BAREKENG: Journal of Mathematics and Its Applications
March 2026 Volume 20 Issue 1 Page 0841–0852
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol20iss1pp0841-0852

ISLAMIC MORTGAGE FINANCING: DETERMINATION OF AN OPTIMAL *NISBAH* OF PROFIT-SHARING SCHEME UNDER *MUSHARAKAH MUTANAQISAH* CONTRACT

Wulan Nurul Kamilah[1*], **Novriana Sumarti**[2], **Kuntjoro Adji Sidarto**[3]

[1] *Mathematics Doctoral Study Program, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung*
[2,3] *Industrial and Financial Mathematics Research group, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung*
[2,3] *Center for Mathematical Modeling and Simulation, Institut Teknologi Bandung*
Jln. Ganesha 10 Bandung, Jawa Barat, 40132, Indonesia

*Corresponding author's e-mail: *wulannurulkamilah@gmail..com*

Article Info

Article History:

Received: 16th June 2025
Revised: 15th July 2025
Accepted: 29th July 2025
Available online: 24th November 2025

Keywords:

Islamic home financing;
Musharakah Mutanaqisah;
Nisbah;
Profit-sharing scheme.

ABSTRACT

In an Islamic investment using a profit-sharing scheme, one of the important issues to be determined is the optimal value of nisbah, so that all the parties are fairly treated. This study aims to determine the optimal constant nisbah for an Islamic home financing under musharakah mutanaqisah (MMQ) contract, to ensure a fair arrangement for both parties: the Islamic Bank (IB) and the customer. Under MMQ, the model combines partnership (Musharakah) and a leasing agreement, with gradual ownership transfer from the IB to the customers using an installment payment. For describing its dynamic, the customer's income is assumed to follow the Geometric Brownian Motion (GMB), and the defined objective function is solved using the simulated data of payments of rental fee and ownership installment. Based on the simulation results, it will show that the middle value of a parameter of the defined objective function α as a biased portion, with $0 < \alpha < 1$, is not always chosen. The objective function, which shows a parabolic plot trend, can be interpreted as a fair situation between the IB and the customer. With the selected α value, the maximum objective function value will be sought. The determined nisbah can inform policy decisions in Islamic financial institutions.

How to cite this article:

W. N. Kamilah, N. Sumarti, and K. Adji Sidarto, "ISLAMIC MORTGAGE FINANCING: DETERMINATION OF AN OPTIMAL NISBAH OF PROFIT-SHARING SCHEME UNDER MUSHARAKAH MUTANAQISAH CONTRACT", *BAREKENG: J. Math. & App.*, vol. 20, iss. 1, pp. 0841-0852, Mar, 2026.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · **Open Access**

1. INTRODUCTION

Islamic finance has emerged as a viable alternative to conventional financial systems, driven by its emphasis on ethical investment, risk-sharing, and compliance with Shariah principles. Among the various sectors within Islamic finance, Islamic housing finance stands out for its rapid growth and its potential to meet the essential human need for shelter while adhering to Islamic legal and moral norms. However, non-performing loans (NPLs) in this sector have risen from 2.5% to over 3% between mid-2023 and April 2025 [1]. This trend needs close monitoring and proactive mitigation by both regulators and banks, especially in the home mortgage market.

Unlike interest-based mortgage systems, Islamic housing finance relies on asset-backed and partnership-based contracts such as *musharakah mutanaqisah* (MMQ), which eliminate riba (interest) and promote real economic activity through equitable transactions [2], [3]. The MMQ contract is a hybrid model that combines joint ownership with gradual equity transfer and leasing. MMQ structures involve a co-ownership agreement between the IB and the client, where the client incrementally buys the financier's share while paying rent on the IB's portion of the asset. This arrangement not only aligns with Islamic principles of risk-sharing and asset-backing but also offers a more equitable alternative to conventional debt financing [4].

In Islamic finance, the *nisbah*, also known as a profit-sharing ratio, plays a crucial role in determining how profits are distributed between financiers and customers in partnership contracts such as MMQ. Since interest (*riba*) is prohibited, profit-sharing serves as the primary mechanism to incentivize cooperation between parties while ensuring Shariah compliance [5]. The selection of an appropriate *nisbah* is not merely a contractual detail; it significantly influences the efficiency, equity, and risk distribution within Islamic financial systems. Although the study of using profit-sharing ratios in Islamic banking, including examining theoretical mechanisms, practical implementations, and their impact on financial stability and performance, has been extensively researched in [6]-[11], their implementation in home financing remains limited. Furthermore, we can barely find any study that addresses the determination of the optimal *nisbah* in the profit-sharing scheme for home financing. This study aims to fill this research gap by introducing an optimization procedure for determining the optimal *nisbah* in profit-sharing schemes for Islamic home financing under the MMQ. Through this investigation, the research seeks to provide practical insights and policy recommendations for Islamic financial institutions to enhance the effectiveness, fairness, and stability of Islamic home financing.

2. RESEARCH METHODS

2.1 Data Collection

This research is based on secondary data sources taken from the official website of OJK, BPS (*Badan Pusat Statistik*- Central Bureau of Statistics), Bank of Indonesia (BI), BRI, and Islamic Banking in Indonesia, such as BSI and Muamalat Bank. OJK provides data on the NPL in home mortgages per month from 2023 to 2024 [1]. Meanwhile, BPS presents employment trends data from 2023 to 2024. From this data, it is known that around 59-60% of total employment comes from the informal sector, and the rest from formal employment. Formal workers typically have written contracts, enroll in BPJS social security programs, and receive regular wages, while informal workers include self-employed individuals (e.g., small traders, artisans); casual or temporary laborers (without formal contracts or guarantees); unpaid family members helping in businesses; and freelancers without social protections. These roles often lack of steady income, benefits, and legal protections, making them more vulnerable during economic shocks. Based on those facts, for simulation purposes in this study, the customer's income is assumed to fluctuate. Data from several IBs gives information about the MMQ contract and a simple simulation of home financing in each IB. BRI's official website provides data, such as a simulation for home financing. The result of the simulation is used for calculating IB's rate of return (RoR) and the customer's profit [12]. Data of the average income per year provided by goodstat.id [13].

2.2 Profit-Sharing Model

A profit-sharing scheme is a system following Islamic law, consists of procedures for fund providers and fund managers to share business returns. The fund manager's responsibilities under the profit-sharing scheme are ascertained through the profit or loss ratio (*nisbah*) of the enterprise. The fact that undertaking business activities will inevitably involve fluctuations in value is a shared reality between IB and its customers; this is referred to as "justice". When conducting business, justice is predicated on natural and humanized conditions. Consequently, the nominal proportion of profit-sharing under *nisbah* will fluctuate periodically [14].

