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MULTIVARIATE ROBUST MONITORING OF PLASTIC WASTE QUALITY USING PCA-BAYESIAN MEWMA CONTROL CHART

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ABSTRACT

Quality control is essential for ensuring manufacturing processes consistently meet predefined specifications and for minimizing the risks caused by process deviations. The MEWMA control chart is widely used for detecting small shifts in multivariate processes which does not require strict multivariate normality but the performance can be compromised when data contain outliers or high multicollinearity that commonly found in plastic waste processing. This study proposes a robust monitoring approach by integrating PCA to address multicollinearity, Bayesian estimation to improve parameter robustness. The four charts examined in this study are PCA-Bayesian MEWMA (SELF), PCA-Bayesian MEWMA (MSELF), PCA-Bayesian MEWMA (KLF), and PCA-MEWMA using Bootstrap control limit as comparison. These charts are evaluated across 324 simulated scenarios, varying in collinearity levels (0.2, 0.6, 0.95), sample sizes (10, 20, 30), outlier proportions (5%, 10%, 15%), and smoothing parameters (λ = 0.2, 0.5, 0.8). Performance is measured using Average Run Length (ARL), Standard Deviation of Run Length (SDRL), Median Run Length (MRL), and False Alarm Rate (FAR). Results indicate that the PCA-Bayesian MEWMA outperformed PCA-MEWMA using Bootstrap control limit. PCA-Bayesian MEWMA (SELF) excelled under clean data condition, whereas PCA-Bayesian (MSELF) provided stable detection under high correlation, moderate-to-high outlier contamination, and larger smoothing parameters, achieving an average ARL of 3.44, an SDRL of 0.58, an MRL of 3.46, and FAR of 0.03, making it well-suited for monitoring complex industrial plastic waste processes and demonstrating its effectiveness for robust quality monitoring in production.

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1. INTRODUCTION

Quality control is a crucial component in the production and processing stages across various industrial sectors. The primary objective of quality control is to ensure that the products meet predefined specifications, so that the final output delivered to consumers aligns with the expected quality. In addition, early detection of process deviations can minimize the risk of losses incurred by companies [1]. In practice, quality control in production processes often involves multivariate and complex data, which necessitates more adaptive approaches.

One of the most commonly used tools in quality control is the Shewhart control chart. This chart is designed to monitor process stability by detecting changes in the mean or variability of certain parameters [2]. Although effective for univariate cases, the Shewhart chart is not well-suited for handling correlated multivariate data [3]. To overcome this limitation, the Shewhart chart has been further developed into various multivariate control charts. Among these, the T^2-Hotelling control chart is the most widely used. It enables simultaneous monitoring of multiple variables while accounting for their correlations [4]. Unfortunately, this chart lacks sensitivity to small shifts in process parameters because the statistics relies on the sample covariance matrix and gives equal weight to all observations, regardless of how recent they are.

To address this issue, the Multivariate Exponentially Weighted Moving Average (MEWMA) chart was developed. MEWMA is more effective in detecting small and gradual shifts, as it gives greater weight to previous observations [4], [5], [6], [7]. This weighting allows MEWMA to gradually amplify small, persistent changes that would otherwise be overlooked in traditional charts. However, the performance of MEWMA strongly depends on accurate covariance estimation. In practice, manufacturing data rarely follow multivariate normality, often exhibit strong correlations, and may contain outliers, all of which can weaken the chart's ability to detect subtle yet meaningful changes in the process mean vector [8], [9]. Therefore, it is necessary to develop control charts that remain capable of identifying small but important process deviations before they escalate into serious quality problems and effective even when manufacturing data violates many of the traditional assumptions of multivariate control charts.

Several studies have developed robust methods against outliers, such as the use of S-estimators and MAD-estimators [10], MCD [11], Fast-MCD [12], [13], [14], MRCD [15], with integration of modified sample sizes [16], [17], [18], [19] and machine learning [20], [21], [22], [23], [24] . Other studies combine these estimators with Bootstrap techniques to handle violations of multivariate normality. However, some research has shown that these methods are still not effective when applied to contaminated high-dimensional data.

To address these limitations, Principal Component Analysis (PCA) is employed to deal with direct correlations between variables [12], [25], [26]. When variables are highly correlated, the resulting covariance matrix can become singular, preventing the inversion needed to compute T^2 statistics. PCA is expected to enhance modeling efficiency and increase the sensitivity of anomaly detection.

Integrating PCA with the Bayesian estimator into MEWMA is believed to yield a more robust and adaptive quality control tool for recent manufacturing process fluctuations [27], [28], [29], [30]. Considering that several studies have shown that Bootstrap can enhance sensitivity at control limits [15], [31], [32]. Therefore, the control limit estimation for the PCA-MEWMA can be carried out using Bootstrap methods as the opposite of PCA-Bayesian MEWMA. The development of a PCA-Bayesian MEWMA is highly relevant for quality control in the manufacturing and plastic processing sectors, especially in detecting small process shifts with an autocorrelated dataset [33], [34], [35], [36], [37]. Quality control for plastic production waste is critical for monitoring shifts that indicate process deviations. Such deviations may result from the use of substandard materials or errors in production. Faster detection will reduce the risk of loss if the deviations persist.

This study proposes the development of a robust multivariate control chart by integrating PCA and Bayesian estimation into the MEWMA framework, aiming to improve the sensitivity and accuracy of process shift detection, particularly in plastic waste quality monitoring. To demonstrate the effectiveness of the proposed PCA-Bayesian MEWMA approach, it is compared with PCA-MEWMA using the Bootstrap control limit. Simulations are conducted across 324 scenarios including three collinearity levels (low=0.2, medium=0.6, high=0.95), three levels of sample sizes (n=10, 20, 30), three levels outlier proportions (5%, 10%, 15%), and three smoothing parameters (λ = 0.2, 0.5, 0.8). Bootstrap resampling of 500 iterations for the control limit of PCA-MEWMA conventional chart is used in this research. The performance is evaluated using Average Run Length (ARL), Standard Deviation of Run Length (SDRL), Median Run Length (MRL), and False Alarm Rate (FAR). A lower ARL, SDRL, and MRL indicates higher adaptability and sensitivity in detecting small process shift, while lower FAR reflects more accurate identification of out-of-control process.

Despite the extensive development of multivariate control charts, several limitations remain unaddressed, particularly when the monitoring data exhibit strong collinearity, non-normality, and outlier contamination conditions commonly encountered in plastic waste processing industries. Traditional charts such as T^2-Hotelling and conventional MEWMA often lose sensitivity to small shifts because their estimators rely heavily on sample covariance matrices, which become unstable under high-dimensional or contaminated data. Ensuring high sensitivity is essential in plastic waste quality control, where small variations in waste proportions often signal upstream process issues such as machine wear, improper heating, or material degradation. This study addresses these gaps by proposing a PCA-Bayesian MEWMA chart designed to remain robust and sensitive under non-ideal industrial conditions. The novelty lies in integrating Bayesian loss-function–based estimation with PCA dimensionality reduction within the MEWMA structure, forming a comprehensive and adaptive multivariate monitoring framework. Besides, improving these limitations is crucial, since early and accurate detection of process deviations can help reduce quality degradation and minimize material loss.

