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APPLICATION OF COOPERATIVE GAME THEORY: SOLVING THE PROBLEM OF TRAVELING BETWEEN FIVE CITIES IN JAVA

Muhammad Aditya Tri Ariyanto[1], **Rubono Setiawan**[2*]

[1,2]*Mathematics Education Department, Faculty of Teacher Training and Education, Universitas Sebelas Maret*
Jln. Ir. Sutami 36A, Surakarta, Jawa Tengah, 57126, Indonesia

*Corresponding author's e-mail: * **rubono.matematika@staff.uns.ac.id***

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ABSTRACT

Intercity travel between Surabaya, Madiun, Surakarta, Semarang, Cirebon, and Jakarta is increasingly facilitated by integrated infrastructure such as toll roads, railways, and airports. Among travelers, forming informal coalitions with others heading in the same direction has become a practical way to reduce individual travel expenses. This study aims to analyze the cost-efficiency of coalition-based intercity travel compared to solo travel, utilizing cooperative game theory to determine fair contribution values among travelers. This study aims to analyze the cost-efficiency of coalition-based intercity travel compared to solo travel, utilizing cooperative game theory to determine fair contribution values among travelers. A case study was conducted in an urban setting involving individuals who travel intercity using various transportation modes. Data were collected through semi-structured interviews. The study applied the Shapley value method within cooperative game theory to model and evaluate each participant's contribution in a travel coalition. An algorithm was developed to calculate Shapley values for different coalition scenarios. Initial expenditures were: Player A (IDR 210,000), B (IDR 240,000), C (IDR 60,000), D (IDR 400,000), and E (IDR 165,000). First calculation (5-player coalition): A spent IDR 122,750, B IDR 147,750, C IDR 77,750, D IDR -52,250, and E IDR 104,000. Player C opted out, as joining the coalition would cost more than traveling individually (IDR 60,000). Second calculation (4-player coalition): A spent IDR 113,750, B IDR 133,750, D IDR 53,750, and E IDR 98,750. The study's findings are based on a small sample with specific subject criteria and cannot be generalized to broader intercity travel scenarios. This research demonstrates the practical application of game theory—specifically the Shapley value—in modeling travel coalitions and optimizing cost distribution, offering insights for policy makers and transport planners in collaborative travel schemes.

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1. INTRODUCTION

Game theory can mathematically model a form of coalition between parties, such as the problem above. Game theory is a mathematical methodology for analyzing strategic interactions between individuals or groups [1]. Game theory is divided into two types, namely cooperative and non-cooperative [2]. Cooperative game theory provides a mathematical framework for modeling how self-interested agents form alliances and share costs or payoffs. Cooperative game theory focuses on analyzing strategic interactions in which players cooperate to achieve a common goal, often by fairly dividing profits or resources. The players basically form groups, and then they distribute the amount among them. In cooperative games, players distribute their payoffs [3]. To compute their payoff as much as their contribution, in cooperative games can use the core method, the Shapley value, and the ordinal bargaining set of NTU games [4]. In the real world, many cases can be modeled using cooperative game theory with those methods. It's because in the real world, naturally, humans are always bargaining with each other to get the most advantages for themselves. Shapley, nucleolus, and proportional methods in horizontal cargo collaborations, reporting 5–15% savings on shared routes. These works, however, focus on large enterprises or fleet operators rather than small groups of individual travelers [5]. Applications in transportation and logistics illustrate its capacity to reduce expenses and improve operational efficiency. Another previous study has shown that the results of Shapley values in game theory can benefit various parties, such as in the case of sago argo-industry supply chain actors [6] and marketing of local and national bottled drinking water [7]. Here, we want to show whether the results of previous research are also relevant to the case of intercity travel in a group of people that we are studying.

Investments in improving long-distance travel infrastructure are substantial and may have significant impacts on travel demand, the environment, and the economy [8]. Traveling intercity is now supported by various adequate facilities, such as toll roads, trains, and airports that are already connected. So that there are more and more choices that can be used to travel between cities. During the traveling process, the vehicle carrying efficiency is influenced by many factors. In intercity travel, the option to depart with colleagues who have the same destination is a profitable way to minimize expenses and be more efficient [9]. This can be interpreted as a coalition or cooperation between individuals. Coalition, in this case, means an agreement between two or more parties in an activity so that each party gets a more profitable result or payoff than if the activity were carried out individually.

Indonesia, the fourth most populous country globally, has a notably uneven population distribution, with 56.10% of the total population residing on Java Island, which represents only 7% of the country's total land area [10]. Despite the proliferation of ridesharing platforms and the rapid expansion of toll roads, railways, and airports across Java, there remains little research on cooperative cost sharing among passengers on intercity journeys. Addressing this gap carries both practical and academic value: transparent allocation mechanisms can lower per-capita travel expenses, optimize vehicle occupancy, mitigate environmental impacts, and underpin smart-mobility solutions and equitable pricing in emerging ridesharing services.

This study fills that void by developing and applying a Shapley value-based cost-sharing framework for intercity travel in Java. Our contributions are fourfold: (1) We formulate a cooperative game model for cost allocation among up to five passengers traveling from Surabaya to Madiun, Surakarta, Semarang, Cirebon, and Jakarta. (2) We design and implement a novel algorithm that efficiently computes Shapley values for multi-player coalitions, reducing computational complexity. (3) We perform comparative analyses of cost allocations across different coalition structures and route distances. (4) We derive policy recommendations to support practical, sustainable ridesharing schemes and inform regulators and platform developers.

By combining theoretical innovation with computational pragmatism, this research not only enriches the literature on cooperative game theory in transportation but also delivers a ready-to-use toolset for real-world intercity ridesharing initiatives. Therefore, an algorithm was developed to calculate Shapley values for different coalition scenarios.

2. RESEARCH METHODS

This research is a case study on traveling intercity in Java Island, Indonesia. The data that was used in this study is primary data. The sampling technique for determining subjects is purposive sampling. Subjects in this case are people who travel from Surabaya to other cities; data was obtained by interviewing the informants. The questions are about where their destinations are, what vehicle they use on their way to their destination, and how much it costs them to travel with their vehicle. After the sampling process, we got five subjects who traveled from our starting point, Surabaya. Each person traveled to Madiun, Surakarta, Semarang, Cirebon, and Jakarta. We assumed some limitations on these subjects, such as there is no requirement for a specific travel time or route to be taken. This study began with the preparation process in March 2024. Then continued with the process of collecting data from the informants, which was carried out in April 2024. After all data were obtained, it was processed in this study with the computation of cooperative game theory. The steps of this research can be seen in Fig. 1.

