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DESIGN AND IMPLEMENTATION OF ANFIS CONTROLLERS FOR STABILIZING FINANCIAL SYSTEMS: A COMPARATIVE STUDY WITH NONLINEAR FEEDBACK CONTROL

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ABSTRACT

The study revisits the well-known Bouali chaotic financial model, which is characterized by nonlinear dynamics. As a benchmark, the nonlinear feedback control method is implemented and compared with an Adaptive Neuro-Fuzzy Inference System (ANFIS) controller. The ANFIS model is trained using 250 data samples derived from the nonlinear feedback controller and divided into training, validation, and testing subsets. The proposed ANFIS controller demonstrates superior stabilization performance by effectively eliminating chaotic behavior, ensuring stability, and achieving faster convergence than the traditional nonlinear feedback method. Quantitative results confirm this improvement: the ANFIS controller achieved very low Root Mean Square Error (RMSE) values, such as 8.78×10^{-5} for training and 1.37×10^{-4} for validation in the profit control input, highlighting its learning accuracy. Additionally, the ANFIS maintained stability even with a reduced number of controllers, demonstrating robustness and adaptability. These findings emphasize the potential of ANFIS controllers to provide efficient and reliable chaos control in complex financial systems.

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1. INTRODUCTION

The financial system is inherently complex, characterized by a multitude of interacting variables that can lead to unpredictable and chaotic behavior [1], [2]. In recent decades, the study of chaos in financial systems has garnered significant attention due to its implications for market stability, risk management, and economic forecasting [3]. Chaotic behavior in financial models can manifest as erratic fluctuations in stock prices, interest rates, and other economic indicators, making it challenging to predict future trends [4]. The application of chaos theory using ANFIS in financial systems is crucial due to the inherently nonlinear, complex, and dynamic nature of financial markets [5]. Chaos theory helps identify hidden patterns and sensitivities in financial data, while ANFIS effectively models these complexities by combining the learning ability of neural networks with the interpretability of fuzzy logic [6]. This synergy allows for more accurate prediction, robust risk assessment, and a deeper understanding of volatile behaviors such as price fluctuations, bubbles, and crashes [7]. By leveraging chaos-based ANFIS models, financial analysts can enhance forecasting performance and decision-making in an environment where traditional linear models often fail [8].

Various control methods have been developed to mitigate chaos in dynamic systems, particularly in financial contexts [9]-[12]. Xu *et al.* [13] proposed a time-delayed feedback controller for the new fractional-order financial model. Hołyst and Urbanowicz [14] investigated Pyragas time-delayed feedback control for the economic models. Hegazi *et al.* [15] investigate the stability conditions in a fractional-order financial system using the fractional Routh-Hurwitz criteria via the linear feedback control method. Xin and Zhang [16] investigated chaotic financial market confidence and designed a controller using Finite-time stabilizing.

Traditional approaches such as linear feedback control have been widely used due to their simplicity and ease of implementation [17]-[20]. However, these methods often fall short in handling highly nonlinear and complex systems where chaos is prevalent. Nonlinear feedback control has emerged as a more effective alternative, offering improved performance in stabilizing chaotic behavior [21]-[23]. Despite these advances, the search for more adaptive and efficient control mechanisms continues, as the complexity of financial systems evolves and the need for robust control strategies becomes increasingly apparent.

One promising approach to controlling chaos in financial systems is the use of ANFIS [24]. The ANFIS combines the learning capabilities of neural networks with the reasoning power of fuzzy logic, making it well-suited for managing nonlinear systems with uncertainty and complexity [25], [26]. By leveraging the strengths of both neural networks and fuzzy logic, ANFIS can adapt to changes in system dynamics and provide more precise control over chaotic behavior [27]. The use of ANFIS in finance significantly enhances the modeling and prediction capabilities for complex financial phenomena [28]. ANFIS can capture nonlinear relationships and uncertainties in financial data, making it highly effective for tasks such as stock market prediction, credit scoring, portfolio optimization, and risk management [29]. This paper explores the application of ANFIS in stabilizing chaotic financial systems, comparing its performance with traditional nonlinear feedback control methods through numerical simulations and analysis.

The contribution of this paper lies in the development and implementation of an ANFIS controller specifically tailored to stabilize chaotic behavior in financial systems. This study provides a comprehensive comparison between the proposed ANFIS controller and traditional nonlinear feedback control methods, demonstrating the superior performance of ANFIS in terms of faster convergence, reduced control complexity, and enhanced stability under varying initial conditions. Additionally, the paper introduces a novel approach to reducing the number of control inputs required for stabilization, thereby offering a more cost-effective and adaptable solution for managing chaos in complex, nonlinear financial systems. This research advances the field by showcasing the potential of ANFIS as a robust alternative to conventional control strategies, with significant implications for financial system stability and economic forecasting.

This paper is organized as follows: Section 1 introduces the concept of chaos in financial systems, discussing its implications and the challenges it presents for market stability and forecasting. It also proposed the use of ANFIS as a potential solution. Section 2 presents the methodology of numerical simulation and the ANFIS controller. Section 3 presents the mathematical modeling of chaotic financial systems, focusing on the Bouali model as the foundational system for this study. In addition, we have designed the ANFIS controller, including the architecture, training process, and the specific techniques used to stabilize the chaotic system. Finally, Section 4 concludes the paper by summarizing the key findings, outlining the contributions of the research.

2. RESEARCH METHODS

2.1 Mathematical Models of Financial Firms

Financial firms play a crucial role in the economic development of countries by facilitating the efficient allocation of resources, enabling capital formation, and supporting business growth. In developed economies such as the United States, Germany, and Japan, financial firms provide a wide range of services, including investment banking, insurance, asset management, and lending. These services fuel innovation, entrepreneurship, and international trade by providing access to credit and investment opportunities. The stability and sophistication of financial firms in these countries also contribute to investor confidence and robust financial markets, which are essential for sustained economic growth.

In emerging economies like Indonesia, India, and Brazil, financial firms are vital for accelerating development and reducing poverty. As these countries experience rapid urbanization and industrialization, financial institutions help mobilize domestic savings, promote small and medium-sized enterprises (SMEs), and support infrastructure projects through structured finance. However, challenges such as limited financial literacy, regulatory weaknesses, and market volatility often hinder their effectiveness.

Bouali's financial firm model is a nonlinear dynamical system designed to represent the complex behavior of financial markets using a set of differential equations. It incorporates key economic variables such as interest rate, investment demand, and price index to simulate the dynamic interactions within a financial firm. Through phase portraits and bifurcation analysis, Bouali's model demonstrates how small changes in economic inputs can lead to significant variations in system behavior, highlighting the importance of nonlinear dynamics in understanding financial instability and guiding policy or investment decisions.

