BAREKENG: Journal of Mathematics and Its Applications
June 2026 Volume 20 Issue 2 Page 1557–1568
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol20iss2pp1557-1568

WAVELET-BASED COMPUTATIONAL FRAMEWORK FOR THE SOLOW-SWAN ECONOMIC MODEL

Jay Kishore Sahani[1], **Pankaj Sharma**[2], **Nikhil Khanna**[3*], **Ajay Kumar**[4]

[1]*Department of Mathematics, D.A.V. PG College*
Shanti Nagar, Siwan,Bihar, 841226, India.

[2]*Department of Mathematics, Pondicherry University*
R.V. Nagar, Kalapet, Puducherry - 605014, India

[3]*Department of Mathematics, College of Science, Sultan Qaboos University*
P. O. Box 36, Al-Khoud 123, Muscat, Sultanate of Oman

[4]*Department of Science and Computation, Shri Vishwakarma Skill University*
Dudhola, Palwal, Haryana, 121102, India

*Corresponding author's e-mail: ** **n.khanna@squ.edu.om**

Article Info

Article History:

Received: 22nd June 2025
Revised: 31st July 2025
Accepted: 24th September 2025
Available Online: 26th January 2026

Keywords:

Approximate solutions;
Economic growth:
Haar Wavelets;
Numerical Methods;
Solow-Swan Model.

ABSTRACT

In this paper, we introduce an innovative numerical technique for addressing the classical Solow-Swan economic growth model through the application of the Haar wavelet approach. The Solow-Swan model, a cornerstone of neoclassical economics, elucidates long-run economic growth influenced by capital accumulation, labor, and technological advancements. Although various computational methods have been utilized to study its behavior, the use of wavelet-based techniques, specifically Haar wavelets, has been largely overlooked. The Haar wavelet method provides distinct benefits, such as computational simplicity and adaptability to piecewise continuous functions. By transforming the Solow-Swan model into a set of algebraic equations using Haar wavelet expansion, we showcase the method's ability to accurately capture growth dynamics. We present numerical results to substantiate the efficacy of this approach and compare it with conventional numerical techniques, underscoring the advantages of wavelet-based solutions. This study offers a fresh perspective on economic modeling, emphasizing the potential of wavelet theory in the numerical analysis of growth equations.

How to cite this article:

J. K. Sahani, P. Sharma, N. Khanna and A. Kumar., "WAVELET-BASED COMPUTATIONAL FRAMEWORK FOR THE SOLOW-SWAN ECONOMIC MODEL", *BAREKENG: J. Math. & App.*, vol. 20, no. 2, pp. 1557-1568, Jun, 2026.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*
Research Article · Open Access

1. INTRODUCTION

Developed independently in 1956 by Robert Solow and Trevor Swan [1], [2], the Solow-Swan model forms a key pillar of neoclassical economics. It investigates sustained economic growth by focusing on the interactions among capital accumulation, labor, and technological advancements. The model underscores how these components drive an economy toward a steady-state equilibrium, assuming diminishing returns to both capital and labor, with technological progress treated as an exogenous factor. This framework remains vital for analyzing economic development and growth trends in various settings.

The model has been explored, utilized, and elucidated across various scholarly works by numerous researchers [3], [4], [5], [6], [7], [8], [9], [10]. Numerous researchers have addressed the Solow-Swan model using a variety of analytical and numerical approaches. González-Parra et al. [11] employed an explicit finite difference scheme to compute the numerical solution of the spatial Solow model, applying it to smuggling issues observed in Venezuela. Bohner et al. [12] examined the model within the framework of time scales, analyzing its stability properties. Cangiotti and Sensi [13] explored an analytical solution for the Solow-Swan model with non-constant returns to scale and provided numerical simulations to illustrate their findings. Brunner et al. [14] utilized an optimization technique, specifically the simulated annealing method, to estimate the model's parameters. Ureña and Vargas [15] investigated the numerical solution of the Solow model incorporating spatial diffusion, using a generalized finite difference method, and evaluated the scheme's convergence. Later in [16], they developed numerical solutions for the Solow-Swan model with spatial diffusion by applying the finite difference method and analyzing the convergence properties of the approach. To our knowledge, prior studies have not utilized the Haar wavelet method to derive numerical solutions for the Solow-Swan model.

Beyond the mathematical formulation, the Haar wavelet approach also provides meaningful economic insights into long-run growth dynamics. Unlike conventional numerical methods that often smooth out local variations, Haar wavelets are well-suited for capturing discontinuities and localized shocks in the Solow–Swan framework. This feature allows the model to reflect how sudden policy interventions, changes in savings rates, or technological disruptions influence convergence toward the steady state, thereby enriching the interpretation of resilience and adjustment in growth paths. Moreover, our contribution is situated within a growing body of work that applies wavelet-based techniques in economics and applied differential equations. For instance, Chen and Hsiao [17] established an early Haar-wavelet framework for lumped and distributed parameter systems, providing a methodological foundation for wavelet-based dynamics; more recent applications span nonlinear differential equations, e.g., solutions of Bessel equation of zero order using Wilson wavelets [18] and non-linear Liénard-type equations using Haar wavelets [19]—as well as linear Fredholm integral equation systems treated with Legendre multi-wavelets [20].

