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A COMPARATIVE STUDY OF IBNR CLAIM RESERVE ESTIMATION USING BENKTANDER, WALTER NEUHAUS, AND OPTIMAL CREDIBILITY LOSS RATIO APPROACHES

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ABSTRACT

Reinsurance plays a crucial role in risk transfer for insurance companies, particularly in managing large and volatile losses. One of the key challenges in reinsurance is the accurate estimation of Incurred But Not Reported (IBNR) claim reserves, especially for nonproportional assumed property business, which is characterized by high claim volatility and delayed reporting patterns. This study provides an empirical comparison of credibility-based reserving methods—namely the Benktander and Walter Neuhaus approaches—using reported claims and earned premium data from United States reinsurance companies for the period 2010–2019. Unlike most existing studies that focus on proportional or direct insurance portfolios, this research evaluates the performance of these methods in a nonproportional reinsurance context and benchmarks them against the Optimal Credibility Loss Ratio method, which minimizes Mean Squared Error (MSE). Claim reserves are estimated using run-off triangle techniques, loss development factors, and credibility weighting schemes, and the accuracy of each method is assessed through MSE ratios. The results show that the Benktander method produces reserve estimates that are consistently closer to the optimal benchmark, with an average MSE ratio of 1.0265, compared to 1.4184 for the Walter Neuhaus method. These findings indicate that the Benktander approach offers a more stable and statistically efficient reserve estimation for immature and volatile nonproportional reinsurance data. The study contributes to actuarial reserving literature by providing empirical evidence on the relative effectiveness of credibility-based methods and offering practical insights for actuaries in selecting appropriate IBNR reserving techniques under high uncertainty.

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1. INTRODUCTION

In life, every individual or entity inevitably faces various risks that may lead to financial losses. To mitigate such risks, insurance mechanisms are widely used to transfer uncertainty from the insured to the insurer. Insurance represents a contractual agreement in which the insurer receives premiums in exchange for providing compensation for losses, damages, or legal liabilities arising from uncertain events [1]. As financial intermediaries, insurance institutions play a crucial role in maintaining economic stability by pooling risks and providing financial protection to policyholders [2]. Insurance companies are required to have readily available funds, known as reserve funds, to meet obligations to the insured. To fulfill these obligations, insurers are required to establish adequate reserve funds to ensure their ability to settle future claims and maintain operational continuity [3], [4].

In the insurance industry, particularly in long-tailed lines of business such as liability insurance and reinsurance, a major challenge arises from uncertainty and delays in claim reporting and settlement. Outstanding claim liabilities consist of Reported But Not Settled (RBNS) claims and Incurred But Not Reported (IBNR) claims, the latter representing losses that have already occurred but have not yet been reported [5]. Accurate estimation of IBNR reserves is essential, as underestimation may lead to solvency risk and regulatory concerns, while overestimation can result in inefficient capital allocation and reduced underwriting capacity. Consequently, actuarial reserving requires statistical methods that are capable of balancing accuracy, stability, and responsiveness under incomplete and immature information.

A wide range of reserving methods has been developed for this purpose, including the Chain-Ladder, Bornhuetter–Ferguson, Benktander, and Walter Neuhaus approaches. Deterministic methods such as Chain-Ladder are widely used due to their simplicity, while Bornhuetter–Ferguson incorporates prior expectations to reduce volatility when historical data are limited [6]. However, traditional deterministic methods such as Chain-Ladder or Bornhuetter–Ferguson often fail to capture variability and uncertainty inherent in long-tail claim developments. More advanced credibility-based methods, such as Benktander and Walter Neuhaus, extend these frameworks by explicitly weighting historical experience and expected losses. The Benktander method combines Chain-Ladder and Bornhuetter–Ferguson estimates through credibility weights and has been shown to reduce Mean Squared Error (MSE) in certain reserving contexts, particularly during early development periods [7]. The Walter Neuhaus method further refines this approach by introducing credibility theory into loss development factors, allowing greater flexibility in handling volatile and heterogeneous claim patterns [8]. This method was developed as an enhancement to the Chain Ladder method, adjusting development factors using stochastic components. It is more flexible in accommodating variations in claim settlement patterns and is suitable for claim data with high volatility.

Over the past decade, several studies have examined and developed credibility-based reserving methods, including the Benktander, Walter Neuhaus, and Optimal Credibility Loss Ratio approaches. One of the most notable contributions is by Hürlimann [8], who introduced an optimal credibility framework that combines Chain-Ladder and Bornhuetter-Ferguson estimates using a single weight designed to minimize both mean squared error (MSE) and reserve variance. This Optimal Credibility Loss Ratio method has been shown to produce more stable and accurate reserve estimates and is especially useful when balancing the reliability of historical claims with expected future losses. In a similar context, Triana et al. [9] explored a modified version of the Benktander method, incorporating optimal credibility weights to provide reserve estimates that lie between the Chain-Ladder and Bornhuetter-Ferguson outcomes. Their findings emphasized that such a hybrid approach reduces estimation bias and improves reserve adequacy. Meanwhile, a study by IPB University [10] applied the Walter Neuhaus method in a practical setting and reported a relative error of approximately 11.5% when compared to actual claim development, suggesting reasonable accuracy when the data is less mature but prior loss ratios are available. Additionally, in his 2015 review, Hürlimann [11] revisited the foundations of IBNR estimation and unified these methods within a single credibility-based IBNR reserving framework, emphasizing their conceptual consistency. Despite these advances, empirical comparisons using real nonproportional reinsurance data remain limited, as most studies focus on proportional or direct insurance portfolios with more stable claim development. Given the higher severity, volatility, and reporting delays inherent in nonproportional assumed property reinsurance, the relative performance of credibility-based methods remains an open question. This study addresses this gap by applying the Benktander and Walter Neuhaus methods to U.S. reinsurance claim data from 2010–2019 and benchmarking them against the Optimal Credibility Loss Ratio approach.

Furthermore, many existing studies remain largely application-oriented, focusing on identifying which method produces higher or lower reserve estimates on a given dataset. Less attention has been given to understanding how and why specific credibility structures perform differently under particular data conditions, such as varying levels of data maturity, volatility, and reporting delay. In particular, empirical insights into the behavior of the Benktander method relative to theoretically optimal credibility benchmarks in nonproportional reinsurance settings are still scarce. This study addresses these gaps by conducting a systematic empirical comparison of the Benktander and Walter Neuhaus methods using United States nonproportional assumed property reinsurance data from 2010 to 2019 [12]. This period captures diverse claim development environments, including multiple underwriting cycles and heterogeneous loss emergence patterns, making it particularly suitable for evaluating credibility-based reserving methods. The Optimal Credibility Loss Ratio method is employed as a benchmark due to its well-established theoretical optimality in minimizing Mean Squared Error [8], [11]. By analyzing reserve estimates and MSE ratios across accident years with varying data maturity, this research not only compares numerical performance but also provides insights into the practical behavior of credibility-based reserving methods under high uncertainty. As such, the study contributes both empirical evidence and decision-relevant insights for actuarial reserving practice in nonproportional reinsurance.

2. RESEARCH METHODS

2.1 Data Source

The type of data used in this study is secondary data, which is data that has been collected and published by others, such as official reports, statistics, or previous research, and is then reused for analysis in a new study. The data used in this research is historical reported claims and earned premium data from a reinsurance company in the United States for the period 2010-2019 for Nonproportional Assumed Property. This data comes from the report "Statistical Compilation of Annual Statement Information for Property/Casualty Insurance Companies in 2019" published by the National Association of Insurance Commissioners in the United States.