The profit-sharing system is regarded as the most ideal method of financing from an Islamic perspective [14], [15]. Regrettably, profit-sharing, being a product within an Islamic financial institution, does not completely follow Sharia compliance [4], [6], [16]. Sharia financial institutions typically offer *murabaha* contracts, similar to conventional mortgages, as they are more cost-effective and allow IBs to transfer risks to customers [17]. Several studies also indicate that MMQ contract financing is limited and has not been effectively applied in Sharia financial institutions, since customers are unaware of it, there is a limitation of competent human resources, and the financing process is complicated. Earlier studies, such as those by Meera and Razak [4], have identified that one of the difficulties in modeling mortgage profit-sharing finance using MMQ is calculating rental pricing (*ujrah*), taxation, land regulation, and other variables. Smolo and Hassan [16] also made similar statements.

2.3 Nisbah

The definition of *nisbah* in the fatwa of the National Sharia Council of the Indonesian Ulama Council No. 114/DSN-MUI/IX/2017 states that *nisbah* is a ratio expressed in numbers, such as a percentage for profit-sharing. There are two types of *nisbah*, proportional-*nisbah* and agreed-*nisbah*. Proportional-*nisbah* is based on the capital share of the parties in a business partnership, used as the basis for dividing profits and losses. Meanwhile, agreed-*nisbah* is a *nisbah* based on mutual agreement used as the basis for dividing profits [18], which is the main objective of this study to determine the optimal *nisbah*. In this study, the optimal agreed-*nisbah* will be referred to as the optimal *nisbah*.

Islamic Sharia does not specifically regulate the determination of the *nisbah*. However, the MUI requires that the determination of *nisbah* must be stated in proportional form and be fair to the parties involved in the transaction. The *nisbah* allocated between the parties can be equal or not, depending on mutual agreement. The only thing not allowed in Sharia is determining a fixed profit amount or percentage for any party involved in the transaction related to capital [16]. This means that the capital provider must not set a return on capital or investment loan in the form of a percentage of the capital, as this may include riba [19].

2.4 Generate the Monthly Income Following Geometric Brownian Motion (GBM)

This study assumed that the income of the customer fluctuates for simulation purposes. Similar to research conducted by Sumarti [19], [20] for a profit-loss sharing scheme on small capital investments. The source of daily profit data is the traders' original data collected for several periods. The traders must record the profits obtained every day. Furthermore, the collected profit data is analyzed by predicting the type of distribution, allowing for the generation of a set of simulation data at any time. Based on the analysis of the rate of increase or decrease in daily profits lognormal distributed. This net profit is considered similar to the dynamics of stock price returns, whose values are always positive, randomly fluctuating based on market conditions, cost, and demand.

However, in this study, the data is going to be generated monthly rather than daily. Assume that the monthly profit data follows a lognormal distribution. Net income is presumed to be uncertain. The net income for $t = 1,2,\cdots,n$ is defined by

$$g(t+\Delta t) = g(t)e^{\left(\mu-\frac{1}{2}\sigma^2\right)\Delta t+\sigma\sqrt{\Delta t}Z}, \qquad Z \sim N(0,1), \tag{1}$$

where $g(t)$ is the customer's income at time t, (μ,σ) is the mean and the standard deviation return of the original monthly profit, and $\Delta t = \frac{n}{l}$, with l is the number of sub-intervals from the interval $[0,n]$. Based on the regulation of major Islamic Banks in Indonesia, for example in [21], the amount of potential installment of the mortgage is about 40% of the minimum income, so we assume the fixed amount of 60% of the minimal

income is used only for supporting the monthly living expenses of the customer's family. Consequently, the minimum income of the customer is

$$g(0) = \frac{I_b^{mmq}}{0.4}, \tag{2}$$

where I_b^{mmq} is the principal installment. The net income used for installments is therefore denoted by

$$\omega(t) = g(t) - 0.6g(0). \tag{3}$$

The following sections refer to $\omega(t)$ as the net income of the customer.

2.5 Islamic Mortgage Financing under *Musharakah Mutanaqisah* (MMQ)

A mortgage, known as property rights or property claims, is a long-term debt instrument in which the debtor provides mortgage rights in the form of immovable objects or property to the financier as collateral for his/her obligations. The debtor repays the loan in installments and interest until it is completed at the agreed time. In this case, the debtor could use and utilize the property during the installment time and own it as the obligation is completed. However, in an Islamic mortgage, if the debtor fails to meet his obligations, the IB may propose to sell the owned property. It can be sold by either the IB or the debtor. If there is any money left over after the debt has been reduced, it becomes the debtor's. The Islamic mortgage is regulated by the fatwa of the National Sharia Board-Indonesian Council of Ulama (DSN-MUI), including home financing [22].

One type of contract for Islamic home financing generally used by IBs in Indonesia is *musharakah mutanaqisah* (MMQ) contracts. An MMQ contract is a form of cooperation between two or more parties for the ownership of an object or asset, whereas the ownership of one party is reducing while the other party's ownership is increasing [19], [23]-[25]. However, there are several problems while modeling the MMQ contract, such as determining the rental rate, taxation, land laws, and others [4], [16], [26], [27]. As for rental cost, this study used a straight-line depreciation.

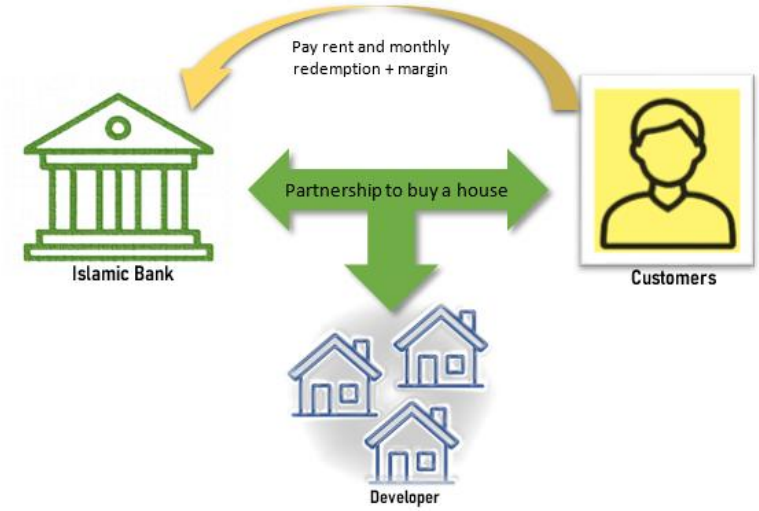

Figure 1. Islamic Mortgage Financing Scheme with MMQ contract

A Sharia mortgage financing scheme with an MMQ contract is shown in Fig. 1. MMQ involves three contracts, those are *musharakah* (joint venture), *ijarah* (lease), and *bay'* (sale) [23], [28]. The customer selects and identifies the desired property or house and submits the MMQ contract to the IB. After approval, the IB and the customer jointly purchase the property from the developer with an agreed share ownership ratio. Because the IB leases back the property, the customer must pay the rental cost (*ujrah*) to the IB and gradually purchase the IB property ownership until the customer has the entire portion. However, because MMQ consists of a *bay'* contract, then the IB can demand the margin from the customers. The margin must be agreed upon by both parties. In the end, the IB gives the ownership of its property to the customer.

