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A COMPOUND CYCLIC POISSON STOCHASTIC MODEL FOR PREMIUM DETERMINATION IN WEATHER INDEXED AGRICULTURAL INSURANCE: CASE STUDY IN SOUTH SULAWESI, INDONESIA

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ABSTRACT

The agricultural sector in developing countries is highly susceptible to significant losses due to weather variability and seasonal risks. Existing premium calculation methods often rely on homogeneous risk assumptions, which fail to account for claim patterns that are highly dependent on agricultural seasonality. This limitation often leads to mispriced premiums, deterring farmer participation in crucial insurance schemes. To address this, our study proposes and analyzes a compound cyclic Poisson model designed to estimate agricultural insurance premiums under weather-dependent shocks. The model explicitly integrates seasonal variations in claim frequency and severity, aligning premium calculation with actual agricultural risk profiles. Our approach uses a quantitative, stochastic modeling method based on a compound cyclic Poisson process, which effectively captures cyclical claim patterns that correspond with planting and harvesting seasons. As a case study, the research was conducted in South Sulawesi province, an ideal representation of an agrarian region with high weather risk intensity. The weather index used in this study combines rainfall and temperature indicators to better represent climate-induced risks. Through simulations, we found that the insurance premium, derived from our model, ranges from IDR 36,796 during low weather index conditions to IDR 328,713 during high weather index conditions, approximately 20-80% below the fixed AUTP market premium of IDR 180,000. This flexible pricing range allows farmers to choose the most suitable policy for their risk level and empowers insurance companies to set fair and financially sustainable premiums, ultimately encouraging broader participation in agricultural insurance. The originality of this study lies in the integration of a compound cyclic Poisson process to model seasonal claim dynamics in agricultural insurance. This approach contributes to the literature by providing a stochastic framework that bridges theoretical modelling and practical premium calibration under real world weather variability.

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1. INTRODUCTION

Agricultural sectors in developing countries are highly vulnerable to substantial losses caused by weather variability and seasonal risks, including droughts, floods, and pest outbreaks, which directly affect crop yields and farmers' incomes [1]. The unpredictable nature of weather exacerbates economic uncertainty, making an accurately priced agricultural insurance system essential for risk mitigation [2]. However, traditional premium setting methods often fail to capture recurring seasonal risk patterns, leading to premiums that do not fully reflect the actual risk exposure [3]. Over the past two decades, literature studies on agricultural insurance have highlighted significant progress in developing innovative approaches to enhance farmers' resilience. These studies demonstrate that risk management and climate change adaptation have become central themes in agricultural research [4], reflecting the increasing urgency to address climate-induced threats to food security and rural livelihoods.

Most existing premium calculation methods assume homogeneous risk, where the claim intensity is considered constant over time [5]. In the context of agricultural risk management in Indonesia, the Asuransi Usaha Tani Padi (AUTP) program, launched in 2015 as part of the government's farmer protection policy, adopts a uniform coverage and premium structure that does not account for spatial or seasonal variation in risk [6]. While the fixed coverage, flat rate premium structure of AUTP ensures simplicity and broad accessibility, it contrasts sharply with variable premium schemes and index-based insurance approaches widely adopted in other agricultural insurance programs globally. In a variable premium system, the premium is adjusted based on quantifiable risk factors such as historical yield variability, local climatic conditions, and hazard frequency, ensuring that farmers in high-risk areas pay higher premiums while those in low-risk zones pay less [7]. This tailoring of premiums can improve financial sustainability for insurers while providing fairer cost distribution among farmers.