In 2002, Bouali [30] described a chaotic financial firm model, which is described first in our paper. Bouali [30] noted that the capital of a financial firm is at the origin of the profit creation, and is composed of reinvestments R and financed by debts F. If we denote the profit of the financial firm by P, then the rate of change of the profit can be described by the following differential Eq. (1):

$$\frac{dP}{dt} = c(R + F), \tag{1}$$

where c is the coefficient that represents the rate of profit.

Bouali [30] also observed that the reinvestments of the financial firm are made of a fraction of profits according to the proportion m and of the capitalization of these investments, which are reevaluated annually at the rate n. Thus, Bouali [30] also derived the second differential Eq. (2) as

$$\frac{dR}{dt} = mP + n(1 - P^2)R. \tag{2}$$

According to Hunt's hypothesis [30], the financial firm chooses an increase in its capital by means of borrowing according to the debt rate b proportional to self-financing. The net capital inflow is obtained by deducting the refunding of the borrowings according to the interest rate a to the profit amount. Thus, Bouali [30] obtained the third differential Eq. (3) as

$$\frac{dF}{dt} = -aP + bR. \tag{3}$$

Combining the differential Eqs. (1), (2), and (3), the Bouali's financial firm model can be expressed in a system form as follows:

$$\begin{cases} \dot{P} = c(R + F), \\ \dot{R} = mP + n(1 - P^2)R, \\ \dot{F} = -aP + bR. \end{cases} \tag{4}$$

In the model, Eq. (4), a, b, c, m, n are positive constants. We use the notation $X = (P, R, F)$ to represent the 3-D state of the Bouali's financial firm model in Eq. (4).

In [30], Bouali showed that the financial firm model Eq. (4) is chaotic for the choice of parameter values

$$a = 0.1, b = 0.6, c = 0.25, m = 0.04, n = 0.02\,. \tag{5}$$

For the parameter values as in Eq. (5) and the initial state $X(0) = (0.02, 0.02, 0.02)$, the Lyapunov exponents for the Bouali financial system, Eq. (4), are calculated for $T = 1E5$ seconds as follows:

$$L_1 = 0.01956, L_2 = 0, L_3 = -0.30571. \tag{6}$$

2.2 Theoretical Background ANFIS

This study proposes an ANFIS controller to stabilize the chaotic Bouali financial model, a nonlinear dynamic system characterized by sensitive dependence on initial conditions. The mathematical formulation of the Bouali model serves as the foundation for designing both the nonlinear feedback controller and the ANFIS-based intelligent controller. To ensure effective stabilization, the ANFIS controller is developed as a data-driven system trained using inputs and outputs from the nonlinear feedback control.

The nonlinear feedback control law is first derived using the Lyapunov stability theorem [31], [32]. The control inputs $u_1(t), u_2(t)$ and $u_3(t)$ are designed to regulate the system states by minimizing the error $e(t) = x_d(t) - x(t)$, where $x_d(t)$ is the desired state vector. A Lyapunov function $V = \frac{1}{2}e^T e$ is proposed to assess stability, and its derivative $\dot{V} = -ke^T e$ guarantees global asymptotic convergence for positive gain values $k > 0$. The feedback control signals generated from this formulation serve as training targets for the ANFIS model, enabling it to learn how to produce similar control behavior autonomously.

The ANFIS architecture integrates the learning capability of neural networks with the human-like reasoning style of fuzzy logic. In this implementation, the controller utilizes three input variables corresponding to the error in *P*, *R*, and *F*, and three output control signals. Each input and output is mapped using four generalized bell-shaped (gbell) membership functions. Based on the defined membership functions, the ANFIS structure automatically generates a set of Takagi–Sugeno fuzzy rules. Since each of the three input variables (profit error e_P , reinvestment error e_R , and debt error e_F) is represented by four generalized bell-shaped membership functions, the total number of fuzzy rules is $4^3 = 64$. Each rule represents a local linear model of the system dynamics. For instance, one rule can be expressed as: *If* e_P *is Low and* e_R *is Medium and* e_F *is High, then* $u = a_1 e_P + a_2 e_R + a_3 e_F + b$. These rules collectively capture the nonlinear interactions in the chaotic financial system.

The dataset used for training consists of 250 samples obtained from the nonlinear feedback controller's behavior. These data are first divided into training, checking (validation), and testing subsets. The training dataset serves as the input to the ANFIS model, while the checking and testing subsets are used to evaluate its performance. After the training and validation processes, the trained ANFIS replaces the nonlinear feedback controller in the closed-loop system. The structure of this process is depicted in Fig. 1. Although the dataset consists of only 250 samples (83 for training, 83 for validation, and 84 for testing), this window was selected because it represents the stabilization phase of the nonlinear feedback controller. Despite the limited dataset, the ANFIS model achieved very low RMSE values during both training and validation (Table 2), confirming its ability to learn the essential dynamics of the system. Future work will extend the simulation length to increase the dataset size and further validate the robustness of the proposed ANFIS controller.

To visualize the control scheme, a block diagram is constructed to depict the training and deployment architecture of the ANFIS controller. Initially, the chaotic system generates outputs that are compared with desired values to compute the error signal. This error is processed by a nonlinear feedback controller, and the resulting control signals are used to train the ANFIS. Once trained, the ANFIS controller replaces the nonlinear controller in the closed-loop system. This hybrid training structure allows ANFIS to emulate the behavior of the nonlinear feedback controller while benefiting from improved adaptability and reduced computational complexity. Simulation results validate the ANFIS controller's superior performance in eliminating chaotic dynamics and achieving system stabilization rapidly. The Theoretical framework of this study can be seen in Fig. 1.

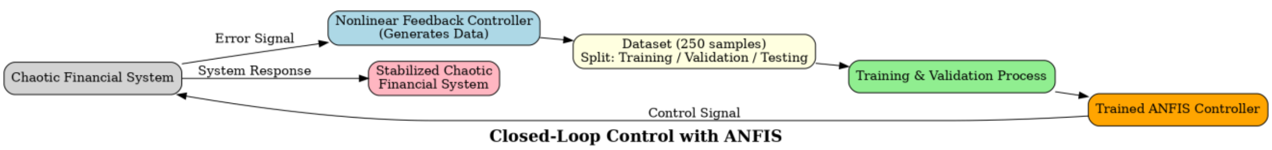

Figure 1. Framework of ANFIS Training and Validation for Chaotic Financial System Control

Additionally, to demonstrate the system's efficiency, a controller reduction experiment is conducted by removing one of the control inputs, specifically $u_3(t)$, from the ANFIS controller. Despite this reduction, the system maintains its ability to stabilize all three state variables, showcasing the intelligent adaptability of the ANFIS architecture. Compared to traditional feedback control, the reduced ANFIS setup achieves faster convergence and better error suppression with fewer control resources. These results highlight the potential of ANFIS to serve as a robust and cost-effective solution for managing chaos in nonlinear financial systems.

