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ENHANCING FUZZY TIME SERIES FORECASTING WITH REVISED HEURISTIC KNOWLEDGE: A CASE STUDY ON TUBERCULOSIS IN SABAH

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ABSTRACT

Accurate forecasting of tuberculosis (TB) cases is essential for effective public health planning, particularly in regions such as Sabah, Malaysia, where TB remains a significant and persistent health concern. This study aims to improve the accuracy of fuzzy time series models by refining the construction of Fuzzy Logical Relationship Groups using a revised heuristic framework. The proposed approach embeds domain-informed rules to dynamically adjust the formulation of fuzzy relationships. It was applied to monthly tuberculosis case data from 2012 to 2019 and evaluated against both the original fuzzy time series model and a heuristic-based variant. The revised heuristic model achieved the best forecasting accuracy, recording a Mean Squared Error of 1315.0160, a Root Mean Square Error of 36.2631, a Mean Absolute Error of 0.0566, and a Mean Absolute Percentage Error of 0.0138 percent. These consistently lower error values confirm the superiority of the revised model compared to the benchmarks. The study demonstrates that incorporating refined heuristic strategies enables fuzzy time series models to capture the dynamic nature of disease data more effectively. However, the analysis is limited to univariate data (monthly tuberculosis cases), and future work should consider multivariate and hybrid approaches. This research contributes to the understanding by demonstrating that revised heuristic knowledge significantly enhances the predictive capability of fuzzy time series models. The findings provide more reliable forecasts for tuberculosis trends and establish a foundation for broader applications in infectious disease forecasting and healthcare analytics.

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1. INTRODUCTION

Tuberculosis (TB) remains one of the world's leading infectious disease killers. According to the World Health Organization (WHO), an estimated 10.6 million people fell ill with TB in 2022, and 1.3 million deaths were recorded globally, making TB the second leading infectious cause of death after COVID-19 [1]. Despite ongoing global control efforts, TB continues to pose serious public health challenges, particularly in low- and middle-income countries where social determinants such as poverty, overcrowding, and limited healthcare access exacerbate disease transmission.

In Malaysia, TB continues to be a significant public health concern. In 2022, the Ministry of Health reported 25,391 new TB cases nationwide, corresponding to an incidence rate of approximately 77.8 per 100,000 population, marking a 17% increase from the previous year [2]. Sabah recorded 1,944 cases as of early May 2024, accounting for 21.9% of the national total [3]. Moreover, in 2023, Sabah had the highest state-level TB incidence with 5,814 reported cases, surpassing Selangor and Sarawak [4]. These figures underscore the pressing need for precise and timely forecasting to support healthcare planning and targeted interventions, particularly in regions with dynamic trends in TB. Tuberculosis (TB) continues to be a major global health threat, and in Malaysia, particularly in the state of Sabah, the disease remains endemic and challenging to control due to various socioeconomic and geographical factors. Accurate forecasting of TB cases plays a crucial role in enabling the government and healthcare stakeholders to plan resource allocation, execute targeted interventions, and minimize disease transmission.

Traditional forecasting methods, such as the Autoregressive Integrated Moving Average (ARIMA) and other statistical models, have long been used in epidemiological studies [5]. However, these models often struggle when dealing with data characterized by imprecision, uncertainty, or vagueness common features in real-world healthcare datasets. Consequently, researchers have turned to soft computing techniques such as Fuzzy Time Series (FTS), initially introduced by Song and Chissom [6], as a more suitable tool for forecasting such uncertain data.

The FTS model has been widely adopted and extended due to its strength in handling linguistic variables and imprecise data [7], [8]. Subsequent developments have introduced enhancements in aspects such as interval partitioning, fuzzification, rule generation, and forecasting mechanisms [9], [10], [11], and [12]. Huarng [13] addressed the challenge of determining an appropriate interval length in fuzzy time series forecasting. He emphasized that the intervals should be carefully selected; overly large intervals may obscure important fluctuations in the data, while overly small intervals may reduce the meaningfulness of the fuzzy representation. To overcome this, Huarng proposed both distribution-based and average-based approaches for interval determination, and his findings showed that the average-based method yielded higher forecasting accuracy. In a related development, Li and Chen [14] introduced the 3-4-5 partitioning rule, a heuristic method that applies natural number partitions to systematically define interval lengths, thereby improving forecasting performance.

One of the most influential aspects in improving FTS forecasting accuracy is the construction of Fuzzy Logical Relationship Groups (FLRGs**)** **[15]**, **[16]**, and **[17]**. The way in which intervals and rules are designed significantly impacts the predictive capability of the model. Several studies have applied heuristic knowledge to improve the interval formulation and FLRG structure, yielding promising results in various domains, including the forecasting of students enrollment in Alabama University **[18]**, air quality management **[19]**, monthly rainfall forecasting in Semarang City, Indonesia **[20]**, and the forecast of the Indonesian Islamic stock index (ISSI) **[21]**. Notably, Singh et. al. **[22]**, and Shafi et. al. **[23]** demonstrated the potential of fuzzy time series forecasting based on hesitant fuzzy sets. Chung-Ming Own **[24]** contributes foundational insights by introducing two enhanced models: a weighted heuristic model that fortifies Huarng's method, and a knowledge-based high-order fuzzy time series (FTS) model that addresses the memory and ambiguity issues inherent in Chen's high-order FTS design. Own's experimental results applied to university enrollment and futures indices demonstrate that these knowledge-augmented models produce more accurate forecasts compared to both baseline methods.