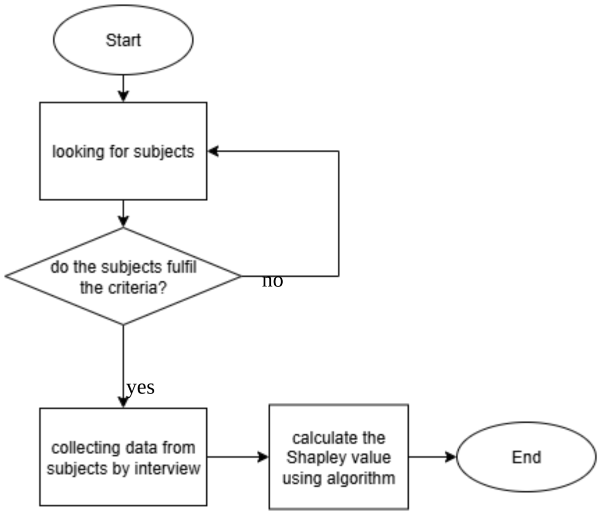

Figure 1. Flowchart of The Research

In this research, we use the Shapley value method. The computation of the Shapley value was assisted by an algorithm using the Python Language. The steps in this algorithm are organized as follows. First, input data from sources such as the initial spending of each player and their destination. The next step is to input the data on the payoff value for each strategy. The last step is computing the Shapley value between players. If each player gets a profit, then all players agree to join the coalition. It means the strategy that was agreed upon is the most optimal. If there is one or more players who suffer a loss, the losing players cancel the coalition. By following the rational rules of the cooperative game theory [11], if there is no benefit gained in the coalition, there are no intentions to join the coalition. And so, the Shapley value will re-find without including the losing players until all the coalition players gain benefits. Lastly, combine all the final strategy choices as a final decision agreed upon by each player. The disadvantage of this algorithm is that it can only calculate Shapley values for up to five players. The flowchart of the algorithm can be seen in Fig. 2.

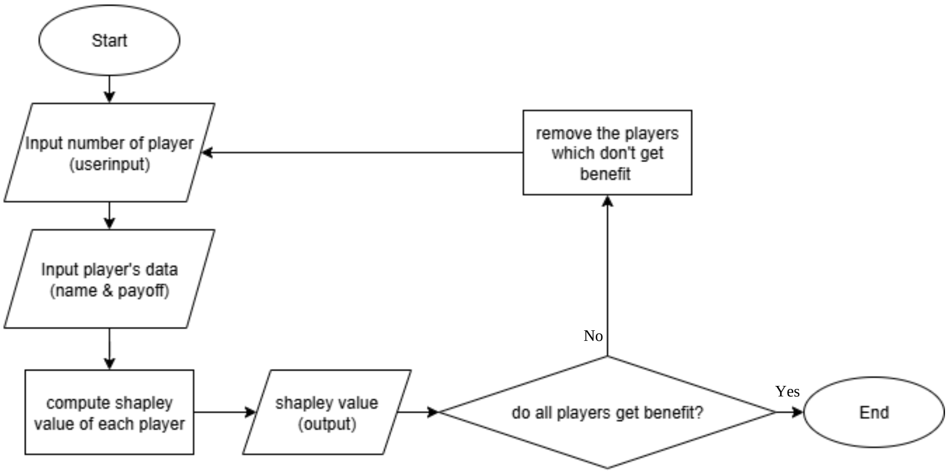

Figure 2. Flowchart of Shapley Value Algorithm

2.1 Game Theory

Game theory is the study of conflict and cooperation between interacting controllers, where the identities (who are the controllers?) and preferences (what are they trying to achieve?) depend on the specific setting [12]. Game theory analyzes the dynamics of conflict and cooperation among rational decision-makers, referred to as players [13]. A game in game theory is defined as a situation in which the intelligent decisions of individual players are interdependent. In this situation, the utility payoff of a single player depends upon her strategy and also the strategy of other agents [14]. Game theory is a mathematical framework that analyzes interactions in situations where individuals or groups have conflicting interests [15]. Game theory has conditions for the existence of games to be fulfilled. Each player has to logically analyze the best way to achieve their ends. In other words, rational play assumes rational opponents [16].

Game theory can be defined as a theory used in deciding on a competitive activity or rational competition, meaning that each player competes as human logic works. Each player is not allowed to deliberately lose so that the competition can continue to run rationally. The competition does not only occur between two people but can also occur between several people. Game theory is often used to make decisions based on the optimal strategy. The strategy taken by each player will mutually influence the strategy or condition of the other players.

Game theory can be divided into two types, namely cooperative games and non-cooperative games [17]. Cooperative games are a type of game where participants or players work together to achieve a common goal. Meanwhile, non-cooperative games are games where each player does not cooperate or agree on their strategy. Each player tries to maximize their profit using their strategy.

2.2 Payoff

The payoff in cooperative game theory is a value that indicates the expectation of a player's victory in a particular coalition. The characteristic function is used in solving n-player games by allowing cooperation between players. For example, the number of players in the game is denoted as n, with $n \geq 2$, and N is the set of players, $N = \{1, 2, \dots, n\}$. A coalition between players is symbolized as S, with $v(S)$ being the value of the coalition S. It should be noted that S is a subset of N. If $S = N$, then S is a grand coalition, which is a coalition involving all players. Furthermore, there is $\emptyset$ which is an empty coalition, and $\emptyset \in S$. The number of coalitions between n-players can be found by the formula 2^N. It is given three definitions related to the characteristic function [18], including:

Definition 1. *Let $N = \{1, 2, \dots, n\}$ be the set of all players, for an n-player game, any non-empty subset of N is called cooperation.*

Definition 2. *The payoff of an n-player game is a function of real numbers v defined on a subset of the set of players N, where for each $S \subset N$ is the maximin value of a two-player game played between S and $N - S$ cooperation.*

Definition 3. *An n-player game in characteristic function form is a function of real numbers v defined on a subset of players N that satisfies:*

1. $v(\emptyset) = 0$,
2. If $S \cap T = \emptyset$, then $v(S \cup T) \geq v(S) + v(T)$ with $S, T \subset N$.