In this work, we propose and apply the Haar wavelet method as an innovative numerical technique to address the Solow-Swan economic growth model, showcasing its precision and effectiveness in capturing the model's dynamic behavior.

2. RESEARCH METHODS

The present study is entirely focused on deriving numerical solutions for the model through mathematical techniques. Here, we assume the production function exhibits either increasing or decreasing returns to scale. Consequently, the model is formulated as a first-order non-autonomous differential equation, for which we seek an approximate solution.

Let $F: \mathbb{R}^2 \to \mathbb{R}$ represent a family of twice continuously differentiable functions. Consider a production function $F(x_1, x_2)$ that adheres to the Inada conditions (refer to [21], [22], [23], [24], [25]). Denote x_1 and x_2 as K and L, respectively. If the rate of change of K is proportional to F, and the labor force grows exponentially, then

$$\frac{dK}{dt} = sF(K, L), \frac{dL}{dt} = \gamma L, \tag{1}$$

where, $s, \gamma > 0$ are scalars. Now, let $k = \frac{K}{L}$ be the capital-labour ratio and let $f(k) = F(k, 1)$, the classic Solow-Swan model is written as:

$$\frac{dk}{dt} = sf(k) - \gamma k. \quad (2)$$

Drawing from [7], the Solow-Swan model can be expressed as

$$\frac{dk}{dt} = sf(k) - \big(\delta + \gamma(t)\big)k, \quad (3)$$

where s is the fraction of output, which is saved, δ is the depreciation rate, f is a production function, and $\gamma(t)$ is the ratio $\frac{dL}{dt}/L$. Further, from [4], the above equation can be written as

$$\frac{dk}{dt} = sL_0^{(n-1)}e^{(n-1)\gamma t}k^{\alpha} - \gamma k, \quad (4)$$

where $0 < \alpha \leq 1$ $0 < \beta \leq 1$ and $\alpha + \beta = n$.

2.1 Haar Wavelets

Wavelets are a collection of functions derived from a single mother wavelet through processes of dilation and translation. For continuous parameters a (translation) and b (dilation), the family of continuous wavelets, as described in [26], is defined as

$$\psi_{a,b}(x) = |a|^{-\frac{1}{2}}\psi\left(\frac{x-b}{a}\right), a, b \in \mathbb{R}, a \neq 0.$$

For discrete values of the parameters a and b, defined as $a = a_0^{-m}, b = nb_0a_0^{-m}$, with $a_0, \mathrm{b}_0 > 1$, and n, m being natural numbers, the family of discrete wavelets is expressed as

$$\psi_{n,m}(x) = |a_0|^{\frac{m}{2}}\psi(a_0^m x - nb_0),$$

where $\psi_{n,m}$ serves as a basis (or wavelet basis) of $L^2(\mathbb{R})$. When $a_0 = 2$ and $b_0 = 1$, the functions $\psi_{n,m}(x)$ constitute an orthonormal basis.

Alfred Haar first introduced the Haar function in 1910. The Haar wavelet family defined over the interval $[0,1)$ comprises the following functions:

$$h_0(x) = \begin{cases} 1, 0 \leq x < 1 \\ 0, \text{ otherwise}, \end{cases} \quad (5)$$

and

$$h_i(x) = \begin{cases} 1, & \frac{k}{m} \leq x < \frac{k+0.5}{m} \\ -1, & \frac{k+0.5}{m} \leq x < \frac{k+1}{m} \\ 0, & \text{otherwise}, \end{cases} \quad (6)$$

where $m = 2^j, j = 0,1,2,3, \ldots J, k = 0,1,2, \ldots m-1, i = m+k+1$. Here m is the dilation parameter, k is the shift parameter and J is the level of resolution. The function $h_0(x)$ is referred to as the scaling function, while $h_1(x)$ serves as the mother wavelet of the Haar wavelet family. A few Haar wavelets are given below:

$$h_0(x) = \begin{cases} 1, & 0 \leq x < 1 \\ 0, & \text{otherwise} \end{cases}, \quad h_1(x) = \begin{cases} 1, & 0 \leq x < 0.5 \\ -1, & 0.5 \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$

$$h_2(x) = \begin{cases} 1, & 0 \leq x < \frac{0.5}{2} \\ -1, & \frac{0.5}{2} \leq x < \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}, h_3(x) = \begin{cases} 1, & \frac{1}{2} \leq x < \frac{1.5}{2} \\ -1, & \frac{1.5}{2} \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$

$$h_4(x)=\begin{cases}1, & 0\le x<\frac{0.5}{4}\\ -1, & \frac{0.5}{4}\le x<\frac{1}{4}\\ 0, & \text{otherwise}\end{cases}, h_5(x)=\begin{cases}1, & \frac{1}{4}\le x<\frac{1.5}{4}\\ -1, & \frac{1.5}{4}\le x<\frac{2}{4}\\ 0, & \text{otherwise}\end{cases}$$

$$h_6(x)=\begin{cases}1, & \frac{2}{4}\le x<\frac{2.5}{4}\\ -1, & \frac{2.5}{4}\le x<\frac{3}{4}\\ 0, & \text{otherwise}\end{cases}, h_7(x)=\begin{cases}1, & \frac{3}{4}\le x<\frac{3.5}{4}\\ -1, & \frac{3.5}{4}\le x<1\\ 0, & \text{otherwise}\end{cases}.$$