Recent studies have explored advanced fuzzy time series and hybrid forecasting approaches across various domains, such as mixed-order FTS models for robust forecasting performance (Wu et al. [25] and Egrioglu et al. [26]) and heuristic-optimized FTS applied to air quality management (Bhagat et al. [27]). Additionally, comparisons of heuristic versus standard FTS in economic projections Utami et al. [28] and reviews of TB prediction models highlight the methodological gaps this study aims to address. However, many heuristic approaches still rely on static or overly generalized rules, which may not sufficiently capture the trends and anomalies in real health data. Recognizing this limitation, our earlier study introduced a revised heuristic knowledge framework in an average-based interval fuzzy time series model to forecast monthly TB cases in Sabah [29]. This revised method demonstrated a noticeable improvement in forecast accuracy compared to models using conventional heuristics.

Despite numerous studies applying fuzzy time series (FTS) models in forecasting, existing approaches often struggle to capture the complex and irregular patterns found in epidemiological data such as tuberculosis (TB). Conventional partitioning strategies and heuristic frameworks, while useful, may lack the flexibility to adapt to dynamic case trends in high-burden regions such as Sabah. This highlights the need for refined methodologies that integrate domain-specific knowledge into the modeling process. The main contribution of this study lies in refining the construction of Fuzzy Logical Relationship Groups through revised heuristic knowledge, designed to improve forecasting accuracy. By systematically comparing the proposed model with both the original fuzzy time series and conventional heuristic-based approaches, the study provides empirical evidence of improved predictive performance. The results are expected to benefit public health authorities by offering more reliable forecasting tools for TB case trends, thus supporting data-driven decision-making, efficient resource allocation, and timely intervention planning.

2. RESEARCH METHODS

2.1 Fuzzy Time Series

The concept of fuzzy time series (FTS) was first introduced by Song and Chissom [6], [30] as a forecasting method for problems where historical data are better represented using linguistic terms rather than precise numerical values. In FTS, raw data are transformed into linguistic variables through fuzzification, allowing the model to effectively handle uncertainty and imprecision.

Let U be the universe of discourse, $U = \{u_1, u_2, u_3, \dots, u_n\}$. A fuzzy set A_i of U is defined by

$$A_i = \frac{f_{A_i}(u_1)}{u_1} + \frac{f_{A_i}(u_2)}{u_2} + \cdots + \frac{f_{A_i}(u_n)}{u_n}, \tag{1}$$

where f_{A_i} is the membership function of fuzzy set A_i, $f_{A_i}: U \to [0,1]$. The element of fuzzy sets A_i is defined as u_k, and $f_{A_i}(u_k)$ is the degree of belongingness of u_k to A_i.

Definition 1. *Let $\boldsymbol{Y(t)} \subseteq \mathbb{R}$ be a time series. If each $\boldsymbol{Y(t)}$ is associated with a fuzzy set $\boldsymbol{f_i(t)}$, then $\boldsymbol{F(t)}$ is called a fuzzy time series on $\boldsymbol{Y(t)}$.*

In traditional time series, each data point is a crisp (exact) numerical value. However, in fuzzy time series, each value is converted into a fuzzy set, which is essentially used to express the data in linguistic terms (e.g., "high", "medium", "low") with associated membership grades. This enables better handling of vagueness and imprecision, which is particularly useful in real-world applications such as forecasting health data or economic indicators.

Definition 2. *If there exists a fuzzy relation $\boldsymbol{R(t-1,t)}$, such that $\boldsymbol{F(t) = F(t-1) \times R(t-1,t)}$, then $\boldsymbol{F(t)}$ is said to be caused by $\boldsymbol{F(t-1)}$, and the relationship can be denoted as $\boldsymbol{F(t-1) \to F(t)}$.*

This definition introduces the idea of causality in fuzzy time series. The symbol $\times$ represents a fuzzy operation (often composition). This formulation says that the fuzzy state at time $\boldsymbol{t}$ is influenced by the fuzzy state at time $\boldsymbol{t-1}$ through a fuzzy relational rule. This is analogous to how traditional time series might model $\boldsymbol{Y(t)}$ as a function of $\boldsymbol{Y(t-1)}$, but now with fuzzy logic.

Definition 3. *Suppose $\boldsymbol{F(t-1) = A_i}$ and $\boldsymbol{F(t) = A_j}$, the fuzzy logical relationship (FLR) is denoted as $\boldsymbol{A_i \to A_j}$. Meanwhile, repeated FLRs are typically removed.*

This notation captures the transition of fuzzy states over time. If, at time $\boldsymbol{t-1}$, the data belongs to fuzzy set $\boldsymbol{A_i}$ and at time $\boldsymbol{t}$, it transitions to fuzzy set $\boldsymbol{A_j}$, then the FLR describes this mapping. In practice, these rules are extracted from historical data and later used for forecasting.

Definition 4. *A fuzzy logical relationship group (FLRG) is formed by grouping all FLRs that share the same left-hand side as $A_i \to A_{j1}, A_{j2}, A_{j3}, \dots, A_{jk}$.*

In many cases, the same fuzzy set A_i might lead to multiple outcomes $A_{j1}, A_{j2}, A_{j3}, \ldots,$. Instead of treating each as a separate rule, they are grouped into a single FLRG. This structure allows for a more compact and flexible model, which is crucial for generating future forecasts using techniques such as defuzzification. This grouping also enables the application of heuristic knowledge in selecting the most appropriate outcomes from the group. For implementation, the modelling and analysis were conducted using the R programming language within Jupyter Notebook through the Anaconda Navigator environment. This setup was used to perform data preprocessing, fuzzification, formulation of fuzzy logical relationships, and evaluation of forecasting accuracy.

