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DEVELOPMENT OF A MATHEMATICAL MODEL FOR SMARTPHONE ADDICTION USING A DETERMINISTIC COMPARTMENTAL APPROACH

Amalina Amalina[1], **Nor Azah Samat**[2*], **Ahmad Sabri**[3]

[1,2]*Department of Mathematics, Faculty of Science and Mathematics, Universiti Pendidikan Sultan Idris Tanjong Malim, Perak, 35900, Malaysia*

[1,3]*Department of Mathematics, Faculty of Science and Mathematics, Universitas Islam Negeri Imam Bonjol Jln. Prof. Mahmud Yunus Lubuk Lintah, Anduring, Kota Padang, Sumatera Barat, 25153, Indonesia*

*Corresponding author's e-mail:** **norazah@fsmt.upsi.edu.my**

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ABSTRACT

Smartphone usage is increasing with technological advancements. However, excessive use can lead to addiction, negatively impacting an individual's physical health, mental well-being, communication skills, and cognitive development. While previous studies have discussed the psychological and social aspects of smartphone addiction, there is still a lack of mathematical models that capture its spread at the population level. To address this gap, this study develops a deterministic compartmental model to describe the dynamics of smartphone addiction, representing real-world addiction scenarios through mathematical analysis. The novelty of this study lies in formulating a theoretical framework that identifies equilibrium points, conducts sensitivity analysis for each parameter, and employs numerical simulations to demonstrate the role of transmission rate and initial conditions in shaping addiction dynamics. The findings highlight that both transmission rate and initial conditions significantly influence the persistence of addiction, underscoring the importance of early interventions to reduce its long-term impact on the population.

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1. INTRODUCTION

The use of smartphones has increased significantly along with technological advancements over the past few decades. These devices not only facilitate communication but also provide various services such as internet access, social media, entertainment applications, and much more. Despite their significant benefits, excessive smartphone use can lead to various health problems, including physical, mental, communication, and cognitive development issues [1]. One of the main problems that arises is smartphone addiction, which is increasingly found in various age groups, including children and teenagers.

Smartphone addiction is a serious issue that requires special attention. It involves a constant attachment and dependence on smartphones to perform various activities, which leads to the emergence of behavioral problems resulting from this addiction. These behavioral problems include withdrawal from social interactions, impulsivity, aggression, and difficulty performing daily activities [2]. The behavior resulting from smartphone addiction is more about being unable to control oneself to engage in daily activities or control the desire to constantly use a smartphone, leading to negative behaviors [3]. Smartphone addiction also affects the relationship between life satisfaction, sleep quality, and academic achievement, with excessive use negatively impacting these factors [4], [5]. It has been shown to adversely affect academic performance, mental health, and physical well-being [6]. Specifically, smartphone addiction is associated with decreased sleep quality, which can negatively impact academic performance [7].

Among university students, smartphone addiction correlates with lower academic performance, physical health issues, and mental health problems [8]. Smartphone addiction is associated with negative effects on emotional, cognitive, and educational dimensions in university students [9]. It is also linked to emotional and mental disorders in adolescents [10], as well as higher levels of stress, depression, and anxiety [11]. The prevalence of smartphone addiction among adolescents is significant, with strong associations to depression [12], negatively affecting mental health and social interactions [13]. Additionally, it is positively correlated with loneliness in adolescents [14] and associated with sleep disturbances, depression, and anxiety [15]. Not only adults and adolescents but also young children can experience smartphone addiction. Smartphone addiction in early childhood can have negative impacts on mental health, including increased anxiety, reduced attention, and disrupted sleep [16]. The negative impacts of smartphone addiction include problems in mental, physical, communication, and cognitive development in children [1]. Given the seriousness of smartphone addiction, understanding its dynamics is crucial for developing effective prevention strategies.

Research related to smartphone addiction is increasing. These studies use various methods to model smartphone addiction. Among them is the research conducted by Zou *et al.*, which shows a potential correlation between smartphone addiction and hypertension among junior high school students in China [17]. Another study conducted by Xin *et al.* explores the influence of parental bonding on smartphone addiction among Chinese medical students. Using binary logistic regression, their study reveals that overprotective maternal care is positively associated with smartphone addiction, while paternal care shows a negative relationship [18]. Additionally, Lei *et al.* investigate the interaction between smartphone addiction, psychological distress, and neuroticism among medical students. Their cross-sectional study identifies common smartphone addiction, showing a moderate positive correlation between smartphone addiction and psychological distress, as well as a weak positive correlation with neuroticism. Linear regression analysis supports these findings, highlighting the relationship between smartphone addiction, psychological well-being, and neuroticism [19]. Furthermore, the study conducted by Lin *et al.* uses the Technology Acceptance Model to investigate the impact of smartphone addiction on behavioral intention among undergraduate students. This research employs Structural Equation Modeling to analyze the relationships between variables [20]. Cheng *et al.* use a methodological approach rooted in the theory of planned behavior (TPB) and social cognitive theory (SCT) to investigate smartphone addiction among senior high school students. Path analysis, specifically Model 14 of the SPSS PROCESS-macro, is used to conduct moderation mediation analysis and test hypotheses [21].

While these studies provide valuable insights into the psychological, social, and behavioral factors of smartphone addiction, they remain limited to empirical analyses based on surveys and regression models. Such approaches are useful for identifying correlations and predictors, but do not adequately explain the dynamics of addiction transmission within a population over time. Mathematical modeling through a deterministic compartmental framework offers a systematic way to analyze how addiction emerges, persists, or diminishes under different conditions. Nevertheless, research that specifically applies this approach to smartphone addiction is still very limited. This study addresses the gap by developing a deterministic compartmental model that analyzes equilibrium points and the basic reproduction number, conducts sensitivity analysis of key parameters, and discusses their practical implications. The novelty of this work lies in providing a theoretical foundation for understanding the spread of smartphone addiction at the population level, which can serve as a basis for future empirical validation and policy interventions.