3. RESULTS AND DISCUSSION

3.1 ANFIS Controller

Unlike linear controllers that assume linearity or rely on linear approximations, nonlinear feedback control is specifically designed to manage nonlinearities in the system dynamics. Our goal in this section is to design an adaptive neural fuzzy system. We will use a dataset to train this adaptive neural fuzzy controller. This data will be obtained from a non-linear feedback controller. Therefore, a general method of designing nonlinear feedback control is presented first.

a. Nonlinear Feedback Controller

A nonlinear feedback controller is a control strategy used to regulate the behavior of nonlinear dynamic systems by continuously adjusting the input based on the difference between the desired and actual output [33], [34]. Unlike linear controllers, which assume system behavior can be approximated linearly, nonlinear feedback controllers account for the system's inherent nonlinearities, making them more effective for complex, real-world systems such as robotics, power systems, and financial models. By incorporating nonlinear functions into the feedback loop, these controllers can stabilize chaotic systems, reject disturbances, and improve performance across a wider range of operating conditions. Their design often involves techniques like Lyapunov stability theory or backstepping, ensuring that the controlled system remains stable even in the presence of nonlinear dynamics and uncertainties. The nonlinear feedback controller applied to the finance system is expressed as follows:

$$\begin{cases} \dot{P} = c(R+F) + u_{NLF(p)}, \\ \dot{R} = mP + n(1-P^2)R + u_{NLF(R)}, \\ \dot{F} = -aP + bR + u_{NLF(F)}, \end{cases} \tag{7}$$

where $u_{NLF(p)}, u_{NLF(R)}, u_{NLF(F)}$ represent the controllers of each line of Eq. (7). See the block diagram in Fig. 2. The figure illustrates a control loop architecture designed to stabilize a chaotic system using a nonlinear feedback control approach. At the core of this setup is a comparison block (represented by the summation circle), which computes the error between the desired point (target behavior or reference signal) and the actual output of the chaotic system. This error is processed by a nonlinear feedback controller, which generates a control signal tailored to the system's nonlinear characteristics. The controlled input is then applied to the chaotic system, aiming to steer it toward the desired behavior. A nonlinear feedback block monitors the system's output and continuously feeds it back into the loop, enabling real-time adjustment and stabilization.

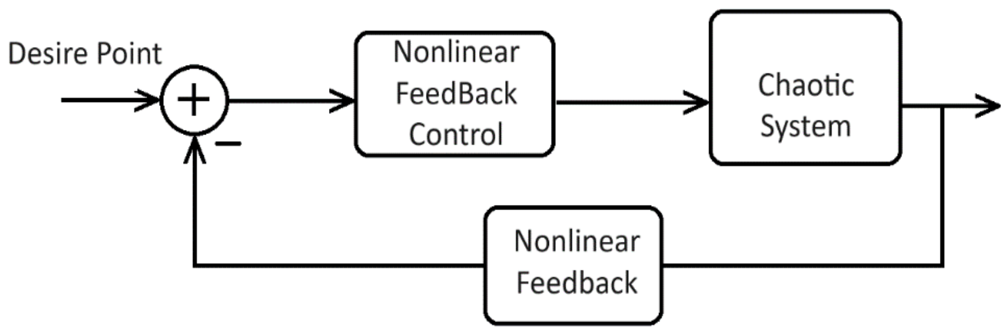

Figure 2. Chaotic System Control Block Diagram

The first step is the calculation of the error, defined as follows:

$$\begin{cases} e_P = P - \hat{P} \\ e_R = R - \hat{R} \\ e_F = F - \hat{F} \end{cases} \tag{8}$$

Where $\hat{P}, \hat{R}, \hat{F}$ are desire points. The derivative of Eq. (8) becomes:

$$\begin{cases} \dot{e}_P = \dot{P} - \dot{\hat{P}} \\ \dot{e}_R = \dot{R} - \dot{\hat{R}} \\ \dot{e}_F = \dot{F} - \dot{\hat{F}} \end{cases} \tag{9}$$

By substituting Eq. (7) in Eq. (9)

$$\begin{cases} \dot{e}_P = c(R+F) + u_{NLF(p)} - \dot{\hat{P}} \\ \dot{e}_R = mP + n(1-P^2)R + u_{NLF(R)} - \dot{\vec{R}} \\ \dot{e}_F = -aP + bR + u_{NLF(F)} - \dot{\vec{F}} \end{cases} \tag{10}$$

Theorem 1. *For Eq. (10) by nonlinear feedback controls $u_{NLF\,(p)}, u_{NLF\,(R)}, u_{NLF\,(F)}$ such that:*

$$\begin{cases} u_{NLF(P)} = -c(R+F) + \dot{\vec{P}} + \beta_P e_P \\ u_{NLF(R)} = -mP - n(1-P^2)R + \dot{\vec{R}} + \beta_R e_R \\ u_{NLF(F)} = aP - bR + \dot{\vec{F}} + \beta_F e_F \end{cases} \tag{11}$$

Then, Eq. (10) can be controlled by nonlinear feedback control.

Proof 1: Consider the candidate Lyapunov function as follows:

$$V(e) = \frac{1}{2}\sum_{i=1}^{3} e_i^2 > 0 \tag{12}$$

By derivation from the above equation, and with substituting Eq. (10)

$$\dot{V} = e_1\left(c(R+F) + u_{NLF(p)} - \dot{\vec{P}}\right) + e_2\left(mP + n(1-P^2)R + u_{NLF(R)} - \dot{R}\right) + e_3\left(-aP + bR + u_{NLF(F)} - \dot{\vec{F}}\right) \tag{13}$$

Finally, by placing the nonlinear feedback controller:

$$\dot{V} = e_1\left(\beta_P e_P - \dot{\vec{P}}\right) + e_2\left(\beta_R e_R - \dot{\vec{R}}\right) + e_3\left(\beta_F e_F - \dot{\vec{F}}\right) < 0 \tag{14}$$

$$\Rightarrow \beta_P e_P^2 + \beta_R e_R^2 + \beta_F e_F^2 < 0. \tag{15}$$

Therefore $\dot{V} < 0$ if $\beta_P = \beta_R = \beta_F < 0$. Considering that our goal is to eliminate chaotic behavior in the system, therefore the values of $\hat{P} = \hat{R} = \hat{F} = 0$ and $\dot{\vec{P}} = \dot{\vec{R}} = \dot{\vec{F}} = 0$. That the values of $\beta_P, \beta_R, \beta_F$ represents the gains of the system.

b. Simulation Results for Nonlinear Feedback Control

Nonlinear feedback controllers can stabilize systems that exhibit chaotic or unstable behavior by shaping the control law to suit the system's nonlinear response, leading to faster convergence and more precise tracking of desired outputs. In the numerical simulation, the 4th order Runge-Kutta method has been used to solve the chaotic differential Eq. (7) under the nonlinear feedback controller. The initial conditions are $P_0 = 0.02, R_0 = 0.02, F_0 = 0.02$ and the controller gain values are $\beta_P = \beta_R = \beta_F = -2$.