The first integral of Haar family function [6] is defined as

$$p_{1,i}=\int_0^x h_i(t)dt=\begin{cases}x-\frac{k}{m}, & \frac{k}{m}\le x<\frac{k+0.5}{m}\\ \frac{k+1}{m}-x, & \frac{k+0.5}{m}\le x<\frac{k+1}{m}.\\ 0, & \text{otherwise}\end{cases} \tag{7}$$

The first integral of a few Haar wavelets is given below:

$$p_{1,0}=\int_0^x h_0(x)dx=\begin{cases}x, & 0\le x\le 1\\ 0, & \text{otherwise}\end{cases}, \quad p_{1,1}=\int_0^x h_1(x)dx=\begin{cases}x-0, & 0\le x<0.5\\ 1-x, & 0.5\le x<1\\ 0, & \text{otherwise}\end{cases}$$

$$p_{1,2}=\int_0^x h_2(x)dx=\begin{cases}x-0, & 0\le x<\frac{0.5}{2}\\ \frac{1}{2}-x, & \frac{0.5}{2}\le x<\frac{1}{2}\\ 0, & \text{otherwise}\end{cases}, \quad p_{1,3}=\int_0^x h_3(x)dx=\begin{cases}x-\frac{1}{2}, & \frac{1}{2}\le x<\frac{1.5}{2}\\ 1-x, & \frac{1.5}{2}\le x<1\\ 0, & \text{otherwise}\end{cases}$$

$$p_{1,4}=\int_0^x h_4(x)dx=\begin{cases}x-0, & 0\le x<\frac{0.5}{4}\\ \frac{1}{4}-x, & \frac{0.5}{4}\le x<\frac{1}{4}\\ 0, & \text{otherwise}\end{cases}, \quad p_{1,5}=\int_0^x h_5(x)dx=\begin{cases}x-\frac{1}{4}, & \frac{1}{4}\le x<\frac{1.5}{4}\\ \frac{2}{4}-x, & \frac{1.5}{4}\le x<\frac{2}{4}\\ 0, & \text{otherwise}\end{cases}$$

$$p_{1,6}=\int_0^x h_6(x)dx=\begin{cases}x-\frac{2}{4}, & \frac{2}{4}\le x<\frac{2.5}{4}\\ \frac{3}{4}-x, & \frac{2.5}{4}\le x<\frac{3}{4}\\ 0, & \text{otherwise}\end{cases}, p_{1,7}=\int_0^x h_7(x)dx=\begin{cases}x-\frac{3}{4}, & \frac{3}{4}\le x<\frac{3.5}{4}\\ 1-x, & \frac{3.5}{4}\le x<1\\ 0, & \text{otherwise}\end{cases}.$$

Every continuous function $f(x)\in L^2(\mathbb{R})$ defined on $[0,1)$ can be expressed as:

$$f(x)=\sum_{i=0}^{\infty} c_i h_i(x), \tag{8}$$

where, $c_i=\left(2^{j-1}\right)\int_0^1 u(x)h_i(x)dx$ are unknown coefficients and h_i are Haar basis functions. If Eq. (8) is truncated, then it can be expressed as:

$$f(x)=\sum_{i=0}^{2^{J-1}} c_i h_i(x), \tag{9}$$

where C and $h_i(x)$ are $2^{\theta-1}\times 1$ matrices given by:

$$C=\left[c_{10},c_{11},\dots c_{1j-1},c_{20},\dots,c_{2j-1},\dots,c_{2^{\theta-1}0},\dots,c_{2^{\theta-1}j-1}\right]^T,$$

$$h(x)=\left[h_{10}(x),h_{11}(x),\dots,h_{1j-1}(x),h_{20}(x),\dots,h_{2j-1}(x)\dots,h_{2^{j-1}0}(x),\dots,h_{2^{\theta-1}j-1}(x)\right]^T.$$

For further insights into wavelets and Haar wavelets, refer to [17], [18], [19], [20], [27], [28], [29], [30], [31].

3. RESULTS AND DISCUSSION

3.1 Methodology with Numerical Simulations

Consider the Solow-Swan model $k'(t) = sL_0^{(n-1)} e^{(n-1)\gamma t} k(t)^{\alpha} - \gamma k(t)$ with initial value: $k(0) = k(0)$. Let

$$k'(t) = \sum_{i=0}^{2^{J-1}} c_i h_i(t), \tag{10}$$

where c_i are unknown coefficients and $h_i(t)$ are Haar wavelet basis. Integrating the above equation with limit from 0 to t, we get

$$k(t) = \sum_{i=0}^{2^{J-1}} c_i p_{1,i}(t) + k(0). \tag{11}$$

Therefore, Eq. (4) becomes

$$\sum_{i=0}^{2^{J-1}} c_i h_i(t) - sL_0^{(n-1)} e^{(n-1)\gamma t} \left(\sum_{i=0}^{2^{J-1}} c_i p_{1,i}(t) + k(0) \right)^{\alpha} + \gamma \sum_{i=0}^{2^{J-1}} c_i p_{1,i}(t) + k(0) = 0. \tag{12}$$

Now, for the collocation points $2^{j-1}, j = 1,2, \dots, N$, we get a set of 2^{j-1} algebraic equations with 2^{j-1} unknown coefficients c_i. Solving these equations using Newton-Raphson method in MATLAB, we can get the value of the unknown coefficients c_i. Finally, substitute these values of c_i into Eq. (12), we can obtain the Haar wavelet approximate solution for the given problem.