Therefore, this study develops a deterministic model to describe the spread of smartphone addiction within a population. The deterministic model was chosen because it provides a systematic framework for analyzing transmission dynamics, including the identification of equilibrium points, the calculation of the basic reproduction number, and the examination of stability conditions that are essential for understanding long-term behavioral patterns. In addition, the model is used to perform numerical simulations that illustrate how addiction can spread in society while predicting population changes over time. With its simple structure and ability to capture temporal dynamics, the deterministic model offers a comprehensive perspective on the mechanisms of smartphone addiction. Through this mathematical approach, deeper insights into the dynamics of smartphone addiction are expected to be obtained, while also providing a basis for designing more effective prevention strategies. The focus of the deterministic model on overall population dynamics makes it particularly suitable as an initial step in establishing a theoretical foundation. This highlights the importance of adopting a deterministic framework as the basis for future empirical research and the development of more complex models.

2. RESEARCH METHODS

This research is based on a quantitative research approach using compartmental modeling. This section outlines the theoretical foundations of the study, including the use of differential equations and systems of differential equations to model the dynamics of smartphone addiction. It also incorporates equilibrium points and the Jacobian matrix to analyze the stability of the system, as well as sensitivity analysis to identify the key parameters that most significantly influence the model outcomes. These theoretical components provide a robust mathematical framework for evaluating and predicting the transition patterns of smartphone usage within the population.

2.1 Differential Equation and System of Differential Equations

Determining differential equations and systems of differential equations in compartmental models aims to describe the dynamics of changes in the number of individuals in each compartment over time quantitatively. This allows for modeling and analyzing phenomena by understanding how interactions between compartments affect transitions and the overall behavior of the system.

Differential Equation is a branch of mathematics that is closely related to daily life problems. The differential equation is an equation involving derivatives of one or more dependent variables with respect to one or more independent variables [22]. There are two types of differential equations: ordinary differential equations and partial differential equations. This study only discusses ordinary differential equations.

An ordinary differential equation is a differential equation involving ordinary derivatives of one independent variable. The order of a differential equation is determined by the highest derivative in the given differential equation. An ordinary differential equation of order n with the independent variable t and the dependent variable y is said to be linear if it is in the form:

$$p_0(t)\frac{d^n y}{dt^n} + p_1(t)\frac{d^{n-1} y}{dt^{n-1}} + \cdots + p_{n-1}(t)\frac{dy}{dt} + p_n(t)y = f(t). \tag{1}$$

If $f(t) = 0$, it is called a homogeneous linear ordinary differential equation. Conversely, if $f(t) \neq 0$, it is called a non-homogeneous linear ordinary differential equation. Nonlinear ordinary differential equations are ordinary differential equations like equation Eq. (1), but their dependent variables are raised to a power, multiplied by their derivatives, or multiplied by each other [23].

According to Boyce and DiPrima [23], another classification of differential equations involves the presence of one function and two or more unknown functions. If there is only one function to be determined, a single equation is sufficient. However, if there are two or more unknown functions, a system of equations is required. The general form of a first-order system of ordinary differential equations is as follows:

$$x_1' = p_{11}(t)x_1 + \cdots + p_{1n}(t)x_n + f_1(t),$$
$$\vdots$$
$$x_n' = p_{n1}(t)x_1 + \cdots + p_{nn}(t)x_n + f_n(t). \tag{2}$$

If $f(t) = 0$ in Eq. (2), then the resulting system is called a homogeneous system of linear differential equations. However, if $f(t) \neq 0$, then it is called a nonhomogeneous system of linear differential equations.

A system of differential equations is called linear if the differential equations that constitute its components are linear differential equations. Conversely, if there is a nonlinear component within the system, it is referred to as a nonlinear system of differential equations.

2.2 Equilibrium Points and Jacobian Matrix

Determining equilibrium points and the Jacobian matrix in compartmental models is crucial for understanding the system's stability and dynamics. Equilibrium points indicate conditions where the system remains unchanged over time, allowing for the analysis of stability. The Jacobian matrix provides insights into how small perturbations around these equilibrium points affect the system, helping to assess whether the system will return to equilibrium or shift to a new state. This analysis is essential for studying local behavior, evaluating the system's response to disturbances, and understanding the impact of parameter changes on stability. Eq. (2) with $g(t) = 0$ can be written in:

$$\frac{dx}{dt} = Ax, \tag{3}$$

with $A = \begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{pmatrix}$ and $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$. The equilibrium point of the system in Eq. (3) is $\tilde{x}$ and is unique (the only critical point) if $\det(A) \neq 0$. A point $\tilde{x}$ is said to be an equilibrium point if it satisfies $A\tilde{x} = 0$ [24]. Eq. (3) is also known as an autonomous system.

The local stability of Eq. (3) can be determined by the eigenvalues obtained from matrix A. There are three types of stability from the form $(A - \lambda I) = 0$ [25]:

1. The equilibrium point $\tilde{x}$ is asymptotically stable if and only if the real part of $\lambda_i < 0$ for each $i = 1,2,\ldots,k$ with $k \leq n$.
2. The equilibrium point $\tilde{x}$ is stable if and only if the real part of $\lambda_i < 0$ for each $i = 1,2,\ldots,k$ with $k \leq n$, and if there is an eigenvalue λ_i located on the imaginary axis, then the algebraic multiplicity must equal the geometric multiplicity for that eigenvalue.
3. The equilibrium point $\tilde{x}$ is said to be unstable if and only if there is a real part of $\lambda_i > 0$ for $i = 1,2,\ldots,k$ with $k \leq n$.