In the agricultural context, claim intensity is strongly influenced by planting and harvesting cycles [8]. Recent studies emphasize that stochastic models are powerful tools for capturing uncertainty in dynamic systems [9]. This insight is highly relevant to agricultural insurance, where weather risks inherently follow stochastic and seasonal patterns. Consequently, adopting a stochastic modeling approach that incorporates seasonal fluctuations is essential to ensure that premiums are not only accurate in reflecting true risk exposure but also equitable for farmers [10]. The compound Poisson model has long been employed in actuarial science to jointly model claim frequency and claim severity [11]. This flexibility makes the compound Poisson framework highly suitable for agricultural insurance, where losses are heterogeneous and vary considerably across seasons and locations. The main limitations of homogeneous Poisson models are their inability to represent claim patterns that are highly dependent on agricultural seasonality [8]. This limitation often results in mispriced premiums, which in turn can discourage farmer participants in agricultural insurance schemes [1]. Without appropriately reflecting seasonal variability, premium schemes may either overcharge low-risk farmers or underfund the pool for high-risk periods. Further developments have introduced the cyclic Poisson process, in which claim intensity varies as a cyclic function of time, such as annual or seasonal cycles [12]. This extension is particularly relevant for agricultural insurance, as crop loss risks typically peak during certain stages of the planting and harvesting cycle. By adopting a nonhomogeneous Poisson process with cyclic intensity, it is possible to construct a more realistic risk representation compared to homogeneous assumptions [13].

Several recent studies have sought to integrate the compound and cyclic Poisson structures to capture seasonal dependencies in claim data [14]. Such integration enables more precise premium estimation by jointly considering seasonal variation in claim frequency and the variability in claim amounts [15]. Building on this, the cyclic Poisson process introduces time-varying claim intensity following seasonal patterns highly relevant for agriculture, where loss risks peak during certain crop stages. Combining compound and cyclic Poisson structures enables more accurate premium estimation by accounting for both seasonal frequency variation and claim severity variability. Despite extensive applications in actuarial science, particularly in life [16], health [17], motor [18], and property insurance [5], few studies have integrated compound cyclic Poisson models into agricultural premium pricing frameworks. This gap presents an opportunity for methodological innovation tailored to agricultural risk, especially under increasing climate variability. By integrating weather index parameters such as weather anomalies and planting and harvesting cycles into the intensity function, premiums can better reflect localized and seasonal risks, improving actuarial fairness and financial sustainability.

The purpose of this study is to formulate and analyze a compound cyclic Poisson model for estimating agricultural insurance premiums under weather-dependent shocks. The model explicitly incorporates seasonal variation in claim frequency and severity, aligning premium calculation with actual agricultural risk profiles. The key contributions of this study are twofold. First, it introduces a stochastic framework that captures the temporal dynamics of weather-related agricultural risk with greater fidelity than traditional models [19]. Second, it demonstrates the implications of this approach for equitable premium setting in agricultural insurance, with potential benefits for insurers, policymakers, and farming communities in developing regions. Unlike the standard Poisson model, which assumes a constant event intensity over time, the compound cyclic Poisson model accounts for the periodic nature of paddy cultivation, where planting and harvesting cycles coincide with seasonal weather patterns, thereby providing a more realistic representation of loss occurrence and improving premium accuracy. The results are expected to inform the design of sustainable risk transfer mechanisms that support agricultural resilience and rural livelihoods.

2. RESEARCH METHODS

2.1 Research Data and Variables

This study utilizes two primary categories of data, namely weather index data and production loss data. The climate-related data are secondary in nature, obtained from the Meteorology, Climatology, and Geophysical Agency (BMKG) and the European Center for Medium-Range Weather Forecasts (ECMWF). The dataset covers climate conditions in South Sulawesi Province recorded over the past ten years. The data are essential for qualifying seasonal weather variability and identifying anomalies in climate patterns that significantly affect agricultural yield. Accurate weather indices derived from credible meteorological sources are crucial for stochastic premium estimation models [20]. Paddy production data are sourced from the BPS-Statistics Indonesia and complemented by field surveys to estimate the financial impact of crop failure. These estimates are critical for calibrating the model's loss parameters. Reliable loss measurement is a prerequisite for deriving stable and unbiased premium estimators in index-based agricultural insurance [21].

The compound cyclic Poisson stochastic model is employed to capture the periodic and random nature of agricultural losses caused by seasonal weather variability. In this model, the occurrence of loss events is assumed to follow a Poisson process whose intensity varies cyclically with time, reflecting seasonal climatic fluctuations. The aggregate loss X_i in each period is modeled as the sum of random claim amounts associated with each weather event, expressed as Eq. (1).

$$X_i = \sum_{i=1}^{N_i} Y_t \,, \tag{1}$$

where N_i follows a periodic Poisson distribution and Y_t represents the individual loss magnitude. The expected value and variance of X_i are then derived to estimate the fair premium. The variables employed in this study are presented in Table 1, which outlines the key parameter and their respective descriptions.

