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COMPARISON OF COPULA FAMILY (GAUSSIAN, ARCHIMEDEAN, AND REGRESSION) IN A CASE STUDY OF COMPOSITE STOCK PRICE INDEX ON INDONESIA STOCK EXCHANGE

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ABSTRACT

Stocks are one of the most popular financial market instruments. On the other hand, stocks are an investment instrument that is widely chosen by investors because stocks are able to provide attractive profit levels. Investment is an effort to postpone consumption in the present. Comparing copula families is crucial for selecting the model that best fits the observed data dependency structure. This helps produce more accurate analysis and more meaningful interpretations. This study analyzes different types of copula relationships using the Tau Kendall method, applying it to the movement of the Composite Stock Price Index (IHSG) on the Indonesia Stock Exchange (IDX). The data used are secondary monthly data of IHSG as a response variable, while the explanatory variables are inflation (%), exchange rate (Rp/USD), and interest rate (%) in 2010-2014. The results show the pattern of the relationship between IHSG and its macroeconomic factors on the IDX using copula parameter estimation with the Tau Kendall approach, with the largest log-likelihood fitting results showing a relationship pattern following the Gumbel copula, namely IHSG with inflation, interest rates with IHSG following the Clayton copula, and exchange rates following the Frank copula. Meanwhile, using the regression copula has better interpretation results compared to the Gaussian and Archimedean copula, with an MAPE value of 0.122 with a correlation of 70.63%.

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1. INTRODUCTION

Correlation analysis is used to examine the linear relationship between two variables, represented by a correlation coefficient. The most commonly used method is the Pearson correlation, which produces reliable results when the data follows a normal distribution. This study presents a novel contribution by integrating the copula regression approach into the analysis, which not only models the dependency structure but also allows for the prediction of the dependent variable value based on information from the independent variables within the copula framework. Copula regression is a relatively new and more flexible method because it is able to capture complex and asymmetric dependency patterns, which cannot be adequately explained by traditional linear regression models. If the relationship between all variables can be calculated, then the method that can be used is copula analysis. A copula is a statistical approach that describes the relationship between variables without relying heavily on strict distribution assumptions, and it effectively captures dependency, especially at extreme values. This method is capable of capturing the dependency structure between variables with different marginal distributions and can also model their tail dependencies. The estimation results indicate that the conditional expectation formula of copulas in higher dimensions can effectively estimate cases involving more than two variables. Furthermore, the visualizations demonstrate that the estimated values are clustered around the actual values [1]. In addition, a copula can describe dependencies at extreme points clearly. Several studies use the copula approach, image modeling using a Gaussian copula with a partitioning method [2]. Analysis of rainfall relationship and El Niño–Southern Oscillation indicator with copula approach [3], Modeling the Indonesia Stock Exchange Composite Index using Gaussian copula with marginal regression [4], and regression copulas for multivariate responses [5].

In general, cases in the financial sector often encounter data whose distribution is not normal or even forced with the assumption of normal distribution. One of the uncertain conditions in the financial sector is stocks. Stocks are one of the investment instruments whose selling value is based on the performance of the company issuing the shares and are traded on the stock exchange with a certain return depending on the type of stock. Meanwhile, the Composite Stock Price Index (IHSG) is a value that functions as a measurement of the performance of a composite stock on the stock exchange [6]. Uncertain market movements give rise to risks that usually come from the internal company issuing the shares, such as financial reports or the company’s financial condition, or from external markets, such as market sentiment, political and social developments in the country, and others. So that good management is absolutely necessary in managing the risks that may arise.

Gaussian Copula is used to link data that is correlated over time and across different datasets, specifically the SP100 and SP600 stock return data in this context [7]. The Gumbel copula is considered the most suitable model, as it more effectively captures heavy tails, as indicated by the resulting Value at Risk [8]. The Clayton Copula model is the most appropriate for forecasting future aggregate losses at the insurance company PT XYZ. [9]. Comparing copula families in their current state is important due to the need for more flexible, accurate modeling that suits today’s complex data structures. With the development of methods, data, and applications, this comparison is not only important but an essential part of the modern statistical modeling process. In contemporary studies, the relationship between one asset and another can be modeled by a function called a copula. In economics, marginal function is a function that describes the risk of an asset, both the risk of profit and the risk of loss, but usually, analysts focus on the risk of loss. Because a copula combines two or more existing marginal functions, a copula is very appropriate to be used to see the relationship between existing variables. Therefore, this study will examine the analysis of the relationship between the IHSG and macroeconomic factors through the Gaussian copula, Archimedean, and copula regression approaches. The novelty of this study examines other methods using the regression copula approach. This research has a positive impact in using a new method to determine the relationship pattern between variables and prediction with copula regression.