To enhance the accuracy of fuzzy time series models, heuristic knowledge can be leveraged to refine the selection of appropriate fuzzy sets within the fuzzy logical relationship groups (FLRGs). As initially proposed by Chen [9], heuristic functions serve as a mechanism to guide the identification of the most relevant fuzzy sets for forecasting, based on predefined rules or contextual input variables. This study builds upon these foundational ideas by introducing a revised heuristic framework that incorporates domain-specific knowledge reflective of observed trends in tuberculosis (TB) case data. The revised approach aims to better capture the dynamic patterns of increase, decrease, or stability in TB incidence, thereby improving the model's ability to produce more accurate and responsive forecasts.

2.2 Methodological Framework

2.2.1 Data Description

The dataset used in this study comprises monthly tuberculosis (TB) case records from Sabah, Malaysia, obtained through the Sabah State Health Department. Specifically, the data was provided by the Clinical Research Centre (CRC) at Hospital Queen Elizabeth (HQE) and the Communicable Disease Unit, Sabah State Health Department. The records cover the period from January 2010 to December 2020. This time series data is used to model and forecast future TB cases using fuzzy time series techniques. The data is presented in Fig. 1.

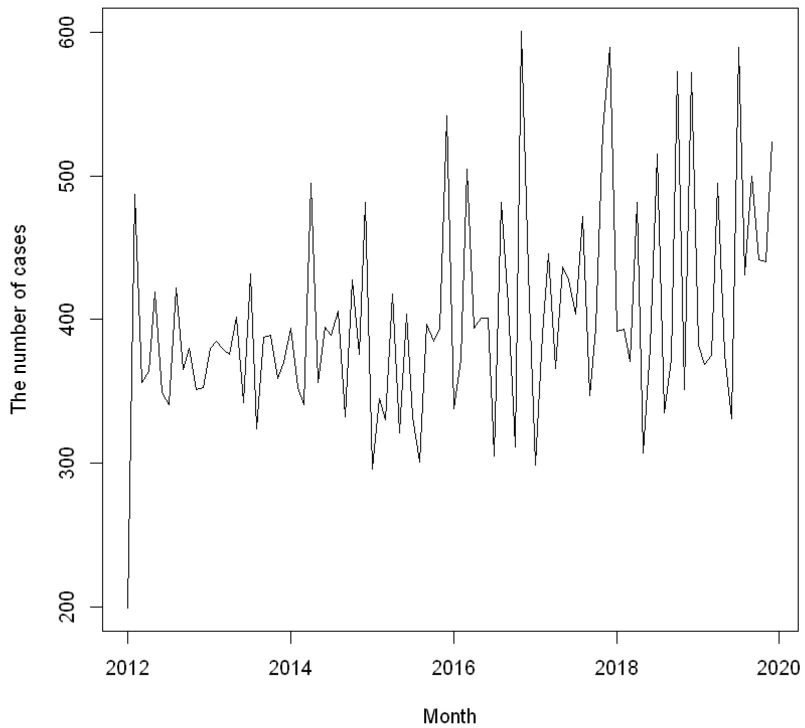

Figure 1. Raw Data of Tuberculosis Cases Reported in Sabah by Monthly from the Year 2012 until 2020

2.2.2 Fuzzy Time Series Framework

The fuzzy time series (FTS) forecasting process in this study follows the framework proposed by Song and Chissom [6], with modifications to incorporate heuristic knowledge. The general steps include:

1. Fuzzification: Partitioning the universe of discourse and mapping historical data into fuzzy sets.

2. Establishment of Fuzzy Logical Relationships (FLRs): Based on the fuzzified data.
3. Grouping FLRs into Fuzzy Logical Relationship Groups (FLRGs).
4. Defuzzification – Producing crisp forecasts from fuzzy sets.

2.2.3 Original Heuristic Knowledge Construction

In the original model, heuristic rules are incorporated during the FLRG formation stage to enhance the forecasting logic. This approach uses domain knowledge, such as the assumption that TB trends often follow gradual increases or decreases, to manually adjust the FLRGs. However, the original method lacks a mechanism to dynamically adapt to trend shifts or irregular fluctuations in real-world data [9]. Suppose $F(t-1) = A_j$ and the fuzzy logical relationship group for A_j is $A_j \to A_q, A_r, \dots$. Proper fuzzy sets, $A_{p1}, A_{p2}, \dots, A_{pk}$, can be selected by heuristic functions $h(\,)$, as shown below

$$h\big(x_1, x_2, \dots; A_q, A_r, \dots\big) = A_{p1}, A_{p2}, \dots, A_{pk}, \tag{2}$$

where $x_1, x_2, \dots$ are heuristic variables; $A_{p1}, A_{p2}, \dots, A_{pk}$ are selected from $A_q, A_r, \dots$, by heuristic function. A heuristic fuzzy logical relationship group is obtained below:

$$A_j = A_{p1}, A_{p2}, \dots, A_{pk,} \tag{3}$$

The heuristic fuzzy logical relationship group is then used to forecast $F(t)$. The evaluation of forecasting accuracy is carried out using four widely recognized performance metrics: Mean Squared Error (MSE), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). These metrics are defined as follows:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2 \tag{4}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2} \tag{5}$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|y_t - \hat{y}_t| \tag{6}$$

$$MAPE = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{y_t - \hat{y}_t}{y_t}\right| \tag{7}$$

where y_t denoted the actual number of cases, $\hat{y}_t$ represent the forecasted number of cases, and n is the total number of observations. These metrics provide a comprehensive assessment of each model's predictive capability by quantifying the average magnitude of forecast errors in both absolute terms and relative percentages.