Table 1. Variable Descriptions

Variable		Unit
$\boldsymbol{k_{(t,\tau)}}$	Weather index	mm/day
$\boldsymbol{\tau}$	Paddy crop cycle duration	day
$\boldsymbol{t_r}$	Beginning of the cropping season	day
$\boldsymbol{X_i}$	Extent of the loss	IDR/ton
$\boldsymbol{\alpha}$	Safety factors	%

The methodological process proceeds in several key stages:

1. Data preprocessing, where climate and yield data are cleaned, standardized, and synchronized over the same temporal range.

2. Weather index construction, which involves deriving rainfall, temperature, and sunshine duration indices to quantify climate risk exposure.
3. Model calibration, where parameters such as claim intensity $\widehat{\boldsymbol{\Lambda}}_{\boldsymbol{n}}(\boldsymbol{t_r})$, complementary risk intensity $\widehat{\boldsymbol{\Lambda}}_{\boldsymbol{n}}{}^{\boldsymbol{c}}(\boldsymbol{t_r})$, and mean loss per claim $\widehat{\boldsymbol{\mu}}_{\boldsymbol{n}}$ are estimated using historical data.
4. Simulation of expected claim and variance, where the final index-based premium is computed.

2.2 Model Formulation

The stochastic model employed in this research is rooted in the theoretical foundation established by studies on the asymptotic distribution of estimators for expectation and variance functions within the compound cyclic Poisson process [22], [23]. Building on this foundation, the model is further adapted to capture seasonal agriculture risks. The model parameters are defined as the expected value of claims $\widehat{\psi}_n(t)$, which represents the estimated average loss that farmers might experience, while the variance of claims $\widehat{V}_n(t)$, describes the uncertainty of this estimation due to fluctuations in weather and the volatility of unhusked dry grain prices. The expected value of claims is analyzed using Eq. (2).

$$\widehat{\psi}_n(t) = \left((1 + k_{t,\tau})\widehat{\Lambda}_n(t_r) + k_{t,\tau}\widehat{\Lambda}_n{}^c(t_r)\right)\hat{\mu}_n, \tag{2}$$

whereas the variance of claims is analyzed using Eq. (3).

$$\widehat{V}_n(t) = \left((1 + k_{t,\tau})\widehat{\Lambda}_n(t_r) + k_{t,\tau}\widehat{\Lambda}_n{}^c(t_r)\right)\hat{\mu}_{2,n}, \tag{3}$$

where

$$\widehat{\Lambda}_n(t_r) = \frac{1}{k_{n,\tau}} \sum_{k=0}^{k_{n,\tau}-1} N([k\tau,\ k\tau + t_r])\,, \tag{4}$$

$$\widehat{\Lambda}_n{}^c(t_r) = \frac{1}{k_{n,\tau}} \sum_{k=0}^{k_{n,\tau}-1} N([k\tau + t_r, k\tau + \tau])\,, \tag{5}$$

$$\hat{\mu}_n = \frac{1}{N([0,n])} \sum_{i=1}^{N([0,n])} X_i\,, \tag{6}$$

$$\hat{\mu}_{2,n} = \frac{1}{N([0,n])} \sum_{i=1}^{N([0,n])} X_i{}^2\,. \tag{7}$$

Here, Eq. (4) represents the primary risk claim intensity, while Eq. (5) represents the complementary risk claim intensity. Then Eq. (6) represents the average farmer's loss, while Eq. (7) represents the variance of the farmer's loss. This structure draws from the premium modeling approach that integrates periodicity into compound Poisson models, yielding a more accurate representation of seasonal claim patterns [24]. Furthermore, incorporating weather index parameters directly into premium calculations improves responsiveness to extreme climate events [25]. Following the estimation of claim intensities and loss components described in Eqs. (4)-(7), additional equations are introduced to capture the correlation structure, the relationship between weather indices and potential losses, the computation of cumulative weather variables, and the final premium estimation.