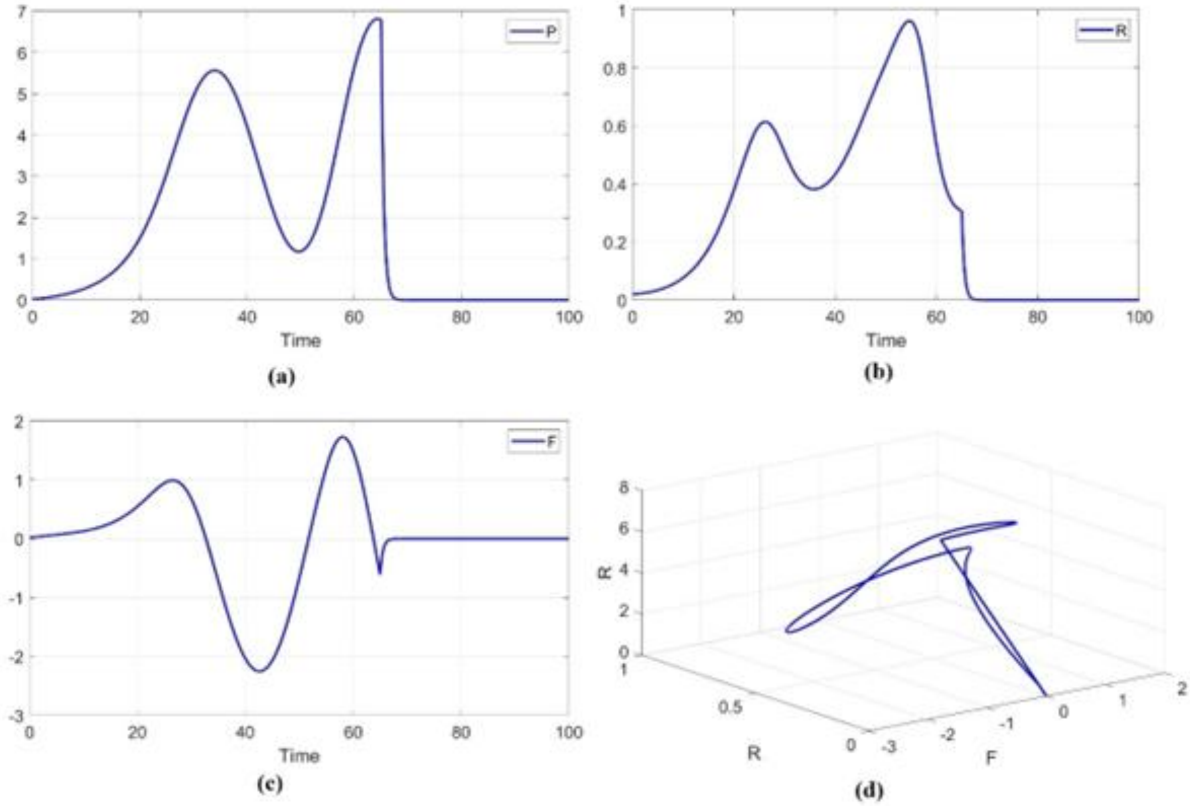

Figure 3. Chaotic System Behavior with Nonlinear Feedback Control: (a) Time Series for Profit of the Financial Firm (P), (b) Time Series for Reinvestments of the Financial Firm (R), (c) Time Series for Financed by Debts (F), and (d) 3D Phase Portrait for Profit, Reinvestments and Financed by Debts

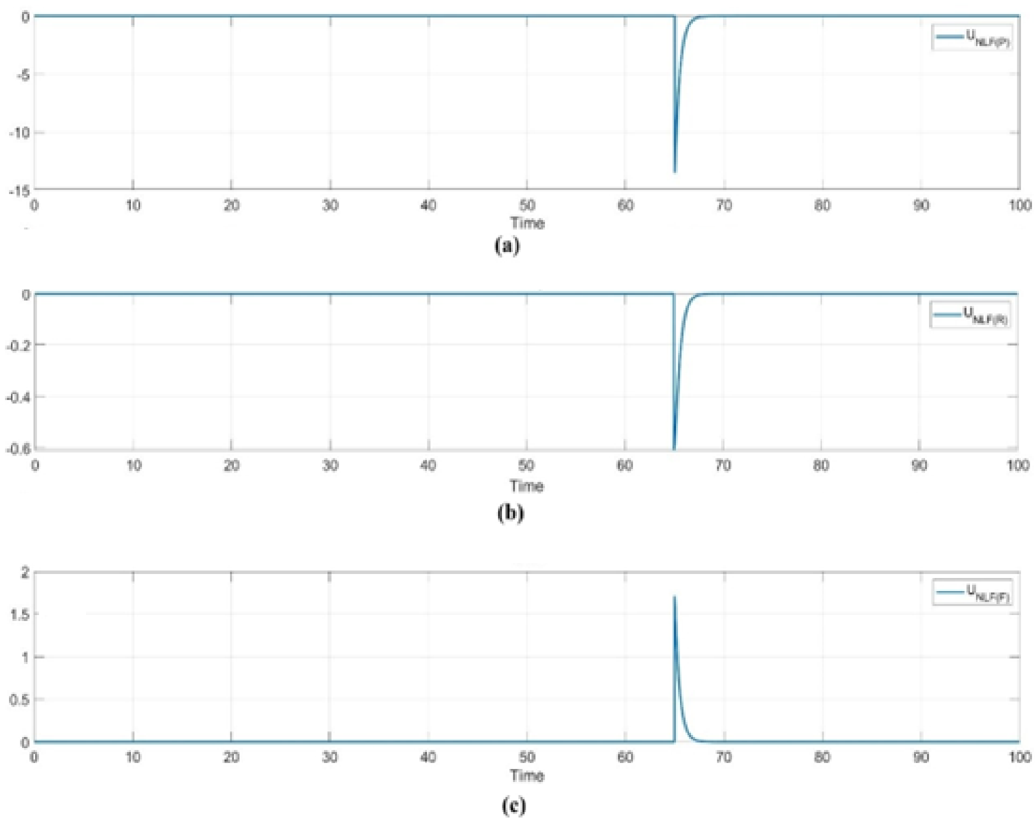

Figure 4. Behavior of Nonlinear Feedback Controller: (a) Time Series for $U_{NLF(P)}$, (b) Time Series for $U_{NLF(R)}$, and (c) Time Series for $U_{NLF(F)}$