Problem 1. Consider the Solow-Swan model $k'(t) - sL_0^{(n-1)} e^{(n-1)\gamma t} k(t)^{\alpha} + \gamma k(t) = 0$ with initial values: $k(0) = 1, L_0 = 1, n = 0.85, \alpha = 0.2$ and $\beta = n - \alpha, \gamma = 0.7$.

The analytical solution of this model is given by

$$k(t) = \left(e^{(\alpha-1)\gamma t} \left[s(1-\alpha) L_0^{(n-1)} \frac{e^{\gamma \beta t} - 1}{\gamma \beta} + (k(0))^{1-\alpha} \right] \right)^{\frac{1}{1-\alpha}}. \tag{13}$$

Let

$$k'(t) = \sum_{i=0}^{2^{J-1}} c_i h_i(t). \tag{14}$$

Integrating the above equation with respect to 't' with limit from 0 to t, we have

$$k(t) = \sum_{i=0}^{2^{J-1}} c_i p_{1,i}(t) + k(0). \tag{15}$$

Therefore, the Eq. (4) reduces to the following form:

$$\sum_{i=0}^{2^{J-1}} c_i h_i(t) - 0.4 e^{0.53t} \left(\sum_{i=0}^{2^{J-1}} c_i p_{1,i}(t) + k(0) \right)^{0.6} + 0.7 \left(\sum_{i=0}^{2^{J-1}} c_i p_{1,i}(t) + k(0) \right) = 0. \tag{16}$$

Now, we apply the suggested method for $i = 4$. Therefore, we get a set of 4 nonlinear equations. Solving these equations by Newton-Raphson method, we get the unknown coefficients c_i. The approximate solution of Problem 1 obtained by the suggested method and analytical solution are compared in Table 1. Also, the solution plots and their error estimates are given in Fig. 1 and Fig. 2.

Table 1. Approximate Solutions and Error Estimation of Problem 1

t	Exact Solution	RK45	HWS at $N = 16$	|Exact - RK45|	|Exact-HWS|
0.0313	0.9907	1.0000	0.9908	0.93e − 02	0.6812e − 04
0.0938	0.9725	0.9814	0.9726	0.90e − 02	0.6389e − 04
0.1563	0.9548	0.9634	0.9549	0.86e − 02	0.5988e − 04
0.2188	0.9377	0.9460	0.9377	0.83e − 02	0.5608e − 04
0.2813	0.9210	0.9290	0.9211	0.80e − 02	0.5249 e - 04
0.3438	0.9048	0.9125	0.9049	0.77e − 02	0.4910 e - 04
0.4063	0.8891	0.8965	0.8892	0.74e − 02	0.4589e − 04
0.4688	0.8738	0.8809	0.8739	0.71e − 02	0.4286 e - 04
0.5313	0.8590	0.8658	0.8590	0.68e − 02	0.3999e − 04
0.5938	0.8445	0.8511	0.8446	0.66e − 02	0.3729 e - 04
0.6563	0.8305	0.8368	0.8305	0.63e − 02	0.3473e − 04
0.7188	0.8168	0.8229	0.8169	0.61e − 02	0.3231e − 04
0.7813	0.8035	0.8094	0.8036	0.59e − 02	0.3003e − 04
0.8438	0.7906	0.7963	0.7906	0.56e − 02	0.2787e − 04
0.9063	0.7780	0.7835	0.7780	0.54e − 02	0.2584e − 04
0.9688	0.7658	0.7710	0.7658	0.52e − 02	0.2392e − 04

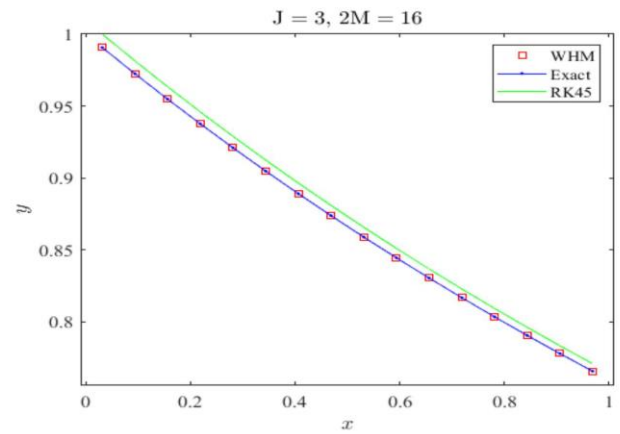


Figure 1. Haar Wavelet Solution for Problem 1

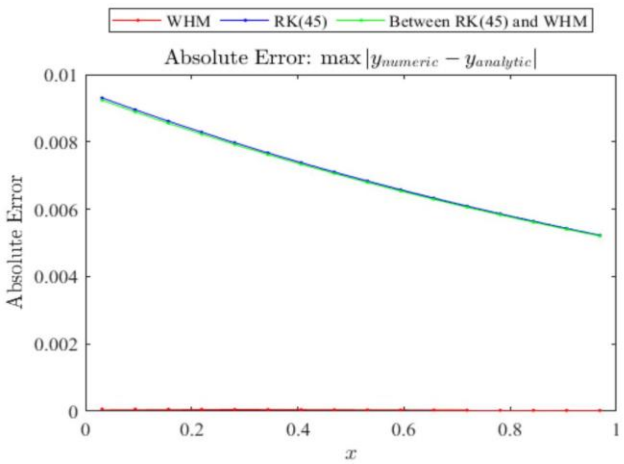


Figure 2. Error Analysis of Haar Wavelet Solution for Problem 1

Problem 2. Suppose that the Eq. (4) is of the form:

$$k'(t) - sL_0^{(n-1)}e^{(n-1)\gamma t}k(t)^{\alpha} + \gamma k(t) = 0,$$

where $k(0) = 1, L_0 = 1, n = 1.5, \alpha = 0.6$ and $\beta = n - \alpha, \gamma = 0.7$.