To assess the influence of weather on paddy yields, Pearson correlation coefficients were computed between annual paddy production (P) band three primary climate variables: temperature (T), rainfall (R), and sunshine duration (S). The formula is expressed as Eq. (8).

$$\rho_{X,Y} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}. \tag{8}$$

The results indicate that:

$\rho_{T,Y}$ = correlation between annual paddy production (q/ha) and max temperature $(°C)$,
$\rho_{R,Y}$ = correlation between annual paddy production (q/ha) and average rainfall during growing season (mm),
$\rho_{S,Y}$ = correlation between annual paddy production (q/ha) and mean sunshine duration (hours/day).

These relationships provide a quantitative basis for designing index-based insurance models.

The simulation of index-based insurance premiums in this study begins with the weather index data $k_{t,\tau}$, which serves as the primary driver of potential losses. To establish a realistic link between climate conditions and financial damages, we assume that higher values of the weather index correspond to larger potential losses. The relationship is expressed in a linear form as Eq. (9).

$$X_i = \beta \cdot k_{t,\tau} + \varepsilon_i, \tag{9}$$

where $\beta > 0$ is a scaling parameter that translates the weather index into economic losses and $\varepsilon_i \sim N(0,5)$ represents normally distributed random noise capturing natural variability. For the baseline simulation, we set $\beta = 10$, meaning that every one unit increase in the weather index adds, on average, 10 units of loss. The selection of $\beta = 10$ is based on an empirical calibration approach rather than an arbitrary assumption. This value was determined by considering the proportional relationship between variation in the weather index $k_{t,\tau}$ and the average actual loss experienced by farmers X_i. Specifically, since the weather index in the dataset ranges approximately from 0 to 200 and the maximum observed or assumed economic loss reaches around IDR 6 million per hectare, the ratio of loss change to weather index change is roughly consistent with the chosen scaling factor. Thus, setting $\beta = 10$ ensures that the model reflects a realistic elasticity of loss relative to weather fluctuations and allows the simulated premiums to align closely with actual market premiums.

Since losses cannot be negative, all simulated values X_i are truncated at zero. To accurately assess the total moisture availability during a specific growing season, we aggregated the daily weather data over a period of τ days. The cumulative weather for the entire crop cycle was then calculated using Eq. (10).

$$k_{t,\tau} = \frac{1}{\tau} \sum_{i=t_r}^{t_r+\tau} CH(i)\,. \tag{10}$$

This process involves summing the daily weather amounts, represented by $CH(i)$, from the start of the reference period (t_r) to the end $t_r + \tau$. The sum is then divided by the total number of days in the cycle τ to derive the average weather $k_{t,\tau}$. This metric provides a crucial indicator of the total water supply available to the crop throughout its lifecycle, offering a robust basis for evaluating agricultural outcomes under hydrological conditions.

The next step is to compute the statistical properties of these simulated losses. The mean loss in Eq. (6) and the mean squared loss in Eq. (7) are calculated as fundamental descriptors of the distribution. These statistics allow us to estimate the expected value of claims, $\hat{\psi}_n(t)$, using the formulation of Eq. (2). At the same time, the variability of claims is quantified by the variance, $\hat{V}_n(t)$, derived from Eq. (3). These two quantities together describe both the average magnitude and the uncertainty of potential payouts.

With these values in hand, the insurance premium is determined according to the principle of risk loading. Specifically, the premium is given by Eq. (11).

$$\pi_n(t) = \hat{\psi}_n(t) + \alpha\sqrt{\hat{V}_n(t)}, \tag{11}$$

where the safety loading factor α is set at 1%. In this formulation, the premium equals the expected claim plus a small margin proportional to the standard deviation of claims, ensuring coverage for both average and extreme outcomes. To further refine the analysis, claim intensity is calculated for different categories of risks. Eqs. (3) and (4) are estimated over the time window $\tau = 90 - 130$ days (according to the paddy planting cycle), with the baseline period set to $t_r = 0$. These intensities reflect the likelihood of adverse events occurring within critical stages of the agricultural season, thereby anchoring the simulated losses to realistic agronomic time horizons.

Altogether, this simulation framework integrates climate indices with stochastic loss modelling and actuarial premium calculation. By combining observed weather data, a linear climate-loss relationship, and formal insurance pricing formulas, the method produces a dynamic and transparent estimate of index insurance premiums. This allows for an intuitive mapping of how premiums adjust in response to varying weather patterns while maintaining statistical rigor and actuarial soundness.