Due to the presence of *musharakah* contact in MMQ, any profit or loss must be divided, if applicable, between the IB and the customer. Profit on home financing through MMQ is derived from the *ujrah*, and the ratio of profit-sharing is called *nisbah*. The fatwa of DSN-MUI No. 114/DSN-MUI/IX/2017 specifies two

types of *nisbah*, namely the proportional-*nisbah* and the agreed-*nisbah*. The first *nisbah* will alter as ownership change occurs. While the second *nisbah*, the profit-sharing, is based on the agreed-upon ratio, which can be changed during the term of the contract if both parties agree. This study only uses one type of *nisbah*, namely, agreed-*nisbah*. It is because the main purpose of this study is to determine the optimal value of agreed-*nisbah*, which will later be referred to as optimal *nisbah*.

2.5.1 Construct the Model of Islamic Mortgage Financing under MMQ

Let the customers submit home mortgages with the MMQ contract to IB. The price of the house can be written as

$$\boldsymbol{C_D = B_{bs} + B_n}. \tag{4}$$

$\boldsymbol{C_D}$ is the house price, $\boldsymbol{B_{bs}}$ is the IB's initial funds, and $\boldsymbol{B_n}$ is the customer's initial funds. Furthermore, $\boldsymbol{B_{bs}}$ must be redeemed periodically by the customer in the amount of $\boldsymbol{K}$, which is calculated by the formula discussed in the *musharakah mutanaqisah partnership* (MMP) model developed by Meera and Razak [4], [28] as follows,

$$\boldsymbol{K = \frac{x(C_D - (1-x)^n)B_N(0)}{(1-x)^n - 1}} \quad \text{with } \boldsymbol{x = \frac{u_b}{C_D}}. \tag{5}$$

Where: $\boldsymbol{K}$ is the monthly installments to redeem IB's share of the house, $\boldsymbol{u_b}$ is the *ujrah* (the basic rent price), $\boldsymbol{n}$ is the total period,$\boldsymbol{B_N(t)}$ is the portion of home ownership owned by the customer at time $\boldsymbol{t}$, and $\boldsymbol{x}$ is the rental rate. The *ujrah* is the revenue of the *ijarah* (leasing) that will be shared between the IB and the customer. *Ujrah* is determined using the straight-line depreciation method [29] with an assumption of zero residual price, which can be calculated by

$$\boldsymbol{u_b = \frac{C_D}{n}}, \tag{6}$$

where $\boldsymbol{u_b}$ is the basic *ujrah* (the basic rent price), $\boldsymbol{C_D}$ is the house price from the developer, and $\boldsymbol{n}$ is the total period.

Due to the *bay*' (sale) contract involved in MMQ, IB can collect a margin from customers. Hence, the principal installments that the customer must pay monthly are

$$\boldsymbol{I_b^{mmq} = K + u_b + m^*}, \tag{7}$$

where $\boldsymbol{I_b^{mmq}}$ is the principal installment, $\boldsymbol{K}$ is the monthly installment to redeem IB's share of the house, $\boldsymbol{u_b}$ is the basic *ujrah*, and $\boldsymbol{m^*}$ is the monthly margin owned by IB and calculated using the formula:

$$\boldsymbol{m^* = \left((1+m)^{\frac{1}{12}} - 1\right)B_{bs}}, \tag{8}$$

where $\boldsymbol{m}$ is the yearly margin. In consequence of the dynamics of customers' net income and $\boldsymbol{B_{bs}}$ is the IB's initial funds. There is a possibility that the transferred ownership could reach less than 100% or more than 100% at the end of the period, which is not desirable. This event will cause varying amounts of the *ujrah* installment, whether it is sufficient to fully pay the basic *ujrah* ($\boldsymbol{u_b}$) or not. Therefore, the monthly payment of the *ujrah* installment, denoted by $\boldsymbol{u(t)}$, for $\boldsymbol{t = 1, 2, \cdots, n}$, is

$$\boldsymbol{u(t)} = \begin{cases} \boldsymbol{u_b}, & \text{if } \boldsymbol{\omega(t) \geq I_b^{mmq}}, \\ \boldsymbol{\omega(t) - K - m^*}, & \text{if } \boldsymbol{u_b < \omega(t) < I_b^{mmq}}, \\ \boldsymbol{\dfrac{C_D - K - B_N(t-1)}{p_a}}, & \text{if } \boldsymbol{B_N(t-1) + K + u_c > C_D} \text{ and } \boldsymbol{\omega(t) \geq I_b^{mmq}} \\ \boldsymbol{0}, & \text{otherwise}, \end{cases} \tag{9}$$

where $\boldsymbol{u_b}$ is the basic *ujrah*, $\boldsymbol{\omega(t)}$ is the customer's net income at time $\boldsymbol{t}$, $\boldsymbol{I_b^{mmq}}$ is the principal installment, $\boldsymbol{m^*}$ is the monthly margin, $\boldsymbol{C_D}$ is the house price from developer, $\boldsymbol{K}$ is the monthly installments to redeem IB's share of the house, $\boldsymbol{B_N(t)}$ is the portion of home ownership held by the customer, and $\boldsymbol{p_a}$ is the *nisbah* of profit-sharing scheme that is contingent upon the agreement established at the beginning of the contract between the IB and the customer. On the right-hand side of Eq. (9), row 1 is the desirable and standard payment. The second row of Eq. (9) expresses the amount of *ujrah* that the customer can pay because his

income is insufficient. Meanwhile, the third row is designed to make sure the portion owned by the customer exactly equal to $\boldsymbol{C_D}$. The varying amount of *ujrah* also make the varying amount of monthly installments, thus the monthly installment for $\boldsymbol{t}$-th month, denoted by $\boldsymbol{I(t)}$ $(\boldsymbol{t = 2, 3, \cdots, n})$, is

$$\boldsymbol{I(t) = K + u(t) + m^*}. \tag{10}$$

where $\boldsymbol{K}$ is the monthly installment to redeem IB's share of the house, $\boldsymbol{u(t)}$ is the *ujrah* fee paid at time $\boldsymbol{t}$, and $\boldsymbol{m^*}$ is the monthly margin.