The stability of a nonlinear differential equation system can be determined by first linearizing the given system. Suppose $x' = f(x)$ with $f(x) = (f1(x),\ldots,fn(x))$ and $f(x)$ is differentiable. The matrix $\boldsymbol{Jf(\tilde{x})}$ is called the Jacobian matrix of f at point $\tilde{x}$.

$$\boldsymbol{Jf(\tilde{x})} = \begin{pmatrix} \dfrac{\partial f_1(\tilde{x})}{\partial x_1} & \cdots & \dfrac{\partial f_1(\tilde{x})}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_n(\tilde{x})}{\partial x_1} & \cdots & \dfrac{\partial f_n(\tilde{x})}{\partial x_n} \end{pmatrix} \tag{4}$$

The system $\boldsymbol{x' = Jf(\tilde{x})(x - \tilde{x})}$ is the linearization of the nonlinear system $\boldsymbol{x' = f(x)}$ around the point $\boldsymbol{\tilde{x}}$ [24]. According to the theorem explained in Wiggins [25], if given the Jacobian matrix, namely $\boldsymbol{Jf(\tilde{x})}$ of a nonlinear differential equation system $\boldsymbol{x' = f(x)}$ with eigenvalues $\boldsymbol{\lambda}$ as the result of the characteristic polynomial $\boldsymbol{|Jf(\tilde{x}) - \lambda I| = 0}$, then two conditions are obtained: (i) Locally asymptotically stable if all the real parts of the eigenvalues ($\boldsymbol{\lambda}$) of the Jacobian matrix resulting from the linearization (matrix $\boldsymbol{Jf(\tilde{x})}$ are negative. (ii) Unstable if there is at least one eigenvalue ($\boldsymbol{\lambda}$) of the Jacobian matrix $\boldsymbol{Jf(\tilde{x})}$ whose real part is positive.

2.3 Sensitivity Analysis

The values of parameters in a mathematical model are often not precisely determined due to limitations in available data, such as incomplete data. Therefore, to identify the most influential parameters in a resulting mathematical model, sensitivity analysis can be utilized.

Definition 1. *Let V be a variable and p be a parameter. The normalized sensitivity index of the variable V with respect to the parameter p is defined as* [26] *:*

$$C_p^V = \frac{\partial V}{\partial p} \times \frac{p}{V},$$

where $\boldsymbol{V}$ *represents the variable analyzed with respect to the parameter* $\boldsymbol{p}$.

3. RESULTS AND DISCUSSION

This section presents the research findings, including the analysis of the deterministic model, equilibrium points, model stability, and parameter sensitivity affecting the dynamics of smartphone addiction.

3.1 Deterministic Model

Before constructing a mathematical model to understand the dynamics of smartphone addiction, there are some definitions and assumptions to help the reader understand the model. We divide the human population into four subpopulations: citizens (C), individuals using smartphones (S), those addicted to smartphones (A), and those who have recovered from smartphone addiction (R), where $N(t) = C(t) + S(t) + A(t) + R(t)$. The total number of population (N) is assumed to be constant. According to the assumptions and definitions, the deterministic model of smartphone addiction, called CSAR, is shown in Fig. 1.

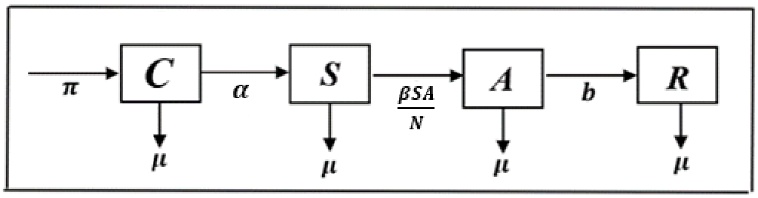


Figure 1. CSAR Model

Fig. 1 describes the transitions of individuals from C to S to A to R. Each arrow pointing towards the inside of the compartment represents a positive term in the differential equation, while the arrow pointing away from the compartment introduces a negative term. All parameters, π, α, μ, β and b, are assumed to be non-negative constants with the following interpretations: π represents the number of individuals entering the citizens class, α represents the rate between humans and humans using smartphones, μ represents the natural death rate, β represents the transmission rate between humans and addicted individuals, and b represents the recovery rate from smartphone addiction. In this case, a compartmental model provides a functional means of gaining a proper understanding of the dynamics [27]. The system of differential equations for the compartmental model in Fig. 1 is given by:

$$\begin{aligned} \frac{dS}{dt} &= -\mu S - \frac{\beta SA}{N} + \alpha C, \\ \frac{dA}{dt} &= -\mu A - bA + \frac{\beta SA}{N}, \\ \frac{dC}{dt} &= \pi - \mu C - \alpha C, \\ \frac{dR}{dt} &= -\mu R + bA. \end{aligned} \tag{5}$$

The domain of the solution from Eq. (5) is $\Omega = \{(C, S, A, R) \epsilon R^4 | 0 < C + S + A + R \leq N\}$ with initial conditions $C(0) > 0, S(0) \geq 0, A(0) \geq 0$, and $R(0) \geq 0$. Note that all parameters; π, μ, α, β, and b are non-negatives. See that,

$$\frac{dN}{dt}=\frac{dC}{dt}+\frac{dS}{dt}+\frac{dA}{dt}+\frac{dR}{dt}=\pi-\mu(C+S+A+R)=\pi-\mu N.$$

Since the total population is constant over time, so $\lim_{t\to\infty} N(t)=\frac{\pi}{\mu}$.

3.2 Equilibrium Point and Stability

The qualitative features of model Eq. (5) can be explained through two distinct equilibrium points, each representing the dynamic conditions of the population in the context of smartphone addiction. The first equilibrium point, E_0, represents the state before the occurrence of the "smartphone addiction outbreak," where the source population (citizens) remains largely unaffected by addiction. Meanwhile, the second equilibrium point, E_1, illustrates the state after the "smartphone addiction outbreak," where a significant portion of individuals has transitioned into the addiction compartment (A), thereby influencing the overall population dynamics.

3.2.1 Establishment of the Source Population before "Smartphone Addiction Outbreaks" (E_0)

In this article, the term "smartphone addiction outbreaks" is interpreted in the same way as a disease epidemic, where the number of individuals in the addicted sub-population $A(t)$ reaches a threshold, leading to an outbreak. Thus, the meaning of the Establishment of the source population before "smartphone addiction outbreaks" is the determination or initial identification of the population before a smartphone addiction outbreak occurs. This population serves as the foundation for studying how addiction develops from a state of zero addiction ($A=0$) to reach the threshold that triggers the outbreak. The equilibrium point $E_0=(C(t),S(t),A(t),R(t))$ is represented by:

$$E_0=\left[C=\frac{\pi}{\alpha+\mu},S=\frac{\pi\alpha}{\mu(\alpha+\mu)},A=0,R=0\right]. \tag{6}$$

As shown above, positivity of E_0 is always guaranteed without any terms.