2.3 Analytic Properties

The analytical assessment of the model includes:

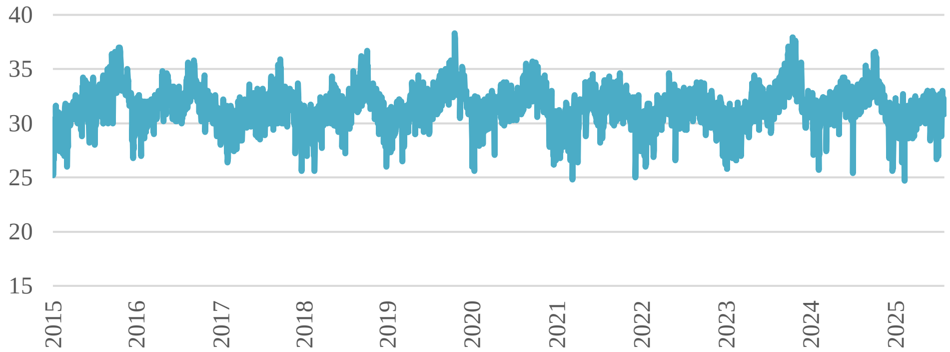
1. Unbiasedness, ensuring that the expected claim estimator $\hat{\psi}_n(t)$ is not systematically over or underestimated. Bias in premium estimators can lead to systemic mispricing, particularly in agricultural insurance contexts [20].
2. Estimator consistency verifying that parameter estimates converge to their true values as the sample size grows, following standard asymptotic properties of stochastic estimators.
3. Parameter sensitivity analysis evaluating how variations in $k_{t,\tau}$, $\hat{\mu}_n$, $\hat{\mu}_{2,n}$, and α affect the premium outcome. Sensitivity analysis of premium models to index parameters provides insight into their robustness under volatile climate conditions [25].

3. RESULTS AND DISCUSSION

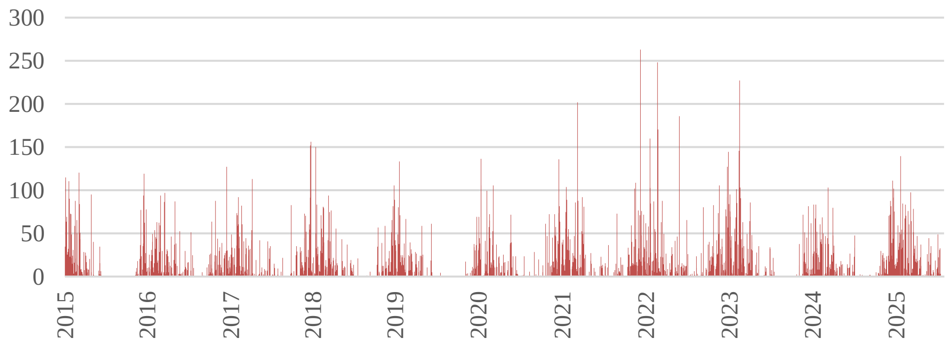
3.1 Historical Climate Pattern Analysis

We begin by analysing daily historical data from January 1, 2015, to July 31, 2025, covering temperature, rainfall, and sunshine duration. The time series reveals multiple extreme weather points where rainfall exceeded 100 mm/day, indicating potential flooding events, daily average temperature spiked above seasonal norms, raising crop stress levels, and sunshine duration surpassed 11 hours/day, which, combined with high temperatures, can accelerate evapotranspiration and trigger drought conditions. A preliminary visual inspection of the rainfall data shows seasonal patterns interspersed with extreme spikes. These events represent high-risk periods for paddy farmers due to significant yield losses.