We solve Eq. (4) by the suggested method for $i = 4$ as discussed in Problem 1. The approximate solution of the Solow-Swan model by the suggested method is compared with the analytical solution. The subsequent Table 2 shows the outcomes. Also, the solution plots and their error estimates are given in Fig. 3 and Fig. 4.

Table 2. Approximate Solutions and Error Estimation of Problem 2

t	exact Solution	RK45	HWS at $N = 16$	|exact - RK45|	|exact - HWS|
0.0313	0.9908	1.0000	0.9909	0.92e − 02	0.1325e − 03
0.0938	0.9737	0.9821	0.9732	0.90e − 02	0.1262e − 03
0.1563	0.9564	0.9652	0.9566	0.87e − 02	0.1203e − 03
0.2188	0.9408	0.9493	0.9409	0.85e − 02	0.1148e − 03
0.2813	0.9261	0.9344	0.9263	0.83e − 02	0.1097e − 03
0.3438	0.9124	0.9205	0.9125	0.81e − 02	0.1049e − 03
0.4063	0.8997	0.9075	0.8998	0.78e − 02	0.1004e − 03
0.4688	0.8878	0.8955	0.8879	0.77e − 02	0.0962e − 03
0.5313	0.8768	0.8843	0.8769	0.75e − 02	0.0923e − 03
0.5938	0.8667	0.8740	0.8668	0.73e − 02	0.0887e − 03
0.6563	0.8575	0.8646	0.8576	0.71e − 02	0.0854e − 03
0.7188	0.8491	0.8560	0.8491	0.69e − 02	0.0823e − 03
0.7813	0.8415	0.8483	0.8416	0.68e − 02	0.0794e − 03
0.8438	0.8347	0.8413	0.8348	0.66e − 02	0.0768e − 03
0.9063	0.8287	0.8352	0.8288	0.65e − 02	0.0745e − 03
0.9688	0.8235	0.8299	0.8236	0.64e − 02	0.0723e − 03

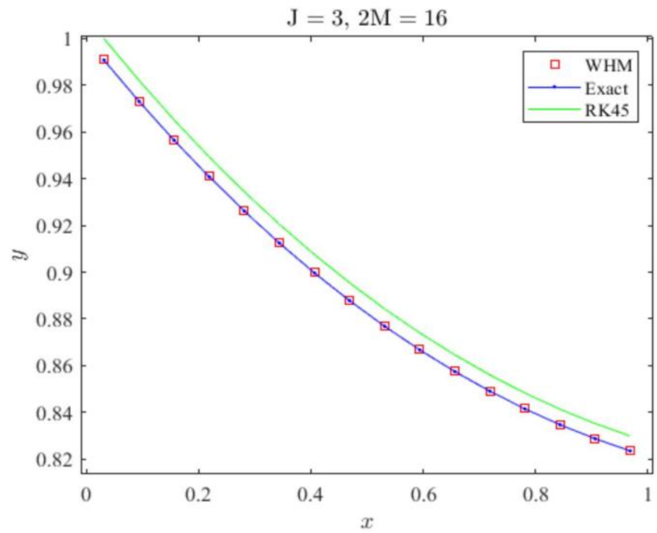


Figure 3. Haar wavelet solution for the Problem 2

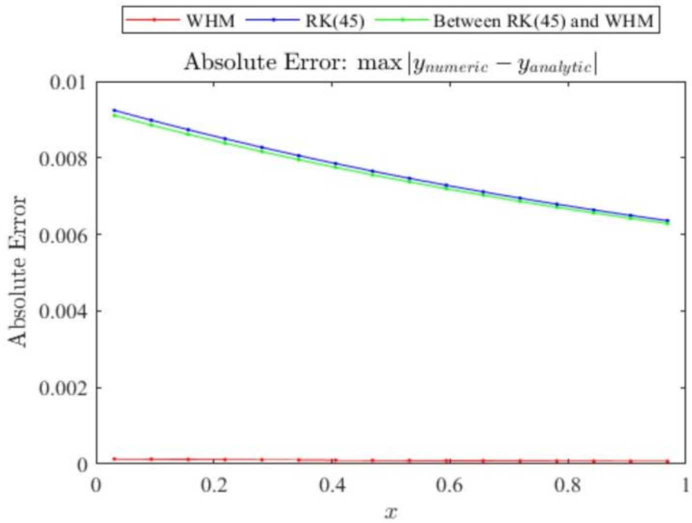


Figure 4. **Error Analysis of Haar wavelet solution for Problem 2**

3.2 Normalization Process for General Solution of Solow-Swan Model within $[\mathbf{0}, \mathbf{1}]$

Given the Haar function is defined on $[0,1]$, we need a transformation which maps the domain $[A, B]$ to $[0,1]$. For this purpose, we consider the solution of the Solow-Swan differential equation as $k(t)$ defined over the interval $[A, B]$, subject to the initial condition $k(A) = 1$. We then introduce a new variable $t_1 = \frac{t-A}{B-A}$. The corresponding differential equation, expressed in terms of this new variable, is formulated as

$$\frac{dk}{dt} = \frac{dk}{dt_1}\frac{dt_1}{dt} = \frac{1}{(B-A)}\frac{dk}{dt_1}. \tag{17}$$