To investigate the local stability criteria of E_0, first, we introduce the Jacobian matrix of the system in Eq. (5) in E_0, namely J_0, which is given by

$$J_0=\begin{bmatrix}-\mu-\alpha & 0 & 0 & 0\\ \alpha & -\mu & -\dfrac{\alpha\beta}{\alpha+\mu} & 0\\ 0 & 0 & \dfrac{\alpha\beta-(\alpha+\mu)(b+\mu)}{\alpha+\mu} & 0\\ 0 & 0 & b & -\mu\end{bmatrix}. \tag{7}$$

The characteristic polynomial of J_0 to determine their eigenvalues is given by

$$P_0(\lambda,\omega)=\frac{(\lambda+\mu)^2(\lambda+\alpha+\mu)(\alpha b-\alpha\beta+\alpha\lambda+\alpha\mu+b\mu+\lambda\mu+\mu^2)}{\alpha+\mu},$$

with λ is the eigenvalue and ω are all parameters in Eq. (5). Thus, Eq. (7) has three different eigenvalues,

$$\lambda_1=-\mu,\qquad \lambda_2=-\alpha-\mu,\ \lambda_3=-\frac{\mu^2+(\alpha+b)\mu+\alpha(b-\beta)}{\alpha+\mu}.$$

A system of differential equations will always be locally asymptotically stable if all its eigenvalues are negative [25]. It can be seen that E_0 has three distinct eigenvalues, where λ_1 and λ_2 are always negative, while λ_3 will be negative if $\frac{\alpha\beta}{(\alpha+\mu)(b+\mu)}<1$. Therefore, E_0 will be stable if this condition is met.

The stability of this system depends on the condition indicated by the third eigenvalue. If the system is stable, meaning it satisfies $\frac{\alpha\beta}{(\alpha+\mu)(b+\mu)}<1$, the population will remain in this state without the spread of addiction; if unstable, the population at this equilibrium point will be disrupted, causing the system to move towards the spread of smartphone addiction.

3.2.2 Establishment of the source population after "smartphone addiction outbreaks" (E_1)

The last equilibrium point, where all sub-populations are positive, represents a condition in the system where each compartment in the model has a non-zero number of individuals. This type of equilibrium is crucial for understanding the long-term behavior of the system, as it indicates the presence of a persistent and ongoing addiction within the population. The equilibrium points of this condition, $E_1 = (C(t), S(t), A(t), R(t))$ is represented by:

$$E_1 = (C^*, S^*, A^*, R^*), \tag{8}$$

with

$$C^* = \frac{\pi}{\alpha + \mu},$$
$$S^* = \frac{(b + \mu)\pi}{\beta\mu},$$
$$A^* = -\frac{(\alpha b - \alpha\beta + \alpha\mu + b\mu + \mu^2)\pi}{(b + \mu)(\alpha + \mu)\beta},$$
$$R^* = -\frac{(\alpha b - \alpha\beta + \alpha\mu + b\mu + \mu^2)b\pi}{(\alpha + \mu)(b + \mu)\mu\beta}.$$

To investigate the local stability criteria of E_1, we introduce the Jacobian matrix of the system in Eq. (5) in E_1 which is given by:

$$J1 = \begin{bmatrix} -\alpha - \mu & 0 & 0 & 0 \\ \frac{\alpha(\alpha + 2\mu)}{\alpha + \mu} - Z_1 & -\frac{Z_2\beta}{(b + \mu)} + Z_2 - Z_1 & -\frac{\alpha b + b\mu + \mu^2}{\alpha + \mu} - Z_1 & Z_2 - Z_1 \\ -Z_2 + Z_1 & \frac{Z_2\beta}{(b + \mu)} - \frac{(2\alpha + \mu)\mu}{\alpha + \mu} + Z_1 & -Z_2 + Z_1 & -Z_2 + Z_1 \\ 0 & 0 & b & -\mu \end{bmatrix}, \tag{9}$$

with $Z_1 = \frac{\mu(b+\mu)}{\beta}$, and $Z_2 = \frac{\alpha\mu}{\alpha+\mu}$.

The characteristic polynomial of J_1 to determine their eigenvalues is given by

$$P_1(\lambda, \omega) = -\frac{(\lambda + \mu)(\lambda + \alpha + \mu)\left(-(b + \mu)(\alpha + \mu)\lambda^2 - \alpha\beta\lambda\mu + \mu(b + \mu)(\alpha b - \alpha\beta + \alpha\mu + b\mu + \mu^2)\right)}{(\alpha + \mu)(b + \mu)},$$

with λ is the eigenvalue and ω are all the parameters in equation Eq. (5). All the eigenvalues should be negative to guarantee the local stability of E_1. Hence, the existence of an endemic equilibrium is satisfied if $\frac{\alpha\beta}{\mu(\alpha+b+\mu)+\alpha b} > 1$. Thus, if this condition is met, an endemic state will occur, indicating that smartphone addiction will persist and not disappear from the population.

Based on both cases, the source population before and after "smartphone addiction outbreaks", the stability criteria are dependent on the value of $R_0 = \frac{\alpha\beta}{(\alpha+\mu)(b+\mu)}$. The non-addicted, representing the state before "smartphone addiction outbreaks", will be stable when $R_0 < 1$ and representing the state after "smartphone addiction outbreaks", will be stable if $R_0 > 1$. Therefore, the next section will discuss numerical simulations to illustrate the dynamics of each strategy and situation, using variations in initial conditions and parameters with $R_0 < 1$ and $R_0 > 1$.

The next section will discuss the sensitivity analysis to identify the parameters that influence the stability of both cases in this study, specifically the variable R_0.