Fig. 3 illustrates the response of the chaotic system described by Eq. (7) when regulated by the nonlinear feedback controller defined in Eq. (11), along with its corresponding phase space. The controller is activated at Time = 10. It is evident from the figure that the system achieves stability and suppresses chaotic behavior within a suitable timeframe. Additionally, Fig. 4 presents the output dynamics of the nonlinear feedback control signals.

The duration of stability—defined as the period during which the system maintains zero error—is significantly influenced by the gain of the nonlinear controller in this design method. A higher gain typically enhances the controller's responsiveness, enabling faster convergence to the desired point, while a lower gain may lead to slower or less stable behavior. The tuning of this gain is therefore critical, as it directly affects how quickly and effectively the controller can suppress chaotic oscillations and drive the system toward stability.

As illustrated in the figure, the designed nonlinear feedback control successfully stabilizes all three system variables, with the zero-error condition being achieved around Time = 2.5. This indicates that the chosen gain value is sufficient to counteract the chaotic dynamics within that timeframe. Maintaining such stability is essential in applications like financial systems, where even small deviations can lead to significant long-term consequences. The result underscores the effectiveness of nonlinear feedback control in achieving rapid and robust stabilization.

3.2 Adaptive Neural Fuzzy Controller Architecture

Fig. 5 illustrates a control system integrating nonlinear feedback and ANFIS to manage a Lintang Chaotic System. The control begins with a Desire Point, which represents the system's target state. The difference between this desired state and the current output is fed into a Nonlinear Feedback Control module. This module adjusts its output to drive the chaotic system towards the desired state. The Chaotic System represents a nonlinear dynamic system known for unpredictable and complex behavior, making it a suitable testbed for advanced control strategies.

To optimize control, the system uses ANFIS, which blends fuzzy logic with neural networks to adaptively learn control rules. The ANFIS module includes Training and Check stages. Training adjusts parameters based on the nonlinear feedback to minimize error, and the Check phase validates this training before outputting to ANFIS Control, which fine-tunes the final control signal. This feedback structure, combining traditional nonlinear control with adaptive intelligence, aims to stabilize and accurately control the chaotic Lintang system despite its complex dynamics.

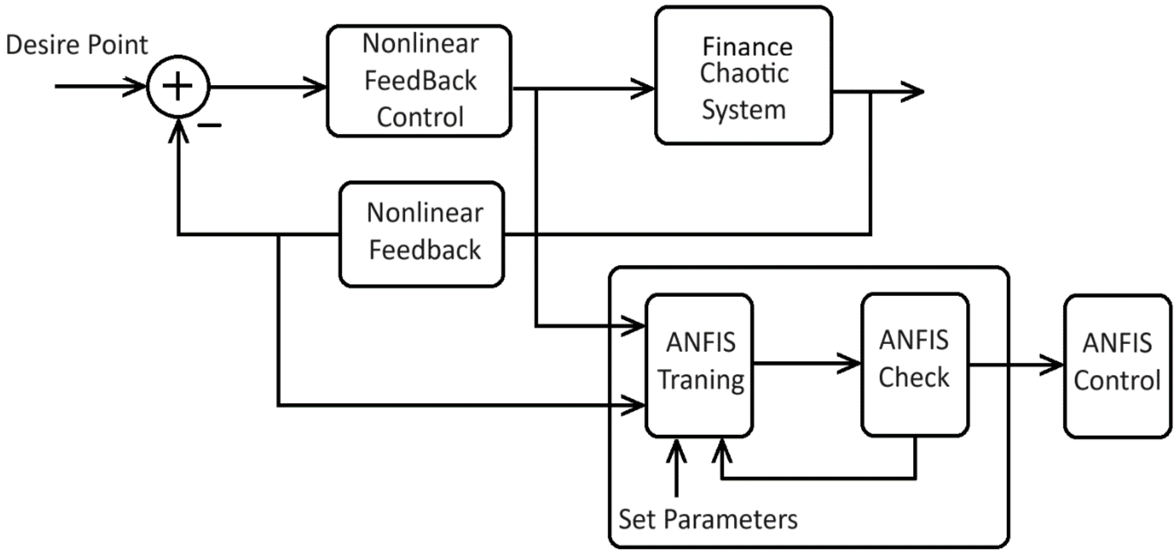

Figure 5. Training Architecture Block Diagram

As seen in Fig. 5, nonlinear feedback and controller data have been used to train the ANFIS controller.

Table 1. **ANFIS Controller Architecture**

ANFIS Parameters	Number
Total number of training data	250
Number of training data	83
Number of check data	83
Number of test data	84
Number of Membership Function (Input/Output)	4
Type of Membership Function (Input/Output)	gbell
Epoch (5)	70
Training error	0

Table 1 outlines the architecture and configuration of the ANFIS controller used in the control system. It specifies that a total of 250 data points were used, divided equally into 83 for training and 83 for checking (validation), with 84 data points reserved for testing the model's performance. The system uses 4 generalized bell-shaped (gbell) membership functions for both input and output variables, which help map fuzzy inputs to precise outputs. The training process was conducted over 70 epochs, indicating the number of iterations used to optimize the model. Notably, the training error reached 0, suggesting that the ANFIS model successfully learned the training data without error.

As mentioned before, the duration of stability in the nonlinear feedback control method is about Time = 2.5. Therefore, ANFIS training data is selected from Time = 65 to Time = 67.5. This time is for non-linear feedback data and the controller.