2.3 Coalition

Cooperative game theory, also known as coalitional game theory, is designed to model situations in which players form groups (i.e., coalitions) rather than acting individually. A central notion in coalitional game theory is the notion of the core. The core is the set of payoff allocations that guarantees that no group of players has an incentive to leave its coalition to form another coalition [19]. Coalition or cooperation in cooperative game theory is one type of strategy carried out by several players that aims to create a joint decision or agreement that provides more benefits than when the players act individually.

In cooperative games, there is no single winner. Instead, each player tries to work together and help each other to achieve a predetermined goal. The motivation is that each player can likely obtain better utility by forming groups and controlling their power cooperatively rather than individually. The joint agreement will contain how the players receive the final payoff that must be shared between coalition members. Cooperative games can be represented by forming a group of players involved in a coalition or collaboration, with the aim that the players gain more benefits than when they do not work together.

Here are the definitions of a coalition game:

Definition 4 [20]**.** *The coalition game with transferable utility is a pair (N, v) where N is a finite set of players and $v: 2n \to R$ is a characteristic function, with $2n$ were the coalitions that formed, where $S \subset N$*

Definition 5 [20]**.** *The coalition form of a game with n-players is shown by the pair (N, v), where $N = \{1,2, \ldots, n\}$ is a set of players and v is a real function. Let's say Function v is a characteristic function of a game. The function v is defined on the set of 2^N, over all coalitions for which the coalitions are subsets of N.*

The values of the coalition depend only on the characteristics and function of the game [21]. In the simplest cases, let us imagine a partition of the set N of players into a priori unions,

$$T = \{T_1, \ldots, T_m\}.$$

Where the unions or coalitions T_k are pairwise disjoint, and their union is N. Let us think of these unions as prior agreements among players [21].

2.4 Shapley Value

The Shapley Value was introduced by Lloyd Shapley in 1953 to solve n-player games. The calculation of the Shapley value uses the basic concept of characteristic functions and Shapley axioms. The Shapley value can also be used to measure the contribution of players in a game. A function ϕ is a function that encapsulates every possible payoff function formed from n-players. The notation $\phi_i(v)$ represents the Shapley value of player i in a game with characteristic function v [21]. Shapley approaches his value axiomatically. We give two definitions first.

Definition 6 [18]**.** *A carrier for a game is a coalition such that, for any S, $v(S) = v(S \cap T)$.*

Definition 4 states that any player who does not belong to a carrier is a dummy — i.e., can contribute nothing to any coalition.

Definition 7 [18]**.** *Let v be an n-person game, and let π be any permutation of the set N. Then, by πv we mean the game u such that, for any $S = \{i_1, i_2, \ldots, i_s\}$*

$$u(\{\pi(i_2), \ldots, \pi(i_s)\}) = v(S).$$

Effectively, the game πs is nothing other than the game v, with the roles of the players interchanged by the permutation π. With these two definitions, it is possible to give an axiomatic treatment. We point out merely that, as games are essentially real-valued functions, it is possible to talk of the sum of two or more games, or of several times a game. There are four axioms of fairness placed on a function ϕ [22].

Axiom 1. *Efficiency is the total amount of payoff divided equally by the value of the grand coalition.*

$$\Sigma_{i \in N} \phi_i(v) = v(N).$$

Axiom 2. *Symmetry, players who contribute equally will get the same payoff. If i and j satisfy*

$$v(S \cup \{i\}) = v(S \cup \{j\}).$$

Axiom 3. *Dummy, the income for an empty player is zero; if the player does not contribute anything, then the player also gets nothing. If i satisfy the conditions:*

$$v(S) = v(S \cup \{i\}).$$

For every coalition of S that does not contain i, then $\phi_i(v) = 0$.

Axiom 4. *Additivity, if u and v are the characteristic functions, then $\phi(u + v) = \phi(u) + \phi(v)$.*

Shapley value in a game is given by the equation $\phi_i = (\phi_1, \phi_2, \ldots, \phi_n)$ which is for each $i = 1, 2, \ldots, n$. Therefore, the equation of the Shapley value is [23]:

$$\phi_i(v) = \Sigma_{S \subset_{i \in S} N} \frac{(|s| - 1)!\,(n - |s|)!}{n!} \big(v(S) - v(S - \{i\})\big).$$

Where:

$\phi_i(v)$ = Shapley value of player-i.
n = Number of players on v.
v = Payoff value from each player.
S = Subset of without player-i.
$|s|$ = Sum of players on S.
$v(S)$ = Payoffs for the coalition of S.

The Shapley value of each player represents the results of the coalition between players. The calculation of the Shapley value can be accepted if the profit obtained by each player after forming a coalition is greater than the profit when the player carries out activities individually.

3. RESULTS AND DISCUSSION

3.1 Results

In this study, we interviewed five people who will travel from Surabaya. Each of them is going to Madiun, Surakarta, Semarang, Cirebon, and Jakarta. Let us say Surabaya is the Point O, Madiun is the Point P, Surakarta is the Point Q, Semarang is the Point R, Cirebon is the Point S, and Jakarta is the Point T. They make a trip to each city by a different mode of transportation, namely, using buses, travel, trains, or a car. Each player has their preferences for making this trip, which are also adjusted to their respective abilities.

Player A travels to the point R by train, with a cost of IDR 210,000. Player B travels to Point S by bus, with a cost of IDR 240,000. Player C travels to Point P using a Shuttle coach, with a cost of IDR 60,000. Player D travels to Point T by car driving, with a cost of IDR 400,000 for the fuel and accommodation. And lastly, Player E travels to Point Q by train, with a cost of IDR 165,000. If the players are traveling together, they will choose the modes of transportation that cost less and reach further cities. An illustration of this route can be seen in Fig. 3.