3. RESULTS AND DISCUSSION

3.1 The Proposed Fuzzy Time Series Models with Revised Heuristic Knowledge Implementation

This study introduces a revised heuristic approach by incorporating domain-specific rules derived from observed behavioral patterns in tuberculosis (TB) case trends. As illustrated in (2), the proposed model emphasizes changes in the variable x_1 (redefined as x), which may exhibit increasing, decreasing, or stable trends. The modification is specifically implemented in Step 6, where the Fuzzy Logical Relationship Groups (FLRGs) are redefined based on the revised heuristic knowledge, as depicted in Fig. 2. These include:

1. Condition 1: If $x = 0$, indication no change in the data, the revised heuristic function selects all fuzzy sets $A_q, A_r, \dots$, associated with the current fuzzy set A_j. This reflects a stable scenario, maintaining consistency in the forecasting model. Thus, the FLRG remains unchanged.

2. Condition 2: If $x \neq 0$, the heuristic function selects the fuzzy sets $A_{p1}, A_{p2}, \dots, A_{pk}$ based on the direction of the trend observed.
   a. If $x > 0$, representing an increasing trend, the heuristic function will select the fuzzy sets corresponding to upward trends.
   b. If $x < 0$, representing a decreasing trend, the heuristic function will select the fuzzy sets corresponding to downward trends.

Thus, the revised FLRG for A_j is $A_j \to A_{p1}, A_{p2}, \dots, A_{pk}$. This means, when $x \neq 0$, the heuristic function adapts the fuzzy set selection to reflect the trend in the data.

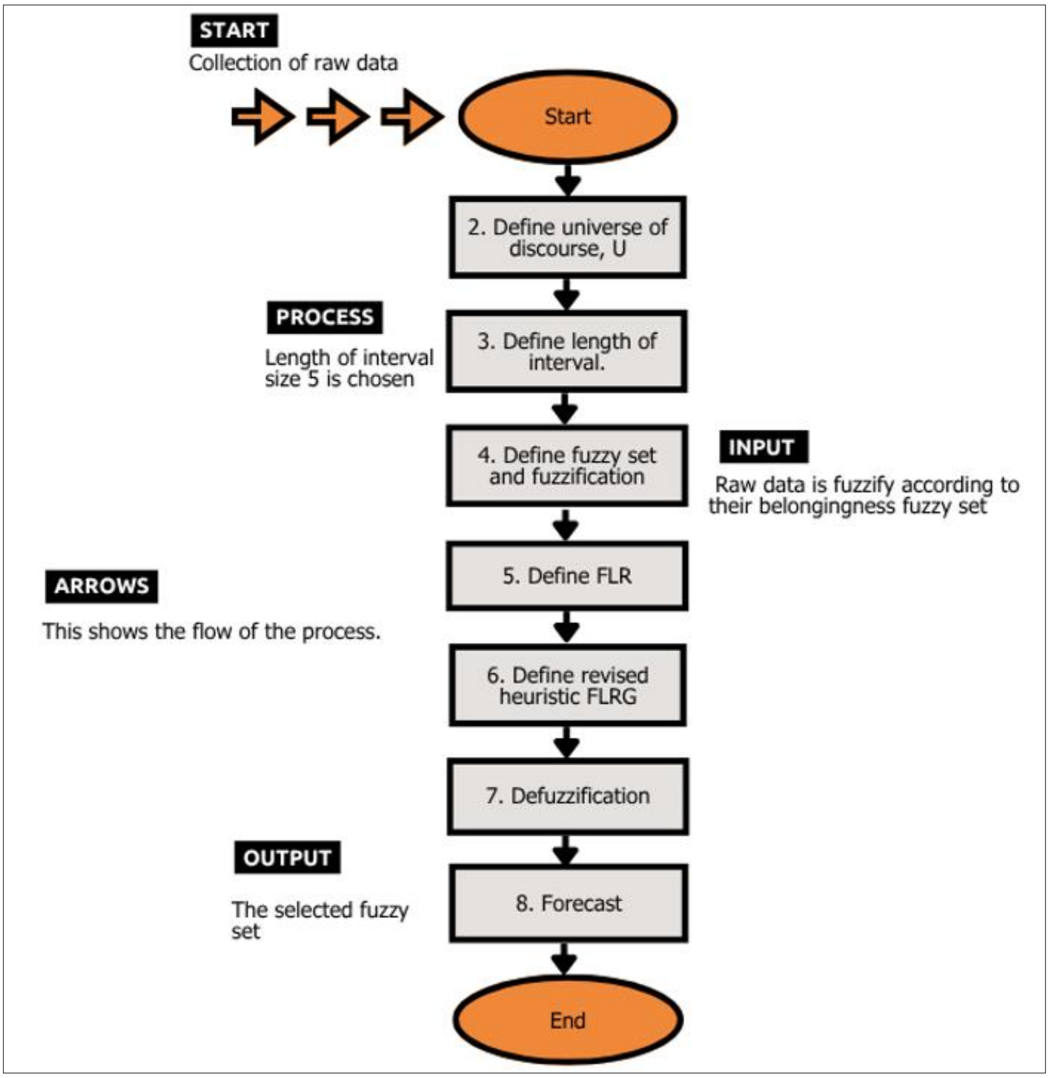

Figure 2. The Flowchart of Fuzzy Time Forecasting Model with Implementation of Revised Heuristic Knowledge.

Fig. 2 illustrates the overall flowchart of the fuzzy time series forecasting model with the implementation of revised heuristic knowledge. The process begins with the collection of raw data, which is then transformed into a structured form through the definition of the universe of discourse and the determination of interval lengths. Next, fuzzy sets are defined, and fuzzification is performed to convert numerical data into linguistic values. The fuzzy logical relationships (FLRs) are then established, followed by the construction of revised heuristic fuzzy logical relationship groups (FLRGs) that integrate domain-specific rules reflecting real TB case dynamics (increase, decrease, or stability). The model proceeds with defuzzification to transform fuzzy outputs into crisp values, and finally, forecasts future case values. This systematic workflow ensures that uncertainty and imprecision in the data are effectively handled while improving forecasting accuracy through revised heuristic knowledge.