Furthermore, if the customer's net income in $\boldsymbol{t}$-th month is insufficient to pay the full installments, or $\boldsymbol{\omega(t) < I_b^{mmq}}$, then the amount of the installment is only the available fund, $\boldsymbol{\omega(t)}$, and the unpaid portion of installments is written in the debt, denoted by $\boldsymbol{H(t)}$. This debt will be paid along with installments in $\boldsymbol{C(t^*)}$, with $\boldsymbol{t^* > t}$. For $\boldsymbol{t = 2, 3, \cdots, n}$, the debt of the installment at the time $\boldsymbol{t}$, denoted by $\boldsymbol{H(t)}$, determined by

$$\boldsymbol{H(t) = tI_b^{mmq} - \sum_{i=1}^{t} I(i) - \sum_{i=1}^{t} C(i)}, \tag{11}$$

$$\boldsymbol{H(1) = I_b^{mmq} - I(1) \quad \text{and} \quad C(1) = 0}, \tag{12}$$

where $\boldsymbol{I_b^{mmq}}$ is the principal installment, $\boldsymbol{I(t)}$ is the monthly installment for $\boldsymbol{t}$-th month, and $\boldsymbol{C(t)}$ is the paid debt at time $\boldsymbol{t}$ and determined by

$$\boldsymbol{C(t) = \begin{cases} H(t-1), & \text{if } \omega(t) - I_b^{mmq} \geq H(t-1), \\ \omega(t) - I_b^{mmq}, & \text{if } 0 < \omega(t) - I_b^{mmq} < H(t-1), \\ 0, & \text{if } \omega(t) - I_b^{mmq} < 0. \end{cases}} \tag{13}$$

Due to periodical payments made every month, the portion of home ownership held by the customer, denoted by $\boldsymbol{B_N(t)}$ will increase. For $\boldsymbol{t = 1, 2, \cdots, n}$, the house ownership held by the customer at the time $\boldsymbol{t}$ is

$$\boldsymbol{B_N(t) = B_N(t-1) + K + u_c(t)}, \tag{14}$$

with $\boldsymbol{B_N(0) = B_n}$ is the customer's initial funds, $\boldsymbol{K}$ is the monthly installment to redeem IB's share of the house, and $\boldsymbol{u_c(t)}$ is the profit share of *ujrah* for customers obtained from

$$\boldsymbol{u_c(t) = p_a\,(u(t) + C(t))}, \text{ with } \boldsymbol{u_c(1) = u_b}. \tag{15}$$

$\boldsymbol{p_a}$ is the *nisbah*, and it can be changed during the term of the contract if both parties agree, $\boldsymbol{u(t)}$ is the *ujrah* fee paid at time $\boldsymbol{t}$, $\boldsymbol{C(t)}$ is debt paid at time $\boldsymbol{t}$, and $\boldsymbol{u_b}$ is the basic *ujrah*. Thus, the total money received by the IB at $\boldsymbol{t}$-th month, denoted $\boldsymbol{S(t)}$, is

$$\boldsymbol{S(t) = I(t) + C(t) - u_c(t) \text{ for } t = 1, 2, \cdots, n}. \tag{16}$$

With $\boldsymbol{I(t)}$ is the monthly installment for $\boldsymbol{t}$-th month, $\boldsymbol{C(t)}$ is the paid debt at time $\boldsymbol{t}$, and $\boldsymbol{u_c(t)}$ is the profit share of *ujrah* for customers.

The rate of return (RoR) for the IB in Islamic mortgage financing with an MMQ contract, denoted by $\boldsymbol{r_{mmq}}$, will satisfy the following equation

$$\boldsymbol{C_D - B_n = \frac{S(1)}{(1 + r_{mmq})} + \frac{S(2)}{(1 + r_{mmq})^2} + \cdots + \frac{S(n) + H(n)}{(1 + r_{mmq})^n}}, \tag{17}$$

with $\boldsymbol{C_D}$ is the house price, $\boldsymbol{B_n}$ is the customer's initial funds, $\boldsymbol{S(t)}$ is the total money received by the IB at $\boldsymbol{t}$-th month, and $\boldsymbol{H(t)}$ is the debt of the installment at time $\boldsymbol{t}$. Meanwhile, the corresponding customer's return, denoted by $\boldsymbol{pd_{mmq}}$ is

$$\boldsymbol{Pd_{mmq} = \frac{\sum_{i=1}^{n} TH(i)}{\sum_{i=1}^{n} \omega(i)}}, \tag{18}$$

where $\boldsymbol{\omega(t)}$ is the customer's net income, $\boldsymbol{S(t)}$ is the total money received by the IB at $\boldsymbol{t}$-th month, and $\boldsymbol{TH(t) = \omega(t) - S(t)}$ is the net profit taken home by the customer at $\boldsymbol{t}$-th month.

2.5.2 The *Nisbah* in Islamic Housing Finance under the MMQ contract

The customer's *nisbah*, denoted as $\boldsymbol{p_a}$, is determined by mutual agreement at the beginning of the contract. The *nisbah* needs to be well calculated so that both the IB and the customer have a fair benefit. Therefore, an objective function for this optimization problem needs to be formed. A standard profit rate needs to be set to establish the objective function [19]. The return on financing for the IB will be compared with the bond yield as in Eq. (19). The data for bond yield is provided by the Bank of Indonesia. Meanwhile, the customer's return is the relative difference in the profit portion taken home, with the profit portion from a conventional mortgage model as in Eq. (20). $\boldsymbol{pd_{conv}}$ is the profit portion from a conventional mortgage model is calculated based on the simulation data taken from BRI's website [12]. The following are the two desired objective functions.

$$\max_{p_a} \boldsymbol{f(p_a)} = \max_{\mathrm{p_a}} \frac{\boldsymbol{r_{mmq}(p_a) - r_{BI}}}{\boldsymbol{f_{max}}}, \tag{19}$$

$$\max_{p_a} \boldsymbol{h(p_a)} = \max_{\mathrm{p_a}} \frac{\boldsymbol{pd_{mmq}(p_a) - pd_{conv}}}{\boldsymbol{h_{max}}}, \tag{20}$$

with $\boldsymbol{f_{max} = \max |r_{mmq}(p_a) - r_{BI}|}$, $\boldsymbol{h_{max} = \max |pd_{mmq}(p_a) - pd_{konv}|}$. $\boldsymbol{r_{mmq}(p_a)}$ is the IB's rate of return using *nisbah* $\boldsymbol{p_a}$, $\boldsymbol{r_{BI}}$ is the syariah bond yields, $\boldsymbol{pd_{mmq}(p_a)}$ is customer's return with *nisbah* $\boldsymbol{p_a}$, and $\boldsymbol{pd_{konv}}$ is the profit portion resulting from the conventional mortgage model with a similar framework. Eqs. (19) and (20) will be written as one multi-objective function with the weights reflecting the importance or fairness to each party, denoted by $\boldsymbol{\alpha}$, as follow

$$\max_{p_a} \boldsymbol{F(p_a)} = \boldsymbol{\alpha}\left(\frac{\boldsymbol{f(p_a)}}{\boldsymbol{f_{max}}}\right)^2 + \boldsymbol{(1-\alpha)}\left(\frac{\boldsymbol{h(p_a)}}{\boldsymbol{h_{max}}}\right)^2, \tag{21}$$