2. RESEARCH METHODS

2.1 Basic Concept of Copula

A copula is a function that connects univariate marginals into multivariate distributions. This function is a joint distribution function of uniform random variables. If there is a random vector $(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p)$ has a marginal cumulative distribution function $F_{x1}, F_{x2}, \ldots, F_{xp}$ with the R domain not descending, namely $F_{xi}(-\infty)) = 0$ and $F_{xi}(\infty)) = 1$ [10], then the distribution with it is like the following Eq. (1),

$$F(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_p})(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_p}) = C(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_p})(\boldsymbol{F_{x1}}(x_1), \boldsymbol{F_{x2}}(x_2), \ldots, \boldsymbol{F_{xp}}(\boldsymbol{x_p})). \quad (1)$$

$\boldsymbol{F_x}$ is a function of multiple variables that is monotonically increasing, where $F(\infty) = 1$. $C(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_p})$ is a copula, for C_x : [0.1] × ... × [0.1] → [0.1]. If the marginal distribution function of $\boldsymbol{F_{xi}}(x_i)$ is continuous then $C(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_p})$ is unique [11] and can be written by the following Eq. (2),

$$C_{(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_p})}(u_1, u_2, \ldots, u_p) = \int_0^{u_1} \ldots \int_0^{u_n} C_{(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_p})}(u_1, u_2, \ldots, u_p) du_1 \ldots du_p. \quad (2)$$

For C is the copula cumulative distribution function and C is the copula density equation.

2.2 Gaussian Copula

One of the copula families is assumed to have a linear relationship, namely the Gaussian copula. The Gaussian copula results are obtained from the transformation of random variables to the standard normal distribution [12]. Random vectors $(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_p})$ has a marginal cumulative distribution function, namely $\boldsymbol{F_{x1}}, \boldsymbol{F_{x2}}, \ldots, \boldsymbol{F_{xp}}$, with $\boldsymbol{U_i} = \boldsymbol{F_{xi}}(\boldsymbol{x_i}) \sim \boldsymbol{U}(\boldsymbol{0}, \boldsymbol{1})$, then each component variable can be transformed into a random variable distributed as follows, Eq. (3).

$$Z_i = F^{-1}_{N(0,1)}(F_x(x_i)) \sim N(0,1) \quad (3)$$

with $i = 1,2, \ldots, p$ and assume that $Z = (Z_1, Z_2 \ldots, Z_p)^T$ follows the standard normal multivariate distribution $N(0, \Sigma)$ with PDF $f_{N(0,\Sigma)}$ and the variance-covariance matrix Σ. The Gaussian copula function is as follows, Eq. (4).

$$C_{(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_p})}(u_1, u_2, \ldots, u_p) = F_{N(0,\Sigma)}\left(F^{-1}_{N(0,1)}(u_1), F^{-1}_{N(0,1)}(u_2), \ldots F^{-1}_{N(0,1)}(u_p)\right), \quad (4)$$

with,

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_1 \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \cdots & \sigma_{pn} \end{bmatrix} = \begin{bmatrix} 1 & \sigma_{12} & \cdots & \sigma_1 \\ \sigma_{21} & 1 & \cdots & \sigma_2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \cdots & 1 \end{bmatrix}.$$

Therefore, the density of the normal copula is like the following Eq. (5),

$$C_{(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_p})}((u_1, u_2, \ldots, u_p) = \frac{\partial}{\partial_{u1}} \cdot \frac{\partial}{\partial_{u2}} \ldots \frac{\partial}{\partial_{up}} \cdot C_X(u_1, u_2, \ldots, u_p)$$
$$= \frac{F_{N(0,\Sigma)}(F^{-1}_{N(0,1)}(u_1), (F^{-1}_{N(0,1)}(u_2), \ldots (F^{-1}_{N(0,1)}(u_n), (F^{-1}_{N(0,1)}}{f_{N(0,1)}(F^{-1}_{N(0,1)}(v) \prod_{j=1}^{n}\left(f_{N(0,1)}\left(F^{-1}_{N(0,1)}(u_j)\right)\right)} \quad (5)$$

If the normal copula is used in a double normal distribution, then it is assumed to have a linear relationship [13].

2.3 Archimedean Copula

The Archimedean copula includes distributions with tail dependence to represent the likelihood of a relationship between two variables in extreme cases [14]. The Archimedean copula family consists of several types and their respective generators, including the Clayton, Frank, and Gumbel copulas, as presented in Table 1 [15].

Table 1. Archimedean Copula Parameter Estimator

Family	Generator $\emptyset(\boldsymbol{u})$
Clayton	$\hat{\theta}_c = \frac{2\tau}{1-\tau}, \theta \epsilon (0, \infty)$
Frank	$\hat{\tau} = 1 - \frac{4(1-D_1(\theta_F))}{\theta_F}, \theta \epsilon\, R \backslash \{0\}$

Family	**Generator $\emptyset(\boldsymbol{u})$**
Gumbel	$\hat{\theta}_G = \frac{1}{1-\tau}, \theta\epsilon[1, \infty)$

The Archimedean copula family has different tail dependencies, including the Clayton copula, which has a tail dependency at the bottom, the Frank copula, which has no tail dependency, and the Gumbel copula, which has a tail dependency at the top [16].