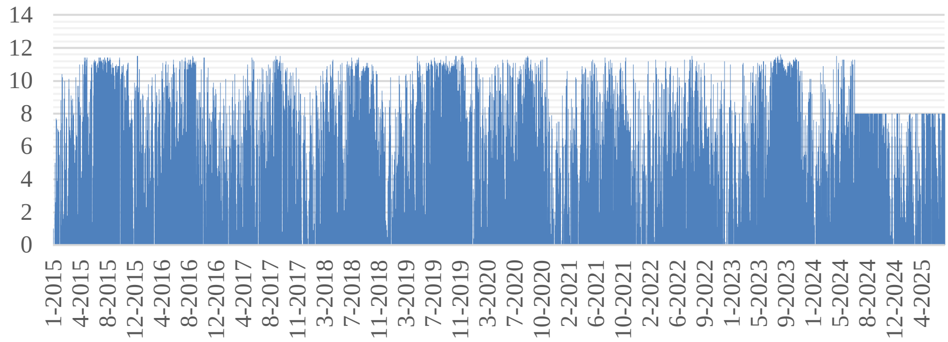
To quantify these risks, the compound cyclic Poisson stochastic model is employed to simulate the stochastic behaviour of weather-induced losses over time. In this framework, weather-related loss occurrences are modelled as a cyclic Poisson process, where the event frequency N_i follows a Poisson distribution with a time-varying rate that captures seasonal fluctuations in weather risk. The total loss during each period is expressed as the sum of individual random losses associated with each event in Eq. (1).



(a)



(b)



(c)

Figure 1. (a) Maximum Daily Temperature (°C), (b) Daily Rainfall (mm), (c) Daily Sunshine Duration (hours) Pattern from January 01, 2015, to July 31, 2025, in South Sulawesi Province

Data source: Meteorology, Climatology, and Geophysical Agency (BMKG) and European Centre for Medium Range Weather Forecasts (ECMWF), with visualization generated using the R application

Fig. 1 shows a distinct seasonal cycle with several extreme weather events identified from the dataset. High temperature episodes surpassing the physiological threshold for paddy growth can lead to reduced panicle formation and grain filling. Heavy rainfall events occur on days with precipitation exceeding 100 mm/day, which are likely to cause flooding and submergence of paddy fields. Excessive sunshine duration occurs on days with more than 8 hours/day of sunshine during the dry season, which can accelerate evapotranspiration and trigger drought stress.

3.2 Correlation Between Weather Variables and Paddy Production

To quantitatively assess the impact of these climatic variables on paddy production, a Pearson correlation analysis was conducted. This analysis paired monthly average data for rainfall, temperature, and sunshine duration with corresponding paddy yields quintal per hectare (q/ha) from 2018 to 2024. To accurately capture the temporal influence of climate on crop growth, the climatic data were lagged by one period, meaning the weather metrics preceding the harvest were correlated with the final production data.

Table 2. Correlation between Climate Variables and Paddy Production

Climate Variable	Correlation Coefficient
Maximum temperature	- 0.3878711
Rainfall	- 0.70013761
Sunshine duration	- 0.6287436

The correlation coefficients presented in Table 2 are used to quantify the degree of dependence between climatic variables and paddy productivity, which directly informs the determination of agricultural insurance premiums. These correlations are incorporated into the stochastic loss function of the compound cyclic Poisson model to calibrate the sensitive parameter β, which links weather indices to expected financial losses. Thus, the correlation analysis serves as the foundation for translating climate variability into measurable financial risk, ensuring that the calculated premiums accurately reflect real-world agricultural vulnerability. Based on the results, the correlation analysis reveals a significant relationship between climatic variables and paddy productivity. A negative correlation between them suggests that excessive climate variable during critical growth stages reduces yields. Rainfall showed a strong negative correlation with paddy productivity ($r = -0.70013761$). This indicates that the excessive rainfall is consistently associated with a decrease in harvest yields. This can be attributed to several factors, such as an increased risk of flooding, physical damage to crops, and a higher prevalence of pests and diseases that thrive in high-humidity conditions. Interestingly, sunshine duration also demonstrated a significant negative correlation with paddy productivity ($r = -0.6287436$). While sunshine is crucial for photosynthesis, this finding suggests that an excessive duration of sunshine, especially without sufficient rainfall, can lead to drought conditions. Drought severely disrupts the water supply, a critical factor for paddy growth. Therefore, both too little and too much water, either from insufficient rain or from extreme heat, are detrimental to paddy productivity. Meanwhile, temperature did not show a strong correlation with paddy productivity ($r = -0.3878711$), suggesting its influence might be less significant than that of rainfall and sunshine duration, or that the temperature in the region remained within the optimal range for the crop.

