, the development and application of the Exponentially Weighted Moving Average Max Multivariate (EWMA Max-Mchart) control chart [38] is a strategic step because this method offers a higher sensitivity in detecting process shifts, both on averages and variability simultaneously in multivariate contexts.

Water sampling is still mostly done using the conventional fixed-interval method (FSI), without paying attention to real-time quality changes. The Variable Sampling Intervals (VSI) approach is now more developed because it can adjust the interval based on process conditions. The VSI-MEWMA CoDa control chart [39] is more effective at detecting uncontrollable conditions through ATS degradation.

4. CONCLUSION

This research confirms that quality control of the clean water production process at PT PP Krakatau Tirta Gresik can be done more effectively by applying the Multivariate Exponentially Weighted Moving Variance (MEWMV) and Multivariate Exponentially Weighted Moving Average (MEWMA) methods. Both methods are proven to sensitively detect variability and mean shifts without relying on the assumption of normal distribution. Statistical control results show that the variability process is statistically under control at the optimal weighting of $\omega = 0.4$ and $\lambda = 0.4$, while the average process is under control after two cause detections using optimal weighting of $\lambda = 0.9$. In addition, the process capability analysis yielded values that far exceeded the minimum limit (> 1), indicating that the company's water production process is in a good category and suitable for consumption.

Author Contributions

Moh. Hafiyusholeh: Conceptualization, Methodology, Validation, and Formal Analysis. Hani Khaulasari: Formal Analysis, Investigation, Visualization, and Funding Acquisition. Fery Firmansyah: Software, Data Curation, Original Draft Writing, and Resources. Nurissaidah Ulinnuha: Supervised, Writing, Review, and Editing Process. All authors reviewed and approved the final manuscript.

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Declarations

This research is carried out jointly according to the division of tasks of each one, without any conflict between authors.

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