2.4 Copula to Uniform Transformation [0.1]

The first step of copula analysis is done by transforming the random variable to uniform [0.1]. If the marginal distribution of the random variable x is unknown [17], then Eq. (6) is as follows,

$$F_X(x) = \frac{1}{n+1}\sum_{j=1}^{n} 1\left(X^{(j)} \leq x\right); x \in R. \tag{6}$$

The transformation process is carried out by creating a rank for each random variable. $R_1^{(j)}, R_2^{(j)}, \ldots, R_p^{(j)}$ is the rank of $X_1, X_2, \ldots, X_p$ which have previously been converted into matrices, each divided by $n+1$ as follows, Eq. (7),

$$\left(\left(\frac{R_1^{(j)}}{n+1}\right), \left(\frac{R_2^{(j)}}{n+1}\right), \ldots, \left(\frac{R_n^{(j)}}{n+1}\right)\right). \tag{7}$$

Therefore, the copula equation with the transformation is as follows, Eq. (8),

$$C(u_1, u_2, \ldots, u_p) = \frac{1}{n}\sum_{j=1}^{n} \mathrm{I}\left(\frac{R_1^{(j)}}{n+1} \leq u_1, \frac{R_2^{(j)}}{n+1} \leq u_2, \ldots, \frac{R_n^{(j)}}{n+1} \leq u_p\right); u_1, u_2, \ldots, u_p \in (0.1) \tag{8}$$

with I(.) in Eqs. (6) and (8) are indicator functions if each $X^{(j)} \leq x$ and $\frac{\boldsymbol{R}_i^{(j)}}{n+1} \leq u_i$; $i = 1, 2, \ldots, p$. Another method used for transformation is through the cumulative distribution function with the following Eq. (9),

$$\begin{aligned} u_1 &= F(x_1) \sim U\,(0.1) \\ u_2 &= F(x_1) \sim U\,(0.1) \\ &\vdots \\ u_p &= F(x_p) \sim U\,(0.1) \end{aligned} \tag{9}$$

2.5 Copula Parameter Estimation

Copula parameters can be estimated using the Maximum Likelihood Estimation (MLE) method. This involves defining the parameters provided by the copula and the marginal distributions, then maximizing the log-likelihood function to obtain the MLE [18]. f the concentration of d-dimensions F with univariate marginals $F_1, F_2, \ldots, F_p$ and univariate density $f_1, f_2, \ldots, f_p$ can be written as follows, Eq. (10);

$$f(x_1, x_2, \ldots, x_p) = C\left(F_1(x_1), F_2(x_2), \ldots F_p(x_p)\right)\prod_{i=1}^{n} f_i(x_i) \tag{10}$$

To $C(u_1, u_2, \ldots, u_p) = \frac{\vartheta^n(u_1, u_2, \ldots, u_p)}{\vartheta_{u_1}\vartheta_{u_2}, \ldots, \vartheta_{u_p}}$ is the density of the d- dimensional copula $C(u_1, u_2, \ldots, u_d; \theta)$.

The parameter estimation for the Archimedean copula can be performed using the Tau Kendall method [19]. This is calculated using Eq. (11).

$$\tau = 1 + 4\int_0^1 \frac{\emptyset(u)}{\emptyset(u)}\, du \tag{11}$$

2.6 Regression Copula

The constructed regression copula model is based on the copula distribution function [20]. To determine the predicted value of Y, the expected value of the conditional probability function is calculated as follows, Eq. (12),

$$Y = E[y|x] + (y - E[y|x]) = E[y|x] + \varepsilon \tag{12}$$

if x_p is a response variable, then find the value x_p prediction can be written with the following Eq. (13),

$$x_p = E\left[x_p|x_1, x_2, \dots, x_{p-1}\right] + \left(x_p - E\left[x_p|x_1, x_2, \dots, x_{p-1}\right]\right) = E\left[x_p|x_1, x_2, \dots, x_{p-1}\right] + \varepsilon \tag{13}$$

For the conditional probability function, four observation variables are used, as in the following Eq. (14),

$$f(y|x) = \frac{f(x,y)}{f(x)},$$
$$f(y|x_1, x_2, x_3) = \frac{f(y, x_1, x_2, x_3)}{f(x_1, x_2, x_3)} = \frac{f_1}{f_2}. \tag{14}$$

Where:

f_1: distribution with 4 variables.
f_2: distribution with 3 variables.

$$\breve{y}|x = E(y|x_1, x_2, x_3) \approx \text{mean}(y|x_1, x_2, x_3). \tag{15}$$

2.7 Data Analysis Procedure

The data analysis steps carried out in this study are:

1. Data description to obtain a general overview.
2. Analysis of relationships between variables
   a. Identifying relationships between variables using Pearson, Tau Kendall, and Spearman correlations.
   b. Creating a distribution of data between variables to identify relationships between variables.
3. Analysis of relationships between variables using copula analysis
   a. Transforming the variables to uniform [0.1].
   b. Estimating copula parameters between variables using the Tau Kendall approach.
   c. Identifying relationships between variables.
4. Analyzing the generated data using the Gaussian copula function to determine the estimated Y value.