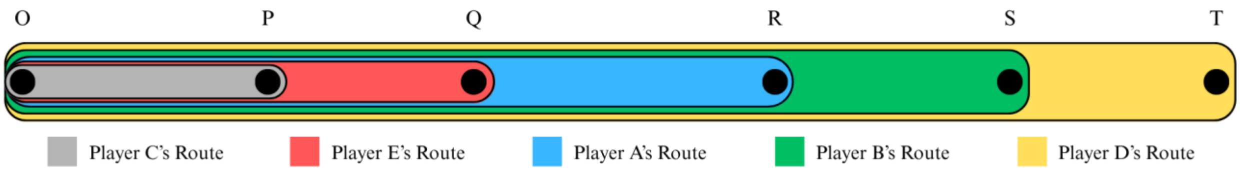

Figure 3. Illustration of The Player's Route

To solve the problem of fair and efficient travel cost sharing, cooperative game theory can be used by applying the Shapley value. The Shapley value will divide the total cost saved by the coalition to each member based on their marginal contribution. The general Shapley value methodology is illustrated, as well as an important particular case in which each participant uses only a portion of the largest participant's asset [24]. In addition, we also need to consider the total individual cost without a coalition. If there is no coalition, then the individual costs incurred by players A, B, C, D, and E are as follows:

Table 1. Individual Cost to Travel from Surakarta to Each Destination

Player	City to Go	Cost (IDR)	Expenses
A	R	210000	Train ticket
B	S	240000	Bus ticket
C	P	60000	Shuttle coach ticket
D	T	400000	Fuel for the car
E	Q	165000	Train ticket

Analytical Construction & Real Modeling Process

Before we calculate using the Shapley value, let's try to make a comparison of what if these players travel together using player $\boldsymbol{D}$'s car (because this mode accommodates all destinations in its route), and then split the cost equally. By using $\boldsymbol{D}$'s car they spent IDR 400,000 in total, and then split it equally among five people. So, they have to spend IDR 80,000 each. But player $\boldsymbol{C}$ spent more money than if he had traveled alone, which would only have cost him IDR 60,000. Because of this disadvantage, we use the Shapley Value method because this is fairer, with each player getting a portion of the profits according to their role in the coalition.

1. Determining the payoff between five players.
   From the problem above, the payoff for the five players is as follows:

Table 2. Payoff from All Combinations between Players A, B, C, D, and E

Strategy	Payoff
v(∅)	0
v(A)	210000
v(B)	240000
v(C)	60000
v(D)	400000
v(E)	165000
v(A,B)	480000
v(A,C)	420000
v(A,D)	400000
v(A,E)	420000
v(B,C)	480000
v(B,D)	400000
v(B,E)	480000
v(C,D)	400000
v(C,E)	330000
v(D,E)	400000
v(A,B,C)	330000
v(A,B,D)	400000
v(A,B,E)	720000
v(A,C,D)	400000
v(A,C,E)	630000
v(A,D,E)	400000
v(B,C,D)	400000
v(B,C,E)	720000
v(B,D,E)	400000
v(C,D,E)	400000
v(A,B,C,D)	400000
v(A,B,C,E)	960000
v(A,B,D,E)	400000
v(A,C,D,E)	400000
v(B,C,D,E)	400000
v(A,B,C,D,E)	400000

2. Calculating the Shapley value of each player
The Shapley value between five players can be calculated by looking at the average sum of marginal contributions between players from all permutations. Here are the computations of the Shapley value for each player:

a. Shapley value of Player A:

$$\begin{aligned}\phi_A\,(5,v) = \frac{1}{5!}\times\Big\{&\Big((0)!\,(4)!\,\big(v(A)\big) + \Big((1)!\,(3)!\,\big(v(AB)-v(B)\big)\Big)\\
&+\Big((1)!\,(3)!\,\big(v(AC)-v(C)\big)\Big) + \Big((1)!\,(3)!\,\big(v(AD)-v(D)\big)\Big)\\
&+\Big((1)!\,(3)!\,\big(v(AE)-v(E)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ABC)-v(BC)\big)\Big)\\
&+\Big((2)!\,(2)!\,\big(v(ABD)-v(BD)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ABE)-v(BE)\big)\Big)\\
&+\Big((2)!\,(2)!\,\big(v(ACD)-v(CD)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ACE)-v(CE)\big)\Big)\\
&+\Big((2)!\,(2)!\,\big(v(ADE)-v(CE)\big)\Big) + \Big((3)!\,(1)!\,\big(v(ABCD)-v(BCD)\big)\Big)\\
&+\Big((3)!\,(1)!\,\big(v(ABCE)-v(BCE)\big)\Big)\\
&+\Big((3)!\,(1)!\,\big(v(ABDE)-v(BDE)\big)\Big)\\
&+\Big((3)!\,(1)!\,\big(v(ACDE)-v(CDE)\big)\Big)\\
&+\Big((4)!\,(0)!\,\big(v(ABCDE)-v(BCDE)\big)\Big)\Big)\Big\}\end{aligned}$$

$$\begin{aligned}\phi_A\,(5,v) = \frac{1}{120}&\\
\times\big\{&\big(24\times(210{,}000)\big) + \big(6\times(480{,}000-240{,}000)\big)\\
&+\big(6\times(420{,}000-60.000)\big) + \big(6\times(400{,}000-400{,}000)\big)\\
&+\big(6\times(420{,}000-165{,}000)\big) + \big(4\times(720{,}000-480{,}000)\big)\\
&+\big(4\times(400{,}000-400{,}000)\big) + \big(4\times(720{,}000-480{,}000)\big)\\
&+\big(4\times(400{,}000-400{,}000)\big) + \big(4\times(630{,}000-330{,}000)\big)\\
&+\big(4\times(400{,}000-400{,}000)\big) + \big(6\times(400{,}000-400{,}000)\big)\\
&+\big(6\times(960{,}000-720{,}000)\big) + \big(6\times(400{,}000-400{,}000)\big)\\
&+\big(6\times(400{,}000-400{,}000)\big) + \big(24\times(400{,}000-400{,}000)\big)\big\}\end{aligned}$$

$$\therefore\ \phi_A\,(5,v) = 122{,}750$$

b. Shapley value of Player B:

$$\begin{aligned}\phi_B\,(5,v) = \frac{1}{5!}\times\Big\{&\Big((0)!\,(4)!\,\big(v(B)\big) + \Big((1)!\,(3)!\,\big(v(AB)-v(A)\big)\Big)\\
&+\Big((1)!\,(3)!\,\big(v(BC)-v(C)\big)\Big) + \Big((1)!\,(3)!\,\big(v(BD)-v(D)\big)\Big)\\
&+\Big((1)!\,(3)!\,\big(v(BE)-v(E)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ABC)-v(AC)\big)\Big)\\
&+\Big((2)!\,(2)!\,\big(v(ABD)-v(AD)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ABE)-v(AE)\big)\Big)\\
&+\Big((2)!\,(2)!\,\big(v(BCD)-v(CD)\big)\Big) + \Big((2)!\,(2)!\,\big(v(BCE)-v(CE)\big)\Big)\\
&+\Big((2)!\,(2)!\,\big(v(BDE)-v(DE)\big)\Big) + \Big((3)!\,(1)!\,\big(v(ABCD)-v(ACD)\big)\Big)\\
&+\Big((3)!\,(1)!\,\big(v(ABCE)-v(ACE)\big)\Big) + \Big((3)!\,(1)!\,\big(v(ABDE)-v(ADE)\big)\Big)\\
&+\Big((3)!\,(1)!\,\big(v(BCDE)-v(CDE)\big)\Big) + \Big((4)!\,(0)!\,\big(v(ABCDE)-v(ACDE)\big)\Big)\Big)\Big\}\end{aligned}$$

$$\phi_B\ (5, v) = \frac{1}{120} \times \{(24 \times (240{,}000)) + (6 \times (480{,}000 - 210{,}000)) + (6 \times (480{,}000 - 60{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (6 \times (480{,}000 - 165{,}000)) + (4 \times (720{,}000 - 420{,}000)) + (4 \times (400{,}000 - 400{,}000)) + (4 \times (720{,}000 - 165{,}000)) + (4 \times (400{,}000 - 400{,}000)) + (4 \times (720{,}000 - 330{,}000)) + (4 \times (400{,}000 - 400{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (6 \times (960{,}000 - 630{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (24 \times (400{,}000 - 400{,}000))\}$$

$$\therefore \phi_B\ (5, v) = 147{,}750$$

c. Shapley value of Player C:

$$\phi_C\ (5, v) = \frac{1}{5!} \times \{((0)!\,(4)!\,(v(C)) + ((1)!\,(3)!\,(v(AC) - v(A))) + ((1)!\,(3)!\,(v(BC) - v(B))) + ((1)!\,(3)!\,(v(CD) - v(D))) + ((1)!\,(3)!\,(v(CE) - v(E))) + ((2)!\,(2)!\,(v(ABC) - v(AB))) + ((2)!\,(2)!\,(v(ACD) - v(AD))) + ((2)!\,(2)!\,(v(ACE) - v(AE))) + ((2)!\,(2)!\,(v(BCD) - v(BD))) + ((2)!\,(2)!\,(v(BCE) - v(BE))) + ((2)!\,(2)!\,(v(CDE) - v(DE))) + ((3)!\,(1)!\,(v(ABCD) - v(ABD))) + ((3)!\,(1)!\,(v(ABCE) - v(ABE))) + ((3)!\,(1)!\,(v(ACDE) - v(ADE))) + ((3)!\,(1)!\,(v(BCDE) - v(BDE))) + ((4)!\,(0)!\,(v(ABCDE) - v(ABDE))))\}$$

$$\phi_C\ (5, v) = \frac{1}{120} \times \{(24 \times (60{,}000)) + (6 \times (420{,}000 - 210{,}000)) + (6 \times (480{,}000 - 240{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (6 \times (330{,}000 - 165{,}000)) + (4 \times (720{,}000 - 480{,}000)) + (4 \times (400{,}000 - 400{,}000)) + (4 \times (630{,}000 - 420{,}000)) + (4 \times (400{,}000 - 400{,}000)) + (4 \times (720{,}000 - 480{,}000)) + (4 \times (400{,}000 - 400{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (6 \times (960{,}000 - 630{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (24 \times (400{,}000 - 400{,}000))\}$$

$$\therefore \phi_C\ (5, v) = 77{,}750$$

d. Shapley value of Player D:

$$\phi_D\ (5,v) = \frac{1}{5!} \times \Big\{\Big((0)!\,(4)!\,\big(v(D)\big) + \Big((1)!\,(3)!\,\big(v(AD) - v(A)\big)\Big) + \Big((1)!\,(3)!\,\big(v(BD) - v(B)\big)\Big) + \Big((1)!\,(3)!\,\big(v(CD) - v(C)\big)\Big) + \Big((1)!\,(3)!\,\big(v(DE) - v(E)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ABD) - v(AB)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ACD) - v(AC)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ADE) - v(AE)\big)\Big) + \Big((2)!\,(2)!\,\big(v(BCD) - v(BC)\big)\Big) + \Big((2)!\,(2)!\,\big(v(BDE) - v(BE)\big)\Big) + \Big((2)!\,(2)!\,\big(v(CDE) - v(CE)\big)\Big) + \Big((3)!\,(1)!\,\big(v(ABCD) - v(ABD)\big)\Big) + \Big((3)!\,(1)!\,\big(v(ABDE) - v(ABE)\big)\Big) + \Big((3)!\,(1)!\,\big(v(ACDE) - v(ACE)\big)\Big) + \Big((3)!\,(1)!\,\big(v(BCDE) - v(BCE)\big)\Big) + \Big((4)!\,(0)!\,\big(v(ABCDE) - v(ABCE)\big)\Big)\Big)\Big\}$$

$$\phi_D\ (5,v) = \frac{1}{120} \times \big\{\big(24 \times (400{,}000)\big) + \big(6 \times (400{,}000 - 210{,}000)\big) + \big(6 \times (400{,}000 - 240{,}000)\big) + \big(6 \times (400{,}000 - 60{,}000)\big) + \big(6 \times (400{,}000 - 165{,}000)\big) + \big(4 \times (400{,}000 - 480{,}000)\big) + \big(4 \times (400{,}000 - 420{,}000)\big) + \big(4 \times (400{,}000 - 420{,}000)\big) + \big(4 \times (400{,}000 - 480{,}000)\big) + \big(4 \times (400{,}000 - 480{,}000)\big) + \big(4 \times (400{,}000 - 330{,}000)\big) + \big(6 \times (400{,}000 - 720{,}000)\big) + \big(6 \times (400{,}000 - 720{,}000)\big) + \big(6 \times (400{,}000 - 630{,}000)\big) + \big(6 \times (400{,}000 - 720{,}000)\big) + \big(24 \times (400{,}000 - 960{,}000)\big)\big\}$$

$$\therefore \phi_D\ (5,v) = -52{,}250$$

e. Shapley value of Player E:

$$\phi_E\ (5,v) = \frac{1}{5!} \times \Big\{\Big((0)!\,(4)!\,\big(v(E)\big) + \Big((1)!\,(3)!\,\big(v(AE) - v(A)\big)\Big) + \Big((1)!\,(3)!\,\big(v(BE) - v(B)\big)\Big) + \Big((1)!\,(3)!\,\big(v(CE) - v(C)\big)\Big) + \Big((1)!\,(3)!\,\big(v(DE) - v(D)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ABE) - v(AB)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ACE) - v(AC)\big)\Big) + \Big((2)!\,(2)!\,\big(v(ADE) - v(AD)\big)\Big) + \Big((2)!\,(2)!\,\big(v(BCE) - v(BC)\big)\Big) + \Big((2)!\,(2)!\,\big(v(BDE) - v(BD)\big)\Big) + \Big((2)!\,(2)!\,\big(v(CDE) - v(CD)\big)\Big) + \Big((3)!\,(1)!\,\big(v(ABCE) - v(ABC)\big)\Big) + \Big((3)!\,(1)!\,\big(v(ABDE) - v(ABD)\big)\Big) + \Big((3)!\,(1)!\,\big(v(ACDE) - v(ACD)\big)\Big) + \Big((3)!\,(1)!\,\big(v(BCDE) - v(BCD)\big)\Big) + \Big((4)!\,(0)!\,\big(v(ABCDE) - v(ABCD)\big)\Big)\Big)\Big\}$$

$$\phi_E\ (5, v) = \frac{1}{120} \times \{(24 \times (165{,}000)) + (6 \times (420{,}000 - 210{,}000)) + (6 \times (480{,}000 - 240{,}000)) + (6 \times (330{,}000 - 60{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (4 \times (720{,}000 - 480{,}000)) + (4 \times (630{,}000 - 420{,}000)) + (4 \times (400{,}000 - 400{,}000)) + (4 \times (720{,}000 - 480{,}000)) + (4 \times (400{,}000 - 400{,}000)) + (4 \times (400{,}000 - 400{,}000)) + (6 \times (960{,}000 - 720{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (6 \times (400{,}000 - 400{,}000)) + (24 \times (400{,}000 - 400{,}000))\}$$

$$\therefore \phi_E\ (5, v) = 104{,}000$$

To summarize the computation, we can use an algorithm. This algorithm calculates the Shapley value of each player and can calculate the value for up to five players. This algorithm quantifies each individual's contribution to the coalition by distributing costs based on fairness principles from cooperative game theory. The algorithm can be written as follows:

Algorithm 1. Shapley Value Algorithm

Input: Number of players, choose between 1 to 5.
Value of each player's payoff, $v(A)$, $v(B)$, $v(C)$, $v(D)$, and $v(E)$.
Coalition value of each player, $v(AB)$, $v(AC)$, $v(AD)$, $v(AE)$, $v(BC)$, $v(BD)$, $v(BE)$, $v(CD)$, $v(CE)$, $v(DE)$, $v(ABC)$, $v(ABD)$, $v(ABE)$, $v(ACD)$, $v(ACE)$, $v(ADE)$, $v(BCD)$, $v(BCE)$, $v(BDE)$, $v(CDE)$, $v(ABCD)$, $v(ABCE)$, $v(ABDE)$, $v(ACDE)$, $v(BCDE)$, and $v(ABCDE)$.
Output: Shapley value of each player

1. Choose the number of players (in this case is 5 players).
2. Compute P_{ij}, for $i = 1; 2; 3; \dots; 120$, $j = 1; 2; 3; 4; 5$.
3. Add up all the values in each column, X_k.
4. Compute the average of each column, $\frac{X_k}{120}$, $i = (1; 2; 3; 4; 5)$.

From the calculation of the algorithm, we can obtain the Shapley value of players A, B, C, D, and E as follows:

Table 3. Shapley value from Player A, B, C, D, and E

	ϕ_A	ϕ_B	ϕ_C	ϕ_D	ϕ_E
Shapley Value	122,750	147,750	77,750	-52,250	104,000

The value obtained from the Shapley value shows the contribution of each player to the distribution of travel accommodation costs. Player A's contribution is IDR122,750, Player B's contribution is IDR 147,750, Player C's contribution is IDR 77,750, Player D's contribution is IDR -52,250, and Player E's contribution is IDR 104,000. The result of player C's contribution to the coalition is IDR 77,250, which is more than $v(C)$ = IDR 60,000. This means that the condition $\forall \phi_i < v_i$ is not fulfilled,

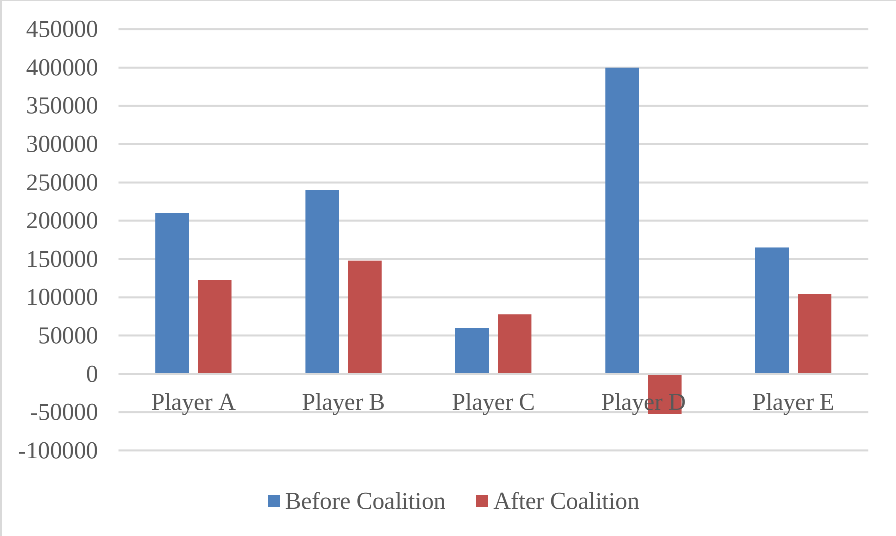

Figure 4. Cost Comparison Before and After the Coalition of 5 Players

Therefore, Player C refuses to join the coalition and chooses to travel by himself using coach travel. Because after forming a coalition, C actually spends more money than before forming a coalition. This decision reflects a real-world scenario where certain individuals may opt out of collaborative cost-sharing if their marginal contribution is negative. Such outcomes highlight the fragility of coalition stability, a critical concept in cooperative game theory. Therefore, we will calculate again the Shapley value involving Player $A, B, D,$ and E.