2.2 Literature Review

2.2.1 Run-Off Triangle

The run-off triangle data is a summary of individual claim data that provides a comprehensive overview of claims in aggregate. It is a primary tool for determining the magnitude of incurred claims, most of which are still unresolved [13]. The run-off triangle includes two aspects: the period of occurrence, referred to as the year of origin, and the delay period, referred to as the development year. Incremental claims refer to the number of claims reported with a reporting delay in a certain year of occurrence, or changes in the number of claims reported in a specific cell [14].

$$\{S_{i,j} \mid i = 1,2,\dots,n; j = 1,2,3,\dots,n\}, \tag{1}$$

which defines the set of incremental claims $S_{i,j}$ for all accident years i and development years j, where both indices range from 1 to n. The run-off triangle can be seen as follows:

Table 1. Incremental Claim's Run-Off Triangle

Underwriting Period	Development Year						
	1	**2**	**⋯**	**j**	**⋯**	$n-1$	n
1	$S_{1,1}$	$S_{1,2}$	⋯	$S_{1,j}$	⋯	$S_{1,n-1}$	$S_{1,n}$
2	$S_{2,1}$	$S_{2,2}$	⋯	$S_{2,j}$	⋯	$S_{2,n-1}$	
⋮	⋮	⋮	⋰	⋮	⋰		
i	$S_{i,1}$	$S_{i,2}$	⋯	$S_{i,j}$			
⋮	⋮	⋮	⋰				
$n-1$	$S_{n-1,1}$	$S_{n-1,2}$					
n	$S_{n,1}$						

Cumulative claims, denoted as $C_{i,j}$, can represent cumulative payments, the total number of claims reported, or incurred claims [15]. Cumulative claims are often referred to as the total number of claims at the end of the year of occurrence or the total number of claims in a certain year. Unlike incremental triangles which show only the claims for each development year, cumulative triangles show how claims build up over time.

$$C_{i,j} = \sum_{k=1}^{j} S_{i,j} \ ; i,j \ = \ 1,2, \dots , n; \tag{2}$$

which defines the cumulative claim amount $C_{i,j}$ for accident year i and development year j, as the sum of incremental claims $S_{i,j}$ from development year $k = 1$ up to j. Run-off triangle of cumulative claims can be seen as follows

Table 2. Cumulative Claim's Run-Off Triangle

Underwriting Period	Development Year						
	1	**2**	**⋯**	$\boldsymbol{j}$	**⋯**	$\boldsymbol{n-1}$	$\boldsymbol{n}$
1	$C_{1,1}$	$C_{1,2}$	⋯	$C_{1,j}$	⋯	$C_{1,n-1}$	$C_{1,n}$
2	$C_{2,1}$	$C_{2,2}$	⋯	$C_{2,j}$	⋯	$C_{2,n-1}$	
⋮	⋮	⋮	⋰	⋮	⋰		
i	$C_{i,1}$	$C_{i,2}$	⋯	$C_{i,j}$			
⋮	⋮	⋮	⋰				
$n-1$	$C_{n-1,1}$	$C_{n-1,2}$					
n	$C_{n,1}$						

This table is widely used for estimating ultimate claims, calculating reserves for incurred but not reported (IBNR) claims and applying methods such as Chain-Ladder, Bornhuetter-Ferguson, including Benktander, Walter Neuhaus and optimal credibility loss ration methods.

2.2.2 Link Ratio

The link ratio is essential in actuarial science for tracking and analyzing claim development from one period to the next. This method helps analysts understand claim patterns over time, crucial for assessing insurance liabilities and financial planning [14]. Applied to a run-off triangle, which displays cumulative claim development across accident and development years, the link ratio modifies it by reducing one column and one row, as each value is calculated based on consecutive periods [16].

$$\gamma_{i,j} \ = \ \frac{C_{i,j+1}}{C_{i,j}}, i; j \ = \ 1,2, \dots , n; \tag{3}$$

The average link ratio for each development year, used as the Loss Development Factor (LDF), measures claim changes from one period to the next. To calculate the average Link Ratio $\hat{\gamma}_j$ for development year j can be calculated as follows [16]

$$\hat{\gamma}_j \ = \ \frac{\sum_{i=1}^{n-j} \gamma_{i,j}}{n-j}, j \ = \ 1,2, \dots , n; \tag{4}$$

Cumulative Development Factors (CDF) estimate the cumulative expected claims at a certain period based on historical patterns [14]. Defined as $x = n - j + 1$, the CDF_x can be calculated as follows:

$$CDF_x \ = \ \prod_{j=j}^{n} \hat{\gamma}_j \ ; \tag{5}$$

2.2.3 Expected Claim Method

The Expected Claim method estimates the expected number of claims over a period using initial estimates or historical data rather than observed experience. It is often applied in the Bornhuetter-Ferguson method for claim reserves in insurance. The Expected Loss Ratio (ELR) estimates expected losses in an insurance claim portfolio as the ratio of total claims to total premiums received [17]. The ELR is determined by calculating the Loss Ratio m_j values for several periods and using their cumulative values.

$$ELR = \sum_{j=1}^{n} m_j \, , j = 1, \dots, n. \tag{6}$$

With the value of m_j determined by the (7).

$$m_j = \frac{\sum_{i=1}^{n-j+1} S_{i,j}}{\sum_{i=1}^{n-j+1} V_i}, i = 1, \dots, n. \tag{7}$$

The Ultimate Claim calculation using this method can be done with the following (8):

$$U_i^0 = V_i \, . ELR, i = 1, \dots, n. \tag{8}$$

The claim reserve estimation method can be optimized by considering various relevant factors. One primary approach is using the loss ratio payout factor, which accounts for the proportion of claims paid in each period. Additionally, the loss ratio reserve factor allows for adjustments to systematic changes in claim and premium patterns [18].

$$p_i = \frac{\sum_{j=1}^{n-i+1} m_j}{\sum_{j=1}^{n} m_j}, i = 1, \dots, n; \tag{9}$$

$$q_i = 1 - p_i, i = 1, \dots, n. \tag{10}$$

2.2.4 Chain-Ladder Method

The Chain-Ladder method is based on the idea that historical claim development patterns can be used to project future claims. It uses historical claim data to build development factors by comparing claims from previous years [19]. These factors are then used to calculate IBNR estimates, representing claims that have occurred but have not been reported at the time of reserve assessment. To calculate Ultimate Claim and Chain-Ladder reserves, the following formula is used:

$$U_i^{CL} = C_{i,n-i+1} \, . \, CDF_x, i = 1, \dots, n; \tag{11}$$

$$R_i^{CL} = U_i^{CL} - C_{i,n-i+1} \, , i = 1, \dots, n. \tag{12}$$

For a reliable claim reserve estimation, it's crucial to use various approaches. The Chain-Ladder method, combined with Cumulative Development Factors (CDF), provides a robust view based on historical claim patterns. Additionally, the loss ratio payout factor offers flexibility for systematic changes and adjustments [20]. The following formula calculates the Ultimate Claims Chain-Ladder using the loss ratio payout factor:

$$U_i^{CL} = \frac{C_{i,n-i+1}}{p_i}, i = 1, \dots, n; \tag{13}$$

$$R_i^{CL} = q_i \, . \frac{C_{i,n-i+1}}{p_i} = \frac{q_i}{p_i} . \, C_{i,n-i+1}. \tag{14}$$