Table 2. **RMSE Training and Check Results for ANFIS Controller**

U	RMSE	Value
UANFIS(P)	Minimal training RMSE	8.78085e-05
	Minimal checking RMSE	0.000137263
UANFIS(R)	Minimal training RMSE	3.31183e-06
	Minimal checking RMSE	1.79072e-05
UANFIS(F)	Minimal training RMSE	9.95086e-06
	Minimal checking RMSE	1.86286e-05

Table 2 presents the RMSE results for both training and checking phases of the ANFIS controller, evaluated under three different control strategies or configurations labeled UANFIS(P), UANFIS(R), and UANFIS(F). RMSE is a measure of prediction accuracy, where lower values indicate better performance. Among the three, UANFIS(R) achieved the lowest training RMSE (3.31183e-06) and also the lowest checking RMSE (1.79072e-05), indicating superior learning and generalization capabilities. UANFIS(F) followed closely with low RMSE values in both phases, while UANFIS(P) had slightly higher RMSEs compared to the other two. These results suggest that UANFIS(R) offers the most accurate and consistent performance for controlling the system among the tested configurations.

a. Simulation Results ANFIS Controller

Now that an acceptable architecture for the adaptive neural fuzzy controller is obtained, the simulation results should be examined. The adaptive neural fuzzy controller is replaced by the nonlinear feedback controller Eq. (16).

$$\begin{cases} \dot{P} = c(R+F) + u_{ANFIS(p)}, \\ \dot{R} = mP + n(1-P^2)R + u_{ANFIS(R)}, \\ \dot{F} = -aP + bR + u_{ANFIS(F)}. \end{cases} \tag{16}$$

If the initial conditions change, the future behavior of the chaotic system will also change. Therefore, the first analysis for the exquisite controller is to change the initial conditions. Therefore, the initial conditions are chosen $P_0 = 0.1, R_0 = 0.2, F_0 = 0.05$. Fig. 6 shows the behavior of the chaotic system with the ANFIS controller. The activation time of the ANFIS controller is equal to Time = 75. Fig. 7 shows the behavior of ANFIS controller.

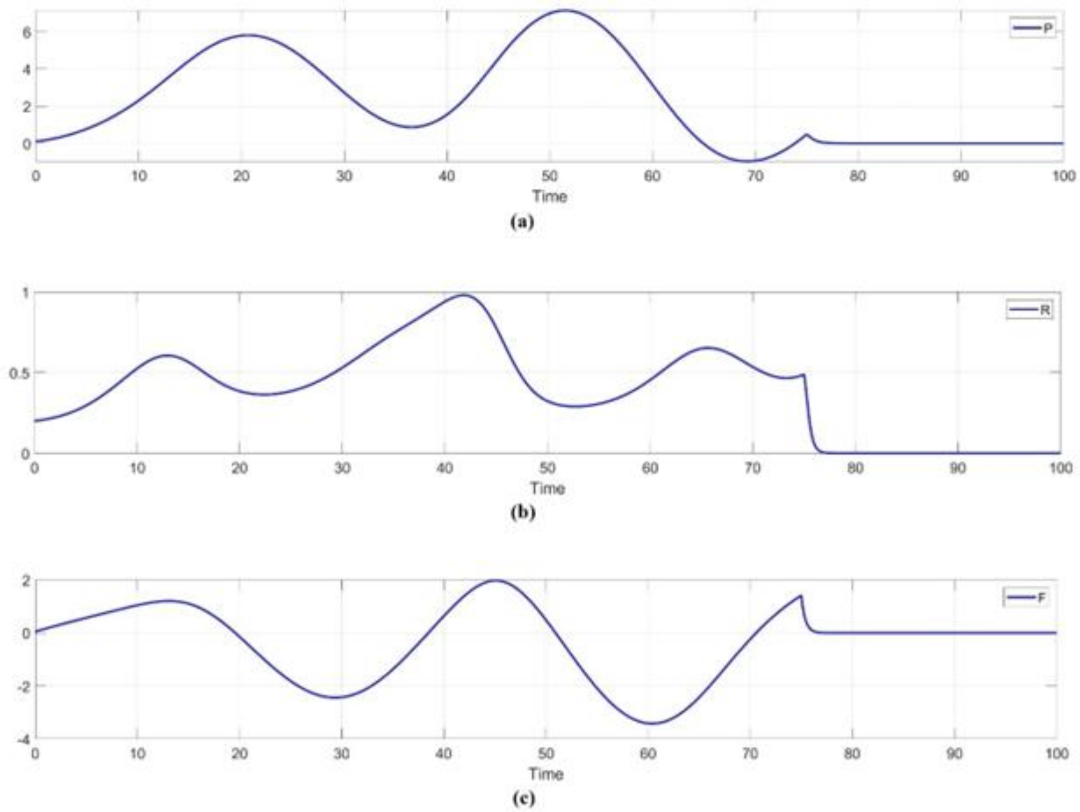

Figure 6. Control and Elimination of Chaos Under the Control of ANFIS: (a) Time Series for Profit of the Financial Firm (P), (b) Time Series for Reinvestments of the Financial Firm (R), (c) Time Series for Financed by Debts (F).