where $\boldsymbol{r_{BI} < r_{conv} < r_{mmq}}$, $\boldsymbol{pd_{conv} < pd_{mmq}}$, and $\boldsymbol{\alpha}$ is a biased portion with $\boldsymbol{0 < \alpha < 1}$. The function $\boldsymbol{f(p_a)}$, as in Eq. (19), represents the payoff received by IB, so it is expected that the distance between $\boldsymbol{r_{mmq}}$ and $\boldsymbol{r_{BI}}$ is as wide as possible, meaning IB receives a large payoff. Meanwhile, the function $\boldsymbol{h(p_a)}$, Eq. (20), represents the payoff received by the customer with the Islamic mortgage financing model under the MMQ contract. By maximizing the objective function $\boldsymbol{F(p_a)}$, it is maximizing the distances between $\boldsymbol{r_{mmq}}$ and $\boldsymbol{r_{BI}}$ and between $\boldsymbol{pd_{mmq}}$ and $\boldsymbol{pd_{conv}}$. If $\boldsymbol{\alpha}$ becomes larger, then the bias towards IB's profit increases, while the customer's profit decreases, and vice versa. To determine the optimum ratio value, the first step that must be taken is to choose an $\boldsymbol{\alpha}$ value that clearly shows the fairness between the two parties, IB and the customer. The simulation will show the selected $\boldsymbol{\alpha}$ value, so that the optimal objective function value will be determined.

3. RESULTS AND DISCUSSION

3.1 Simulation and Analysis of Islamic Housing Finance under MMQ Contract

Choose the house price C_D = IDR 500,000,000, the MMQ mortgage tenure n = 15 years = 180 months, the customer's initial payment B_n = 10% × IDR 500,000,000 = IDR 50,000,000. Therefore, from Eq. (4), the mortgage fund $B_{bs} = C_D - B_n$ = IDR 450,000,000. Using Eq. (5), the customer's monthly redemption of the mortgage fund is K = IDR 1,183,553. If the yearly margin is m = 5.28%, then by Eq. (8), the monthly margin is m^* = $\left((1+m)^{\frac{1}{12}} - 1\right) B_{bs}$ = IDR 1,933,641. From Eq. (6), we have the basic *ujrah* $u_b = \frac{C_D}{n}$ = IDR 2,777,778. The MMQ monthly installment, using Eq. (7), is I_b^{mmq} = IDR 5,894,971. The minimal income of the customer is calculated using Eq. (2), so that $g(0)$ = IDR 14,737,429. Assuming $\mu = 0.2$ and $\sigma = 0.35$, the customer's monthly income from Eq. (1) is

$$g(t) = 14{,}737{,}429e^{(0.2-\frac{1}{2}(0.35)^2)+(0.35)Z}, \; Z \sim N(0,1). \tag{22}$$

Thus, by Eq. (3), the money for paying installments from the income equals to $\omega(t) = g(t) - 0.6g(0)$. The simulation results can be seen in Table 1. We chose the *nisbah* used in the simulation to be $p_a = 47.727\%$ and assumed fixed until the end of the period.

As shown in Table 1, the *ujrah* presented in Column (5) represents the *ijarah* revenue, which is distributed between the IB and the customer according to the *nisbah*, p_a. For example, in the second month, the *ujrah*, $u(2) = IDR\ 2{,}777{,}778$, it is noticed that the amount of customer ownership $B_N(2)$ is increased due to the first payment. Therefore, based on Eq. (15), the customer profit, $u_c(2) = p_a \times \big(u(2) + C(2)\big) = IDR\ 1{,}325{,}739$, while the IB retains the remaining *ujrah*. At the end of the period, the ROR of the IB is 9.37%, while the return of the customer is 45.36%.

Table 1. The Result of Simulation in Home Financing with Profit-Sharing Scheme Under MMQ Contract

n (1)	ω (2)	m^* (3)	K (4)	$u(t)$ (5)	I_{mmq} (6)	H_{mmq} (7)	C_{mmq} (8)
0							
1	5,894,971	1,933,641	1,183,553	2,777,778	5,894,971	0	0
2	12,681,085	1,933,641	1,183,553	2,777,778	5,894,971	0	0
3	3,371,563	1,933,641	1,183,553	254,369	3,371,563	2,523,408	0
4	4,324,259	1,933,641	1,183,553	1,207,065	4,324,259	4,094,121	0
5	11,938,092	1,933,641	1,183,553	2,777,778	5,894,971	0	4,094,121
6	11,978,492	1,933,641	1,183,553	2,777,778	5,894,971	0	0
7	13,110,533	1,933,641	1,183,553	2,777,778	5,894,971	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
178	12,595,853	1,933,641	1,183,553	2,777,778	5,894,971	0	2,043,502
179	11,466,082	1,933,641	1,183,553	2,777,778	5,894,971	0	0
180	8,709,010	1,933,641	836,739	0	2,390,526	3,504,446	0

Table 2. The Result of Simulation in Home Financing with Profit-Sharing Scheme Under MMQ Contract (Continued)

n (1)	p_a (9)	u_c (10)	u_{bs} (11)	B_N (12)	B_{BS} (13)	S_{mmq} (14)	TH_{mmq} (15)
0							
1	0.47727	1,325,739	1,452,039	52,509,292	447,490,708	4,569,233	1,325,739
2	0.47727	1,325,739	1,452,039	55,018,584	444,981,416	4,569,233	8,111,853
3	0.47727	121,402	132,968	56,323,539	443,676,461	3,250,161	121,402
4	0.47727	576,091	630,974	58,083,183	441,916,817	3,748,168	576,091
5	0.47727	3,279,724	3,592,175	62,546,460	437,453,540	6,709,369	5,228,723
6	0.47727	1,325,739	1,452,039	65,055,752	434,944,248	4,569,233	7,409,260
7	0.47727	1,325,739	1,452,039	67,565,044	432,434,956	4,569,233	8,541,301
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
178	0.47727	2,301,033	2,520,247	496,653,969	3,346,031	5,637,441	6,958,412
179	0.47727	1,325,739	1,452,039	499,163,261	836,739	4,569,233	6,896,849
180	0.47727	0	0	500,000,000	0	2,737,340	5,971,670

3.2 Determine the Optimal Agreed-*nisbah*

Incorrectly determining the proportion of the profit-sharing agreement will prevent Sharia transactions from achieving the goal of justice for IB and customers. The following are the steps taken to determine the optimal profit-sharing proportion [30]:

1. Generate the customer's net income.
2. Find pd_{conv} from the simulation home financing of a conventional bank on the website and r_{BI} is the bond yield from the Bank of Indonesia.
3. Generate 500 nisbah within the interval of 0.001 to 0.5 with a sub-interval of 0.001.
4. Each generated nisbah is implemented in the Islamic mortgage financing model in Subsection 2.5.1 to obtain r_{mmq} and pd_{mmq}.
5. Determine the interval of the profit-sharing agreement that satisfies the conditions $r_{mmq} > r_{BI}$ and $pd_{mmq} > pd_{conv}$.