3. RESULTS AND DISCUSSION

3.1 Identification of Relationships between Variables

Correlation aims to measure the strength of the relationship between variables. The results of the correlation analysis of the IHSG with macroeconomic factors. The correlation analysis of the IHSG and interest rates has a real relationship, as well as the IHSG and exchange rates have a real or linear relationship, but the correlation of the IHSG and inflation does not have a real relationship, as shown in Table 2. Furthermore, identifying the relationship between variables with data distribution analysis to see the distribution of the data.

Table 2. Correlation Coefficient Between Variables and p-value

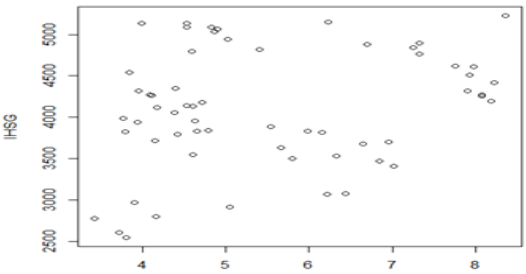
Variable	Pearson		Tau Kendall		Spearman	
	Correlation coefficient	p-value	Correlation coefficient	p-value	Correlation coefficient	p-value
IHSG and Inflation	0.250	0.054	0.134	0.131	0.245	0.059
IHSG and Exchange Rates	0.684**	0.000	0.550**	0.000	0.786**	0.000

Variable	Pearson		Tau Kendall		Spearman	
	Correlation coefficient	***p*-value**	**Correlation coefficient**	***p*-value**	**Correlation coefficient**	***p*-value**
IHSG and Interest Rates	-0.285*	0.027	-0.222*	0.013	-0.322*	0.012

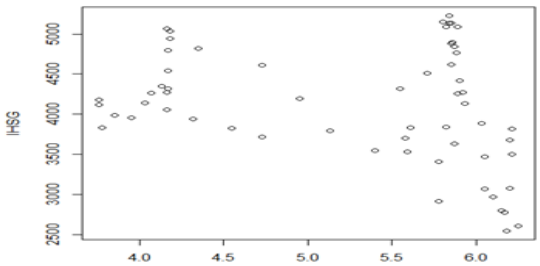
*Information: * real on* $\alpha = 0.05$*; ** real on* $\alpha = 0.01$.

3.2 Data Scatter Plot

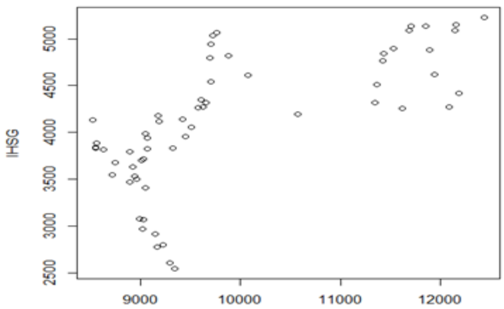
The data distribution aims to provide an overview of the relationship between the Composite Stock Price Index (IHSG) and its macroeconomic factors. The results of the Composite Stock Price Index (IHSG) data distribution plot with its macroeconomic factors. The pattern of the relationship between the Composite Stock Price Index (IHSG) and inflation, interest rates and exchange rates is shown using a data distribution plot forming a non-linear pattern in Fig. 1, making it difficult to identify the relationship between variables, even though based on the results of the correlation analysis in Table 2, the Composite Stock Price Index (IHSG) and inflation and exchange rates have a linear relationship.