3. Determining the payoff between four players.
   From the new problem, the payoff for the four players is as follows:

Table 4. Payoff from All Combinations between Player A, B, D, and E

Strategy	Payoff
$v(\emptyset)$	0
$v(A)$	210000
$v(B)$	240000
$v(D)$	400000
$v(E)$	165000
$v(A,B)$	480000
$v(A,D)$	400000
$v(A,E)$	420000
$v(B,D)$	400000
$v(B,E)$	480000
$v(D,E)$	400000
$v(A,B,D)$	400000
$v(A,B,E)$	720000
$v(A,D,E)$	400000
$v(B,D,E)$	400000
$v(A,B,D,E)$	400000

4. Calculating the Shapley value of each player
   As the computation before, the Shapley value between four players can be calculated by looking at the average sum of marginal contributions between players from all permutations.

a. Shapley value of Player A:

$$\phi_A(4,v) = \frac{1}{4!} \times \Big\{\Big((0)!\,(3)!\,\big(v(A)\big) + \Big((1)!\,(2)!\,\big(v(AB) - v(B)\big)\Big) + \Big((1)!\,(2)!\,\big(v(AD) - v(D)\big)\Big) + \Big((2-1)!\,(4-|2|)!\,\big(v(AE) - v(E)\big)\Big) + \Big((2)!\,(1)!\,\big(v(ABD) - v(BD)\big)\Big) + \Big((2)!\,(1)!\,\big(v(ABE) - v(BE)\big)\Big) + \Big((2)!\,(1)!\,\big(v(ADE) - v(DE)\big)\Big) + \Big((3)!\,(0)!\,\big(v(ABDE) - v(BDE)\big)\Big)\Big)\Big\}$$

$$\phi_A(4,v) = \frac{1}{24} \times \{(6 \times (240{,}000)) + (2 \times (480{,}000 - 240{,}000)) + (2 \times (400{,}000 - 400{,}000)) + (2 \times (420{,}000 - 165{,}000)) + (2 \times (400{,}000 - 400{,}000)) + (2 \times (720{,}000 - 480{,}000)) + (2 \times (400{,}000 - 400{,}000)) + (6 \times (400{,}000 - 400{,}000))\}$$

$$\therefore \phi_A(4,v) = 113.750$$

b. Shapley value of Player B:

$$\phi_B(4,v) = \frac{1}{4!} \times \Big\{\Big((0)!\,(3)!\,\big(v(B)\big) + \Big((1)!\,(2)!\,\big(v(AB) - v(A)\big)\Big) + \Big((1)!\,(2)!\,\big(v(BD) - v(D)\big)\Big) + \Big((1)!\,(2)!\,\big(v(BE) - v(E)\big)\Big) + \Big((2)!\,(1)!\,\big(v(ABD) - v(AD)\big)\Big) + \Big((2)!\,(1)!\,\big(v(ABE) - v(AE)\big)\Big) + \Big((2)!\,(1)!\,\big(v(BDE) - v(DE)\big)\Big) + \Big((3)!\,(0)!\,\big(v(ABDE) - v(ADE)\big)\Big)\Big)\Big\}$$

$$\phi_B(4,v) = \frac{1}{24} \times \{(6 \times (240{,}000)) + (2 \times (480{,}000 - 210{,}000)) + (2 \times (400{,}000 - 400{,}000)) + (2 \times (480{,}000 - 165{,}000)) + (2 \times (400{,}000 - 400{,}000)) + (2 \times (720{,}000 - 420{,}000)) + (2 \times (400{,}000 - 400{,}000)) + (6 \times (400{,}000 - 400{,}000))\}$$

$$\therefore \phi_B(4,v) = 133{,}750$$

c. Shapley value of Player D:

$$\phi_D(4,v) = \frac{1}{4!} \times \Big\{\Big((0)!\,(3)!\,\big(v(C)\big) + \Big((1)!\,(2)!\,\big(v(AD) - v(A)\big)\Big) + \Big((1)!\,(2)!\,\big(v(BD) - v(B)\big)\Big) + \Big((1)!\,(2)!\,\big(v(DE) - v(E)\big)\Big) + \Big((2)!\,(1)!\,\big(v(ABD) - v(AB)\big)\Big) + \Big((2)!\,(1)!\,\big(v(ADE) - v(AE)\big)\Big) + \Big((2)!\,(1)!\,\big(v(BDE) - v(BE)\big)\Big) + \Big((3)!\,(0)!\,\big(v(ABDE) - v(ABE)\big)\Big)\Big)\Big\}$$

$$\phi_D(4,v) = \frac{1}{24} \times \{(6 \times (400{,}000)) + (2 \times (400{,}000 - 210{,}000)) + (2 \times (400{,}000 - 240{,}000)) + (2 \times (400{,}000 - 165{,}000)) + (2 \times (400{,}000 - 480{,}000)) + (2 \times (400{,}000 - 420{,}000)) + (2 \times (400{,}000 - 480{,}000)) + (6 \times (400{,}000 - 720{,}000))\}$$

$$\therefore \phi_D(4,v) = 53.750$$

d. Shapley value of Player E:

$$\phi_E(4,v) = \frac{1}{4!} \times \Big\{\Big((0)!\,(3)!\,\big(v(D)\big) + \Big((1)!\,(2)!\,\big(v(AE) - v(A)\big)\Big) + \Big((1)!\,(2)!\,\big(v(BE) - v(B)\big)\Big) + \Big((1)!\,(2)!\,\big(v(DE) - v(D)\big)\Big) + \Big((2)!\,(1)!\,\big(v(ABE) - v(AB)\big)\Big) + \Big((2)!\,(1)!\,\big(v(ADE) - v(AD)\big)\Big) + \Big((2)!\,(1)!\,\big(v(BDE) - v(BD)\big)\Big) + \Big((3)!\,(0)!\,\big(v(ABDE) - v(ABD)\big)\Big)\Big)\Big\}$$

$$\phi_E(4,v) = \frac{1}{24} \times \{(6 \times (165{,}000)) + (2 \times (420{,}000 - 210{,}000)) + (2 \times (480{,}000 - 240{,}000)) + (2 \times (400{,}000 - 400{,}000)) + (2 \times (720{,}000 - 480{,}000)) + (2 \times (400{,}000 - 400{,}000)) + (2 \times (400{,}000 - 400{,}000)) + (6 \times (400{,}000 - 400{,}000))\}$$

$$\therefore \phi_E(4,v) = 98{,}750$$

To summarize the computation, we can use the same algorithm as before. We can calculate the Shapley value of each player using the new data that we have obtained. In this case, we choose the four players' calculation. The algorithm can be written as follows:

Algorithm 1. Shapley Value Algorithm

Input: Number of players, input between 1 to 5.
Value of each player's payoff, $v(A)$, $v(B)$, $v(D)$, and $v(E)$.
Coalition value of each player, $v(AB)$, $v(AD)$, $v(AE)$, $v(BD)$, $v(BE)$, $v(DE)$, $v(ABD)$, $v(ABE)$, $v(ADE)$, $v(BDE)$, and $v(ABDE)$.
Output: Shapley value of each player

1. Choose the number of players (in this case is 4 players).
2. Compute P_{ij}, for $i = 1; 2; 3; \ldots; 24$, $j = 1; 2; 3; 4$.
3. Add up all the values in each column. X_k.
4. Compute the average of each column, $\frac{X_k}{24}$, $i = (1; 2; 3; 4)$.

From the calculation of the algorithm, we can obtain the Shapley value of the players A, B, D, and E as follows:

Table 5. Shapley Value from Player A, B, D, and E

	ϕ_A	ϕ_B	ϕ_D	ϕ_E
Shapley Value	113,750	133,750	53,750	98,750

The value obtained from the computation of the Shapley value shows the contribution of each player to the distribution of travel accommodation costs. Player A's contribution is IDR 113,750, player B's contribution is IDR 133,750, player D's contribution is IDR 53,750, and player E's contribution is IDR 98,750. The result is $\forall \phi_i < v_i$ with $i = \{A, B, D, E\}$, this means that after forming a coalition, both players A, B, D, and E spend less money than before forming a coalition.

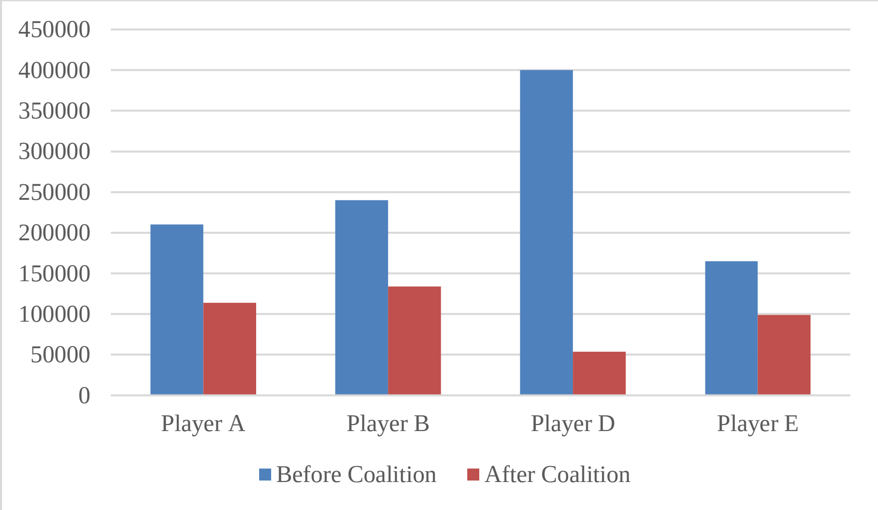

Figure 5. Cost Comparison Before and After the Coalition of 4 Players

As we can see in Fig. 5, the benefit of joining a coalition is that there is a probability that the player can spend less cost than traveling by their self. This is the condition $\forall \phi_i < v_i$. Therefore, the player $A, B, D,$ and E decide to form a coalition and choose to travel together using player D's car. These findings directly address the primary objective of the study: to explore the practical application of cooperative game theory—specifically, the Shapley value—in optimizing intercity travel costs. The algorithm not only provides a transparent cost allocation method but also supports informed decision-making in group travel scenarios.

3.2 Discussion

Based on the previous research [6],[7], Shapley value method can be a solution for some cooperative game theory cases. Here, we want to see whether the Shapley value can also be a solution to the case of intercity travel.

Smart transportation networks may integrate cooperative cost models to enhance both efficiency and user satisfaction. The model holds significant potential for modern transportation systems:

1. Ride-sharing platforms can use such algorithms to ensure fair cost distribution.
2. Logistical planning for group travel or corporate commuting can benefit from predictive coalition dynamics.

The findings contribute to the ongoing development of cooperative game theory by showing that the Shapley value may not guarantee universal participation. The case of Player C illustrates how negative marginal contributions can deter individuals from joining coalitions, raising questions about fairness and incentive design. This suggests opportunities for:

1. Enhancing the Shapley value with adjustment mechanisms for fairness,
2. Investigating alternative stability conditions beyond $\forall \phi_\mathrm{i} < v_\mathrm{i}$,
3. Applying the concept to heterogeneous agent-based travel models.

Player C's refusal is not just a numerical outcome—it represents a strategic behavior rooted in individual utility. Exploring such behaviors helps illuminate the human dimension of cooperative modeling and its limits, offering new perspectives on coalition dynamics and real-world travel decision-making.

4. CONCLUSION

The conclusions of this study are as follows:

1. The cooperative game theory approach, particularly the Shapley value, has proven to be an effective and fair method for managing intercity travel costs across Java through the formation of coalitions using shared vehicles.

2. The cost allocation algorithm developed based on each player's marginal contribution ensures transparency and fairness, and can be adapted for carpooling platforms, ride-sharing systems, and community-based transport planning.
3. This study offers practical insights for developing sustainable and inclusive transportation policies by providing fair cost incentives to encourage participation in shared travel systems.
4. The study is limited by static cost assumptions and a small number of players; future research is encouraged to incorporate dynamic pricing models, more complex coalition structures, and spatiotemporal data.

Author Contributions

M. Aditya Tri Ariyanto: Writing-Original Draft, Data Curation, Software Computation, Analysis. Rubono Setiawan: Analysis, Writing Paper, Editing and Review, Validation, Supervision. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare no competing interests.

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