2.2.5 Bornhuetter-Ferguson Method

The Bornhuetter-Ferguson method combines deterministic and stochastic elements to estimate claim reserves, projecting reported and Incurred but Not Reported (IBNR) claims using historical loss ratios and accumulated premiums. It integrates historical claim data with probability distributions to estimate future unreported claims, providing more accurate estimates by reducing uncertainty [16]. The Ultimate Claim using the Bornhuetter-Ferguson method can be calculated as follows:

$$U_i^{BF} = C_{i,n-i+1} + \left(1 - \frac{1}{CDF_x}\right) U_i^0, \tag{15}$$

$$R_i^{BF} = \left(1 - \frac{1}{CDF_x}\right) U_i^0. \tag{16}$$

Besides using the product of the percentage unreported and expected claims, another way to calculate claim reserves with the Bornhuetter-Ferguson method involves using the loss ratio reserve factor (q_i). This approach calculates claim reserves by multiplying the loss ratio reserve factor with the ultimate burning cost or expected claims [8]. This method allows for considering relevant loss ratio factors, providing more accurate claim reserve estimates based on historical data and future loss projections.

$$R_i^{BF} = q_i . U_i^0 \; ; i = 1, \dots , n. \tag{17}$$

2.2.6 Benktander Method

The Benktander method, introduced by Gunnar Benktander in 1976, combines the Chain-Ladder and Bornhuetter-Ferguson methods to estimate claim reserves. The Chain-Ladder method projects future claims using historical data, while Bornhuetter-Ferguson integrates historical data with future expectations [21]. This combination reduces uncertainty and variability, adapting well to data changes and fluctuations, and often resulting in lower mean square error estimates. To estimate claim reserves using this method, a credibility factor Z_i^{GB} equal to the loss ratio payout factor p_i, is required [22]. The formula to calculate claim reserves using the Benktander method is:

$$R_i^{GB} = p_i\left(R_i^{CL}\right) + (1 - p_i)\left(R_i^{BF}\right), i = 1, \dots n; \tag{18}$$

$$U_i^{GB} = R_i^{GB} + C_{i,n-i+1} \, , i = 1, \dots n. \tag{19}$$

2.2.7 Walter Neuhaus Method

The Walter Neuhaus method, introduced in 1992, combines the Bornhuetter-Ferguson principles with a credibility approach to estimate claim reserves. It adjusts credibility weights based on claim development, integrating historical information with future expectations for more accurate and stable estimates [23]. This method uses a credibility model to adjust weights based on actual claim experience, making the estimates responsive to data changes. Continuous adjustment of credibility weights (Z_i^{WN}) ensures accurate estimates that reflect real claim conditions. The formula to calculate credibility weights and claim reserves using the Walter Neuhaus method is:

$$Z_i^{WN} = p_i . \sum_{j=1}^{n} m_j \, , i = 1, \dots n; \tag{20}$$

$$R_i^{WN} = Z_i^{WN}\left(R_i^{CL}\right) + \left(1 - Z_i^{WN}\right)\left(R_i^{BF}\right), i = 1, \dots n; \tag{21}$$

$$U_i^{WN} = R_i^{WN} + C_{i,n-i+1} \, , i = 1, \dots n. \tag{22}$$

In these equations, $i = 1, \dots , n$ denotes the accident year and $j = 1, \dots , n$the development period. The term Z_i^{WN}represents the credibility weight for accident year i, where $p_i$is a proportionality factor and $m_j$is the development-dependent credibility factor. The quantities R_i^{WN}, R_i^{CL}, and R_i^{BF}denote the reserve estimates obtained using the Walter Neuhaus, Chain-Ladder, and Bornhuetter–Ferguson methods, respectively. Finally, $U_i^{WN}$is the estimated ultimate claim amount and $C_{i,n-i+1}$represents the cumulative claims observed up to development period $n - i + 1$.

2.2.8 Optimal Credibility Loss Ratio Method

After estimating claim reserves using the Benktander and Walter Neuhaus methods, this study applies the Optimal Credibility Loss Ratio method as a benchmark credibility-based approach. Similar to the previous two methods, this approach combines the Chain-Ladder and Bornhuetter–Ferguson reserve estimates through a credibility-weighted structure. However, unlike the Benktander method, which uses a fixed credibility factor, and the Walter Neuhaus method, which allows the credibility weight to vary with claim development, the Optimal Credibility Loss Ratio method determines the credibility weight by explicitly minimizing the mean squared error (MSE) of reserve estimates across accident years [24]. The formula for estimating claim reserves using the Optimal Credibility Loss Ratio method is:

$$R_i^{C} = Z_i^{C}\left(R_i^{CL}\right) + \left(1 - Z_i^{C}\right)\left(R_i^{BF}\right), i = 1, \dots n; \tag{23}$$

$$U_i^{C} = R_i^{C} + C_{i,n-i+1} \, , i = 1, \dots n. \tag{24}$$

The optimal credibility weights Z_i^C that minimize the mean squared error are given by the following formula [14].

$$Z_i^C = \frac{p_i}{p_i + \sqrt{p_i}}, i = 1, \ldots n. \tag{25}$$

The optimal credibility factor Z_i^C is derived to balance the trade-off between historical claim experience and prior expectations embedded in the loss ratio assumption. By minimizing the MSE, this method provides weights that optimally reflect the reliability of observed data, making the resulting reserve estimates more stable and less sensitive to random fluctuations. Consequently, this approach is particularly effective in accommodating systematic changes and variations in claim patterns, especially in reinsurance portfolios where data volatility and immaturity are common.

In this study, the Optimal Credibility Loss Ratio method is implemented using the same reinsurance dataset and accident-year structure as those employed for the Benktander and Walter Neuhaus methods. This consistent application allows for a direct and meaningful comparison of reserve estimates, positioning the Optimal Credibility Loss Ratio approach as a quantitative reference for evaluating the performance of the other credibility-based reserving methods.

2.2.9 Mean Squared Error Ratio

Mean Squared Error (MSE) is a measure used to evaluate the accuracy of an estimator for a parameter. Let $\widehat{\theta}$ be an estimator of the parameter θ, and $\widehat{\theta}$ is an unbiased estimator. The error of the estimation is defined as $\widehat{\theta} - \theta$. A loss function is a function that converts the estimation error into a set of real numbers. The following theorem explains the mean squared error of the Benktander, Walter Neuhaus and Credibility Loss Ratio methods [24]. Assuming that $\frac{C_{i,j-i}}{U_i} | U_{i,j}$ is beta distribution with parameters p_i and q_i, the mean squared error for each method is as follows where $i = 1, \ldots, n$.

$$mse(R_i^{GB}) = E[\alpha_i^2(U_i)] \left(\frac{p_i^2}{p_i} + \frac{1}{q_i} + \frac{(1-p_i)^2}{t_i} \right) q_i^2, \tag{26}$$

$$mse(R_i^{WN}) = E[\alpha_i^2(U_i)] \left(\frac{(Z_i^{WN})^2}{p_i} + \frac{1}{q_i} + \frac{(1-Z_i^{WN})^2}{t_i} \right) q_i^2, \tag{27}$$

$$mse(R_i^{C}) = E[\alpha_i^2(U_i)] \left(\frac{(Z_i^{C})^2}{p_i} + \frac{1}{q_i} + \frac{(1-Z_i^{C})^2}{t_i} \right) q_i^2. \tag{28}$$

The assumption that the normalized cumulative claims proportion $\frac{C_{i,j-i}}{U_i} | U_{i,j}$ follows a beta distribution is commonly adopted in credibility-based loss reserving literature. This ratio represents the proportion of ultimate claims that has emerged by development year $j - i$, and is therefore naturally bounded between 0 and 1. The beta distribution provides a flexible family for modeling such proportions, allowing for a wide range of shapes depending on the underlying claim development variability.