6. Set $\alpha = 0.1, 0.2, \cdots, 1.0$, then substitute into the objective equation of Eq. (21) according to the interval in Step 5 to find $F(p_a)$. The result can be seen in Table 4.

Using the simulation variables in Subsection 3.1, we have Table 3 as the simulation result. The yield bond from the Bank of Indonesia is $r_{BI} = 0.0657$, the customer's return from a conventional mortgage model is $pd_{conv} = 0.1574$, f_{max}= max $|f(p_a)| = 0.0634$, and h_{max}= max $|h(p_a)| = 0.3689$**.** The result will be used to determine the *nisbah* from the objective equation in Eq. (21).

Table 3. The values used for the Objective Function Calculation

p_a	r_{mmq}	pd_{mmq}	$f(p_a)$	$h(p_a)$
0.001	0.1291	0.2577	0.0634	0.1003
0.002	0.1291	0.3062	0.0634	0.1488
0.003	0.1288	0.2569	0.0631	0.0995
0.004	0.1289	0.3419	0.0632	0.1845
0.005	0.1288	0.3154	0.0613	0.1580
⋮	⋮	⋮	⋮	⋮
0.500	0.0761	0.5021	0.0104	0.3447

Table 4. The Values of $F(p_a)$ with Various α and p_a

p_a	α									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.001	0.1665	0.2591	0.3517	0.4443	0.5369	0.6296	0.7222	0.8148	0.9074	1.0739
0.002	0.2463	0.3299	0.4134	0.4970	0.5806	0.6642	0.7478	0.8313	0.9149	1.1612
0.003	0.1644	0.2560	0.3477	0.4393	0.5309	0.6226	0.7142	0.8059	0.8975	1.0619
0.004	0.3244	0.3985	0.4726	0.5468	0.6209	0.6951	0.7692	0.8434	0.9175	1.2419
0.005	0.2639	0.3444	0.4249	0.5054	0.5858	0.6663	0.7468	0.8273	0.9078	1.1717
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.481	0.6220	0.5572	0.4924	0.4276	0.3628	0.2980	0.2332	0.1684	0.1036	0.7256

The plot of the objective function for all α values are shown in Fig. 2, where some trends are monotonically increasing while others are monotonically decreasing. In practice, models may assume that both low and high *nisbah* values could lead to undesirable outcomes, for example, if the *nisbah* is too low, the bank returns suffer, otherwise if the *nisbah* is too high, then the customer burden increases or default risk rises. Therefore, the "right spot" needs to be found to minimize a certain loss or maximize stability. From Fig. 2, the trend of the plot of the objective function tends to form a parabola for α = 0.4. The clear plot for $\alpha = 0.4$ can be seen in Fig. 3. In Murniarti's research [30], it is stated that the optimal type with a parabolic plot is the most ideal compared to the monotonic increasing or monotonic decreasing types, because there is a balance of profits between traders and investors. Moreover, the parabolic trend in Fig. 3 shows that the parabola opens upwards, so the "right spot" in the upward parabola to determine the optimal *nisbah* is at the minimum value that balances profit for both parties.

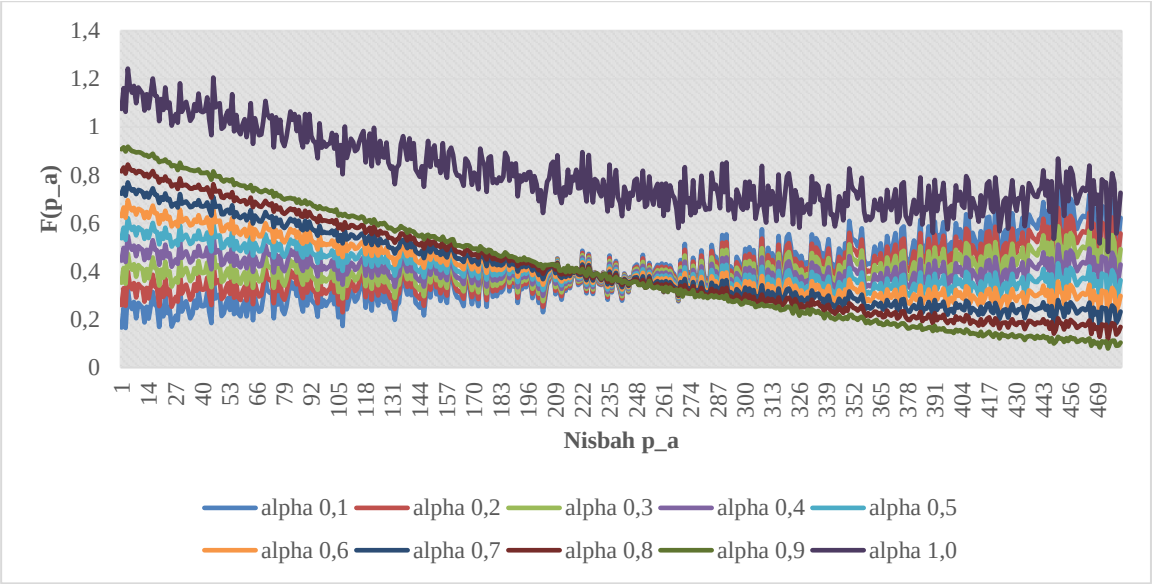

Figure 2. Plot of Objective Function $F(p_a)$ for all values of α

The optimal nisbah p_a should be observed around the minimum point of the plot, but due to stochastic fluctuations in the simulation (as seen in the blue line in Fig. 3), a range of *nisbah* values, approximately between p_{a_min} and p_{a_max}, results in near-optimal outcomes. Based on the fitted parabolic curve in Fig. 3, the minimum value of the objective function $F(p_a)$ occurs near $p_a = 0.265$. However, due to simulation noise, we identify an optimal *nisbah* range between $p_{a\_min} = 0.229$ and $p_{a\_max} = 0.301$. This result provides a flexible range of profit-sharing ratios for the IB and customer, ensuring near-optimal performance even with slight deviations.