The correlation results build the empirical foundation for designing weather-indexed insurance in which payouts are triggered by the derivation of $R, T,$ or S beyond optimal thresholds. These findings suggest that there is an optimal point for both climatic variables. Paddy yields are maximized when rainfall and duration are at balanced levels, rather than at either extreme, too wet or too dry. The results confirm that both recessive rainfall and prolonged dry spells exert a significant adverse effect on production. This aligns with literature that identifies hydrometeorological extremes as key determinants of agricultural loss.

3.3 Index-Based Premium Calculation

The results of the agricultural insurance claim simulation based on the weather index can be seen in Fig. 2, which illustrates the distribution of actual losses X_i, expected claims $\hat{\psi}_n(t)$, and standard deviations $\sqrt{\hat{V}_n(t)}$.

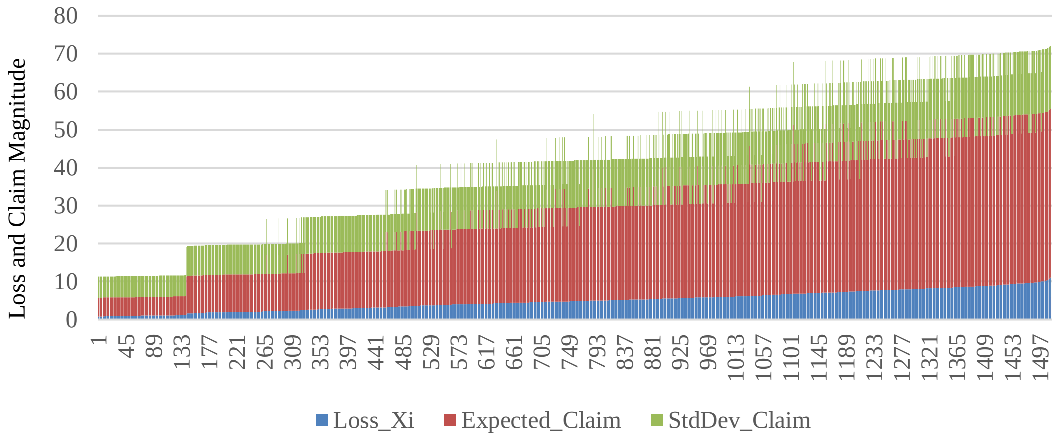


Figure 2. Histogram of Simulated Claims
***Source**: Author's own simulation results using R application (2025)*

The histogram in Fig. 2 shows that actual losses X_i are relatively small and increase gradually as the number of simulated claims grows. In contrast, the expected claim values are higher than the actual losses, indicating that the premium model provides a conservative estimate to ensure greater protection for farmers. Meanwhile, the standard deviation of claims exhibits significant fluctuations with sharp spikes at certain points, reflecting high uncertainty or volatility caused by potential extreme weather events.

The premium is calculated by applying Eq. (11) within this mathematical model. However, the $\pi_n(t)$ value derived from this simulation is a model loss expressed in abstract mathematical units, not in real currency. To make this result relevant to market conditions, the premium must be calibrated to an IDR value. This calibration process is done using Eq. (12),

$$calibration\ factor = \frac{market\ premium\ (IDR)}{model\ premium\ (units)}. \tag{12}$$

This factor acts as a bridge, connecting the model's theoretical results with a price that can be applied in the real market. Based on the AUTP scheme, the established market premium is IDR 180,000/ha/planting cycle (without government subsidies), with a benefit value of IDR 6,000,000/ha/claim. The numerical simulation results are shown in Table 3.

Table 3. Premium Estimates by Calibration Factor and Risk Level

Loss X_i	$\hat{\psi}_n(t)$	$\hat{V}_n(t)$	$\pi_n(t)$ (units)	Premium (IDR)
0.77531	4.93327	30.6921	4.98867	36,796
0.87792	4.93327	30.6921	4.98867	36,796
⋮	⋮	⋮	⋮	⋮
1.25601	4.93327	30.6921	4.98867	36,796
2.45819	9.86653	61.3842	9.94488	73,353