2. RESEARCH METHODS

2.1 Literature Review

2.1.1 Multivariate Normality

Multivariate statistical process control methods such as the T^2-Hotelling and MEWMA charts are based on the assumption that the data follows a multivariate normal distribution, as shown in Eq. (1).

$$\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \tag{1}$$

where $\boldsymbol{X} \in R^p$ is the matrix of multivariate variables, $\boldsymbol{\mu}$ is the process mean vector, and $\boldsymbol{\Sigma}$ is the variance-covariance matrix. This assumption must be verified to ensure that the control chart is built upon the appropriate theoretical distribution, which is crucial for subsequent statistical inferences. The hypothesis for this normality test is [38]:

H_0: $\boldsymbol{X} \sim \boldsymbol{N_p}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, The data follows a multivariate normal distribution
H_1: $\boldsymbol{X} \backslash\sim \boldsymbol{N_p}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, The data does not follow a multivariate normal distribution

Mardia's test is a widely used statistical procedure for assessing multivariate normality based on multivariate skewness and multivariate kurtosis measures [38]. The test evaluates whether the joint distribution of the variables resembles a multivariate Gaussian distribution. The skewness component assesses symmetry, whereas the kurtosis component measures tail heaviness relative to the multivariate normal as shown in Eqs. (2) and (3) [38] [39].

$$b_{1,p} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[(\boldsymbol{x_i} - \boldsymbol{\bar{x}})' \boldsymbol{S^{-1}} (\boldsymbol{x_j} - \boldsymbol{\bar{x}})\right]^3, \tag{2}$$

$$b_{2,p} = \frac{1}{n} \sum_{i=1}^{n} \left[(\boldsymbol{x_i} - \boldsymbol{\bar{x}})' \boldsymbol{S^{-1}} (\boldsymbol{x_i} - \boldsymbol{\bar{x}})\right]^2. \tag{3}$$

where n is the number of samples, x_i and x_j are the i-th and j-th observation vectors, $\bar{x}$ is the mean vector, S is the sample covariance matrix, p donates the number of variables. The skewness statistic $b_{1,p}$ asymptotically follows a chi-square distribution with $1/6p(p+1)(p+2)$ degrees of freedom. For kurtosis, the data is considered to be from a multivariate normal distribution if it satisfies Eqs. (4), (5), and (6) [1].

$$E(b_{1,p}) = p(p+2), \tag{4}$$

$$Var(b_{2,p}) = \frac{8p(p+2)}{n}, \tag{5}$$

$$z = \frac{b_{2,p} - p(p+2)}{\sqrt{\frac{8p(p+2)}{n}}}. \tag{6}$$

If the skewness or kurtosis values significantly deviate from the expected values, if $|z| > 1.96$ or p-value $< \alpha$ for α=0.05, then the null hypothesis of multivariate normality is rejected.

2.1.2 Correlation Coefficient

Karl Pearson stated that the correlation coefficient between two continuous variables can be calculated using Eq. (7) [40].

$$r = \frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{\sqrt{\left[n\sum_{i=1}^{n} X_i^{2} - \left(\sum_{i=1}^{n} X_i\right)^2\right]}\sqrt{\left[n\sum_{i=1}^{n} Y_i^{2} - \left(\sum_{i=1}^{n} Y_i\right)^2\right]}}. \tag{7}$$

Pearson correlation has a value between -1 to 1, meaning that if the value $|r|$ = 1, the relationship is very strong, and if $|r|$=0, then there is no correlation between the two variables. A negative coefficient indicates that the increase of one variable is followed by a decrease in the other variable. A positive coefficient indicates that the increase of one variable is followed by an increase in the other variable. The significance of the relationship can be tested using the *t-test* with the test hypothesis as follows.

H_0: $\rho = 0$ or there is no significant correlation between variables
H_1: $\rho \neq 0$ or there is significant correlation between variables

With t-test statistics can be calculated using Eq. (8).

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}, \tag{8}$$

where t is the Pearson correlation significance test statistic, and r is the Pearson correlation value, and n is the number of observations. The critical region for rejecting H_0 is when the $|t_{stat}|$>$t_{(\alpha,n-2)}$ or p-value $< \alpha$ [41].

2.1.3 Multivariate Exponentially Weighted Moving Average (MEWMA)

The MEWMA chart is an extension of the univariate EWMA chart for multivariate data. The MEWMA statistic at time t is defined as Eq. (9).

$$\boldsymbol{Z_t} = \lambda \boldsymbol{X_t} + (1-\lambda)\boldsymbol{Z_{t-1}}, \qquad \boldsymbol{Z_0} = \boldsymbol{0}, \tag{9}$$

where λ is a a smoothing constant that determines the weight assigned to the most recent observation relative to past observations. It lies within $0 < \lambda \leq 1$ where the smaller values of λ increase sensitivity to small and gradual shifts by placing greater weight on accumulated historical information [1]. The control statistic is calculated using Eq. (10).

$$T_t^2 = \boldsymbol{Z_t^T \Sigma_Z^{-1} Z_t}, \tag{10}$$

with variance using Eq. (11).

$$\boldsymbol{\Sigma_z} = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2t}\right]\boldsymbol{\Sigma}. \tag{11}$$

The process considered out of control if $T_t^2 > \chi^2_{p,1-\alpha}$ [1].

2.1.4 PCA-MEWMA

Principal Component Analysis (PCA) is a dimension reduction technique that transforms correlated variables into a smaller set of uncorrelated (orthogonal) principal components. Integrating PCA into MEWMA aims to reduce dimensionality without significant information loss, eliminate multicollinearity, and enhance anomaly detection efficiency [42]. If X is the matrix of data, PCA involves eigen decomposition using Eq. (12) and the transformation to PC space using Eq. (13).

$$\mathbf{\Sigma} = \mathbf{P\Lambda P}^T, \tag{12}$$

$$\boldsymbol{Y} = \boldsymbol{XP}. \tag{13}$$

The transformed data Y is used as input for the MEWMA chart. PCA-MEWMA is particularly effective when only some variables are correlated and experience minor shifts [43].

2.1.5 Bootstrap for Control Limit

In a conventional PCA-MEWMA chart, the control limit is usually obtained from the chi-square distribution. After reducing the data to k principal components, the MEWMA statistic is compared with the theoretical limit $\chi^2_{k,1-\alpha}$. This approach works only when the PCA scores follow multivariate normality. However, real manufacturing data often violate this assumption, causing the chi-square control limit to become inaccurate. The Bootstrap approach estimates empirical control limits without relying on theoretical distributions, which may be violated in real data. In MEWMA, Bootstrap is used to empirically derive the distribution of control statistics (T^2) and calculate more accurate control limits even though the assumption of the distribution is declined [44]. The average quantile from resampled data is calculated using Eq. (14).

$$\mathrm{CL_B} = \frac{1}{B}\sum_{b=1}^{B} Q_b\,, \tag{14}$$

$\mathrm{CL_B}$ is the bootstrap control limit after B resampling iterations and Q_b is the *b-th* quantile.