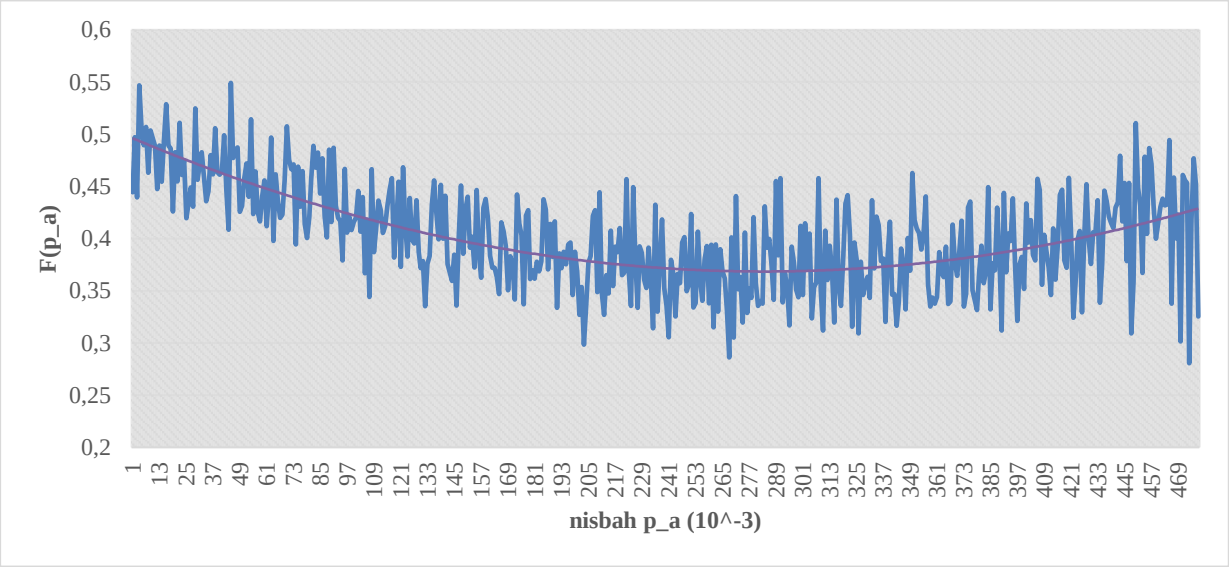

Figure 3. Plot of the Objective Function $F(p_a)$ for $\alpha = 0.4$

By implementing the findings into the IB's calculation procedures, considering the fairness, it can put trust in the customers and potential customers in Islamic Banks in managing their mortgage securely.

4. CONCLUSION

The profit-sharing scheme in Islamic home financing is expected to provide fairness of the transaction for both parties. In addition, this profit-sharing system can be an alternative for customers with dynamic monthly incomes. This scheme can be an alternative strategy to effectively reduce the possibility of loan failure or default. It is also important that the IB and the customer need to achieve fair profit shares. In the MMQ, the construction of a method is needed to determine a fair *nisbah* between the IB and the customer,

so it will be an attractive financial instrument in implementing Sharia regulation, either for Muslims or non-Muslim. The method provided in this study to determine the optimal *nisbah* is done by maximizing the objective function in Eq. (21). The result of this study offers Islamic financial institutions practical implications by identifying an optimal *nisbah* range, allowing flexibility in setting profit-sharing terms while maintaining efficiency and competitiveness. This enables more inclusive and personalized product offerings, aligning with Sharia objectives of fairness and risk-sharing, and reinforcing the ethical foundation of Islamic finance.

Despite its contributions, this study has several limitations. Behavioral and economic assumptions are simplified, and the model lacks empirical calibration with institutional data. These limitations highlight promising opportunities for future research, including empirical validation with real-world data and the incorporation of behavioral, regulatory, and Shariah compliance considerations. Such developments will enhance practical application and deepen analysis in optimizing Islamic housing finance.

Author Contributions

Wulan Nurul Kamilah: Conceptualizing the research, presenting the findings, analysis, and writing and editing the article. Novriana Sumarti: Conceptualizing the research, methodology, analysis, and writing-reviewing the article. Kunjtoro Adji Sidarto: Methodology, analysis, and reviewing the article. All authors discussed the results and contributed to the final manuscript.

Funding Statement

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Acknowledgment

The authors would like to express gratitude to the Head of the Mathematics Doctoral Study Program, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, for their guidance and support during this research. The authors also extend their appreciation to all individuals and institutions who have provided valuable assistance, insight, and encouragement throughout the completion of this study

Declarations

The authors declare no competing interests.

REFERENCES

[1] OJK, "STATISTIK PERBANKAN INDONESIA VOLUME 22 NOMOR 1," 2024.
[2] A. W. Dusuki and N. I. Abdullah, "WHY DO MALAYSIAN CUSTOMERS PATRONISE ISLAMIC BANKS?", *Int. J. Bank Mark.*, vol. 25, no. 3, pp. 142–160, 2007. doi: https://doi.org/10.1108/02652320710739850
[3] A. Kammer, M. Norat, M. Pinon, A. Prasad, C. Towe, and Z. Zeidane, "ISLAMIC FINANCE: OPPORTUNITIES, CHALLENGES, AND POLICY OPTIONS," 2015. doi: https://doi.org/10.5089/9781498325035.006.
[4] A. K. M. Meera and D. A. Razak, "ISLAMIC HOME FINANCING THROUGH MUSHARAKAH MUTANAQISAH AND AL-BAY' BITHAMAN AJIL CONTRACTS: A COMPARATIVE ANALYSIS," *Rev. Islam. Econ.*, vol. 9, no. 2, pp. 5–30, 2005, [Online]. Available: http://irep.iium.edu.my/9713/
[5] A. Salman and H. Nawaz, "ISLAMIC FINANCIAL SYSTEM AND CONVENTIONAL BANKING: A COMPARISON," *Arab Econ. Bus. J.*, vol. 13, no. 2, pp. 155–167, 2018. doi: https://doi.org/10.1016/j.aebj.2018.09.003.
[6] Z. Hasan, "ISLAMIC BANKS: PROFIT SHARING, EQUITY, LEVERAGE LURE AND CREDIT CONTROL," *Munich Pers. RePEc Arch.*, no. 11737, p. 19, 2008, [Online]. Available: http://mpra.ub.uni-muenchen.de/11737/
[7] Z. HASAN, "DETERMINATION OF PROFIT AND LOSS SHARING RATIOS IN INTEREST-FREE BUSINESS FINANCE," *J. Res. Islamic Econ*, vol. 3, no. 1. pp. 13–29, 1985.
[8] N. A. Syahri and D. A. Harjito, "THE EFFECT OF FINANCING USING THE PRINCIPLE OF PROFIT-LOSS SHARING ON PROFITABILITY LEVEL OF COMMERCIAL ISLAMIC BANK REGISTERED IN BANK INDONESIA," *Asian J. Islam. Manag.*, vol. 2, no. 1, pp. 46–58, 2020. doi: https://doi.org/10.20885/ajim.vol2.iss1.art5.
[9] P. S. Chow, Y. Wang, T. M. Choi, and B. Shen, "AN EXPERIMENTAL STUDY ON THE EFFECTS OF MINIMUM PROFIT SHARE ON SUPPLY CHAINS WITH MARKDOWN CONTRACT: RISK AND PROFIT ANALYSIS," *Omega (United Kingdom)*, vol. 57, pp. 85–97, 2015. doi: https://doi.org/10.1016/j.omega.2013.11.007.