From a theoretical perspective, the beta distribution arises naturally within credibility theory as the conjugate prior for binomial or multinomial development processes, leading to closed-form expressions for credibility weights and Mean Squared Error. This assumption has been used extensively in the derivation of credibility-based reserving models, including the Benktander, Walter Neuhaus, and Optimal Credibility Loss Ratio frameworks, and is well documented in the actuarial literature [24], [8], [11].As a result, the beta distribution assumption provides a theoretically justified and widely accepted basis for comparing reserve estimators using MSE.

2.3 Data Analysis Method

For this report's objectives, the author executed a series of steps using Microsoft Excel. The research process is outlined as follows:

1. Process reported claims into cumulative run-off triangle data.
2. Calculate parameters for IBNR reserves: Link ratio, Loss Development Factor, Cumulative Development Factor, Loss Ratio, Expected Loss Ratio, Loss Ratio Payout Factor, and Loss Ratio Reserve Factor.

3. Calculate Expected Claims using the Expected Loss Ratio (ELR) and Estimate IBNR reserves using the Chain-Ladder and Bornhuetter – Ferguson methods.
4. Estimate IBNR reserves with Benktander method using Chain-Ladder and Bornhuetter-Ferguson reserves weighted by loss ratio payout factor.
5. Calculate Walter Neuhaus credibility, then estimate IBNR reserves using Chain-Ladder and Bornhuetter-Ferguson reserves weighted by the credibility.
6. Calculate Optimal Credibility Loss Ratio credibility, then estimate IBNR reserves using Chain-Ladder and Bornhuetter-Ferguson methods.
7. Compute MSE ratio for Benktander-Optimal Credibility Loss Ratio and Walter Neuhaus-Optimal Credibility Loss Ratio methods.
8. Analyze IBNR reserve estimates by comparing MSE ratios to identify the best method.

3. RESULTS AND DISCUSSION

This section will explain the calculation of IBNR claim reserves using the Benktander and Walter Neuhaus methods. Subsequently, a comparative analysis of the claim reserve results from both methods will be conducted to reach conclusions addressing the research problem statements. The first step that will be taken is constructing a Cumulative Run-Off Triangle. Constructing a Cumulative Run-Off Triangle from the available reported claims is a crucial step in identifying claim development patterns over time before calculating claim reserves. The values for the Incremental Run-Off Triangle are shown in Table 3.

Table 3. Incremental Reported Claim's Run-Off Triangle

Accident	Development Year							
Year	**1**	**2**	**3**	**4**	**…**	**8**	**9**	**10**
2010	3,848,494	3,592,471	3,497,520	3,429,712	…	3,383,834	3,395,940	3,392,198
2011	7,681,868	7,328,397	6,969,433	6,845,795	…	6,397,773	6,390,304	
2012	5,228,615	4,647,874	4,312,191	3,948,809	…	3,789,167		
2013	3,939,520	3,556,686	3,202,066	3,055,779	…			
2014	3,420,716	3,051,029	2,890,099	2,803,696	…			
2015	4,077,613	3,740,972	3,512,198	3,498,438				
2016	4,245,923	4,423,279	4,456,226	4,415,862				
2017	7,955,624	8,209,635	8,114,676					
2018	7,424,738	6,944,714						
2019	6,166,707							

Table 3 presents the Incremental Reported Claim's Run-Off Triangle, which shows the development of reported claims over time for each accident year from 2010 to 2019. The rows represent the accident years, indicating when claims were incurred, while the columns represent the development years, showing the amount of claims reported in each subsequent year after the accident year. The values in the table are incremental claim amounts reported in each development year. From Table 3 can be refer that in accident year 2010, claims of 3,848,494 were reported in development year 1 (i.e., in 2010), 3,592,471 in year 2 (2011), and so on, with reporting continuing up to development year 10 (2019). As we move down the rows, the number of filled development years decreases, reflecting more recent accident years that have had less time for claims to be reported. The cumulative run-off triangle can be calculated using (2). The results of these calculations are presented in the Table 4.

Table 4. Cumulative Reported Claim's Run-Off Triangle

Accident	Development Year						
Year	**1**	**2**	**3**	**…**	**8**	**9**	**10**
2010	3,848,494	7,440,965	10,938,485	…	27,963,434	31,359,374	34,751,572
2011	7,681,868	15,010,265	21,979,698	…	54,870,819	61,261,123	
2012	5,228,615	9,876,489	14,188,680	…	33,407,383		
2013	3,939,520	7,496,206	10,698,272	…			
2014	3,420,716	6,471,745	9,361,844	…			
2015	4,077,613	7,818,585	11,330,783	…			
2016	4,245,923	8,669,202	13,125,428				
2017	7,955,624	16,165,259	24,279,935				
2018	7,424,738	14,369,452					
2019	6,166,707						

Based on Table 4, the largest reported claims occurred in 2011 with a cumulative value of 61.261.123, while the smallest cumulative value was in 2019 at 6,166,707. To understand the development of reported claims, it is necessary to calculate the link ratio.

The link ratio shows the relationship between claims reported in a given period and those reported in the previous period. The calculated link ratio for reported claims using (3) is presented in Table 5 below.

Table 5. Run-Off Triangle of Link Ratio

Accident Year	Development Year									
	1	2	3	4	5	6	7	8	9	10
2010	1.9335	1.4700	1.3135	1.2381	1.1913	1.1598	1.1377	1.1214	1.1082	1
2011	1.9540	1.4643	1.3115	1.2359	1.1805	1.1526	1.1320	1.1165	-	
2012	1.8889	1.4366	1.2783	1.2119	1.1740	1.1478	1.1279	-		
2013	1.9028	1.4272	1.2856	1.2204	1.1772	1.1498	-			
2014	1.8919	1.4466	1.2995	1.2273	1.1810	-				
2015	1.9174	1.4492	1.3088	1.2382	-					
2016	2.0418	1.5140	1.3364	-						
2017	2.0319	1.5020	-							
2018	1.9353	-								
2019	-									

Based on Table 5, the annual development values of the reported claims are shown. The highest development value occurred in 2016 at 2.0418, while the lowest was in 2010 during the 9th development period at 1.1082. The next step is to calculate the Loss Development Factor (LDF) and Cumulative Development Factor (CDF). Using (4) for LDF and (5) for CDF, Table 6 presents the LDF and CDF values.

Table 6. Loss Development Factor and Cumulative Development Factor

Development Factor	Development Year									
	1	2	3	4	5	6	7	8	9	10
LDF	1.9442	1.4637	1.3048	1.2286	1.1808	1.1525	1.1325	1.1190	1.1082	1.0000
CDF	1.0000	1.1082	1.2400	1.4043	1.6185	1.9111	2.3480	3.0637	4.4844	8.7185

Table 6 presents the Loss Development Factor (LDF) and Cumulative Development Factor (CDF) for each development year from 1 to 10. The LDF represents the ratio of claims expected to develop from one year to the next, indicating how much the reported claims are projected to grow over time. The CDF is the cumulative product of the LDFs up to each development year and reflects the total expected development from the current point to the ultimate year. For example, the LDF in development year 1 is 1.9442, meaning claims are expected to nearly double from year 1 to year 2. The CDF value of 8.7185 in development year 10 indicates that the claims reported in year 1 are expected to grow by a factor of over 8 by the time they are fully developed. These factors are essential for estimating ultimate claims and for calculating reserves such as IBNR in actuarial projections.

After determining the value of CDF and LDF the next step of this research is to calculate the loss ratio and its components, such as the Expected Loss Ratio, Loss Ratio Payout Factor, and Loss Ratio Reserve Factor, the earned premium value is required as the basis.