The Solow-Swan equation, along with its initial conditions, will now be converted into a new equation based on the new variable t_1 in the domain $[0,1]$. By implementing the technique outlined in Subsection 3.1 on this new equation, we can determine the unknown coefficients c_i. After calculating these coefficients, we transform the new variable into the original variable t. Subsequently, we apply this procedure to the Solow-Swan model over the domain [0,10]. This transformation simplifies the application of Haar wavelets by aligning the solution domain with the wavelet basis defined on [0, 1].

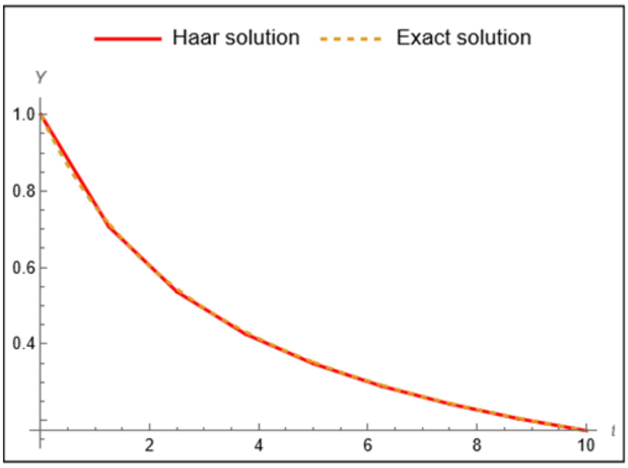
Problem 3. Consider the model $k'(t) - sL_0^{(n-1)}e^{(n-1)\gamma t}k(t)^\alpha + \gamma k(t) = 0$ with initial values:

$$k(0) = 1, L_0 = 1, n = 0.85, \alpha = 0.2 \text{ and } \beta = n - \alpha, \gamma = 0.7, t \in [0,10].$$

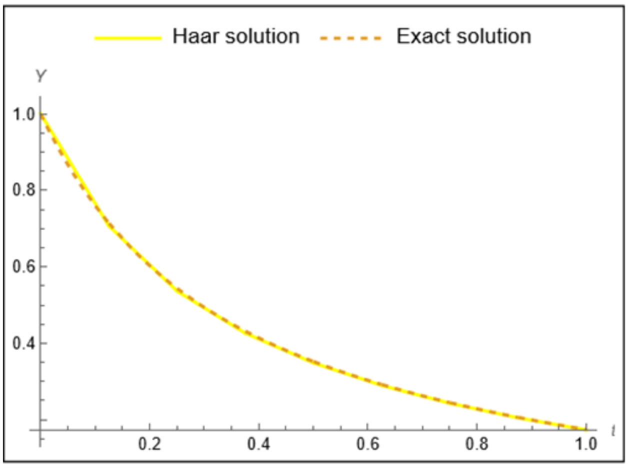
Consider the new variable $t_1 = \frac{t}{10}$. Then the new equation becomes:

$$\frac{1}{10}\frac{dk}{dt_1} - 0.4\exp\left(-0.105 * t_1\right)k(t_1)^{0.2} + 0.7k(t_1) = 0 \tag{18}$$

By applying the suggested method to solve Eq. (18), we obtain an approximate solution for Problem 3 within the interval [0,1]. Additionally, by converting the new variable back to the original one, we derive the approximate solution for the model across the domain [0,10]. Fig. 5 illustrates the solution of the model for both the [0,1] and [0,10] intervals.



(a)



(b)

Figure 5. Comparison between the Haar Wavelet Approximate Solution ("Haar") and the Analytical Solution ("Exact") for Problem 3 in Two Domains: (a) $[0, 1]$ and (b) $[0, 10]$

4. CONCLUSION

The wavelet-based approach presented here demonstrates superior accuracy compared to traditional numerical techniques, as confirmed by benchmark test cases and comparisons with analytical solutions. Notably, this method proves highly effective in approximating the Solow-Swan growth model, achieving reliable results even with a minimal number of grid points, thereby reducing computational requirements while preserving accuracy. Nevertheless, to further improve the precision of the solution, it is advisable to increase the number of collocation points, which enhances the accuracy and robustness of the model's dynamic representation.

Author Contributions

Jay Kishore Sahani: Conceptualization, Formal Analysis, Investigation, Methodology, Software, Writing - Original Draft. Pankaj Sharma: Validation, Writing - Review and Editing. Nikhil Khanna: Project Administration, Supervision, Validation, Visualization, Writing - Review and Editing. Ajay Kumar: Validation, Writing - Review and Editing. All authors discussed the results and contributed to the final manuscript.