(a)



(b)

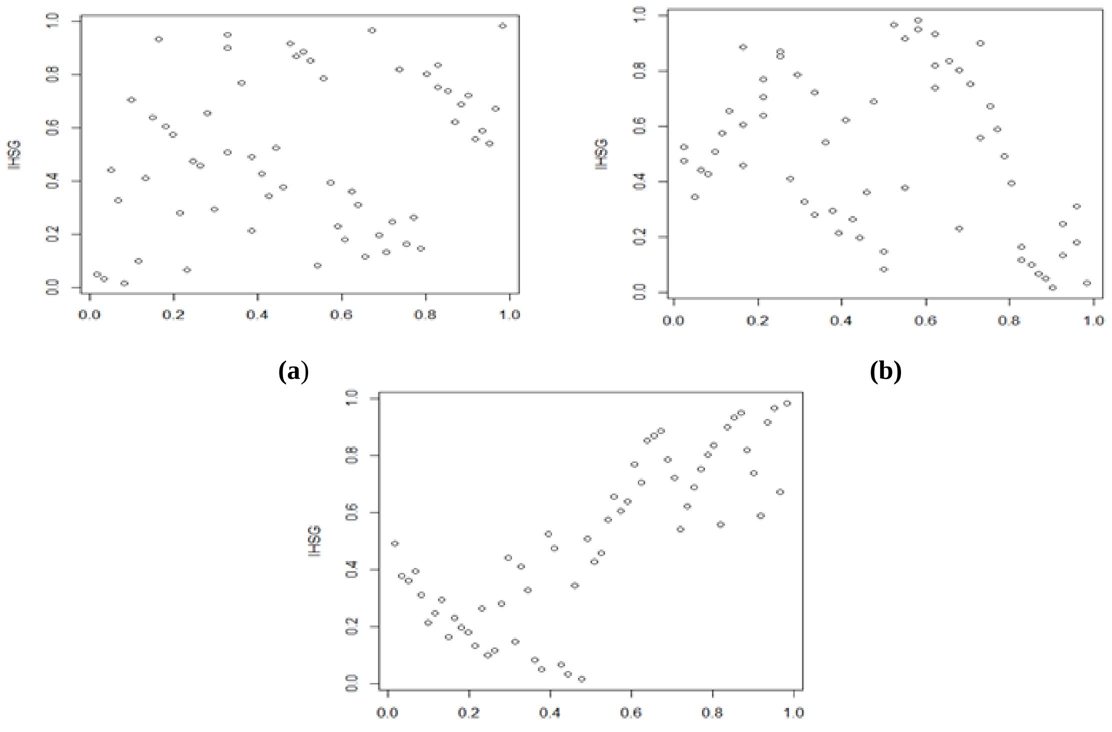


(c)

Figure 1. **Data Scatter Plot IHSG and Inflation (a), Interest Rates (b), and Exchange Rates (c)**

3.3 Identifying the Relationship between Variables and Copula

Some of the advantages of the copula method are that it describes a relationship that is not close to the assumption of the distribution, can explain non-linear relationships, and the distribution of each random variable is unknown. This will not be a problem in this method because all random variables are transformed to uniform [0,1]. The distribution plot of data between variables after being transformed through their cumulative distribution function shows that the observation points are not spread too far. The data from this transformation will be used to estimate the copula parameters in Fig. 2.



(a) (b)

(c)

Figure 2. Scatter Plot of Data Resulting from Transformation between IHSG and Inflation (a), Interest Rates (b), and Exchange Rates (c)

3.4 Copula Parameter Estimator Between Variables

In this study, the copula parameters are estimated to identify the relationship between variables of one of the elliptic copula families, namely the Gaussian copula. If the relationship follows the Gaussian copula, then it shows that there is a linear relationship between the two. In addition, identifying the relationship between variables from the Archimedean copula family, namely the Clayton, Frank, and Gumbel copula. If the relationship follows the Gumbel, Clayton, or Frank copula, it means that there is an extreme event, and there is a relationship at the extreme point. The results of the estimated copula parameters for each copula are listed in Table 3.

Table 3. Copula Parameter Estimator using Tau Kendall Approach

Variable	Copula type	Paramater	p-value
IHSG and Inflation	Gaussian	0.21	0.103
	Frank	1.23**	0.000
	Clayton	0.31	0.165
	Gumbel	0.16**	0.000
IHSG and Exchange Rates	Gaussian	0.76**	0.000
	Frank	6.73**	0.000
	Clayton	2.45**	0.000
	Gumbel	2.22*	0.000
IHSG and Interest Rates	Gaussian	-0.34**	0.006
	Frank	-2.08**	0.000
	Clayton	-0.36**	0.001
	Gumbel	-	-

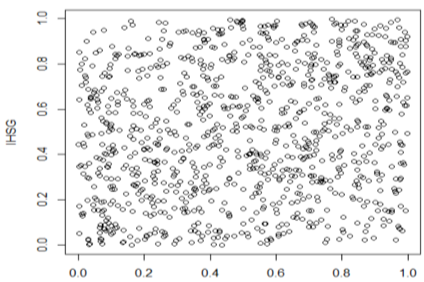
*Information : * real on α =0.05 ; ** real on α =0.01*

In Table 3, it can be seen that with the identified copula, there is a relationship between the IHSG and its macroeconomic factors. If the relationship follows a Gaussian copula, it indicates a linear dependency between the two variables. However, if it follows a Gumbel, Clayton, or Frank copula, it suggests the presence of extreme events and dependency in the tails of the distribution.