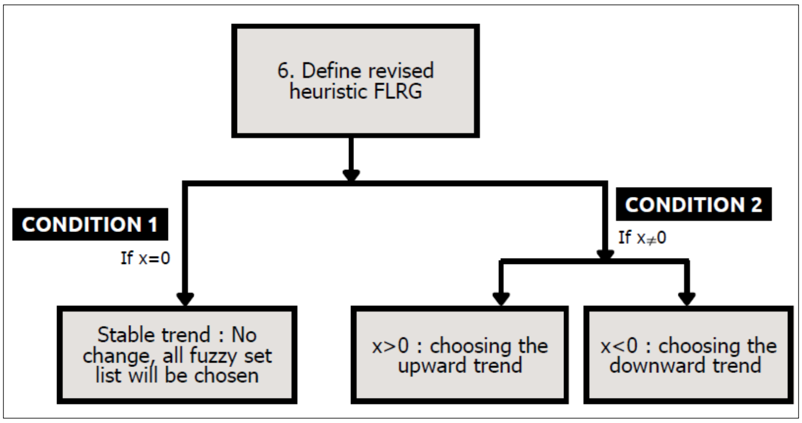

Figure 3. Rules Applied in Step 6 for Defining the Fuzzy Logical Relationship Groups (FLRGs) in the Fuzzy Time Series Forecasting Model with the Implementation of Revised Heuristic Knowledge.

Fig. 3 presents the rules applied in Step 6 of the forecasting process, which involves defining the revised heuristic FLRGs. The rules are designed to reflect the observed dynamics of TB case movements. When the change in cases is stable ($x = 0$), the full set of fuzzy rules is considered without modification. However, when an increasing trend is detected ($x > 0$), the model prioritizes upward fuzzy rules to emphasize the rise in case numbers. Conversely, when a decreasing trend is observed ($x < 0$) the model emphasizes downward fuzzy rules to better capture the decline. These conditions enable the model to incorporate heuristic knowledge into the construction of FLRGs, allowing for more responsive and context-aware forecasting outcomes.

3.2 Output – Defuzzification and Forecast

The forecasted values are computed using Chen's Defuzzification Rules, which consist of three distinct cases:

1. Rule 1: If the current fuzzy set is A_i, and its FLRG is empty, (i.e. $A_i \to \emptyset$), the forecasted value is the midpoint m_i of the corresponding interval u_i.
2. Rule 2: If the current fuzzy set is A_i, and its FLRG contains only a single fuzzy set, (i.e. $A_i \to A_{j1}$), the forecasted value is the midpoint m_{j1} of the corresponding interval u_{j1}.
3. Rule 3: If the current fuzzy set is A_i, and its FLRG contains multiple fuzzy set, (i.e. $A_i \to A_{j_1}, A_{j_2}, A_{j_3}, \dots, A_{j_k}$), the forecasted value is calculated as the average of the midpoint of the corresponding interval $u_{j_1}, u_{j_2}, u_{j_3}, \dots, u_{j_k}$, denoted as $\frac{1}{k}\sum_{r=1}^{k} m_{j_r}$.

3.3 Performance Evaluation

The forecasting performance of the proposed model is evaluated using four standard accuracy metrics: Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Squared Error (MSE), and Mean Absolute Percentage Error (MAPE). These metrics quantify the deviation between actual and predicted values, thereby facilitating a comparative assessment of the revised heuristic model's effectiveness against the original approach.

3.4 Results and Discussion

This section presents the step-by-step implementation of the proposed fuzzy time series forecasting model with revised heuristic knowledge. To illustrate the process clearly and comprehensively, the year 2013 is selected as a representative case study. Each stage of the forecasting procedure, from fuzzification to defuzzification and performance evaluation, is demonstrated using the monthly tuberculosis (TB) case data for that year. The aim is to provide an in-depth understanding of how the revised heuristic rules influence the forecasting outcome compared to the original model.

3.4.1 Defining Universe of Discourse, *U* and Length of Interval

The universe of discourse refers to the complete range of values that the historical data may assume. In this study, the monthly tuberculosis (TB) case data are first analyzed to determine their minimum and maximum values. According to raw data, $D_{min} = 199$ and $D_{max} = 601$ represent the minimum and maximum number of TB cases observed from the year 2012 until 2019 respectively. To ensure all possible values are captured within the defined intervals, the universe of discourse, U, is extended slightly beyond these boundaries. Mathematically, it is defined as:

$$U = [D_{min} - D_1 , D_{max} + D_2] \tag{8}$$

where D_1 and D_2 is a small positive constant added to both ends of the range to provide a margin of safety and prevent boundary-related fuzzification errors. The resulting universe of discourse is then divided into several equal-length intervals. In this study, the interval length is fixed at 5, ensuring sufficient granularity while maintaining interpretability as shown in Table 1.

Table 1. List of intervals and Its Corresponding Midpoints and Fuzzy Sets

Interval	Midpoint, m_i	Midpoint	Fuzzy interval, u_i	*Fuzzy Sets*, A_i
$190 - 195$	m_1	193.5	u_1	A_1
$195 - 200$	m_2	198.5	u_2	A_2
$200 - 205$	m_3	203.5	u_3	A_3
…	…	…	…	…
$595 - 600$	m_{82}	598.5	u_{82}	A_{82}
$600 - 605$	m_{83}	603.5	u_{83}	A_{83}
$605 - 610$	m_{84}	608.5	u_{84}	A_{84}

3.4.2 Fuzzification, Defining FLRs and FRLGs

Each interval is associated with a fuzzy set labelled A_i, for $i = \{1, 2, \dots, 84\}$. These fuzzy sets represent linguistic variables used to describe the level of TB cases. For example, lower-indexed sets correspond to intervals with relatively fewer reported cases, while higher-indexed sets represent intervals with a greater number of cases. This transformation allows the model to capture the underlying patterns of TB incidence in linguistic terms, facilitating more flexible and accurate forecasting.