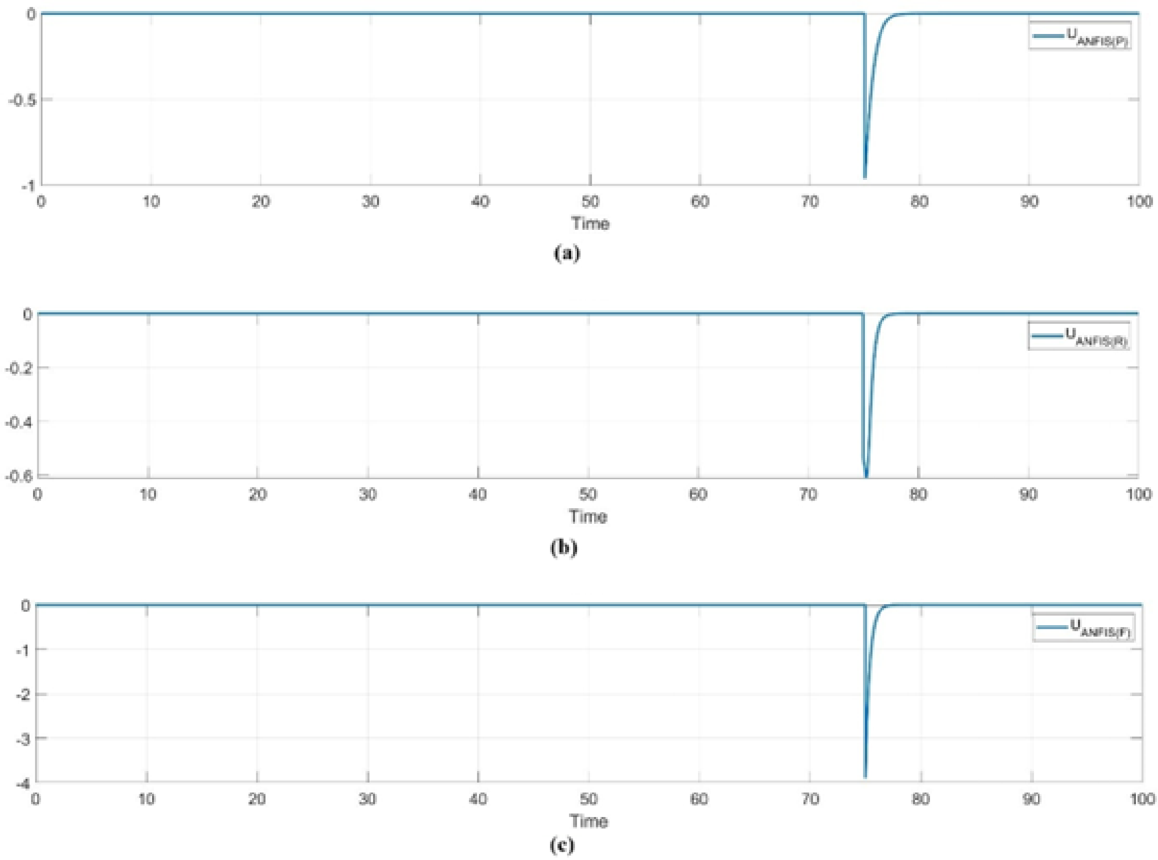

Figure 7. Behavior of ANFIS Controller: (a) Time Series for $U_{ANFIS(P)}$, (b) Time Series for $U_{ANFIS(R)}$, and (c) Time Series for $U_{ANFIS(F)}$.

b. Reducing the number of ANFIS controllers

The three ANFIS controllers have been designed and implemented to regulate the system's dynamic behavior. These controllers work in parallel to manage the complex and nonlinear interactions within the system, ensuring stability and accuracy. Each ANFIS controller is trained to adapt to the system's changing states, leveraging its hybrid learning approach—combining neural networks and fuzzy logic—to respond effectively to chaotic fluctuations. However, given that the ANFIS architecture is inherently based on a nonlinear feedback structure, it is expected to possess intelligent control capabilities. This suggests that with optimized training and configuration, a single or reduced number of ANFIS controllers may be sufficient to stabilize chaotic system Eq. (16). The potential to minimize the number of controllers without compromising performance demonstrates ANFIS's efficiency and adaptability, making it a powerful tool for controlling complex nonlinear systems.

Therefore, consider the chaotic system as follows:

$$\begin{cases} \dot{P} = c(R+F) + u_{ANFIS(p)}, \\ \dot{R} = mP + n(1-P^2)R + u_{ANFIS(R)}, \\ \dot{F} = -aP + bR. \end{cases} \tag{17}$$

The controller on the last line has been removed to simplify the system configuration and avoid redundancy in control logic. This modification was made without altering the structure or parameters of the ANFIS controller, ensuring that its learning mechanism and response characteristics remain consistent with the original design. By eliminating the redundant controller, the overall system becomes more streamlined, allowing for clearer analysis of the ANFIS controller's performance in isolation.

Additionally, the system operates under the same set of initial conditions as previously defined, with $P_0 = 0.1, R_0 = 0.2, F_0 = 0.05$. These values serve as the starting point for the dynamic variables in the simulation and are crucial for maintaining consistency in comparative analyses. By keeping the initial conditions unchanged, the effects of the controller modification can be more accurately observed without the influence of altered system states.

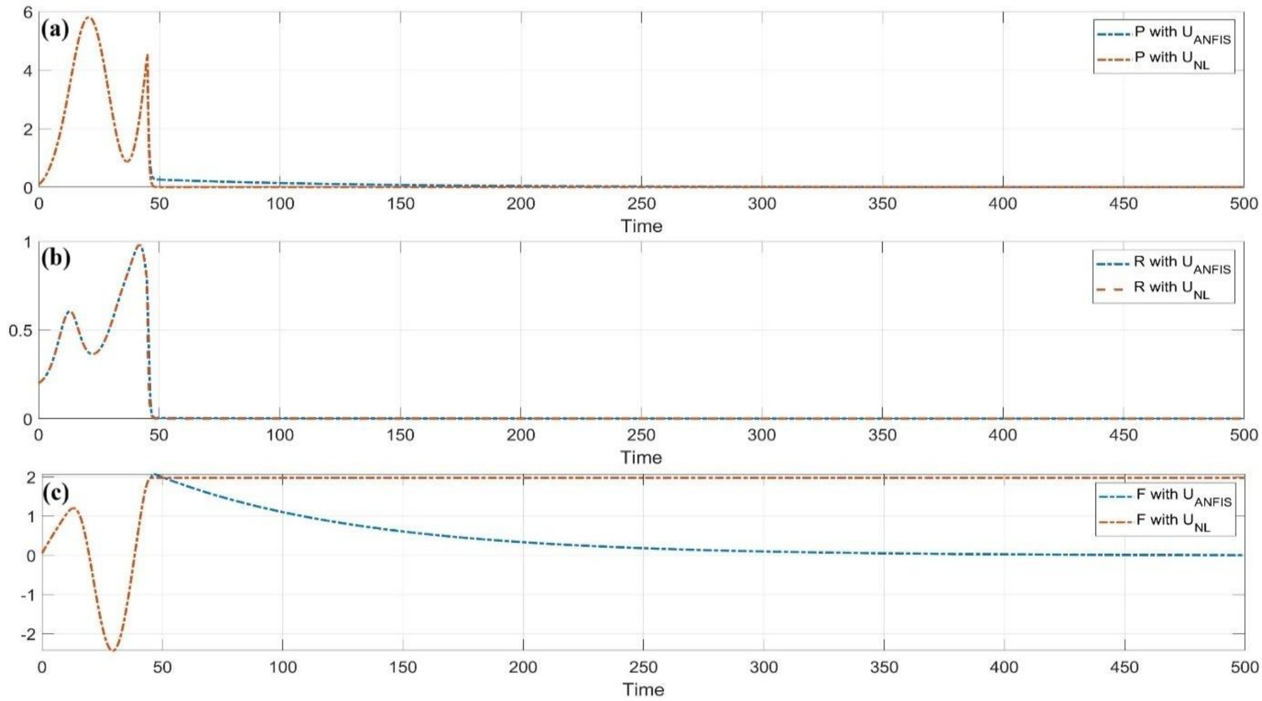

Figure 8. Comparing the Performance of ANFIS Controller and Nonlinear Feedback by Removing the Control in the Third Row of the Chaotic Equations (15): (a) Time series for P with U_{ANFIS} and P with U_{NL}, (b) Time series for R with U_{ANFIS} and R with U_{NL}, (c) Time series for F with U_{ANFIS} and F with U_{NL}.