3.3 Sensitivity Analysis

In this model, sensitivity analysis is conducted using a partial derivative approach on key parameters to calculate the sensitivity index. The sensitivity index measures the proportional change in the model's solution in response to changes in a specific parameter. In this section, a sensitivity analysis will be conducted to identify the parameters that influence the stability of both cases in this study, namely the variable R_0. In this analysis, the parameters examined are α, β, and b. The sensitivity analysis is carried out using Definition [26]. The sensitivity index for the parameter α is obtained from:

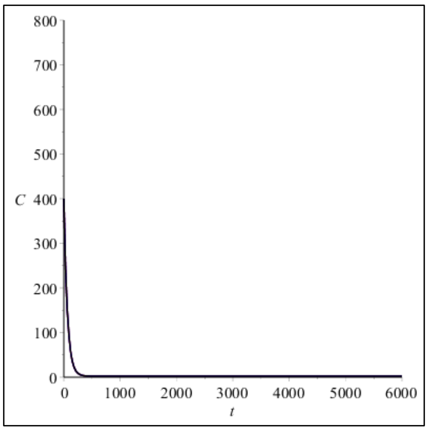
$$C_{\alpha}^{R_0} = \frac{\partial R_0}{\partial \alpha} \times \frac{\alpha}{R_0},$$

$$C_{\alpha}^{R_0} = \frac{\beta - \alpha\beta}{(b+\mu)(\alpha+\mu^2)} \times \frac{\alpha}{\frac{\alpha\beta}{(\alpha+\mu)(b+\mu)}} = \frac{\mu}{\alpha+\mu}.$$

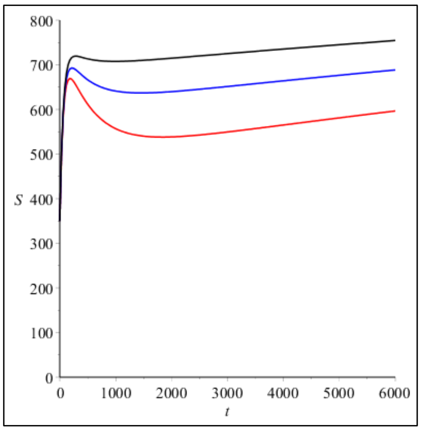
The same process is also applied to the other parameters, yielding the following results: $C_{\beta}^{R_0} = 1, C_{b}^{R_0} = -\frac{b}{b+\mu}$.

The results of the sensitivity analysis indicate the extent to which each parameter influences the basic reproduction number R_0, which serves as a key indicator of the potential spread and persistence of smartphone addiction in the model. The sensitivity index for the parameter α is $C_{\alpha}^{R_0} = \frac{\mu}{\alpha+\mu}$, suggesting that an increase in α has a positive but less than proportional effect on R_0. In other words, a 1% increase in α will result in an increase in R_0 of less than 1%, depending on the relative values of α and μ. This implies that while α, representing the transition rate into the addicted state, influences the dynamics of addiction; its impact is moderated by the recovery or transition-out rate μ. In contrast, the parameter β has a sensitivity index of $C_{\beta}^{R_0} = 1$, indicating a directly proportional relationship: a 1% increase in β leads to a 1% increase in R_0. This highlights β as the most influential parameter in determining the magnitude of R_0, as it governs the rate of contact or transmission of addictive behavior. Meanwhile, the parameter b has a negative sensitivity index, $C_{b}^{R_0} = -\frac{b}{b+\mu}$, indicating that an increase in b will decrease R_0. The larger the value of b, the more substantial the reduction in R_0. This underscores the role of b, which reflects the recovery or cessation rate from addictive smartphone use, as a controlling factor in mitigating the spread within the model.

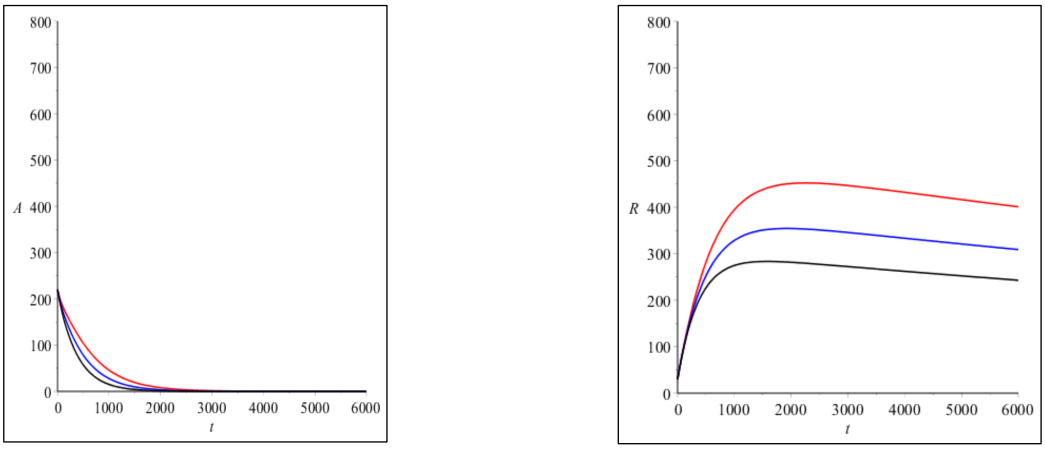
To illustrate the dynamics of each strategy and situation, a simulation was carried out using various parameter values and initial conditions. The first scenario assumes identical initial values but varies the transmission rate between smartphone users and addicted individuals (β), encompassing cases where $R_0 < 1$ and $R_0 > 1$. The second scenario involves varying initial values while keeping the beta value constant, covering conditions where $R_0 < 1$ and $R_0 > 1$. This simulation aims to analyze the impact of parameter variation and initial conditions on the spread of smartphone addiction, providing deeper insights into the effectiveness of different control approaches. The parameter values used in this study were chosen hypothetically for simulation purposes, as there is currently no empirical data available in the literature regarding the transmission dynamics of smartphone addiction. These values are not derived from real-world observations but are instead selected to illustrate the mathematical behavior of the proposed model. Future research is expected to validate and refine these parameters using empirical data once such studies become available.