2.1.6 PCA-Bayesian MEWMA

The integration of PCA and Bayesian estimation within the MEWMA framework offers two main advantages. First, PCA is used to remove multicollinearity by transforming the original correlated variables into uncorrelated principal component (PC) scores. In this study, PCA is not used to reduce the number of variables. All components are retained so that no dimensional information is lost. The purpose of PCA here is only to stabilize the covariance structure before applying Bayesian MEWMA. After centering the data, the eigenvalues and eigenvectors are computed, and the original data matrix is projected onto the eigenvectors to obtain PC scores. These PC scores being uncorrelated linear combinations of the original variables serves as the inputs for the covariance estimated using the Bayesian loss function [33]. Conjugate prior of 3 Loss Function that is commonly used in Bayesian Control Chart are SELF (Self-starting Loss Function), MSELF (Modified Self-starting Loss Function), and KLF (Kullback-Leiber Loss Function). The estimator SELF, MSELF, and KLF are calculated using Eqs. (15), (16), and (17) [33].

$$L(\theta,\hat{\theta}) = \left(\theta-\hat{\theta}\right)^2,\ \hat{\theta}_{SELF} = E(\theta|x), \tag{15}$$

$$L(\theta,\hat{\theta}) = \left(\frac{1-\hat{\theta}}{\theta}\right)^2,\ \hat{\theta}_{MSELF} = \frac{E(\theta^{-1}|x)}{E(\theta^{-2}|x)}, \tag{16}$$

$$L(\theta,\hat{\theta}) = \left(\sqrt{\frac{\theta}{\hat{\theta}}}-\sqrt{\frac{\hat{\theta}}{\theta}}\right)^2,\ \hat{\theta}_{KLF} = \sqrt{\frac{E(\theta|x)}{E(\theta^{-1}|x)}}. \tag{17}$$

Although conjugate priors allow closed-form posterior distributions, the MEWMA chart requires only the posterior point estimates of the mean and covariance. The result of the mean and covariance matrix from the estimation is used to calculate $T^2_{t(Bayes)}$. The Bayesian MEWMA statistic at time t is calculated using Eq. (18).

$$T^2_{t(Bayes)} = \boldsymbol{Z}_t^T \boldsymbol{\Sigma}^{-1}_{\boldsymbol{Z}(\boldsymbol{Bayes})} \boldsymbol{Z}_t. \tag{18}$$

The process considered out of control if $T^2_{t(Bayes)} > \chi^2_{k,1-\alpha}$. This formulation connects the Bayesian estimators directly to the MEWMA statistic. The loss functions determine the posterior covariance used in $T^2_{t(Bayes)}$, and the MEWMA chart then monitors deviations based on this robust Bayesian covariance structure.

2.1.7 ARL, SDRL, MRL, FAR

One of the key metrics in evaluating control charts is the Average Run Length (ARL), which includes ARL_0 (in-control) and ARL_1 (out-of-control). ARL_0 represents the average number of observations before a false alarm occurs, while ARL_1 is the average number of observations before a process shift is successfully detected. A good MEWMA chart should have a high ARL_0 (to minimize false alarms) and a low ARL_1 (to detect anomalies quickly) [4]. The standard deviation (SDRL) and median (MRL) of ARL also can be used to estimate the out-of-control detection. False Alarm Rate (FAR) used to evaluate the chart detect the real out of control process or false out of control rate. The combination of PCA-Bootstrap and Bayesian has been shown to significantly reduce ARL_1 in T^2-Hotelling control charts. It is expected that similar results will be observed for MEWMA charts, even though this may come at the cost of a slightly lower ARL_0 [8]. Assuming the process mean shifts from μ_0 to $\mu_1 = \mu_0 + \delta$, the ARL and FAR can be calculated using Eqs. (19), (20), and (21).

$$\delta^2 = (\boldsymbol{\mu_1} - \boldsymbol{\mu_0})'\Sigma^{-1}(\boldsymbol{\mu_1} - \boldsymbol{\mu_0}), \tag{19}$$

$$ARL_1 \approx \frac{1}{P_{detect}(\delta)}, \tag{20}$$

$$FAR \approx \frac{1}{1 - P_{detect}(\delta)}. \tag{21}$$

2.1.8 Plastic Waste

Every manufacturing company undergoes a production process to meet customer demands. The production process involves transforming inputs (raw materials) into outputs (finished or semi-finished goods) by utilizing all of the company's available resources [45][46]. As business competition and customer demands increase, companies are required to manage their production processes effectively and efficiently [47]. One method that can be used is lean manufacturing, which aims to identify and eliminate wasteful activities that do not add value [48]. Ohno [47], in his book *Toyota Production System: Beyond Large Scale Production*, identified seven types of waste: waste of overproduction, waste of waiting, waste in transportation, waste of over-processing, waste of inventory, waste of motion, and waste of defect.

Recent studies show that a significant portion of global plastic waste is generated not only after use, but also during the manufacturing process due to defects, scrap, and unstable production conditions. Large-scale analyses of plastic-waste research emphasize that production-stage waste is a critical yet often overlooked contributor to overall plastic pollution, and reducing it is essential for improving efficiency and sustainability [49]. Reviews on the circular plastic economy further highlight that minimizing manufacturing waste through better process control supports resource conservation and reduces pressure on downstream recycling systems [50]. In addition, recent assessments of chemical recycling technologies indicate that manufacturing defects and off-spec products currently make up a substantial share of the feedstock for recycling, demonstrating the need to prevent waste at its source [51]. Therefore, monitoring plastic waste in products such as mulch film, irrigation hose, polybag rolls, and cut polybags using robust multivariate control charts is essential for early detection of process deviations and contributes directly to sustainable plastic production. The findings of this research are expected to provide insights for management to help reduce production waste. A waste reduction is expected to lead to increased company profits and a more effective and efficient production process [52][53].

2.2 Data Structure

This study uses simulated data designed to match the characteristics of the real plastic-waste case, which involves four waste variables from mulch film (X_1), irrigation hose (X_2), polybag roll (X_3), and cut-polybag (X_4) production. The data used in this study is simulated data that has been adjusted to match the characteristics of actual production waste percentage data, which falls within the 2–6% interval. The simulated data was generated in such a way that the 4 quality variables follow the structure shown in Table 1. Small sample sizes (n = 10, 20, 30) were chosen because real production batches are relatively limited, and Bayesian methods are particularly suitable for such conditions. Three correlation levels (ρ = 0.2, 0.6, 0.95) were included to represent low, moderate, and high multicollinearity, allowing us to evaluate whether PCA can effectively stabilize the covariance structure across different dependence levels. Outlier proportions of 5%, 10%, and 15% were introduced to test the robustness of the proposed control charts under varying degrees of contamination, which commonly occur in plastic manufacturing due to machine or material disruptions. The smoothing parameters (λ = 0.2, 0.5, 0.8) were selected to examine chart performance under different weighting behaviors such as strong, moderate, and weak emphasis on past observations that representing typical MEWMA settings in monitoring gradual versus sudden shifts. These scenarios were implemented on the 4 types of control charts (PCA-MEWMA Bootstrap, PCA- Bayesian MEWMA (SELF), PCA- Bayesian MEWMA (MSELF), PCA- Bayesian MEWMA (KLF)), resulting in a total of 324 scenarios. The simulation results will be evaluated using the ARL, ADRL, MRL, FAR metric as explained in subsection 2.1.7.