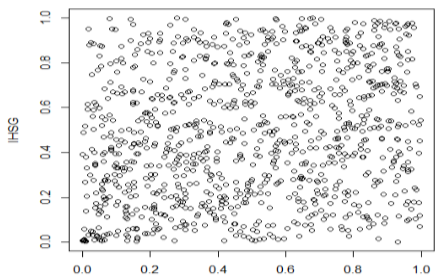
The Gumbel copula captures upper tail dependence, meaning the explanatory variable is strongly related to the response variable only when it takes on high values. When the explanatory variable is low, the relationship between the two variables weakens or becomes negligible. The relationship pattern that follows the Clayton copula illustrates that there is an extreme event at a low value, and there is a relationship between the two variables. When both values are low, the higher the observation value of the variable, the weaker the relationship between the two because this copula has a relationship tail at the bottom.

The relationship between IHSG and variables such as inflation, exchange rates, and interest rates follows the Clayton copula. In contrast, the Frank copula, which lacks both upper and lower tail dependence, visually resembles the Gaussian copula. The Frank copula indicates that a strong relationship between IHSG and inflation, exchange rates, and interest rates only appears when these variables are either very high or very low.

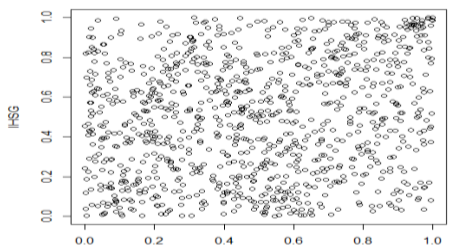
To see the pattern of the relationship between IHSG and its macroeconomic factors, 5000 data points are generated based on the results of the parameter estimates. Like the scatter plot rank copula data $\boldsymbol{n} = \mathbf{1000}$ between IHSG and inflation, exchange rates, and interest rates that follow the pattern of the Gaussian, Clayton, Gumbel, and Frank copula relationships as shown in the following Fig. 3.



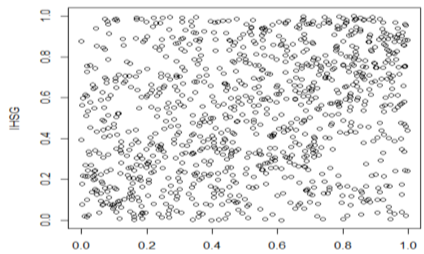
(a)



(b)



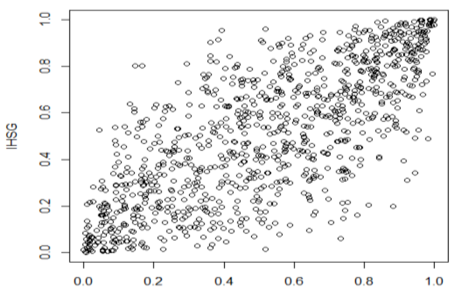
(c)



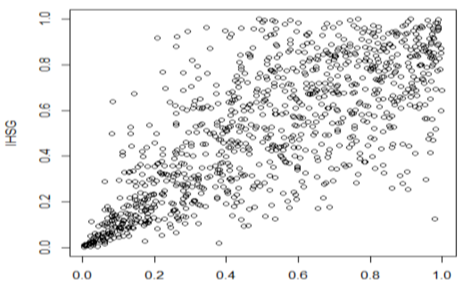
(d)

Figure 3. **Distribution of Rank Copula**

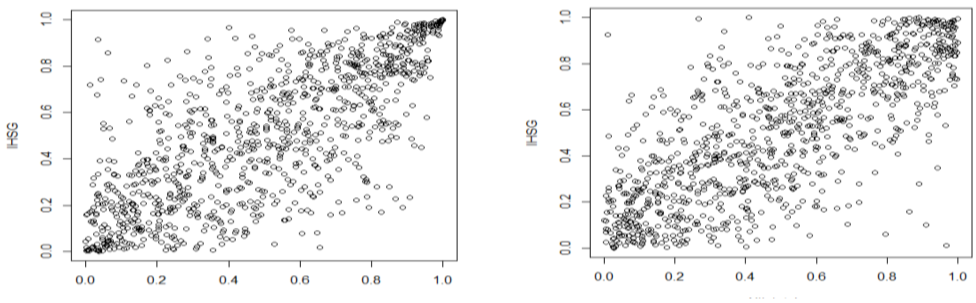
Distribution of rank copula data with $\boldsymbol{n} = \mathbf{1000}$ between the IHSG and inflation following Gaussian (a), Clayton (b), Gumbel (c), and Frank copulas, is shown in Fig. 3. The interpretation of Fig. 3 indicates that the scatter plot of the rank copula data ($\boldsymbol{n} = \mathbf{1000}$) does not clearly exhibit a relationship pattern consistent with any specific copula family