(a)



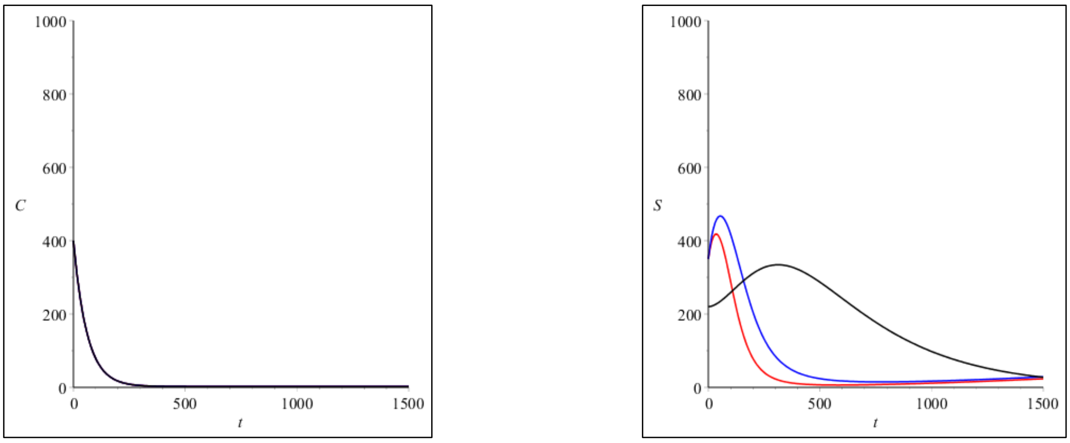
(b)



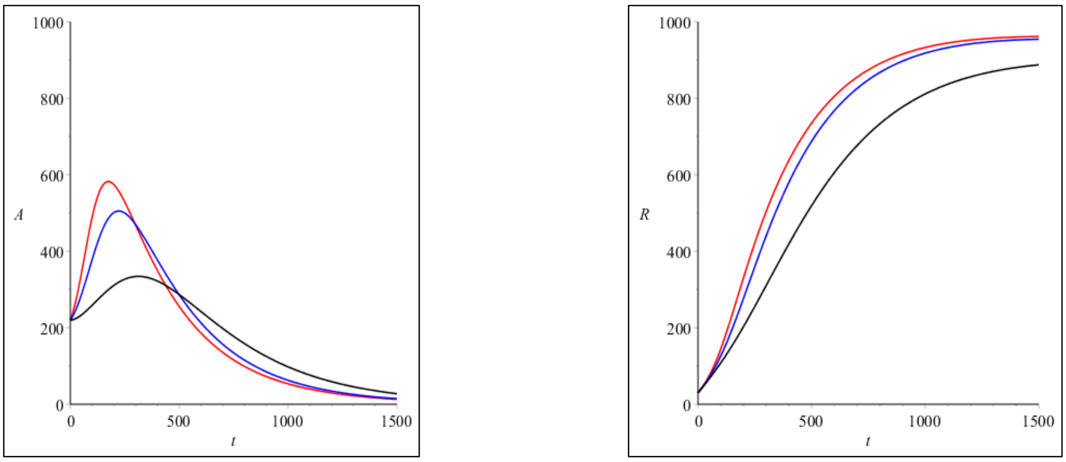
(c) (d)

Figure 2. Human Dynamics of Smartphone Addiction with Identical Initial Value but Different Addiction Rate (β) for $R_0 < 1$. The Initial Condition Is Given by $C(0) = 400, S(0) = 350, A(0) = 220, R(0) = 30$, Shown Respectively in Subfigures: (A) $C(t)$, (B) $S(t)$, (C) $A(t)$, and (D) $R(t)$

Fig. 2 illustrates the dynamics of the population of citizens, including non-active smartphone users $C(t)$, active smartphone users $S(t)$, addicted individuals $A(t)$, and recovered individuals $R(t)$, under the condition $R_0 < 1$, indicating that addiction does not spread widely. Variations in the transmission rate (β) show that, with the same initial value, a lower β keeps the population of addicted individuals $A(t)$ consistently low. Conversely, a higher β results in a temporary spike in $A(t)$, but the number of addicted individuals eventually decreases over time. This indicates that controlling the transmission rate, for example by reducing interactions between "healthy" smartphone users and addicted individuals, can prevent spikes in cases and accelerate recovery. Therefore, under $R_0 < 1$, controlling transmission is highly effective in managing the spread of addiction.



(a) (b)