Table 1. Research Data Structure

n	ρ	Distribution
10	low (0.2)	Multivariate Normal
10	$medium$ (0.6)	Multivariate Normal
10	$high$ (0.95)	Multivariate Normal
20	low (0.2)	Multivariate Normal
20	$medium$ (0.6)	Multivariate Normal
20	$high$ (0.95)	Multivariate Normal
30	low (0.2)	Multivariate Normal
30	$medium$ (0.6)	Multivariate Normal
30	$high$ (0.95)	Multivariate Normal

2.3 Steps of the Analysis

The step of the analysis is conducted as follows:

1. Simulating multivariate plastic waste quality data from a normal multivariate distribution consisting of 10, 20, 30 sample sizes and 0.2, 0.6, 0.95 collinearities. Then add 3 level outlier contamination (5%, 10%, 15%) to the existing normal multivariate generated data.
2. Testing the multivariate normality assumption on each dataset using Mardia's multivariate normality test and correlation using Pearson statistics.
3. Implementing the MEWMA control chart on the original data using the recursive formulation.
4. Applying PCA transformation to reduce the collinearity of the data, extracting principal component scores for a cumulative explained variance threshold = 100%. MEWMA is then applied in this space.
5. Estimating robust covariance matrix using the Bayesian loss function on the PCA scores to obtain robust mean and covariance parameters for MEWMA monitoring.
6. Calculating MEWMA control limit using a nonparametric Bootstrap approach:
   a. Resampling the PCA-transformed data with replacement B=500 times
   b. Calculating MEWMA T^2 statistics for each bootstrap sample
   c. Estimating the upper control limit (UCL) as the average quantile (e.g., 99.73%) across all B bootstrap samples
   d. Calculating the mean of the UCL.
7. Combining PCA and Bayesian into a single robust monitoring scheme (PCA-Bayesian MEWMA), which performs:
   a. PCA transformation
   b. SELF, MSELF, KLF estimation
   c. MEWMA statistic computation
   d. Computing the chi-square based UCL with $k = 4$ degrees of freedom under the 3σ condition (16.25).
8. Simulating control chart performance for each scenario using the following parameters:
   a. Lambda values = {0.2, 0.5, 0.8}
   b. Methods: PCA-MEWMA Bootstrap, PCA- Bayesian MEWMA (SELF, MSELF, KLF).
9. Evaluating detection performance using ARL, SDRL, MRL, FAR for each scenario.
10. Comparing the effectiveness of MEWMA variations.
11. Applying the method in real plastic waste quality process.
12. Concluding the best performing method for monitoring the plastic waste quality process.

3. RESULTS AND DISCUSSION

3.1 Multivariate Normality Test

The initial examination to determine whether multivariate data can be used for constructing an MEWMA control chart is the multivariate normality test. Table 2 shows that out of the all datasets to be used, all meet the skewness criterion but do not satisfy the kurtosis criterion. This indicates that the data are symmetric but may contain outliers. These results indicate that most simulated datasets do not satisfy multivariate normality, especially for larger sample sizes and higher outlier proportions. This finding is important because traditional multivariate control charts rely heavily on the multivariate normality assumption for accurate estimation of covariance structures and control limits. When this assumption is violated, these charts may become less reliable, produce inflated false-alarm rates, or fail to detect true shifts.

Table 2. Multivariate Normality Test of Simulated Data

Datasets n=10	**Normality Test**	**Datasets** n=20	**Normality Test**	**Datasets** n=30	**Normality Test**
ρ=0.2, out=5%	Fail to reject H_0	ρ=0.2, out=5%	Reject H_0	ρ=0.2, out=5%	Reject H_0
ρ=0.2, out=10%	Fail to reject H_0	ρ=0.2, out=10%	Reject H_0	ρ=0.2, out=10%	Reject H_0
ρ=0.2, out=15%	Fail to reject H_0	ρ=0.2, out=15%	Reject H_0	ρ=0.2, out=15%	Reject H_0
ρ=0.6, out=5%	Fail to reject H_0	ρ=0.6, out=5%	Reject H_0	ρ=0.6, out=5%	Reject H_0
ρ=0.6, out=10%	Fail to reject H_0	ρ=0.6, out=10%	Reject H_0	ρ=0.6, out=10%	Reject H_0
ρ=0.6, out=15%	Fail to reject H_0	ρ=0.6, out=15%	Reject H_0	ρ=0.6, out=15%	Reject H_0
ρ=0.95, out=5%	Fail to reject H_0	ρ=0.95, out=5%	Reject H_0	ρ=0.95, out=5%	Reject H_0
ρ=0.95, out=10%	Fail to reject H_0	ρ=0.95, out=10%	Reject H_0	ρ=0.95, out=10%	Reject H_0
ρ=0.95, out=15%	Fail to reject H_0	ρ=0.95, out=15%	Reject H_0	ρ=0.95, out=15%	Reject H_0

3.2 Correlation Coefficients

Multivariate quality data are typically interrelated. This aligns with the concept that if there is no correlation between variables, then constructing a multivariate control chart is unnecessary—the process can be monitored using univariate control charts. However, if the correlation is too strong, the inverse of the covariance matrix—which is essential for calculating the T^2 statistic—cannot be computed. The results of the correlation test among the 4 variables are presented similarly to the simulation results, both for low (0.2), moderate (0.6), and high (0.95) collinearity. The correlation matrices for the low, medium, and high collinearity settings are shown as follows.

$$\boldsymbol{\rho}_{low} = \begin{pmatrix} 1 & 0.2 & 0.2 & 0.2 \\ 0.2 & 1 & 0.2 & 0.2 \\ 0.2 & 0.2 & 1 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1 \end{pmatrix},$$

$$\boldsymbol{\rho}_{medium} = \begin{pmatrix} 1 & 0.6 & 0.6 & 0.6 \\ 0.6 & 1 & 0.6 & 0.6 \\ 0.6 & 0.6 & 1 & 0.6 \\ 0.6 & 0.6 & 0.6 & 1 \end{pmatrix},$$

$$\boldsymbol{\rho}_{high} = \begin{pmatrix} 1 & 0.95 & 0.95 & 0.95 \\ 0.95 & 1 & 0.95 & 0.95 \\ 0.95 & 0.95 & 1 & 0.95 \\ 0.95 & 0.95 & 0.95 & 1 \end{pmatrix}.$$

These correlation settings illustrate that the variables are strongly correlated, especially under the medium and high collinearity scenarios. High correlation can make the covariance matrix nearly singular, which reduces the reliability of classical multivariate control charts that require matrix inversion. Therefore, PCA is applied to remove multicollinearity before constructing the MEWMA charts.

3.3 Simulation Study

The simulation study conducted in this research aims to compare four control charts: PCA-MEWMA using Bootstrap control limit, PCA-Bayesian MEWMA (SELF), PCA-Bayesian MEWMA (MSELF), PCA-

Bayesian MEWMA (KLF), under 3 sample sizes (10, 20, 30) with ρ= 0.2, 0.6, 0.95. Nine types of datasets were used, consisting 3 level outlier proportions (5%, 10%, 15%), 3 level of smoothing parameter λ (0.2, 0.5, 0.8). Table 2 summarizes the performance of the four control charts under all simulation scenario. In general, ARL values tend to increase with smaller sample sizes, greater collinearity, higher outlier proportions, and larger λ values.

Across all settings, the PCA-MEWMA with Bootstrap control limits consistently produces very small ARL values. However, this is accompanied by noticeably higher FAR, indicating that the chart frequently signals false alarms and is therefore unreliable for practical monitoring. In contrast, all PCA-Bayesian MEWMA charts demonstrate FAR values close to zero, showing strong robustness across the entire simulation space. Their relative performance varies depending on the data conditions. The SELF version often achieves smaller ARL values when sample sizes are large, correlations are low, or outlier contamination is minimal. The KLF version also performs competitively in several scenarios and occasionally produces the smallest ARL among the Bayesian charts, although its improvements appear irregular. The MSELF version performs well across all settings and tends to show more stable ARL, SDRL, and MRL values, particularly when the data exhibit moderate-to-high correlation or higher outlier proportions. Overall, none of the Bayesian loss functions dominates all scenarios. Instead, each performs best under different data characteristics, while all three consistently outperform the Bootstrap approach in terms of false alarm control. Fig. 1 provides an illustration of representative performance differences among the four charts.