Fig. 8 presents a performance comparison between the nonlinear feedback controller and the ANFIS controller. As illustrated, the ANFIS controller—when implemented with two control layers—demonstrates a superior capability to ensure the stability of the chaotic system. This highlights the robustness and adaptability of the ANFIS approach in managing complex dynamics. The figure also reveals the system's response under the influence of the reduced infinite controller in conjunction with the nonlinear feedback controller. Notably, the reduced ANFIS controller exhibits better regulation and stabilization characteristics than the nonlinear feedback controller alone, indicating its effectiveness in controlling chaotic behaviors.

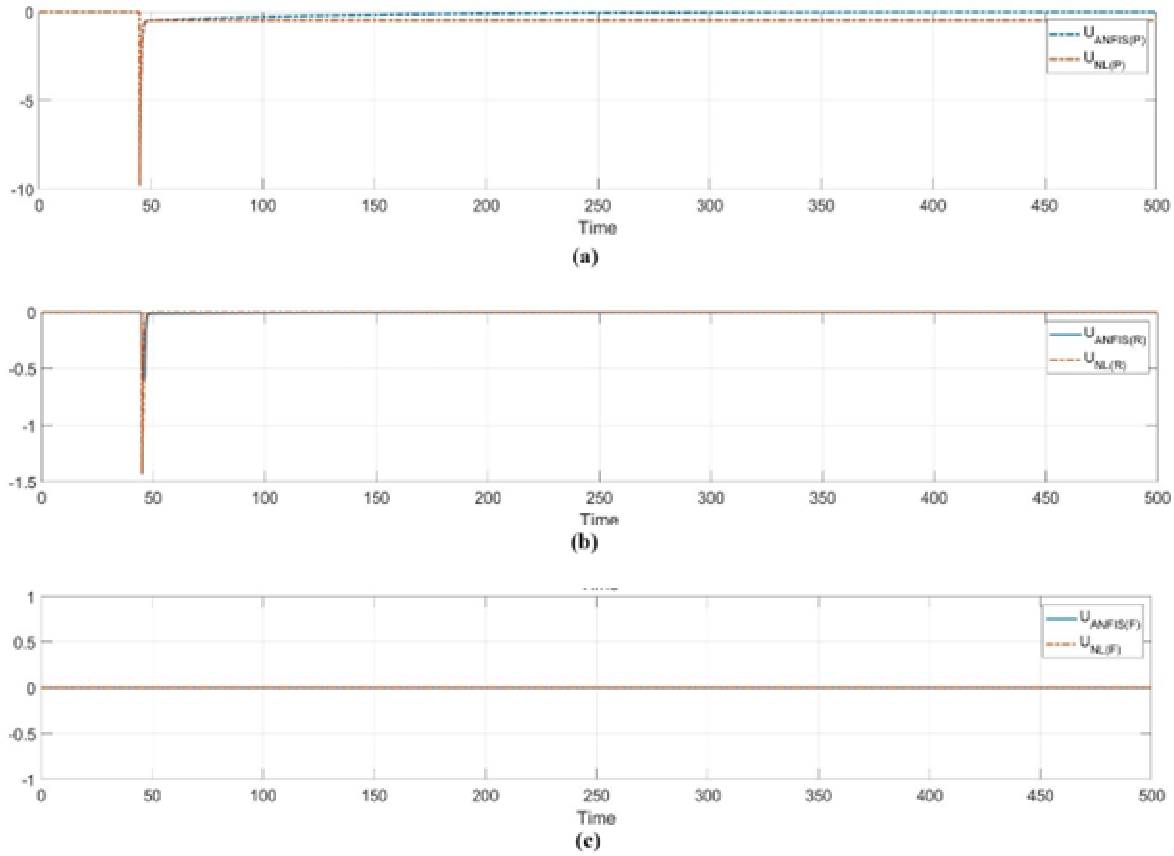

Figure 9. Comparison of the Behavior of ANFIS and Nonlinear Feedback Controller: (a) Time Series for $U_{ANFIS(P)}$ and $U_{NL(P)}$, (b) Time Series for $U_{ANFIS(R)}$ and $U_{NL(R)}$, (c) Time Series for $U_{ANFIS(F)}$ and $U_{NL(F)}$.

For a more detailed analysis and visual comparison of the behaviors of both controllers, refer to Fig. 9. This figure further emphasizes the improved performance of the ANFIS controller relative to the nonlinear feedback method. To avoid redundancy and focus solely on the system's response characteristics, the control signals $u_{\text{ANFIS }(F)}$ and $u_{\text{NLF }(F)}$ have been omitted from the display. This omission enables a clearer interpretation of the system's intrinsic dynamics and the controllers' influence without the added complexity of visualizing the control input functions.

4. CONCLUSION

In this paper, we have proposed and implemented an ANFIS controller to stabilize chaotic behavior in financial systems, using the Bouali financial model as a case study. The performance of the ANFIS controller was rigorously compared with the traditional nonlinear feedback control method. Through extensive numerical simulations, the ANFIS controller demonstrated good performance in eliminating chaos and ensuring system stability, achieving faster convergence and greater adaptability. One of the key findings of this study is that the ANFIS controller can achieve effective stabilization with fewer control inputs than the traditional approach, highlighting its potential for reducing the complexity and cost of practical implementations. The ability of the ANFIS system to adapt to varying initial conditions and maintain stability further underscores its robustness and flexibility in managing nonlinear dynamic systems. The results of this study suggest that ANFIS controllers offer a promising alternative to conventional control methods, particularly in applications where system dynamics are complex and highly nonlinear, such as in financial systems. Future work could explore the application of this approach to other chaotic systems and investigate the potential of integrating ANFIS with other advanced control strategies to further enhance system performance.

Author Contributions

Lintang Patria: Writing - Original Draft, Investigation, Supervision, Software. Seyed Mohamad Hamidzadeh: Methodology, Investigation, Software, Writing - Original Draft. Mohamad Afendee Mohamed: Formal Analysis, Investigation, Validation, Writing - Review and Editing. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare no conflicts of interest to report study.

Declaration of Generative AI and AI-assisted Technologies

Generative AI tools (e.g., ChatGPT) were used solely for language refinement, including grammar, spelling, and clarity. The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. All final text was reviewed and approved by the authors.

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