Table 2 shows some list of intervals according to the length of intervals size 5, the midpoint and their belongingness to their fuzzy sets group for the year 2013.

Table 2. The Lists of Fuzzy Sets, A_i, FLR and FLRG Showing Specially for the Year 2013

Month	Raw data	Fuzzy sets, A_i	FLR	FLRG
…	…	…	…	…
January	380	A_{39}	$A_{33} \to A_{39}$	$A_{33} \to A_{31}, A_{33}, A_{39}, A_{77}$
February	385	A_{40}	$A_{39} \to A_{40}$	$A_{39} \to A_{33}, A_{36}, A_{40}, A_{66}$
March	379	A_{38}	$A_{40} \to A_{38}$	$A_{40} \to A_{34}, A_{38}, A_{40}, A_{41}, A_{44}$
April	376	A_{38}	$A_{38} \to A_{38}$	$A_{38} \to A_{29}, A_{38}, A_{43}, A_{59}, A_{62}$
May	402	A_{43}	$A_{38} \to A_{43}$	$A_{38} \to A_{29}, A_{38}, A_{43}, A_{59}, A_{62}$
June	342	A_{31}	$A_{43} \to A_{31}$	$A_{43} \to A_{24}, A_{29}, A_{31}, A_{43}, A_{57}$
July	432	A_{49}	$A_{31} \to A_{49}$	$A_{31} \to A_{47}, A_{49}, A_{62}$
August	324	A_{27}	$A_{49} \to A_{27}$	$A_{49} \to A_{22}, A_{27}, A_{63}$
September	388	A_{40}	$A_{27} \to A_{40}$	$A_{27} \to A_{40}, A_{43}$
October	389	A_{40}	$A_{40} \to A_{40}$	$A_{40} \to A_{34}, A_{38}, A_{40}, A_{41}, A_{44}$
November	359	A_{34}	$A_{40} \to A_{34}$	$A_{40} \to A_{34}, A_{38}, A_{40}, A_{41}, A_{44}$
December	370	A_{37}	$A_{34} \to A_{37}$	$A_{34} \to A_{35}, A_{37}, A_{42}$
…	…	…	…	…

Data source: *Raw data of reported TB cases obtained from the Communicable Disease Unit, Sabah State Health Department, Hospital Queen Elizabeth (HQE).*

3.4.3 Define FLRG and revised FLRG

To better illustrate the differences in forecasting approaches, this section compares the fuzzy logical relationship groups (FLRGs) generated by the original method, the heuristic method, and the revised heuristic method. Table 2 presents the list of original FLRGs, while Table 3 provides a comparison of the FLRG lists generated using heuristic knowledge and revised heuristic knowledge for the year 2013.

Table 3. List of FLRG, Implementing Heuristic FLRG and The Revised Heuristic FLRG Respectively

Month	FLR	*Heuristic Knowledge*	*Heuristic FLRG*	*Revised FLRG (Proposed method)*
…	…	…	…	…
January	$A_{33} \rightarrow A_{39}$	↑	$A_{33} \rightarrow A_{33}, A_{39}, A_{77}$	$A_{33} \rightarrow A_{33}, A_{39}, A_{77}$
February	$A_{39} \rightarrow A_{40}$	↑	$A_{39} \rightarrow A_{40}, A_{66}$	$A_{39} \rightarrow A_{40}, A_{66}$
March	$A_{40} \rightarrow A_{38}$	↓	$A_{40} \rightarrow A_{34}, A_{38}, A_{40}$	$A_{40} \rightarrow A_{34}, A_{38}, A_{40}$
April	$A_{38} \rightarrow A_{38}$	*No change*	$A_{38} \rightarrow A_{29}, A_{38}$	$A_{38} \rightarrow A_{29}, A_{38}, A_{43}, A_{59}, A_{62}$
May	$A_{38} \rightarrow A_{43}$	↑	$A_{38} \rightarrow A_{38}, A_{43}, A_{59}, A_{62}$	$A_{38} \rightarrow A_{38}, A_{43}, A_{59}, A_{62}$
June	$A_{43} \rightarrow A_{31}$	↓	$A_{43} \rightarrow A_{24}, A_{29}, A_{31}, A_{43}$	$A_{43} \rightarrow A_{24}, A_{29}, A_{31}, A_{43}$
July	$A_{31} \rightarrow A_{49}$	↑	$A_{31} \rightarrow A_{47}, A_{49}, A_{62}$	$A_{31} \rightarrow A_{47}, A_{49}, A_{62}$
August	$A_{49} \rightarrow A_{27}$	↓	$A_{49} \rightarrow A_{22}, A_{27}$	$A_{49} \rightarrow A_{22}, A_{27}$
September	$A_{27} \rightarrow A_{40}$	↑	$A_{27} \rightarrow A_{40}, A_{43}$	$A_{27} \rightarrow A_{40}, A_{43}$
October	$A_{40} \rightarrow A_{40}$	*No change*	$A_{40} \rightarrow A_{40}, A_{41}, A_{44}$	$A_{40} \rightarrow A_{34}, A_{38}, A_{40}, A_{41}, A_{44}$
November	$A_{40} \rightarrow A_{34}$	↓	$A_{40} \rightarrow A_{34}, A_{38}, A_{40}$	$A_{40} \rightarrow A_{34}, A_{38}, A_{40}$
December	$A_{34} \rightarrow A_{37}$	↑	$A_{34} \rightarrow A_{35}, A_{37}, A_{42}$	$A_{34} \rightarrow A_{35}, A_{37}, A_{42}$
…	…	…	…	…

The key differences can be observed in the *"No change"* column under the heuristic knowledge section, where the adjustments introduced in the revised version are highlighted. As an illustration, the results for January 2013 to April 2013 are examined in detail.