Funding Statement

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Acknowledgment

The authors thank the anonymous referees for their insightful comments and constructive suggestions, which have helped to improve the quality of this work.

Declarations

The authors declare that they have no conflict of interest in reporting study.

Declarations of Generative AI and AI-assisted technologies

The authors declare that no generative AI or AI-assisted technologies were used in the preparation of this manuscript, including for writing, editing, data analysis, or the creation of tables and figures.

REFERENCES

[1] R. M. Solow, "A CONTRIBUTION TO THE THEORY OF ECONOMIC GROWTH," *The Quarterly Journal of Economics*, vol. 70, no. 1, pp. 65-94, February 1956, doi: https://doi.org/10.2307/1884513

[2] T. W. Swan, "ECONOMIC GROWTH AND CAPITAL ACCUMULATION," *Economic Record*, vol. 32, no. 2, pp. 334-361, November 1956, doi: https://doi.org/10.1111/j.1475-4932.1956.tb00434.x

[3] O. Bajo-Rubio, "A FURTHER GENERALIZATION OF THE SOLOW GROWTH MODEL: THE ROLE OF THE PUBLIC SECTOR," *Economics Letters*, vol. 68, no. 1, pp. 79-84, July 2000, doi: https://doi.org/10.1016/S0165-1765(00)00220-2

[4] A. Dohtani, "A GROWTH-CYCLE MODEL OF SOLOW-SWAN TYPE, I," *Journal of Economic Behavior & Organization*, vol. 76, no. 2, pp. 428-444, November 2010, doi: https://doi.org/10.1016/j.jebo.2010.07.006

[5] R. E. A. Farmer, *MACROECONOMICS*, 3rd ed., Mason, OH: South-Western College Publishing, 2008.

[6] G. Gandolfo, *ECONOMIC DYNAMICS*, Heidelberg: Springer-Verlag, 1997.

[7] L. Guerrini, "THE SOLOW-SWAN MODEL WITH A BOUNDED POPULATION GROWTH RATE," *Journal of Mathematical Economics*, vol. 42, no. 1, pp. 14-21, February 2006, doi: https://doi.org/10.1016/j.jmateco.2005.05.001

[8] V. Halsmayer, "FROM EXPLORATORY MODELING TO TECHNICAL EXPERTISE: SOLOW'S GROWTH MODEL AS A MULTIPURPOSE DESIGN," History of Political Economy, vol. 46, Supplement 1, pp. 229-251, December 2014. doi: https://doi.org/10.1215/00182702-2716181

[9] D. A. Kulikov, "*THE GENERALIZED SOLOW MODEL*," Journal of Physics: Conference Series, vol. 1205, pp. 012033, 2019, doi: https://doi.org/10.1088/1742-6596/1205/1/012033

[10] N. L. P. Lundström, "HOW TO FIND SIMPLE NONLOCAL STABILITY AND RESILIENCE MEASURES," Nonlinear Dynamics, vol. 93, pp. 887-908, April 2018, doi: https://doi.org/10.1007/s11071-018-4234-x

[11] G. González-Parra, B. Chen-Charpentier, A. J. Arena, and M. Díaz-Rodríguez, "MATHEMATICAL MODELING OF PHYSICAL CAPITAL DIFFUSION USING A SPATIAL SOLOW MODEL: APPLICATION TO SMUGGLING IN VENEZUELA," *Economies*, vol. 10, no. 7, pp. 164, July 2022, doi: https://doi.org/10.3390/economies10070164

[12] M. Bohner, J. Heim, and A. Liu, "QUALITATIVE ANALYSIS OF A SOLOW MODEL ON TIME SCALES," *Journal of Concrete and Applicable Mathematics*, vol. 13, no.'s 3-4, pp. 183-197, 2015.

[13] N. Cangiotti and M. Sensi, "EXACT SOLUTIONS FOR A SOLOW-SWAN MODEL WITH NON-CONSTANT RETURNS TO SCALE," Indian Journal of Pure and Applied Mathematics, vol. 54, pp. 1278-1285, December 2023, doi: https://doi.org/10.1007/s13226-022-00341-7

[14] N. Brunner, G. Mayrpeter, and M. Kühleitner, "PARAMETER ESTIMATION OF THE SOLOW-SWAN FUNDAMENTAL DIFFERENTIAL EQUATION," Heliyon, vol. 8, no. 10, pp. 1-11, October 2022, doi: https://doi.org/10.1016/j.heliyon.2022.e10816

[15] N. Ureña and A. M. Vargas, "ON THE NUMERICAL SOLUTION TO A SOLOW MODEL WITH SPATIAL DIFFUSION AND TECHNOLOGY-INDUCED MOTILITY," Engineering Analysis with Boundary Elements, vol. 157, pp. 541-552, December 2023, doi: https://doi.org/10.1016/j.enganabound.2023.09.026

[16] N. Ureña and A. M. Vargas, "NUMERICAL SOLUTION TO A PARABOLIC ODE SOLOW-SWAN MODEL WITH SPATIAL DIFFUSION AND TECHNOLOGY-INDUCED MOTILITY," Journal of Computational and Applied Mathematics, vol. 447, pp. 115913, September 2024, doi: https://doi.org/10.1016/j.cam.2024.115913