[10] M. Rohansyah, Y. Yanti, A. W. Mongkito, A. Toar, and N. A. Bin Abdul Rahman, "THE EFFECT OF INTEREST RATES AND PROFIT SHARING RATIOS ON THE COLLECTION OF THIRD PARTY FUNDS," *Al-Kharaj J. Islam. Econ. Bus.*, vol. 5, no. 4, pp. 517–529, 2023, doi: https://doi.org/10.24256/kharaj.v5i4.4689.
[11] A. Amiludin, "PERBANDINGAN SISTEM BAGI HASIL DAN BUNGA SERTA IMPLEMENTASI PADA BIAYA," *AKSY J. Ilmu Akunt. dan Bisnis Syariah*, vol. 3, no. 2, pp. 75–90, 2021. doi: https://doi.org/10.15575/aksy.v3i2.14053.
[12] BRI Admin, "KPR BRI." Accessed: Oct. 12, 2024. [Online]. Available: https://bri.co.id/
[13] A. A. Putri, "RATA-RATA PENDAPATAN GAJI DI INDONESIA 2023 - GOODSTATS DATA." Accessed: May 08, 2024. [Online]. Available: https://data.goodstats.id/statistic/rata-rata-pendapatan-gaji-di-indonesia-2023-68Yxg
[14] S. M. Hanafi, "THE CONTRIBUTION OF PROFIT-SHARING CHARACTERISTICS TO THE PERFORMANCE OF ISLAMIC BANKS," *Shirkah J. Econ. Bus.*, vol. 6, no. 1, pp. 52–65, 2021. doi: https://doi.org/10.22515/shirkah.v6i1.378.
[15] A. U. F. Ahmad, "SHARI'AH PARAMETERS OF MUSHARAKAH MUTANAQISAH IN ISLAMIC FINANCE: THE EXPERIENCE OF AUSTRALIAN INSTITUTIONS OFFERING ISLAMIC FINANCIAL SERVICES," *ISRA Int. J. Islam. Financ.*, vol. 3, no. 2, pp. 7–40, 2011. doi: https://doi.org/10.55188/ijif.v3i2.130.
[16] E. Smolo and M. Kabir Hassan, "THE POTENTIALS OF MUSHĀRAKAH MUTANĀQISAH FOR ISLAMIC HOUSING FINANCE," *International Journal of Islamic and Middle Eastern Finance and Management*, vol. 4, no. 3. Emerald Group Publishing Ltd., pp. 237–258, Aug. 30, 2011. doi: https://doi.org/10.1108/17538391111166476.
[17] A. Asadov, Z. Bin Muhamad Sori, S. Mohamad Ramadilli, Z. Anwer, and S. V. Shamsudheen, "MUSHARAKAH MUTANAQISAH HOME FINANCING: ISSUES IN PRACTICE," *J. Islam. Account. Bus. Res.*, vol. 9, no. 1, pp. 91–103, 2018. doi: https://doi.org/10.1108/JIABR-08-2015-0036.
[18] DSN MUI, "FATWA DEWAN SYARIAH NASIONAL NO. 114/DSN-MUI/IX/2017 TENTANG AKAD SYIRKAH," *J. Chem. Inf. Model.*, vol. 53, no. 9, pp. 6–9, 2017.
[19] N. Sumarti, *MATEMATIKA KEUANGAN SYARIAH*. Bandung: ITB Press, 2019.
[20] N. Sumarti, V. Fitriyani, M. Damayanti, and A. D. Marendri, "A MATHEMATICAL MODEL OF PROFIT-LOSS SHARING SCHEME OF SMALL INVESTMENT FOR TRADITIONAL MARKET TRADERS USING THE SEMI-FUZZY LOGIC APPROACH," *Procedia - Soc. Behav. Sci.*, vol. 115, no. 2, pp. 131–137, 2017. doi: https://doi.org/10.21098/jimf.v2i2.650.
[21] B. Admin, "PEMBIAYAAN KPR SESUAI PRINSIP SYARIAH." Accessed: Feb. 02, 2025. [Online]. Available: https://www.bankbsi.co.id/news-update/edukasi/pembiayaan-kpr-sesuai-prinsip-syariah
[22] A. Budiono, "PENERAPAN PRINSIP SYARIAH PADA LEMBAGA KEUANGAN SYARIAH," *Law Justice*, vol. 2, no. 1, pp. 54–65, 2017. doi: https://doi.org/10.23917/laj.v2i1.4337.
[23] R. Haneef, S. Kunhibava, and E. Smolo, "MUSHĀRAKAH MUTANĀQIṢAH AND LEGAL ISSUES: CASE STUDY OF MALAYSIA," *ISRA Int. J. Islam. Financ.*, vol. 3, no. 1, pp. 93–122, 2011. doi: https://doi.org/10.55188/ijif.v3i1.120.
[24] A. M. Naim, "PURCHASE UNDERTAKING ISSUES IN MUSHĀRAKAH MUTANĀQIṢAH HOME FINANCING," *ISRA Int. J. Islam. Financ.*, vol. 3, no. 1, pp. 25–47, 2011. doi: https://doi.org/10.55188/ijif.v3i1.12.
[25] S. A. Mikail and M. S. M. Rani, "SHARĪʿAH CONTRACTS UNDERPINNING MUSHĀRAKAH MUTANĀQIṢAH FINANCING: A CONCEPTUAL ANALYSIS," *ISRA Int. J. Islam. Financ.*, vol. 8, no. 1, pp. 35–66, 2016.
[26] M. R. A. Aziz, S. I. Tamrin, A. Saad, M. Z. Mokhtar, and N. S. N. M. Nor, "MUSYARAKAH MUTANAQISAH: A SYSTEMATIC LITERATURE REVIEW," *Inf. Manag. Bus. Rev.*, vol. 15, no. 3, pp. 163–171, 2023. doi: https://doi.org/10.22610/imbr.v15i3(SI).3470
[27] W. Juliyanti and Y. K. Wibowo, "LITERATURE REVIEW: IMPLEMENTATION OF MUSHARAKAH MUTANAQISAH PARTNERSHIP OVER THE WORLD," *Bukhori Kaji. Ekon. dan Keuang. Islam*, vol. 1, no. 1, pp. 1–10, 2021. doi: https://doi.org/10.35912/bukhori.v1i1.196.
[28] A. K. M. Meera and D. A. Razak, "HOME FINANCING THROUGH THE MUSHARAKAH MUTANAQISAH CONTRACTS: SOME PRACTICAL ISSUES," *J. King Abdulaziz Univ. Islam. Econ.*, no. 1, pp. 3–25, 2009. doi: https://doi.org/10.4197/Islec.22-1.1
[29] N. Sumarti, "A MATHEMATICS MODEL FOR DETERMINATING THE VALUE OF IJARAH CONTRACT," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 288, no. 1, 2018, doi: https://doi.org/10.1088/1757-899X/288/1/012111.
[30] W. Murniati, SIMULASI VARIASI JUMLAH DAN PERIODE INVESTASI DALAM MODEL PROFIT AND LOSS SHARING DENGAN DANA TABARRU', Institut Teknologi Bandung, 2015.