January 2013: The FLR is identified as $A_{33} \rightarrow A_{39}$. According to Eq. (2), since $x \neq 0$, this indicates an increase in the value x. Referring to Fig. 3, this condition corresponds to Condition 2. Therefore, by definition, the list of FLRG is determined as $A_{33} \rightarrow A_{33}, A_{39}, A_{77}$. When applying the revised heuristic knowledge, the resulting list of FLRGs remains unchanged.

February 2013: The FLR is identified as $A_{39} \rightarrow A_{40}$. According to Eq. (2), since $x \neq 0$, this indicates an increase in the value x. Referring to Fig. 3, this condition corresponds to Condition 2. Therefore, by definition, the list of FLRG is determined as $A_{39} \rightarrow A_{40}, A_{66}$. When applying the revised heuristic knowledge, the resulting list of FLRGs remains unchanged.

March 2013: The FLR is identified as $A_{40} \rightarrow A_{38}$. According to Eq. (2), since $x \neq 0$, this indicates a decrease in the value x. Referring to Fig. 3, this condition corresponds to Condition 2. Therefore, by definition, the list of FLRG is determined as $A_{40} \rightarrow A_{34}, A_{38}, A_{40}$. When applying the revised heuristic knowledge, the resulting list of FLRGs remains unchanged.

April 2013: The FLR is identified as $A_{38} \rightarrow A_{38}$. According to Eq. (2), since $x = 0$, this indicates a stable trend in the value x. Referring to Fig. 3, this condition corresponds to Condition 1. Based on the definition and noting that the raw data shows a decrease compared to the previous month, the list of fuzzy logical relationship groups (FLRGs) is determined as as $A_{38} \rightarrow A_{29}, A_{38}$. When the revised heuristic knowledge is applied, the list of FLRGs is expanded to include additional relevant fuzzy sets, resulting in $A_{38} \rightarrow A_{29}, A_{38}, A_{43}, A_{59}, A_{62}$.

The same approach is applied to the following months, with fuzzy logical relationships and FLRGs identified according to the conditions and revised heuristic rules.

3.4.4 Defuzzification and Forecast

The defuzzification stage converts the fuzzy outputs into crisp forecast values, allowing for direct comparison with actual tuberculosis (TB) case data. For this study, defuzzification was performed following Chen's rules, as described in Section 3.2.

Table 4. Actual and Forecasted TB Cases (Showing Specially for January–December 2013) using Original and Revised Heuristic Models

Month	Raw data	Forecasted values (original model)	Forecasted values (heuristic knowledge)	Forecasted values (proposed model)
…	…	…	…	…
January	380	412.500	435.833	435.833
February	385	406.250	452.500	452.500
March	379	384.500	374.167	374.167
April	376	418.500	355.000	418.500
May	402	418.500	440.000	440.000
June	342	371.500	346.250	346.250
July	432	450.833	450.833	450.833
August	324	374.167	310.000	310.000
September	388	395.000	395.000	395.000
October	389	384.500	395.833	384.500
November	359	384.500	374.167	374.167
December	370	377.500	377.500	377.500
…	…	…	…	…

The process was applied on a month-by-month basis for the period from January 2013 to December 2013, using both the original heuristic model and the revised heuristic model. Table 4 presents the actual TB cases alongside the forecasted values produced by both models. The revised heuristic model generally provides forecasts that align more closely with the actual data, particularly during months with fluctuations in case numbers. Table 5 presents the forecast accuracy metrics for the original fuzzy time series model, the model incorporating heuristic knowledge, and the model enhanced with revised heuristic knowledge over the period from 2012 to 2019.

Table 5. Forecast Accuracy Metrics Comparison between Original and Revised Heuristic Models for the year 2012 until the year 2019

Accuracy metrics	Original model	Implementing heuristic knowledge model	Implementing revised heuristic knowledge model
MSE	2487.9090	1449.2020	1315.0160
RMSE	49.8790	38.0684	36.2631
MAE	0.0830	0.0567	0.0566
MAPE	0.0212	0.0139	0.0138

The evaluation results indicate that the revised heuristic model consistently produces the most accurate forecasts across all performance metrics. For instance, the Mean Absolute Percentage Error (MAPE) recorded at 0.0138% demonstrates that, on average, the model's forecasts deviate from the actual values by less than 0.02%, which is considered highly accurate in time series forecasting. Similarly, the very low Mean Absolute Error (MAE = 0.0566) shows that the predicted TB case counts differ only slightly from the observed values on an absolute scale. The Root Mean Squared Error (RMSE = 36.2631) and Mean Squared Error (MSE = 1315.0160) further validate the model's robustness, as smaller values in these metrics correspond to higher precision and stability in forecasting. Collectively, these results confirm that the integration of revised heuristic knowledge enhances the predictive reliability of fuzzy time series models, thereby strengthening their applicability in epidemiological forecasting.

The reduction in RMSE and MAE further confirms that the revised heuristic model not only improves long-term predictive accuracy but also minimizes short-term fluctuations in forecast errors. This stability is essential for ensuring reliable monthly forecasts, which directly inform interventions such as case detection campaigns, medication stock management, and targeted awareness programs. This improvement stems from the refined construction of Fuzzy Logical Relationship Groups (FLRGs). In conventional models, FLRGs are formed based on observed transitions between fuzzy sets without deeper analysis of the context or trend direction (e.g., whether case numbers are increasing, decreasing, or stable). Basic heuristic models attempt to guide this process using predefined rules; however, these are often too rigid or generalized to effectively handle the dynamic nature of real-world data. In contrast, the revised heuristic model incorporates rule-based adjustments that reflect domain knowledge, specifically behavioral patterns observed in TB transmission data. Specifically, the model analyzes the direction and magnitude of change from one time point to the next and refines the list of FLRGs accordingly. This dynamic adjustment ensures that the rules used in forecasting are more representative of actual epidemic patterns, rather than being based solely on statistical transitions. As a result, the revised model captures subtle or irregular fluctuations that traditional models might overlook, especially during months with mild, inconsistent, or nonlinear variations in case numbers.