(a)



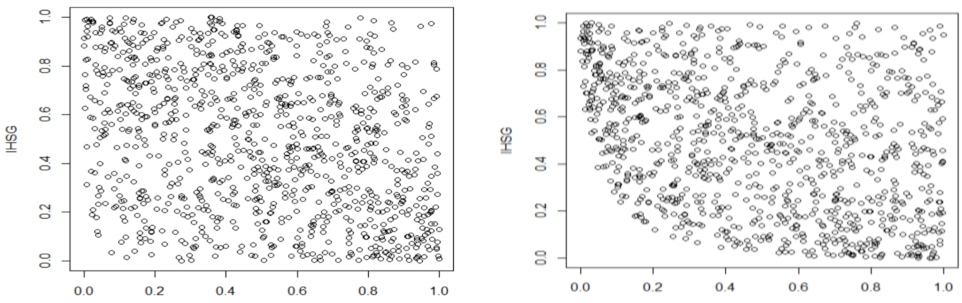
(b)



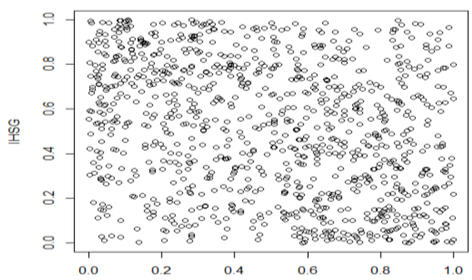
(c) **(d)**

Figure 4. Distribution of Rank Copula

Distribution of rank copula data with $n = 1000$ between IHSG and exchange rates following the Gaussian copula (a), Clayton (b), Gumbel (c), and Frank (d) is shown in Fig. 4. The interpretation of Fig. 4 indicates that the scatter plot of rank copula data ($n = 1000$) shows the scatter plot of the relationship pattern of the image is clearly visible following the copula family.



(a) **(b)**



(c)

Figure 5. The IHSG and Interest Rates Following Gaussian (a), The IHSG and Interest Rates Following Clayton (b), and The IHSG and Interest Rates Following Frank (c)

Interpretation of Fig. 5 indicates that the scatter plot of rank copula data $n = 1000$ shows the scatter plot of the relationship pattern of the image is not clearly visible following the copula family.

Table 4. Copula Fitting Results with MLE

Variable	Types of Copula	Estimator	Log Possibility
IHSG and Inflation	Gumbel	1.225*	2.746
	Clayton	-	-
	Frank	1.523	1.816
	Gaussian	-	-
IHSG and Exchange Rates	Gumbel	2.263	25.310
	Clayton	0.946	8.102
	Frank	7.552*	25.380
	Gaussian	0.722	19.790

Variable	Types of Copula	Estimator	Log Possibility
IHSG and Interest Rates	Gumbel	-	-
	Clayton	-0.339*	4.566
	Frank	-1.928	3.006
	Gaussian	-0.339	3.779

*Information: *: shows the largest possible log value*

The results of copula fitting with MLE for each significant copula are shown in Table 4. The best model for each pair of variables is selected based on the fitting results with the largest log likelihood. The relationship model between IHSG and Inflation follows the Gumbel copula because the log likelihood value is the largest compared to other copulas. This shows that inflation only has a relationship with IHSG when inflation is very high, while when inflation is low, the closeness of the relationship between the two is low (no relationship). Similarly, the relationship between IHSG and interest rates follows the Clayton copula, which indicates the presence of extreme events at lower values. This suggests a stronger relationship between the two variables when both are low, while the relationship weakens as the values increase, consistent with the Clayton copula's lower tail dependence, also seen in the IHSG-inflation and IHSG-interest rate relationships. In contrast, the relationship between IHSG and exchange rates is best described by the Frank copula, as indicated by the highest log-likelihood value. This implies that a strong association between IHSG and exchange rates occurs when both variables are at very high or very low levels.

3.5 Identify Marginal Distribution for Each Variable

Identification of the marginal distribution of the appropriate distribution for each random variable using Cullen and Frey graph observations in the R program, obtained around several distributions that fit each random variable, and selected the distribution with the smallest AIC value. To determine the average estimated Y model, the distribution of each variable from the data on the Indonesian stock exchange is needed. The next step after selecting four appropriate distributions, including lognormal, normal, gamma, and Weibull, from these four distributions in estimating the parameters. The selected distribution that approaches the distribution of each variable is determined by the smallest AIC value, as shown in Table 5.