Loss X_i	$\hat{\psi}_n(t)$	$\hat{V}_n(t)$	$\pi_n(t)$ (units)	Premium (IDR)
3.94132	14.7998	92.0763	14.8958	109,870
4.86127	19.7331	122.768	19.8437	146,367
6.49263	24.5996	153.460	24.7902	182,851
7.47795	29.5996	184.153	29.7353	219,326
8.46191	34.5329	214.845	34.6794	255,793
9.98815	39.4661	245.537	39.6228	292,255
11.0082	44.3994	276.229	44.5656	328,713

This mathematical framework in Table 3 ensures that the premium is endogenously linked to climate variability. As the weather index increases, the model predicts higher potential losses due to excessive or insufficient rainfall, which are major determinants of paddy yield reduction. Consequently, the premium rises sharply under adverse weather conditions to compensate for higher expected payouts, while remaining relatively low when the weather is within favorable ranges. From an economic perspective, the premium's variability mirrors the risk exposure faced by farmers. At low indices, with favorable weather, the premium is modest $\approx$ IDR 36,796, or about 20% of the current market premium AUTP, indicating minimal risk of loss. However, under extreme rainfall scenarios corresponding to high weather indices, the premium escalates beyond IDR 328,713, which is around 182% of the AUTP rate, signaling substantial potential crop damage and the need for greater financial protection. This comparison underscores that the proposed model not only reflects the dynamic relationship between climate risk and insurance pricing but also provides a flexible and risk-responsive alternative to the fixed rate structure of existing agricultural insurance schemes.

The simulation results demonstrate a clear divergence in premium values between high and low seasonal risk scenarios, represented by variations in the seasonal index parameter $k_{t,\tau}$. Under high-risk conditions where the seasonal index reflects elevated weather variability and a higher probability of extreme weather events, the model produced premium estimates higher than in low-risk conditions. This sensitivity aligns with designing a dynamic pricing scheme that responds directly to seasonal climate signals rather than applying uniform rates. The simulations also confirmed that as $k_{t,\tau}$ approaches its lower bound (indicative of stable weather patterns), the model converges to premiums close to those of flat rate schemes, thereby maintaining competitiveness during low-risk seasons.

4. CONCLUSION

The simulation shows that the weather index insurance premium, derived from the model $X_i = \beta \cdot k_{t,\tau} + \varepsilon_i$; $\pi_n(t) = \hat{\psi}_n(t) + \alpha\sqrt{\hat{V}_n(t)}$ ranges from IDR 36,796 under low weather index conditions to IDR 328,713 under high weather index conditions. These values are strongly influenced by the calibration coefficient $\beta = 10$, which empirically represents the sensitivity of farmers' losses to changes in the weather index. Each one unit increase in the index corresponds to an average rise of ten units in potential loss, reflecting the proportional impact of adverse weather on paddy yields. This variation emerges directly from the model's mathematical structure, where losses X_i are expressed as a linear function of the weather index $k_{t,\tau}$, scaled by a sensitivity parameter β, and adjusted by stochastic noise $\varepsilon_i \sim N(0, \sigma^2)$ that captures natural uncertainty. Compared to the fixed market premium of IDR 180,000 per season, the model generates lower premiums under favorable weather, indicating underpricing relative to the market, and significantly higher premiums under extreme weather, implying possible overpricing. This divergence highlights a key tension: while the market offers a uniform, averaged risk price, the mathematical model yields climate-sensitive, risk-adjusted premiums that escalate with increasing weather index values. In summary, the model demonstrates that weather index insurance premiums are strongly influenced by weather conditions, capturing actuarial fairness, but may diverge from the fixed market rate depending on the severity of climatic risk.

Author Contributions

Ika Reskiana Adriani: Conceptualization, Formal Analysis, Methodology, Project Administration, Software, Writing-Original Draft, Review and Editing-Manuscript. Miftahulkhairah: Data Curation, Resources and Validation. Gemala Hardinasinta: investigation, Resources, and Formal Analysis. Hafidzah: Data Curation and Visualization. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare that they have no competing interests.

Declaration of Generative AI and AI-Assisted Technologies

Generative AI (e.g, ChatGPT) was used exclusively for language editing and stylistic improvement. The authors take full responsibility for the content and confirm that all analyses, results, and interpretations are their own. The final manuscript was additionally reviewed for linguistic accuracy by an English-language expert.

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