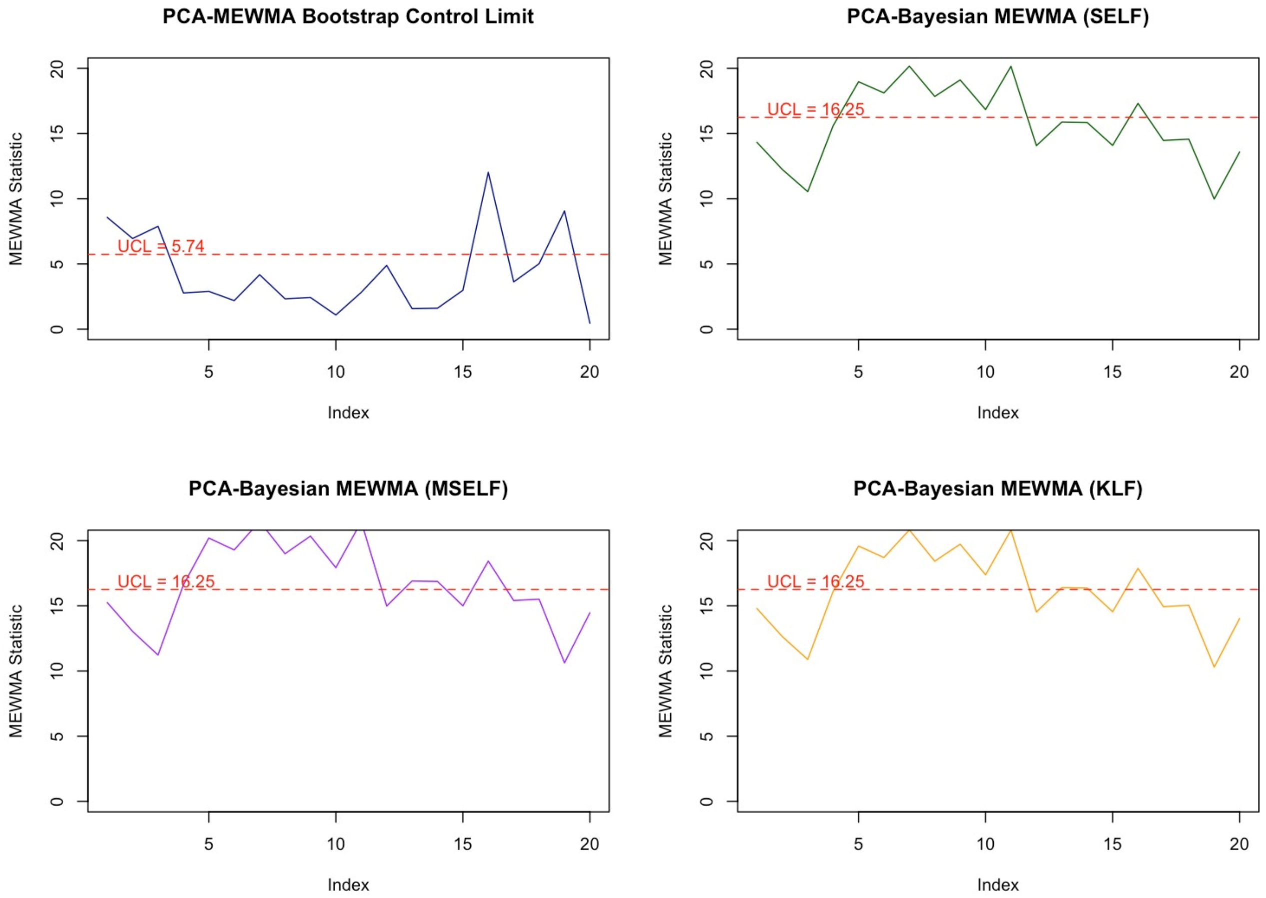

Figure 1. Control Charts Comparation Through n=20, ρ=0.95, Outlier=10%
(Source: Author's analysis using RStudio)

3.4 Real Case Study

The real case study was applied to monitor the quality detection process of plastic production waste (4 variables: % waste of mulch film production (X_1), % waste of hose production (X_2), % waste of polybag roll production (X_3), and % waste of cut-polybag production (X_4)). The results of the multivariate normality test of real case study shown in Table 3.

Table 3. Multivariate Normality Test of Real Case Study

Test	Statistic	p-value	Distribution
Skewness	99.093	<0.05	Not Multivariate
Kurtosis	3.495	<0.05	Normal

It is indicated that the plastic waste data were not multivariate normal. Table 4 shows the correlation test of the real case study of plastic waste quality. The correlation results show that the real plastic-waste variables exhibit a mix of low, medium, and medium-to-high correlations. For example, mulch waste and cut-polybag waste (ρ = 0.139), hose waste and cut-polybag show low correlation (ρ = -0.114). Mulch waste and hose waste (ρ = 0.212), mulch waste and polybag-roll waste (ρ = 0.261), polybag-roll waste and cut-polybag waste (ρ = 0.297) show medium correlation, while hose waste and polybag-roll waste present a strong medium-to-high correlation (ρ = 0.732). This combination reflects the same conditions used in the simulation study and confirms that the real data contain multicollinearity at different levels. Because classical MEWMA relies on a stable covariance matrix, such varying correlation strengths can reduce its reliability. Therefore, applying PCA before Bayesian MEWMA is appropriate in this case, as PCA removes multicollinearity and ensures that the monitoring chart remains stable and accurate for detecting process shifts.

Table 4. Correlation Test of Real Case Study

Variables	ρ	p-value	Correlation Test
X_1, X_2	0.212	0.101	Fail to reject H_0
X_1, X_3	0.261	**0.043**	Reject H_0
X_1, X_4	-0.114	0.382	Fail to reject H_0
X_2, X_3	0.732	**<0.05**	Reject H_0
X_2, X_4	0.139	0.282	Fail to reject H_0
X_3, X_4	0.297	**0.020**	Reject H_0

The analysis of real case plastic waste results are shown in Fig. 2. In the real case, the PCA-MEWMA with Bootstrap control limits signals an out-of-control condition at the very first observation, which is unlikely to reflect a true process shift and is consistent with the high false-alarm tendency observed in the simulation study. More importantly, this chart fails to detect the abnormal pattern around observation 28, even though this point shows clear signs of process instability.