Table 7. Earned Premium

Year	Earned Premium
2010	8,253,587
2011	8,253,587
2012	11,049,206
2013	11,040,805
2014	10,378,846
2015	11,917,896
2016	10,443,139
2017	10,733,712
2018	10,899,249
2019	11,594,062

Table 7 presents the earned premium figures from the year 2010 to 2019. Earned premium refers to the portion of written premiums that corresponds to the expired part of the policy period and therefore represents the income the insurer has "earned" for the coverage provided. the earned premium shows a general upward trend with some minor declines, reflecting growth in insurance business over the decade with some volatility. After determining the Earned Premium values for each year, the next step is to calculate the loss ratio using Eq. (7). The Loss Ratio values will be displayed in Fig. 1.

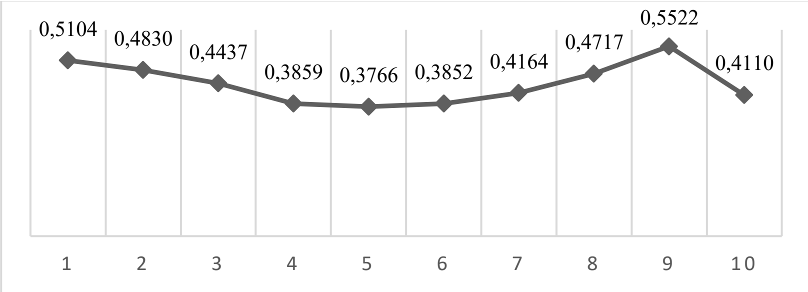

Figure 1. Loss Ratio in Development Period

Fig. 1 illustrates the Loss Ratio over the Development Period, showing how the ratio of losses to premiums evolves across development years 1 to 10. The graph shows a declining trend in the early years, with the loss ratio dropping from 0.5104 in year 1 to its lowest point at 0.3766 in year 5. This indicates reduced claim costs relative to earned premiums during this period. From year 6 onward, the loss ratio begins to increase gradually, peaking at 0.5522 in year 9, before slightly decreasing to 0.4110 in year 10. This U-shaped trend suggests that losses may initially stabilize or improve but tend to increase again in later development years, possibly due to delayed reporting or higher-than-expected late claims. After determining the loss ratio values, the next step is to find the Expected Loss Ratio (ELR) by calculating the cumulative loss ratio over the research period. Using Eq. (6), the value obtained is 4.4362 . The next step is to calculate the Loss Ratio Payout Factor and Loss Ratio Reserve Factor using Eqs. (8) and (9), respectively. The results are shown in Table 8.

Table 8. Loss Ratio Payout Factor and Loss Ratio Reserve Factor

Loss Ratio	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Payout Factor	1.0000	0.9074	0.7829	0.6765	0.5827	0.4958	0.4110	0.3240	0.2239	0.1151
Reserve Factor	0.0000	0.0926	0.2171	0.3235	0.4173	0.5042	0.5890	0.6760	0.7761	0.8849

The loss ratio payout factor indicates the percentage of claims that have been paid, while the loss ratio reserve factor shows the percentage of claims that still need to be reserved. By using these factors, claim estimations can be improved and employed as variables in calculating the Mean Squared Error (MSE) for claim reserve predictions. Subsequently, the calculation for the expected claim will be carried out using Eq. (10) based on the information obtained, such as an ELR of 4.4362 and the earned premium. Table 9 will display the results of the Expected Claims over the 10-year study period.

Table 9. Value of Expected Claim

Accident Year	Earned Premium	Expected Claim
2010	8.253.587	36.614.418
2011	9.468.408	42.003.586
2012	11.049.206	49.016.294
2013	11.040.805	48.979.026
2014	10.378.846	46.042.455
2015	11.917.896	52.869.962
2016	10.443.139	46.327.671
2017	10.733.712	47.616.706
2018	10.899.249	48.351.058
2019	11.594.062	51.433.375

The expected claim results show a consistent upward trend in earned premiums and expected claims from 2010 to 2019, with earned premiums increasing from 8,253,587 to 11,594,062 and expected claims rising from 36,614,418 to 51,433,375. This increase in collected premiums and expected claims indicates the insurance company anticipates significant claims alongside business growth or increased risk. The next step is to calculate the ultimate claim and claim reserves using the Chain Ladder method with Eqs. (11) and (12). Table 10 will display the results of these reserve calculations.

Table 10. Claim Reserve Chain-Ladder Method Estimation

Accident year	Cumulative Reported Claim	CDF	Ultimate Claim	IBNR
2010	34,751,572	1.0000	34,751,572	0
2011	61,261,123	1.1082	67,887,845	6,626,722
2012	33,407,383	1.2400	41,424,836	8,017,453
2013	22,719,588	1.4043	31,905,701	9,186,113
2014	17,632,349	1.6185	28,537,178	10,904,829
2015	18,361,292	1.9111	35,090,021	16,728,729
2016	17,541,290	2.3480	41,186,941	23,645,651
2017	24,279,935	3.0637	74,385,845	50,105,910
2018	14,369,452	4.4844	64,438,680	50,069,228
2019	6,166,707	8.7185	53,764,603	47,597,896
			Total	222,882,531

Table 10 presents the claim reserve estimation using the Chain-Ladder method across accident years 2010–2019. A clear increasing trend in the cumulative development factors (CDF) is observed for more recent accident years, reflecting lower claim maturity and a higher proportion of unreported or unsettled claims. Consequently, accident years closer to 2019 exhibit substantially higher IBNR values compared to earlier years. For older accident years such as 2010 and 2011, the CDF values are close to unity, indicating that most claims have already been reported and developed, resulting in negligible or relatively small IBNR reserves. In contrast, accident years from 2016 onward show significantly larger ultimate claim estimates and IBNR amounts, driven by higher CDFs that amplify the reported claims to account for expected future development. These differences arise primarily from variations in claim development maturity rather than changes in underlying exposure, highlighting the Chain-Ladder method's strong dependence on historical development patterns and its sensitivity to immature data. As a result, while the method performs well for older, well-developed accident years, it tends to produce larger and more volatile reserve estimates for recent years where claim information is still incomplete. The next step is to calculate the claim reserves using the Bornhuetter-Ferguson method based on the obtained information on expected claims. This calculation will be performed using Eqs. (15) and (16). The results will be presented in Table 11.

Table 11. Claim Reserve Bornhuetter-Ferguson Method Estimation

Accident Year	Cumulative Reported Claim	Expected Claim	Percentage Unreported	Ultimate Claim	IBNR
2010	34.751.572	36.614.418	0.00%	34,751,572	0
2011	61.261.123	42.003.586	9.76%	65,361,210	4,100,087
2012	33.407.383	49.016.294	19.35%	42,894,103	9,486,720
2013	22.719.588	48.979.026	28.79%	36,821,358	14,101,770
2014	17.632.349	46.042.455	38.21%	35,226,419	17,594,070
2015	18.361.292	52.869.962	47.67%	43,566,386	25,205,094
2016	17.541.290	46.327.671	57.41%	44,138,262	26,596,972
2017	24.279.935	47.616.706	67.36%	56,354,295	32,074,360
2018	14.369.452	48.351.058	77.70%	51,938,504	37,569,052
2019	6.166.707	51.433.375	88.53%	51,700,763	45,534,056
				Total	212,262,182

Table 11 reports the IBNR reserve estimates obtained using the Bornhuetter–Ferguson method, which combines cumulative reported claims with expected claims derived from prior loss ratio assumptions. A clear upward trend in the percentage of unreported claims is observed for more recent accident years, increasing from nearly 0% for 2010 to over 88% for 2019. This pattern reflects the decreasing level of claim maturity in recent years, where only a small portion of ultimate claims has been reported. Consequently, the IBNR amounts increase substantially for later accident years, with the highest reserves observed from 2016 onward. Unlike the Chain-Ladder method, the Bornhuetter–Ferguson approach produces more stable ultimate claim estimates across accident years, as these estimates are largely driven by expected claims rather than purely by historical development patterns. The differences in IBNR across years are therefore primarily attributable to variations in reported-to-expected claim proportions, rather than fluctuations in cumulative reported claims alone. This highlights the strength of the Bornhuetter–Ferguson method in mitigating volatility for immature data, while also underscoring its reliance on the accuracy of the assumed expected loss ratios. After determining the claim reserve values using the Chain Ladder and Bornhuetter-Ferguson methods, the IBNR reserves can be estimated for the Benktander and Walter Neuhaus methods. These two methods utilize the claim reserve information from Bornhuetter-Ferguson and Chain Ladder in their estimations. Next, the IBNR reserve calculation for the Benktander method will be performed using Eqs. (18) and (19), with the results presented in Table 12.