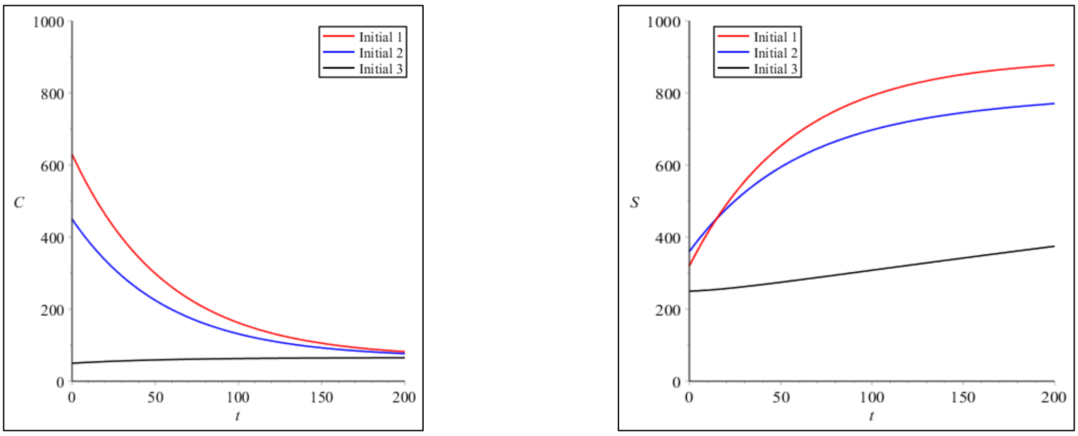


(c) **(d)**

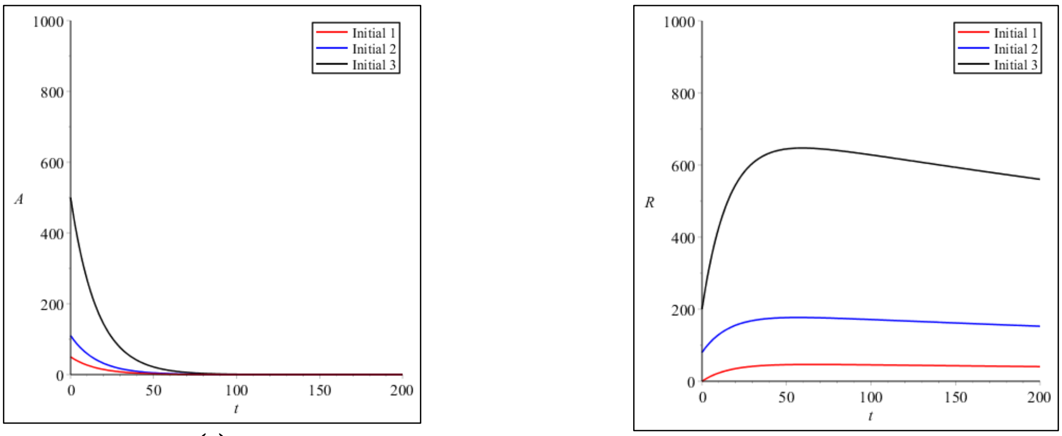
Figure 3. Human Dynamics of Smartphone Addiction with Identical Initial Value but Different Addiction Rate (β) for $R_0 > 1$. The Initial Condition Is Given by $C(0) = 400, S(0) = 350, A(0) = 220, R(0) = 30$, Shown Respectively in Subfigures: (a) $C(t)$, (b) $S(t)$, (c) $A(t)$, and (d) $R(t)$

Fig. 3 illustrates the population dynamics under the condition $R_0 > 1$, where, with the same initial value, smartphone addiction has the potential to spread widely within the population. Variations in the transmission rate (β) show that the higher the transmission rate, the faster and more significantly the population of addicted individuals $A(t)$ reaches its peak. This emphasizes the importance of reducing β to prevent addiction from becoming endemic within the population.

Fig. 2 and Fig. 3 also show that an increase in β can cause the spike to occur more quickly, meaning the peak addiction phase happens earlier and individuals in the recovery category $R(t)$ also increase faster after the peak is reached. This suggests that while a higher β increases the risk of addiction spreading, the dynamic mechanism in the model indicates that the recovery phase also begins sooner. This could be due to increased pressure or awareness that encourages addicted individuals to enter the recovery stage immediately after the peak of addiction has passed. However, it is important to note that a higher peak of addiction can have serious consequences for the population before the recovery phase occurs, so controlling β is crucial to prevent excessive impacts.



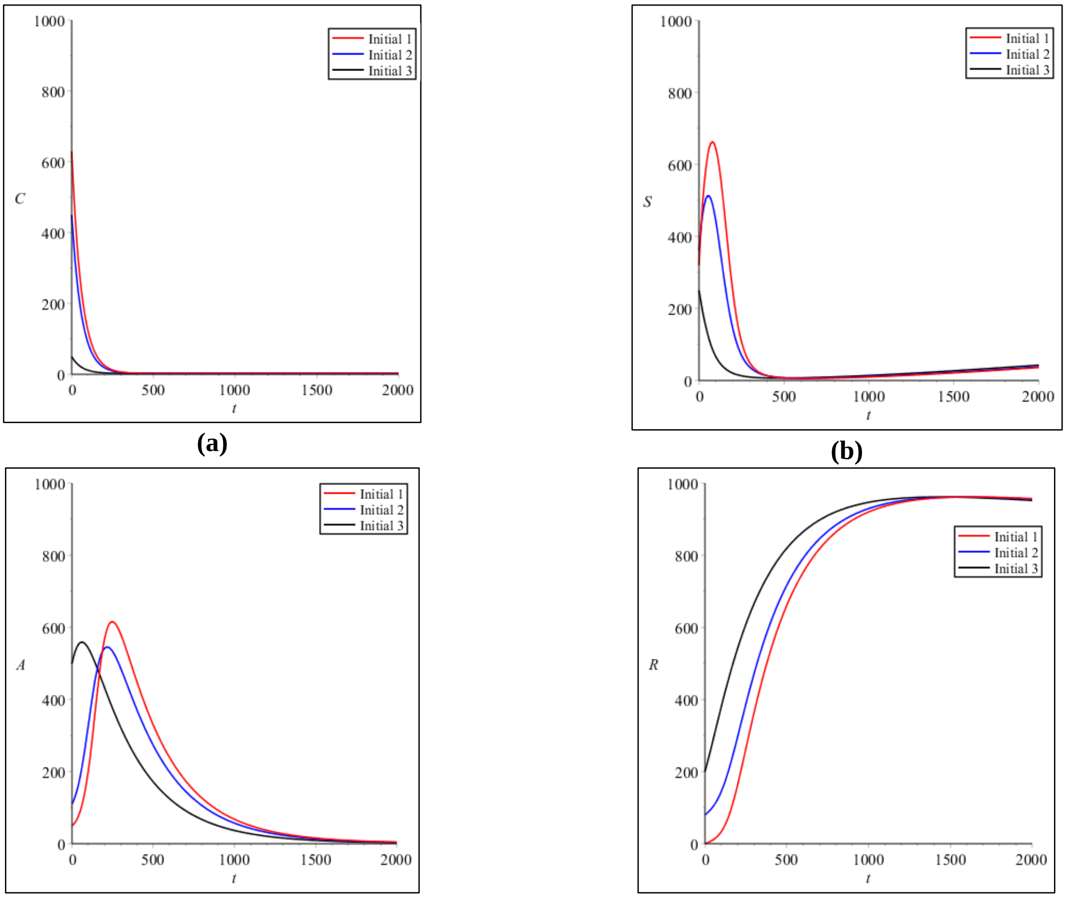
(a) **(b)**



(c) **(d)**

Figure 4. **Human Dynamics with Identical Parameter Values but Different Initial Values, All Resulting in $R_0 < 1$. Initial Values Are Given as $(630, 320, 50, 0)$, $(450, 360, 110, 80)$, $(50, 250, 500, 200)$ for Initial 1, Initial 2, and Initial 3, Respectively. Show in Subfigures : (a) $C(t)$, (b) $S(t)$, (c) $A(t)$, and (d) $R(t)$**