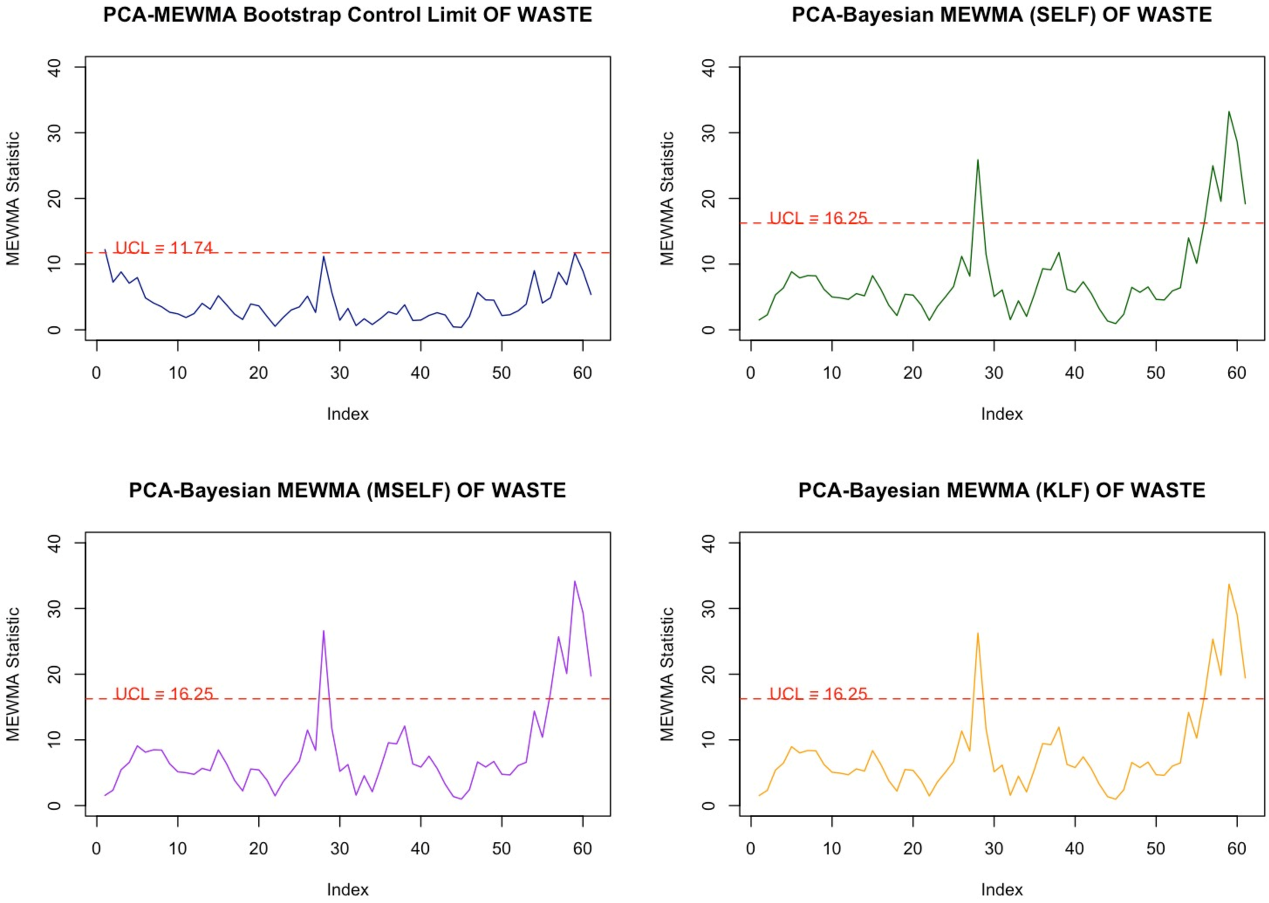

Figure 2. Control Charts Comparison of Plastic Waste Quality Data
(Source: Author's analysis using RStudio)

By contrast, all PCA-Bayesian MEWMA charts using the chi-square control limit are able to correctly identify observation 28 as out of control. Although the differences among the SELF, MSELF, and KLF loss functions are small—as also seen in the simulation results—they all remain more stable and sensitive than the conventional PCA-MEWMA. This finding supports that the Bayesian approach provides more reliable detection performance in practice, especially when the data contain moderate correlation and occasional irregularities.

3.5 Discussion

This study evaluates four multivariate control charts: PCA-MEWMA with Bootstrap limits and PCA-Bayesian MEWMA (SELF, MSELF, KLF) under different sample sizes, correlations, outlier levels, and smoothing parameters. Bootstrap MEWMA showed very low ARL values (0.1–1), suggesting rapid detection, but it also generated extremely high false alarms. Its performance was insensitive to sample size, correlation, outliers, or λ, signaling almost constantly regardless of the process state. Therefore, Bootstrap MEWMA is unsuitable for practical monitoring, as it cannot distinguish real shifts from normal variation.