Table 12. Claim Reserve Benktander Method Estimation

Accident Year	Cumulative Reported Claim	Credibility (p_i)	R_i^{CL}	R_i^{BF}	Ultimate Claim	IBNR
2010	34.751.572	1.0000	0	0	34,751,572	0
2011	61.261.123	0.9074	6,626,722	4,100,087	67,653,761	6,392,638
2012	33.407.383	0.7829	8,017,453	9,486,720	41,743,851	8,336,468
2013	22.719.588	0.6765	9,186,113	14,101,770	33,495,672	10,776,084
2014	17.632.349	0.5827	10,904,829	17,594,070	31,328,662	13,696,313
2015	18.361.292	0.4958	16,728,729	25,205,094	39,363,389	21,002,097
2016	17.541.290	0.4110	23,645,651	26,596,972	42,925,414	25,384,124
2017	24.279.935	0.3240	50,105,910	32,074,360	62,195,812	37,915,877
2018	14.369.452	0.2239	50,069,228	37,569,052	54,737,805	40,368,353
2019	6.166.707	0.1151	47,597,896	45,534,056	51,938,217	45,771,510
					Total	209,643,462

Table 12 illustrates how the credibility factor p_i varies systematically across accident years and governs the relative influence of the Chain-Ladder and Bornhuetter–Ferguson estimates in the Benktander method. Higher credibility values are observed for older accident years, such as 2010 and 2011, reflecting a high level of claim maturity and a greater reliance on observed claim development. As the accident years become more recent, the credibility factor declines steadily, reaching its lowest value in 2019, which indicates increased uncertainty due to incomplete claim reporting and development. This declining credibility pattern shifts greater weight toward the Bornhuetter–Ferguson component, which relies on prior expectations rather than historical development. Consequently, the Benktander method produces ultimate claim and IBNR estimates that transition smoothly between the Chain-Ladder-dominated results for mature years and the Bornhuetter–Ferguson-dominated results for immature years. This adaptive weighting mechanism explains the observed differences in reserve estimates across accident years and highlights the Benktander method's ability to balance data credibility with prior assumptions in the presence of varying claim maturity.

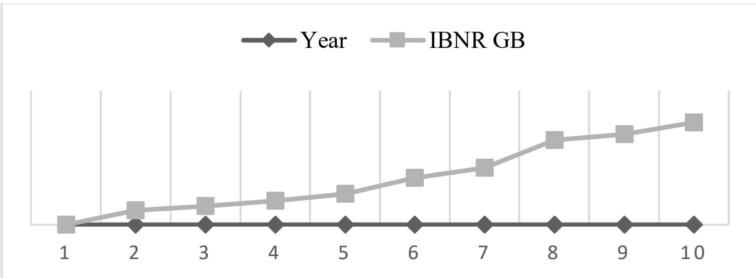

Figure 2. Benktander Claim Reserve's Trend
Source: Calculated using Microsoft Excel based on Benktander Claim Reserve (2010–2019)

Fig. 2 displays the trend of IBNR claim reserves estimated using the Benktander method over the period from 2010 to 2019. The horizontal axis represents the accident year, while the vertical axis shows the corresponding IBNR reserve values. The graph reveals a gradual and steady increase in reserve amounts across the years, indicating that the Benktander method reflects a conservative and consistent reserve estimation approach. There are no sharp spikes or fluctuations, suggesting that this method applies moderate credibility to past claims data, resulting in stable reserve projections. This trend aligns with the nature of the Benktander method, which typically yields lower and smoother reserve estimates compared to other methods. After determining the reserve values using the Benktander method, the next step is to calculate the reserve values using the Walter Neuhaus method. First, the credibility value must be calculated using Eq. (20). Calculating credibility using the Walter Neuhaus method can result in values greater than 1 if the obtained ELR is quite large. To keep the values within the range of 0 to 1, normalization is performed to adjust the credibility values, ensuring consistent and realistic results in claim reserve estimation [13]. The results will be displayed in Table 13.

Table 13. Walter Neuhaus Credibility

Accident Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Z_i^{WN}	4.4362	4.0252	3.4730	3.0013	2.5849	2.1997	1.8231	1.4371	0.9934	0.5104
Z_i^{WN} Normalization	1	1	1	1	1	1	1	1	0.9934	0.5104

The reserve values will be calculated using Eqs. (21) and (22), and the results will be presented in Table 14.

Table 14. Claim Reserve Walter Neuhaus Method Estimation

Accident Year	Cumulative Reported Claim	$Z_i^{WN(norm)}$	R_i^{CL}	R_i^{BF}	Ultimate Claim	IBNR
2010	34,751,572	1	0	0	34,751,572	0
2011	61,261,123	1	6,626,722	4,100,087	67,887,845	6,626,722
2012	33,407,383	1	8,017,453	9,486,720	41,424,836	8,017,453
2013	22,719,588	1	9,186,113	14,101,770	31,905,701	9,186,113
2014	17,632,349	1	10,904,829	17,594,070	28,537,178	10,904,829
2015	18,361,292	1	16,728,729	25,205,094	35,090,021	16,728,729
2016	17,541,290	1	23,645,651	26,596,972	41,186,941	23,645,651
2017	24,279,935	1	50,105,910	32,074,360	74,385,845	50,105,910
2018	14,369,452	0.9934	50,069,228	37,569,052	64,356,711	49,987,259
2019	6,166,707	0.5104	47,597,896	45,534,056	52,754,152	46,587,445
					Total	221,790,111

Table 14 presents the claim reserve estimates obtained using the Walter Neuhaus method and allows for a direct comparison with the Benktander results reported in Table 12. Both methods employ a credibility-based framework to blend the Chain-Ladder and Bornhuetter–Ferguson estimates; however, they differ in how credibility evolves across accident years. In the Walter Neuhaus method, the normalized credibility factor remains equal to one for fully developed accident years, indicating complete reliance on the Chain-Ladder estimates, which results in reserve outcomes identical to those of the Benktander method for mature years. Differences emerge for more recent accident years, where the Walter Neuhaus credibility decreases only when claim development becomes substantially incomplete. Consequently, for years such as 2018 and 2019, the Walter Neuhaus method assigns higher credibility to observed claims than the Benktander method, which applies a more rapidly declining credibility factor. This leads to slightly higher ultimate claim and IBNR estimates under the Walter Neuhaus approach for these years. Overall, while both methods produce similar reserve levels for older accident years, the Walter Neuhaus method exhibits greater responsiveness to partial claim development in moderately immature years, whereas the Benktander method places stronger emphasis on prior expectations. This comparison highlights how alternative credibility specifications can materially affect reserve estimates, particularly in recent accident years with incomplete data.