In Fig. 4, when $R_0 < 1$, variations in initial conditions show that even if the population of addicted individuals $A(t)$ initially high, their number will gradually decrease until it reaches a very low level. Differences in initial conditions also affect the time required to achieve stability. Higher initial conditions result in a larger addiction peak, but the final trend indicates that the population of addicted individuals can still be controlled in the long term. This demonstrates that $R_0 < 1$ is very effective in preventing the spread of addiction, regardless of the high initial conditions. Early intervention remains crucial to accelerate the population stabilization process, especially in situations with poor initial conditions.



(a) **(b)**

(c) **(d)**

Figure 5. **Human Dynamics with Identical Parameter Values but Different Initial Values, All Resulting in $R_0 > 1$. Initial Values Are Given as $(630, 320, 50, 0)$, $(450, 360, 110, 80)$, $(50, 250, 500, 200)$ for Initial 1, Initial 2, and Initial 3, Respectively. Show in Subfigures : (a) $C(t)$, (b) $S(t)$, (c) $A(t)$, and (d) $R(t)$**

Fig. 5 shows that in the condition where $R_0 > 1$, variations in the initial conditions have a significant impact on the spread of addiction. When the initial population of addicts $(A(t))$ is high, the number of addicted individuals reaches a larger peak and takes longer to decrease. Furthermore, recovery $R(t)$ is also slower in populations with high initial conditions. This indicates that in situations where $R_0 > 1$, poor initial conditions can exacerbate the long-term impact of addiction. Therefore, more aggressive interventions are necessary to reduce R_0 and mitigate the negative impact of high initial conditions, such as educational campaigns, access restrictions, or therapy-based interventions.

Based on the four figures, it can be concluded that the dynamics of smartphone addiction spread are heavily influenced by the value of R_0, the parameter value such as transmission rate (β), and the initial conditions of the population. When $R_0 < 1$, the spread of addiction can be controlled even with poor initial conditions, causing the number of addicted individuals $A(t)$ to gradually decrease until the population stabilizes. In contrast, when $R_0 > 1$, addiction spreads rapidly and can affect a large portion of the population, especially if the β value is high, or if the initial conditions indicate a large number of addicted individuals. The value of β plays a significant role in determining the speed of spread and the peak number of addicted individuals, where the higher the β value, the greater its impact on the population. Additionally, poor initial conditions at $R_0 > 1$ worsen the spread of addiction and prolong the time until stability is achieved. Moreover, unfavorable initial conditions when $R_0 > 1$ not only accelerate the spread but also prolong the time required for the system to stabilize. These findings indicate that effective interventions should focus on reducing β, for example through education, awareness campaigns, or restrictions on excessive smartphone use, as well as on improving initial conditions by providing early preventive measures. By doing so, the value of R_0 can be maintained below unity, ensuring that the spread of smartphone addiction remains under control and its long-term impact on the population can be minimized. Therefore, early interventions such as limiting interactions between smartphone users and those addicted are crucial to reduce β and improve initial conditions. With these measures, the spread of addiction can be effectively controlled, minimizing the negative impact on the population.

These results reinforce that the long-term behavior of the system is determined by whether the population moves toward an addiction-free equilibrium when $R_0 < 1$ or toward an endemic equilibrium when $R_0 > 1$. In practical terms, this means that maintaining R_0 below unity should be the central objective of intervention strategies. Reducing the transmission rate (β) through education, awareness campaigns, or restrictions on excessive smartphone use, together with improving initial conditions through preventive measures, can significantly decrease the peak number of addicted individuals and accelerate recovery. Such efforts not only limit the spread of addiction but also reduce the social and health consequences that arise before the recovery phase is reached [28]-[30]. Strengthening these preventive strategies, therefore, provides a clear pathway for policymakers and stakeholders to control smartphone addiction at the population level and ensure long-term stability.

4. CONCLUSION

This study developed a deterministic compartmental model to analyze the dynamics of smartphone addiction within a population, incorporating key parameters such as the transmission rate (α), addiction rate (β), and recovered rate (b). The results indicate that the basic reproduction number R_0 serves as a crucial indicator in determining whether addiction remains under control or spreads widely. When $R_0 < 1$, the number of addicted individuals tends to decrease until the system reaches stability. However, when $R_0 > 1$, there is a significant increase in the addicted population, especially when β is high or the initial population conditions are unfavorable. Sensitivity analysis shows that β has the greatest influence on increasing R_0, while b contributes to reducing it, highlighting the importance of early intervention. One effective strategy is to reduce the interactions between addicted and vulnerable individuals to lower the likelihood of vulnerable individuals imitating addictive smartphone use patterns. This can be achieved through education, the promotion of positive role models, access restrictions to limit excessive use, and awareness campaigns highlighting the risks of addiction. These findings offer a theoretical contribution to understanding smartphone addiction as a population-level phenomenon and provide a foundation for more effective prevention and recovery policies. Future research is recommended to validate this model using empirical data and to develop stochastic approaches that better capture individual behavioral variability.

Author Contributions

Amalina Amalina: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Writing, Editing. Nor Azah Samat: Conceptualization, Methodology, Formal analysis, Supervision. Ahmad Sabri: Investigation, Resources. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare no conflicts of interest to report study.

Declaration of Generative AI and AI-assisted Technologies

Generative AI tools (e.g., ChatGPT) were used solely for language refinement, including grammar, spelling, and clarity. The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. All final text was reviewed and approved by the authors.

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