Table 5. ARL, SRDL, MRL, FAR of The Simulation Study

n	ρ	out	λ	UCL	Bootstrap				UCL	Bayesian (SELF)				Bayesian (MSLF)				Bayesian (KLF)			
					ARL	SDRL	MDRL	FAR		ARL	SDRL	MDRL	FAR	ARL	SDRL	MDRL	FAR	ARL	SDRL	MDRL	FAR
10	low	5%	0.2	2.61	1.5	0.85	1	0.20	16.25	5.7	0.95	6	0.00	5.7	0.95	6	0.00	5.1	1.45	6	0.00
10	low	10%	0.2	2.63	1.1	0.32	1	0.10	16.25	2.7	0.48	3	0.00	2.5	0.53	2.5	0.00	2.7	0.48	3	0.00
10	low	15%	0.2	2.97	1.1	0.32	1	0.10	16.25	4.2	1.55	4.5	0.00	3.8	0.42	4	0.00	3.9	0.57	4	0.00
10	medium	5%	0.2	2.47	1.1	0.32	1	0.10	16.25	4.9	0.32	5	0.00	5	0	5	0.00	5	0	5	0.00
10	medium	10%	0.2	2.52	1.1	0.32	1	0.10	16.25	3	0	3	0.00	3	0	3	0.00	3	0	3	0.00
10	medium	15%	0.2	2.73	1.2	0.42	1	0.10	16.25	2	0	2	0.00	2	0	2	0.00	2	0	2	0.00
10	high	5%	0.2	3.26	1.1	0.32	1	0.10	16.25	3	0	3	0.00	2.9	0.32	3	0.00	3.1	0.32	3	0.00
10	high	10%	0.2	3.42	1	0	1	0.10	16.25	2.7	0.48	3	0.00	2.7	0.48	3	0.00	2.5	0.53	2.5	0.00
10	high	15%	0.2	3.04	1	0	1	0.10	16.25	2	0	2	0.00	2	0	2	0.00	2	0	2	0.00
20	low	5%	0.2	4.53	1.6	0.97	1	0.05	16.25	4	0	4	0.00	4	0	4	0.00	4	0	4	0.00
20	low	10%	0.2	5.38	1.7	0.67	2	0.10	16.25	3.7	1.16	3	0.00	3	0	3	0.00	2.9	0.32	3	0.00
20	low	15%	0.2	4.19	1.4	0.52	1	0.05	16.25	5.5	1.65	6	0.00	5.3	1.83	5.5	0.00	6	1.15	6.5	0.00
20	medium	5%	0.2	5.02	1.7	0.82	1.5	0.05	16.25	4	0	4	0.00	3.7	0.48	4	0.00	4	0	4	0.00
20	medium	10%	0.2	5.35	1.5	0.71	1	0.05	16.25	2.8	0.42	3	0.00	2.9	0.57	3	0.00	3.2	0.42	3	0.00
20	medium	15%	0.2	6.55	1.6	0.84	1	0.05	16.25	5	0	5	0.00	4.6	0.84	5	0.00	4.8	0.63	5	0.00
20	high	5%	0.2	5.03	1	0	1	0.05	16.25	2	0	2	0.00	2	0	2	0.00	2	0	2	0.00
20	high	10%	0.2	5.74	1.1	0.32	1	0.15	16.25	2.8	0.42	3	0.00	2.7	0.48	3	0.00	2.5	0.53	2.5	0.00
20	high	15%	0.2	5.8	1.1	0.32	1	0.05	16.25	3.4	0.52	3	0.00	3.1	0.57	3	0.00	3.5	0.53	3.5	0.00
30	low	5%	0.2	12.4	4	1.41	3	0.03	16.25	6	0	6	0.03	6	0	6	0.03	6	0	6	0.03
30	low	10%	0.2	7.98	2.9	1.73	3	0.03	16.25	6.5	0.53	6.5	0.00	6.4	0.52	6	0.00	6.4	0.52	6	0.00
30	low	15%	0.2	13.52	4.9	2.56	4	0.03	16.25	6.7	0.67	7	0.03	6.4	0.7	6.5	0.03	6.5	0.53	6.5	0.03
30	medium	5%	0.2	4.26	1.6	1.07	1	0.03	16.25	3.6	0.97	3	0.00	3	0	3	0.00	3	0	3	0.00
30	medium	10%	0.2	8.49	2.5	1.08	2.5	0.03	16.25	5.2	0.63	5	0.00	4.5	1.08	4.5	0.00	5	0.82	5	0.00
30	medium	15%	0.2	5.82	2	0.94	2	0.07	16.25	5	0	5	0.00	5	0	5	0.00	5	0	5	0.00
30	high	5%	0.2	8.78	1.5	0.97	1	0.03	16.25	2	0	2	0.00	2.1	0.32	2	0.00	2	0	2	0.00
30	high	10%	0.2	7.22	1.1	0.32	1	0.03	16.25	2.9	0.32	3	0.00	2.8	0.42	3	0.00	3	0	3	0.00
30	high	15%	0.2	7.99	1.5	0.53	1.5	0.07	16.25	3	0	3	0.00	2.9	0.32	3	0.00	2.9	0.32	3	0.00
10	low	5%	0.5	3.58	1.2	0.42	1	0.20	16.25	3.5	1.35	3	0.00	3.1	1.1	3	0.00	2.9	0.32	3	0.00
10	low	10%	0.5	4.87	1	0	1	0.10	16.25	2	0	2	0.00	2	0	2	0.00	2	0	2	0.00
10	low	15%	0.5	5.25	1.2	0.42	1	0.10	16.25	4.5	2.64	4.5	0.00	2	0	2	0.00	3.5	2.42	2	0.00
10	medium	5%	0.5	4.35	1.2	0.42	1	0.10	16.25	4.9	0.32	5	0.00	4.4	1.26	5	0.00	4.7	0.95	5	0.00
10	medium	10%	0.5	4.46	1	0	1	0.20	16.25	2.5	0.85	3	0.00	1.8	1.03	1	0.00	2.2	0.92	2.5	0.00
10	medium	15%	0.5	5.48	1	0	1	0.10	16.25	2	0	2	0.00	2	0	2	0.00	2	0	2	0.00
10	high	5%	0.5	5.15	1.1	0.32	1	0.10	16.25	2.3	0.48	2	0.00	2.1	0.32	2	0.00	2	0	2	0.00
10	high	10%	0.5	5.43	1	0	1	0.20	16.25	2	0	2	0.00	2	0	2	0.00	2	0	2	0.00
10	high	15%	0.5	5.02	1	0	1	0.10	16.25	1	0	1	0.00	1	0	1	0.00	1.1	0.32	1	0.00
20	low	5%	0.5	8.92	1.7	0.67	2	0.05	16.25	4	0	4	0.00	3.6	0.84	4	0.05	4	0	4	0.00
20	low	10%	0.5	10.5	2.3	1.16	2	0.05	16.25	2	0	2	0.05	2	0	2	0.05	2	0	2	0.05
20	low	15%	0.5	9.62	1.3	0.48	1	0.05	16.25	3	2.11	2	0.00	2.5	1.58	2	0.05	2	0	2	0.00
20	medium	5%	0.5	9.42	1.3	0.48	1	0.05	16.25	2.2	0.63	2	0.05	2.2	0.63	2	0.05	2.6	0.97	2	0.05
20	medium	10%	0.5	9.96	1.4	0.7	1	0.10	16.25	2	0	2	0.05	2	0	2	0.05	2	0	2	0.05
20	medium	15%	0.5	11.16	1.9	0.99	2	0.05	16.25	5	0	5	0.05	4.1	1.45	5	0.05	4.1	1.45	5	0.05
20	high	5%	0.5	8.86	1	0	1	0.05	16.25	1.9	0.32	2	0.00	1.8	0.42	2	0.05	1.9	0.32	2	0.00