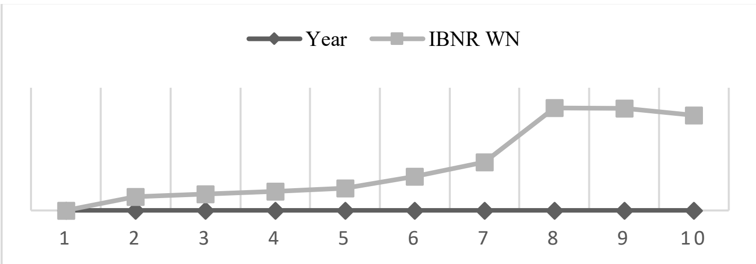

Figure 3. Walter Neuhaus Claim Reserve's Trend
Source: Calculated using Microsoft Excel based on Walter Neuhaus Claim Reserve (2010–2019)

Fig. 3 shows an increasing trend in IBNR estimates under the Walter Neuhaus method for accident years 2010–2019. IBNR levels are low for earlier years due to full claim maturity and credibility values equal to one, resulting in Chain-Ladder–driven estimates. For more recent accident years, IBNR increases as credibility declines and greater weight is assigned to the Bornhuetter–Ferguson component. Compared with the Benktander method, the Walter Neuhaus approach exhibits a smoother increase in IBNR for moderately immature years, reflecting its stronger reliance on observed claim development before shifting toward prior expectations. After determining the reserve values using both methods, the next step is to calculate the reserve values using the reference method, which is the Optimal Credibility Loss Ratio method. This method is used as a reference because it is considered capable of minimizing the MSE. The first step is to calculate the credibility values using Eq. (25), and the results will be displayed in Table 15.

Table 15. Optimal Credibility Loss Ratio Credibility

Accident Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Z_i^C	0.5000	0.4878	0.4694	0.4513	0.4329	0.4132	0.3906	0.3627	0.3212	0.2533

After determining the credibility values, the reserve values will be calculated using Eqs. (23) and (24), and the results will be presented in Table 16.

Table 16. Claim Reserve Optimal Credibility Loss Ratio Method Estimation

Accident Year	Cumulative Reported Claim	Z_i^C	R_i^{CL}	R_i^{BF}	Ultimate Claim	IBNR
2010	34,751,572	0.5000	0	0	34,751,572	0
2011	61,261,123	0.4878	6,626,722	4,100,087	72,973,918	5,332,705
2012	33,407,383	0.4694	8,017,453	9,486,720	50,506,188	8,796,987
2013	22,719,588	0.4513	9,186,113	14,101,770	45,204,269	11,883,279
2014	17,632,349	0.4329	10,904,829	17,594,070	45,791,649	14,698,331
2015	18,361,292	0.4132	16,728,729	25,205,094	60,833,661	21,702,639
2016	17,541,290	0.3906	23,645,651	26,596,972	68,325,391	25,444,081
2017	24,279,935	0.3627	50,105,910	32,074,360	100,854,701	38,614,812
2018	14,369,452	0.3212	50,069,228	37,569,052	96,310,287	41,584,316
2019	6,166,707	0.2533	47,597,896	45,534,056	97,994,275	46,056,793
					Total	214,113,943

Table 16 reports the reserve estimates obtained using the Optimal Credibility Loss Ratio method and reveals a clear and systematic pattern in the optimal credibility weights across accident years. The credibility factor Z_i^C decreases gradually from 0.5000 in 2010 to 0.2533 in 2019, reflecting the declining reliability of observed claim development for more recent accident years. Unlike the Benktander and Walter Neuhaus methods, where credibility is mainly driven by claim maturity or development structure, the Optimal Credibility approach determines Z_i^C by explicitly minimizing the mean squared error (MSE) of reserve estimates across accident years. As a result, the credibility pattern represents an optimal trade-off between bias and variance, with lower credibility values placing greater weight on the prior loss ratio assumptions embedded in the Bornhuetter–Ferguson component. This MSE-based weighting yields smoother and more stable reserve estimates, positioning the Optimal Credibility Loss Ratio method as a statistically grounded benchmark for evaluating the Benktander and Walter Neuhaus results.

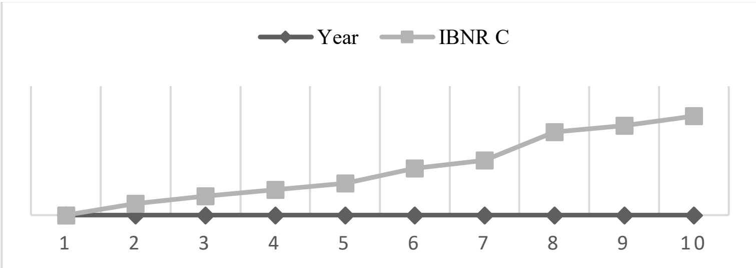

Figure 4. Optimal Credibility Loss Ratio Reserve's Trend
Source: Calculated using Microsoft Excel based on Optimal Credibility Loss Ratio Claim Reserve (2010–2019)

Fig. 4 illustrates the trend of IBNR claim reserves estimated using the Optimal Credibility Loss Ratio method over the period 2010–2019. The horizontal axis represents the accident years, while the vertical axis shows the corresponding reserve values. The graph demonstrates a consistent upward trend in reserve estimates, indicating that this method captures the progressive accumulation of claims over time. Unlike the Walter Neuhaus method, which shows a sharp increase in later years, the Optimal Credibility method displays a smoother and more gradual increase. This reflects its balanced approach that adjusts credibility weights to minimize the mean squared error, combining historical data and recent claim experience in a proportionate manner. The steady rise in reserves suggests that this method is responsive to emerging data while maintaining stability across years.

After determining the Optimal Credibility Loss Ratio claim reserve, the Mean Squared Error (MSE) ratio for both methods will be calculated. This calculation will use the MSE equations derived from Eqs. (26), (27), and (3). Table 17 will present the comparison results of the ratios.

Table 17. MSE Ratio

Accident Year	$mse\left(\frac{R_i^{GB}}{R_i^C}\right)$	$mse\left(\frac{R_i^{WN}}{R_i^C}\right)$
2010	-	-
2011	1.0334	1.0498
2012	1.0454	1.1302
2013	1.0364	1.2158
2014	1.0216	1.3100
2015	1.0083	1.4201
2016	1.0006	1.5599
2017	1.0028	1.7569
2018	1.0229	2.0918
2019	1.0668	1.2310
Average	1.0265	1.4184

Based on the Table 17, Mean Squared Error (MSE) ratios over a 10-year period, it can be concluded which of the two ratios is closer to 1 or more optimal. The MSE ratio $mse\left(\frac{R_i^{GB}}{R_i^C}\right)$ is advantageous, consistently close to 1, with an average of 1.0265. In contrast, the MSE ratio $mse\left(\frac{R_i^{WN}}{R_i^C}\right)$ shows higher values, with an average of 1.4184, and tends to be well above 1. This indicates that the $mse\left(\frac{R_i^{GB}}{R_i^C}\right)$ ratio is better and closer to 1. After determining that $mse\left(\frac{R_i^{GB}}{R_i^C}\right)$ produces superior results, the next step involves analyzing the reserve values obtained. Table 18 will present the claim reserve values from all three methods.