However, it's important to note that the model, like most univariate time series approaches, focuses solely on historical TB case data. In real-world epidemiology, TB incidence is not only influenced by its historical values but also by external factors such as population mobility and urban crowding, which increase exposure risk, climatic and seasonal variations, which may affect bacterial survival and immune responses, socioeconomic status, including access to healthcare or public awareness and public health policies, such as vaccination campaigns, screenings, or case isolation. These variables were not included in the current model, which is a recognized limitation. While the revised heuristic mechanism improved the internal structure of the fuzzy series model, its accuracy might still be impacted in the presence of sudden or unexplained outbreaks driven by these exogenous factors.

Despite this, the dataset used monthly TB case reports from 2012 to 2019 offers a solid foundation for the model. This time period was selected for its data completeness, consistency of reporting, and temporal sufficiency to capture both seasonal cycles and irregular fluctuations. While it does not cover major health policy shifts (e.g., post-COVID implications), the chosen duration reflects a relatively stable epidemiological environment, enabling the model to learn regular patterns effectively. It is important to note, however, that the model relies exclusively on historical TB case data. Real-world epidemiology is influenced not only by past incidence but also by exogenous factors such as population mobility, urban crowding, socioeconomic status, and access to healthcare. For example, sudden outbreaks driven by migration flows or changes in diagnostic policies could introduce abrupt deviations that are not fully captured by the current model. Incorporating such external variables or integrating hybrid approaches (e.g., fuzzy logic combined with machine learning) could further strengthen forecasting performance. In summary, the revised heuristic model represents a significant advancement in TB case forecasting, as it substantially reduces forecast errors across all standard metrics and offers practical benefits for healthcare planning. The model's design demonstrates the importance of embedding domain knowledge into fuzzy time series forecasting, not only improving accuracy but also enhancing its applicability to other infectious diseases characterized by nonlinear and uncertain dynamics.

Moving forward, researchers are encouraged to extend the fuzzy time series approach into multivariate frameworks, where external factors such as climate variables, population mobility, or socioeconomic indicators can be modeled alongside case counts. Unlike previous models which focused on environmental indices or economic indicators, our fuzzy time series model is specifically tailored for TB forecasting using revised heuristic rules. While mixed-order FTS and heuristic-GA models have shown promise Wu et al., [25]; Bhagat et al., [19], the scarcity of domain-refined TB-specific FTS frameworks underscores the novelty of this work. Despite the improved accuracy of the revised heuristic fuzzy time series model, one limitation of the present study is its reliance on univariate data, which may restrict its ability to account for external shocks or sudden epidemiological changes. Addressing this gap, future research could also explore hybrid approaches that integrate fuzzy logic with machine learning techniques such as neural networks or decision trees, enabling the detection of nonlinear patterns and anomalies in TB trends. Such advancements would not only strengthen forecasting accuracy in unpredictable conditions but also contribute new insights into the methodological development of fuzzy time series in epidemiological applications.

4. CONCLUSION

This study introduced a fuzzy time series forecasting model enhanced with revised heuristic knowledge to improve the prediction of tuberculosis (TB) prevalence in Sabah, Malaysia. The main conclusions are as follows:

1. The key contribution lies in embedding domain-informed rules into fuzzy logical relationship groups (FLRGs), which enable the model to more accurately capture directional trends such as increases, decreases, and stability in TB cases.
2. The findings confirm that incorporating heuristic knowledge strengthens the interpretability and forecasting power of fuzzy time series, particularly in dealing with the uncertainty and complexity of health-related data. More importantly, the refinement demonstrates that domain expertise can be systematically integrated into fuzzy modeling to enhance both accuracy and practical relevance.
3. This study contributes to the literature by advancing a knowledge-driven approach that is adaptable to dynamic epidemiological data, offering a more context-sensitive alternative to conventional models.

Nonetheless, the research is limited by its reliance on univariate case data, without considering external factors that may influence TB prevalence. Future studies should therefore explore multivariate fuzzy frameworks or hybrid approaches that combine fuzzy logic with machine learning to capture nonlinear dynamics and detect anomalies more effectively. By addressing these aspects, the proposed model not only extends methodological developments in fuzzy forecasting but also supports more informed and adaptive decision-making in public health.

Author Contributions

Suriana Lasaraiya: Conceptualization, Formal Analysis, Investigation, Methodology, Project Administration, Software, Validation, Visualization, Writing - Original Draft, Writing - Review and Editing. Suzelawati Zenian: Conceptualization, Formal Analysis, Investigation, Methodology, Supervision, Validation, Writing - Review and Editing. Risman Mat Hasim: Conceptualization, Methodology, Supervision. Azmirul Ashaari: Conceptualization, Formal Analysis, Investigation, Methodology, Supervision, Validation. All authors contributed to the interpretation of the results, reviewed the manuscript, and approved the final version for publication.

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Declarations

The authors declare no competing interest.

Declaration of Generative AI and AI-assisted technologies

ChatGPT was utilized only to improve the readability and grammatical structure of the manuscript. No AI tool was used to generate or alter the research data, methodology, results, or interpretations. All content was verified by the authors for accuracy and consistency with the study.

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