n	ρ	out	λ	UCL	Bootstrap				UCL	Bayesian (SELF)				Bayesian (MSLF)				Bayesian (KLF)			
					ARL	SDRL	MDRL	FAR		ARL	SDRL	MDRL	FAR	ARL	SDRL	MDRL	FAR	ARL	SDRL	MDRL	FAR
20	high	10%	0.5	10.21	1.1	0.32	1	0.05	16.25	2.2	0.42	2	0.05	1.9	0.32	2	0.05	2	0.67	2	0.05
20	high	15%	0.5	12.17	1	0	1	0.10	16.25	2.8	0.92	3	0.00	2.4	0.52	2	0.05	2.5	0.53	2.5	0.05
30	low	5%	0.5	17.78	5.2	1.48	5.5	0.03	16.25	6	0	6	0.07	6	0	6	0.07	6	0	6	0.07
30	low	10%	0.5	14.99	3.2	2.49	2.5	0.03	16.25	7.5	1.35	7	0.10	6.9	0.32	7	0.10	7.2	1.03	7	0.10
30	low	15%	0.5	18.04	2.7	1.7	2.5	0.03	16.25	7.7	0.95	8	0.07	7.8	0.42	8	0.07	8	0	8	0.07
30	medium	5%	0.5	9.74	2	0.94	2	0.07	16.25	3	0	3	0.00	2.9	0.32	3	0.00	3	0	3	0.00
30	medium	10%	0.5	13.47	1.7	0.67	2	0.03	16.25	3.4	1.26	3	0.03	3.2	1.03	3	0.03	3.4	1.26	3	0.03
30	medium	15%	0.5	11.65	1.8	0.92	2	0.03	16.25	3.5	1.08	3	0.00	3.2	0.63	3	0.03	3.4	1.17	3	0.00
30	high	5%	0.5	17.16	1.2	0.42	1	0.07	16.25	2	0	2	0.07	2	0	2	0.07	2	0	2	0.07
30	high	10%	0.5	16	1.1	0.32	1	0.07	16.25	2.5	0.71	2	0.10	2	0.47	2	0.10	2.4	0.7	2.5	0.10
30	high	15%	0.5	16.53	1.4	0.52	1	0.03	16.25	2.4	0.52	2	0.10	2.3	0.48	2	0.10	2.3	0.48	2	0.10
10	low	5%	0.8	4.43	1.2	0.42	1	0.10	16.25	4	1.76	3	0.00	2.6	0.52	3	0.00	4.4	1.71	4.5	0.00
10	low	10%	0.8	6.19	1.1	0.32	1	0.10	16.25	2.1	0.32	2	0.00	2	0	2	0.10	2	0	2	0.00
10	low	15%	0.8	6.92	1.1	0.32	1	0.20	16.25	4.5	2.64	4.5	0.00	3.9	2.69	2	0.00	3.4	2.5	2	0.00
10	medium	5%	0.8	5.48	1.2	0.42	1	0.10	16.25	4.7	0.95	5	0.00	3.8	1.55	5	0.00	4.1	1.45	5	0.00
10	medium	10%	0.8	5.9	1.1	0.32	1	0.20	16.25	1.4	0.84	1	0.00	1.2	0.63	1	0.00	1	0	1	0.00
10	medium	15%	0.8	6.88	1.1	0.32	1	0.20	16.25	1.4	0.52	1	0.00	1.5	0.53	1.5	0.00	1.8	0.42	2	0.00
10	high	5%	0.8	5.98	1	0	1	0.10	16.25	2.5	1.08	2	0.00	1.9	0.32	2	0.00	2.1	0.74	2	0.00
10	high	10%	0.8	6.68	1	0	1	0.10	16.25	2	0	2	0.00	2	0	2	0.10	2	0	2	0.10
10	high	15%	0.8	6.22	1	0	1	0.10	16.25	1	0	1	0.00	1	0	1	0.00	1	0	1	0.00
20	low	5%	0.8	11.53	2.2	1.62	1.5	0.05	16.25	4	0	4	0.05	4	0	4	0.05	4	0	4	0.05
20	low	10%	0.8	12.81	2.1	1.45	1.5	0.05	16.25	2.3	0.95	2	0.10	2.3	0.95	2	0.10	2	0	2	0.10
20	low	15%	0.8	12.54	2.1	1.97	1	0.10	16.25	4.5	2.64	4.5	0.15	4.5	2.64	4.5	0.15	5.5	2.42	7	0.15
20	medium	5%	0.8	11.87	1.4	0.7	1	0.05	16.25	2.8	1.03	2	0.05	2.7	1.16	2	0.05	3	1.05	3	0.05
20	medium	10%	0.8	12.99	1.2	0.42	1	0.05	16.25	2	0	2	0.10	2	0	2	0.10	2	0	2	0.10
20	medium	15%	0.8	12.5	1.3	0.67	1	0.05	16.25	4.6	1.26	5	0.15	4.3	1.49	5	0.15	4.6	1.26	5	0.15
20	high	5%	0.8	11.64	1	0	1	0.05	16.25	1.6	0.52	2	0.05	1.2	0.42	1	0.05	1.4	0.52	1	0.05
20	high	10%	0.8	12.13	1.1	0.32	1	0.05	16.25	2.6	1.43	3	0.05	2.3	1.25	2.5	0.05	2.1	1.1	2	0.05
20	high	15%	0.8	15.61	1.3	0.48	1	0.10	16.25	3.1	1.1	3	0.10	3	0.82	3	0.10	3.3	0.95	3	0.10
30	low	5%	0.8	20.23	4.5	2.72	4	0.03	16.25	6.4	1.26	6	0.07	6	0	6	0.07	6.4	1.26	6	0.07
30	low	10%	0.8	18.74	4.6	3.06	3	0.03	16.25	10.1	0.32	10	0.10	10.1	0.32	10	0.10	10	0	10	0.10
30	low	15%	0.8	18.95	2.4	1.96	1.5	0.03	16.25	6.8	1.55	8	0.10	5.9	1.45	5	0.13	7.1	1.45	8	0.13
30	medium	5%	0.8	14.93	1.9	0.99	2	0.03	16.25	2.8	0.42	3	0.07	2.8	0.42	3	0.07	2.8	0.42	3	0.07
30	medium	10%	0.8	17.36	2.4	1.17	2	0.03	16.25	5.6	3.1	8	0.10	3.2	2.3	2	0.10	6.8	2.53	8	0.10
30	medium	15%	0.8	15.49	3.2	3.39	2	0.03	16.25	3.8	1.55	5	0.07	2.6	1.26	2	0.07	2.9	1.45	2	0.07
30	high	5%	0.8	22.17	1.3	0.48	1	0.07	16.25	1.9	0.32	2	0.07	1.8	0.42	2	0.07	1.9	0.32	2	0.07
30	high	10%	0.8	21.54	1.4	0.52	1	0.07	16.25	3.1	1.2	3.5	0.10	2.5	0.97	3	0.10	2.3	1.34	2.5	0.10
30	high	15%	0.8	20.74	1.9	1.37	1	0.03	16.25	3.3	1.06	3	0.13	3.4	1.35	3.5	0.13	3.5	1.18	3	0.13

In contrast, the PCA-Bayesian MEWMA charts demonstrate robust and reliable performance. SELF performs particularly well under clean data conditions—low outlier contamination, low correlation, larger sample sizes, and smaller λ values—producing low ARL values (1.5–6) while maintaining stable SDRL and MRL metrics and keeping FAR near zero, making it suitable for relatively well-behaved processes. MSELF, while not always the fastest, exhibits superior stability under more challenging conditions such as medium-to-high correlation, moderate-to-high outlier contamination, and larger smoothing parameters, with consistent ARL, SDRL, MRL, and FAR values across scenarios, suggesting that its modified loss function effectively adapts to complex posterior distributions. KLF shares some performance characteristics with SELF, occasionally achieving the smallest ARL in specific cases, but its advantage is situational rather than systematic. Applied to a real plastic waste process, PCA-Bayesian MEWMA charts, particularly MSELF, effectively detected shifts in waste percentages for mulch film, hose irrigation, polybag-roll, and cut-polybag production, supporting proactive interventions to reduce waste and improve quality. Theoretically, these findings reinforce the value of integrating dimensionality reduction (PCA) with Bayesian MEWMA charts for high-dimensional, correlated multivariate processes and highlight the importance of adaptive Bayesian loss functions for robust process monitoring, thereby contributing to the development of a more generalizable framework for multivariate statistical process control that bridges Bayesian estimation theory with practical industrial monitoring.

4. CONCLUSION

This study developed and evaluated four multivariate control charts—PCA-MEWMA with Bootstrap limits and PCA-Bayesian MEWMA (SELF, MSELF, KLF)—under varying sample sizes, correlations, outlier levels, and smoothing parameters. Bootstrap MEWMA, despite rapid detection, generated extremely high false alarms and showed no sensitivity to process conditions, making it unsuitable for practical monitoring. In contrast, PCA-Bayesian MEWMA charts performed robustly: SELF excelled under clean data conditions, while MSELF provided stable and reliable detection even under high correlation, moderate-to-high outlier contamination, and larger smoothing parameters. Applied to a real recycled plastic waste process, MSELF effectively detected shifts in waste percentages for mulch film, hose irrigation, polybag-roll, and cut-polybag production, supporting proactive interventions to reduce waste and improve quality. These findings highlight the theoretical and practical benefits of integrating PCA with Bayesian MEWMA charts, demonstrating the importance of adaptive Bayesian loss functions for robust multivariate process monitoring. Future research could explore the performance of these control charts under varying levels of collinearity within a single covariance matrix, various levels of smoothing parameter, develop alternative Bayesian priors or loss functions to improve adaptability, and investigate posterior predictive-based control limits for enhanced robustness. Additionally, further study could examine interactions between dimensionality reduction and control limit construction to optimize monitoring for high-dimensional processes.

Author Contributions

Agis Wahyu Lestari: Conceptualization, Methodology, Writing-Original Draft, Software, Formal Analysis, Data Curation, Resources. Ani Budi Astuti: Supervision, Draft Preparation. Suci Astutik: Supervision, Validation. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare that they have no conflict of interest, either direct or indirect, that could have influenced the outcome initial value.

Declaration of Generative AI and AI-assisted Technologies

ChatGPT was used only to improve the readability and grammatical structure of the manuscript. No AI tool was used to generate or alter the research data, methodology, results, or interpretations. All content was verified by the authors for accuracy and consistency with the study.

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