Table 5. Appropriate Distribution for each Variable

Variable	Selected Distribution	AIC Value
IHSG	Weibull*	958.30
	Gamma	963.54
	Normal	960.59
	Lognormal	965.78
Inflation	Gamma	215.95
	Weibull	222.51
	Normal	221.55
	Lognormal*	214.28
Exchange Rate	Gamma	1021.98
	Normal	1025.96
	Weibull	1036.20
	Lognormal*	1020.19
Interest Rate	Gamma	159.04
	Normal	155.63
	Weibull*	148.01
	Lognormal	161.13

Information:: shows the smallest AIC value*

The results of the smallest AIC value of the four variables that approach the distribution owned and the two distributions selected from the table, namely the Weibull and Lognormal distributions, are shown in

Table 5. These two distributions determine the parameter values of the four variables for the IHSG by following the Weibull distribution, inflation follows the lognormal distribution, the exchange rate follows the lognormal distribution, and the interest rate follows the Weibull distribution.

3.6 IHSG Prediction Using Regression Copula

The prediction of the simulation data is obtained from the results of data generation with 500 thousand rows, and the selected Y estimate is obtained from the distribution of each selected variable; after that, the correlation matrix of all variables is calculated. To find out the measure of the goodness of Y estimate, the MAPE value, and correlation on the actual IHSG data and the results of the estimation are shown in Table 6.

Table 6. MAPE Values and Correlation

Value	Training data (60 observations)	Testing data
MAPE	0.1122	0.0955
Correlation	0.7063	-0.6028

The measure of the goodness of the prediction model using the combined stock price index variable copula regression of simulated data and actual data has an MAPE value of 0.112 with a correlation of 70.63%, as seen in Table 6.

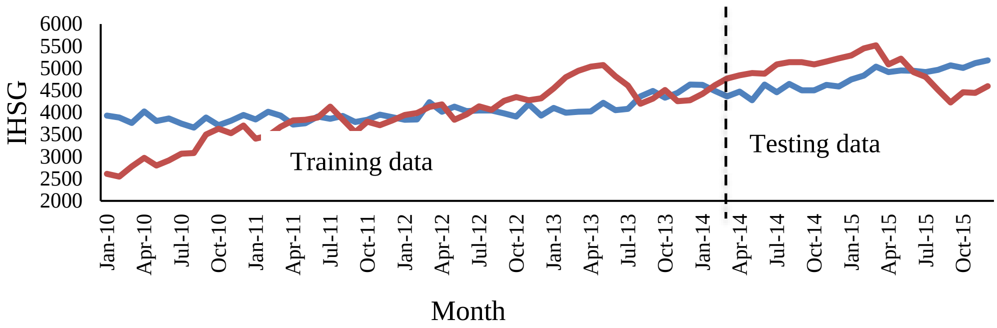


Figure 6. Plot of Actual (Red) and Estimated (Blue) IHSG Time Series

Line plot of IHSG by comparing actual IHSG data and its prediction results. The prediction results of IHSG are close to its actual data. In the training data from September 2010 to February 2011, there is a good prediction, as well as from September 2010 to April 2012 and from August to January 2014, which are close to their actual data, while for the other months, the predicted Y value is still not good. Therefore, the regression copula has the best interpretation results with both Gaussian and Archimedean copulas. While the results of previous research on the relationship between two assets can be modeled with a function called a copula. In economics, the marginal function is a function that describes the risk of an asset, both the risk of profit and the risk of loss, but usually the analysis focuses on the risk of loss. Since a copula combines two or more marginal functions, it is very appropriate to see the relationship between variables. Therefore, this research will further study the analysis of the relationship pattern between IHSG and macroeconomic factors through the copula regression approach to determine the prediction of the composite stock price index with better data interpretation.

4. CONCLUSION

The relationship pattern of IHSG with its macroeconomic factors at BEI uses copula parameter estimation with the Tau Kendall approach, with the results of the largest probability log fitting showing a relationship pattern following the Gumbel copula, namely IHSG with inflation, interest rates following the Clayton copula, and IHSG with exchange rates following the Frank copula. The prediction results using copula regression of the composite stock price index variable of simulation data and actual data have an MAPE value of 0.112 with a correlation of 70.63%. The results of the simulation data indicate that the copula regression has better interpretation results. Copula regression models, with an approach based on different dependency characteristics among macroeconomic factors, provide a more flexible and realistic framework for predicting and understanding the movement of the IHSG. This has a direct impact on risk management practices, investment strategies, and data-driven economic policies.

Author Contributions

Darwis: Conceptualization, Data curation, Funding acquisition, Methodology. Bagus Sartono: Formal analysis, Supervision. Leny Yuliani: Writing - Review and Editing. All authors reviewed and approved the final version of the manuscript.

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Declarations

The authors confirm that there are no conflicts of interest related to the publication of this manuscript. Since the research did not involve human participants or animal testing, ethical approval and participation consent are not required. All authors have reviewed and approved the final manuscript and have agreed to its publication. Research data can be accessed by contacting the corresponding author, subject to a reasonable request. The specific roles and responsibilities of each author are described in the Author Contributions section.

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