[17] C. F. Chen and C. H. Hsiao, "HAAR WAVELET METHOD FOR SOLVING LUMPED AND DISTRIBUTED PARAMETER SYSTEMS," IEE Proceedings - Control Theory and Applications, vol. 144, no. 1, pp. 87-94, January 1997, doi: https://doi.org/10.1049/ip-cta:19970702

[18] M. Devi and B. Yadav, "SOLVING BESSEL EQUATION OF ZERO ORDER USING WILSON WAVELETS," Poincare J. Anal. Appl., vol. 9, no. 2, pp. 239-247, December 2022, doi: https://doi.org/10.46753/pjaa.2022.v09i02.008

[19] J. K. Sahani, P. Kumar, A. Kumar, and N. Khanna, "NUMERICAL SOLUTION OF NON-LINEAR LIÉNARD EQUATION USING HAAR WAVELET METHOD," Poincare J. Anal. Appl., vol. 11, no. 2, pp. 197-210, December 2024, doi: https://doi.org/10.46753/pjaa.2024.v011i02.009

[20] M. Devi, B. Yadav, and P. Vats, "NUMERICAL SOLUTION OF LINEAR FREDHOLM INTEGRAL EQUATIONS SYSTEM BY LINEAR LEGENDRE MULTI-WAVELETS," Poincare J. Anal. Appl., vol. 12, no. 2, pp. 43-59, May 2025, doi: https://doi.org/10.46753/pjaa.2025.v012i02.005

[21] P. Barelli and S. de Abreu Pessôa, "INADA CONDITIONS IMPLY THAT PRODUCTION FUNCTION MUST BE ASYMPTOTICALLY COBB-DOUGLAS," Economics Letters, vol. 81, no. 3, pp. 361-363, December 2003, doi: https://doi.org/10.1016/S0165-1765(03)00218-0

[22] K.-I. Inada, “ON A TWO-SECTOR MODEL OF ECONOMIC GROWTH: COMMENTS AND A GENERALIZATION,” *The Review of Economic Studies*, vol. 30, no. 2, pp. 119-127, June 1963, doi: https://doi.org/10.2307/2295809

[23] A. Litina and T. Palivos, “DO INADA CONDITIONS IMPLY THAT PRODUCTION FUNCTION MUST BE ASYMPTOTICALLY COBB-DOUGLAS? A COMMENT,” Economics Letters, vol. 99, no. 3, pp. 498-499, June 2008, doi: https://doi.org/10.1016/j.econlet.2007.09.035

[24] A. Takayama and T. Akira, *MATHEMATICAL ECONOMICS*, 2nd ed. Cambridge, UK: Cambridge University Press, 1985.

[25] H. Uzawa, “ON A TWO-SECTOR MODEL OF ECONOMIC GROWTH II,” The Review of Economic Studies, vol. 30, no. 2, pp. 105-118, June 1963, doi: https://doi.org/10.2307/2295808

[26] Siraj-ul-Islam, I. Aziz, and B. Šarler, “THE NUMERICAL SOLUTION OF SECOND ORDER BOUNDARY VALUE PROBLEMS BY COLLOCATION METHOD WITH THE HAAR WAVELETS,” Mathematical and Computer Modelling, vol. 52, no.’s 9-10, pp. 1577-1590, November 2010, doi: https://doi.org/10.1016/j.mcm.2010.06.023

[27] M. J. Kheirdeh, A. Askari-Hemmat, and H. Saeedi, “ON s-ELEMENTARY WAVELETS IN $\mathbb{R}$ AND THEIR APPLICATIONS IN SOLVING INTEGRAL EQUATIONS,” Poincare J. Anal. Appl., vol. 11, no. 1, pp. 67-84, June 2024, doi: https://doi.org/10.46753/pjaa.2024.v011i01.005

[28] S. Lal, H. C. Yadav, and Abhilasha, “APPROXIMATION OF FUNCTIONS BELONGING TO $C^{M,\alpha}[0,1)$ CLASS AND SOLUTION OF CHANDRASEKHAR’S WHITE DWARFS AND PANTOGRAPH DIFFERENTIAL EQUATION BY GENOCCHI WAVELETS,” Poincare J. Anal. Appl., vol. 10, no. 3, pp. 171-186, December 2023, doi: https://doi.org/10.46753/pjaa.2023.v010i03.011

[29] M. Singh, S. Sharma, and S. Rawan, “SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS USING OPERATIONAL MATRIX OF BERNOULLI ORTHONORMAL POLYNOMIALS,” Poincare J. Anal. Appl., vol. 7, no. 1, pp. 51-60, June 2020, doi https://doi.org/10.46753/pjaa.2020.v07i01.005

[30] P. Yadav and S. Jahan, “FIBONACCI WAVELETS APPROACH FOR SOLVING NON LINEAR FREDHOLM INTEGRAL EQUATIONS,” Poincare J. Anal. Appl., vol. 10, no. 3, pp. 55-67, December 2023, doi: https://doi.org/10.46753/pjaa.2023.v010i03.004

[31] M. Devi, S. Sharma, and S. Rawan, “NUMERICAL SOLUTIONS OF SYSTEM OF LINEAR DIFFERENTIAL EQUATIONS USING HAAR WAVELET APPROACH,” Poincare J. Anal. Appl., vol. 10, no. 1, pp. 61-73, June 2023, doi: https://doi.org/10. 46753/pjaa.2023.v010i01.005