Table 18. Summary of IBNR Claim Reserve Estimates

Accident Year	IBNR GB	IBNR WN	IBNR C
2010	0	0	0
2011	6,392,638	6,626,722	5,332,705
2012	8,336,468	8,017,453	8,796,987
2013	10,776,084	9,186,113	11,883,279
2014	13,696,313	10,904,829	14,698,331
2015	21,002,097	16,728,729	21,702,639
2016	25,384,124	23,645,651	25,444,081
2017	37,915,877	50,105,910	38,614,812
2018	40,368,353	49,987,259	41,584,316
2019	45,771,510	46,587,445	46,056,793
Total	209,643,462	221,790,111	214,113,943

Based on Table 18, the IBNR claim reserve estimates show significant differences between the Benktander, Walter Neuhaus, and Optimal Credibility Loss Ratio methods. The Optimal Credibility Loss Ratio method, which is designed to minimize the Mean Squared Error (MSE), estimates the reserve at 214,113,943 and is considered the most statistically efficient. These methods demonstrated that optimal credibility methods, which apply weighted combinations of the Chain-Ladder and Bornhuetter-Ferguson models, lead to more accurate claim reserve estimations by minimizing reserve variance and . In comparison, the Benktander method, yields an estimate of 209,643,462, which is 4,470,481 lower than the benchmark with an average deviation of 447,048, indicating strong accuracy and reliability. On the other hand, the Walter Neuhaus method produces a higher estimate of 221,790,111, which is 7,676,168 higher than the benchmark and has an average deviation of 767,616, indicating lower accuracy. These results are consistent with the findings of Shi and Hartman, who emphasized the importance of incorporating dynamic credibility theory into loss reserving models to enhance estimation precision, especially for immature data. Collectively, these studies validate the empirical performance observed in this analysis and highlight the strengths of credibility-based approaches in actuarial claim reserving**.** The comparative results demonstrate that differences in reserve estimates across the Benktander, Walter Neuhaus, and Optimal Credibility Loss Ratio methods are fundamentally driven by the structure of credibility weights and their sensitivity to data maturity. The Benktander method exhibits consistently lower Mean Squared Error (MSE) ratios across almost all accident years, indicating that its credibility mechanism effectively balances historical development information and prior expectations, particularly for nonproportional reinsurance data characterized by high volatility and delayed reporting. In contrast, the Walter Neuhaus method tends to produce higher reserve estimates and larger MSE ratios, especially for more recent accident years. This behavior suggests that the Neuhaus credibility formulation may overreact to early-stage claim development in immature data, leading to increased estimation variance. While this characteristic can be advantageous in portfolios where rapid claim emergence is expected, it may result in overly conservative reserves for nonproportional assumed property business. The Optimal Credibility Loss Ratio method serves as a theoretically grounded benchmark by explicitly minimizing MSE. The empirical proximity of Benktander estimates to this benchmark provides strong evidence that the Benktander method offers a robust and practically efficient alternative when full implementation of optimal credibility is not feasible. From a practical actuarial perspective, these findings highlight that credibility-based reserving methods should not be selected solely on theoretical grounds but must also consider data maturity, volatility, and the risk profile of the underlying reinsurance portfolio.

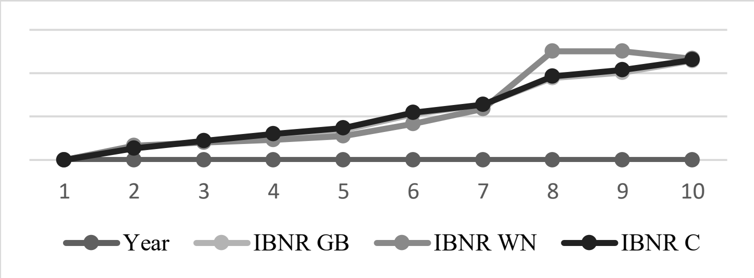

Figure 5. Summary of IBNR Claim Reserve Trends
Source: Calculated using Microsoft Excel based on Summary of IBNR Claim Reserve (2010–2019)

Based on Fig. 5, The horizontal axis represents the year of occurrence, while the vertical axis indicates the estimated IBNR values. The graph shows that the Walter Neuhaus method (IBNR WN) consistently yields the highest reserve estimates in the later years, peaking sharply around year 8 before slightly declining. The Benktander method (IBNR GB), on the other hand, remains relatively flat across all years, indicating more conservative and stable reserve estimates. The Optimal Credibility method (IBNR C) lies between the two, showing a steady increase over time that reflects its balanced approach by combining both historical patterns and credibility adjustments. Overall, the graph emphasizes the varying sensitivities of each method to the progression of claim development across different accident years.

4. CONCLUSION

This study examines the performance of credibility-based methods for estimating Incurred But Not Reported (IBNR) claim reserves in a nonproportional assumed property reinsurance portfolio using United States data from 2010 to 2019. By benchmarking the Benktander and Walter Neuhaus methods against the Optimal Credibility Loss Ratio approach, which is designed to minimize Mean Squared Error (MSE), this research provides an empirical assessment of reserve estimation accuracy under conditions of high volatility and delayed claim reporting. The IBNR claim reserve estimates show significant variation between methods. The Benktander method produces an IBNR reserve of 209,643,462, while the Walter Neuhaus method results in 221,790,111. The Optimal Credibility Loss Ratio method, which minimizes Mean Squared Error (MSE), estimates 214,113,943 and serves as a benchmark. The Benktander method is 4,470,481 lower than the benchmark, while the Walter Neuhaus method is 7,676,168. The results indicate that the Benktander method produces IBNR reserve estimates that are consistently closer to the optimal benchmark, with an average MSE ratio of 1.0265, compared to 1.4184 for the Walter Neuhaus method. This demonstrates that the credibility structure of the Benktander approach achieves a more effective balance between historical claim development and prior expectations, particularly for immature reinsurance data. In contrast, the Walter Neuhaus method tends to generate higher and more variable reserve estimates, reflecting greater sensitivity to early-stage claim development, which may lead to over-reserving in nonproportional reinsurance portfolios. From a practical actuarial perspective, these findings indicate that the Benktander method provides a robust and efficient alternative for IBNR reserving when data maturity is limited and claim volatility is high. This study contributes to the actuarial reserving literature by offering empirical evidence on the relative performance of credibility-based reserving methods in a nonproportional reinsurance setting, an area that has received limited attention in prior research. From an industry standpoint, the results underscore that reserving method selection has important implications for solvency management, capital efficiency, and pricing strategy, particularly in reinsurance portfolios characterized by delayed claim development. Credibility-based approaches that balance stability and responsiveness can support prudent reserve setting without imposing excessive capital requirements. This study is subject to several limitations. The analysis is based on U.S. nonproportional assumed property reinsurance data and may not be directly generalizable to other business lines or jurisdictions. In addition, a deterministic framework is employed, with performance evaluated using Mean Squared Error, without explicitly quantifying reserve uncertainty or variability. Future research may extend this work by applying credibility-based reserving methods to other lines of business and regions, incorporating stochastic or Bayesian frameworks to assess reserve uncertainty, exploring tail risk behavior, or comparing traditional actuarial methods with machine learning–based reserving approaches.

Author Contributions

Dwi Mahrani: Conceptualization, Data Curation, Funding Acquisition, Supervision, Validation, Writing - Review and Editing. Edward Al Faruq Purba: Formal analysis, Methodology, Software, Visualization, Writing - Original Draft. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare no conflicts of interest to report study.

Declaration of Generative AI and AI-assisted Technologies

The authors used generative AI only to assist with language polishing and formatting consistency (e.g., improving wording and ensuring uniform terminology). No AI was used to generate research content, perform analyses, or create/modify figures and tables. The authors reviewed the manuscript in full and remain responsible for its content.

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