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LOGIC MINING FOR TELECOMMUNICATION CHURN CLASSIFICATION: PERMUTATION WEIGHTED RANDOM 2 SATISFIABILITY REVERSE ANALYSIS APPROACH

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ABSTRACT

The telecommunications industry is experiencing rapid transformation, resulting in tense competition and increased customer volatility. Telecom churn, which refers to the discontinuation of services by customers, poses a serious challenge due to its direct impact on revenue and long-term profitability. Addressing this issue requires effective methods for understanding and predicting customer behavior. Hence, a logic mining approach is introduced in this study, namely the Permutation Weighted Random 2 Satisfiability Reverse Analysis Method, to classify customer churn in the telecommunications sector. The proposed method is based on a logical rule known as Weighted Random 2 Satisfiability, which is implemented in the Discrete Hopfield Neural Network. The logical rule facilitates the dynamic allocation of negative literals, contributing to improved logical representation. Furthermore, the Election algorithm is incorporated during the training phase to enhance the accuracy of logical structure interpretation. The proposed method is capable of extracting optimal data patterns and generating induced logic that accurately describes customer churn behaviour. This induced logic not only predicts whether a customer will churn but also provides interpretable insights into the underlying causes. Experimental results demonstrate a strong average accuracy of 85.6%, indicating the effectiveness and scalability of the proposed approach for knowledge discovery. Although the proposed approach achieves strong accuracy, the lower F1-Score and Matthews Correlation Coefficient reveal limitations in churn customer classification, highlighting the need for further improvement in handling class imbalance. This study contributes to the field of data mining by offering a logic-based framework for churn classification and emphasizing its practical relevance in supporting strategic customer retention efforts in a competitive telecommunications sector.

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1. INTRODUCTION

Telecommunication is essential for real-time communication and instant information sharing. Moreover, people can stay connected across the world through telecommunication. The telecom industry has grown rapidly and become competitive. Then, understanding and managing customer churn is required, as customer churn significantly impacts business success. In the dynamic telecommunications industry, customer churn is defined as the action of customers canceling a subscription or switching from one service to another. Annually, the telecommunication industry faces churn rates of 20% to 40%, which leads to a huge loss of revenue [1], [2], [3]. This highlights the importance of effective data analysis to retain customers. Retaining existing customers is often more cost-effective than acquiring new ones due to the high costs associated with marketing and advertising. Logic mining models are a subset of data mining that offer interpretable solutions for churn classification. By applying logic mining techniques, telecom operators can discover the patterns and insights that facilitate churn management strategies. Various reasons that can lead to customer churn, such as pricing strategies and service quality, can be determined. Telecom companies can improve their operational strategies and build long-lasting customer relationships in a competitive market by understanding the factors influencing customer decisions.

The purpose of logic mining is to discover and identify patterns in a dataset using logical rules. Unlike standard data mining, which may use various statistical and machine learning techniques, logic mining emphasizes the application of logic to extract meaningful information and construct a logical rule to explain the behavior of a dataset. The output is presented as logic derived from Satisfiability (SAT), which would be beneficial to various classification tasks [4]. According to Zamri et al. [4], the main components of logic mining are the computational structure of Artificial Neural Network (ANN), symbolic rule based on SAT, and the method of reverse analysis (RA). As supported by Zamri et al. [4], ANN mimics the operational patterns of the human brain, and ANN is composed of interconnected nodes, also known as artificial neurons in the network. Synaptic weights represent the connections between the neurons. Furthermore, ANN is commonly used for classification tasks by extracting the trends of large-dimensional data. Hopfield and Tank [5] proposed a variant of ANN, namely the Discrete Hopfield Neural Network (DHNN). There is no hidden layer in DHNN; however, the black box property of DHNN has become an issue as the internal workings in the network are not visible. Therefore, a symbolic rule in the form of a logical rule is required to govern the neurons in the network. Based on this requirement, Abdullah [6] proposed Horn Satisfiability (HornSAT) as the symbolic rule in the DHNN. HornSAT has the ability to navigate the overall behaviour of neuron states in the network. Moreover, the RA technique is another key aspect of logic mining, which was initially presented by Sathasivam and Abdullah [7] to generate the logical rules based on education datasets. The WA method was used in the proposed logic mining to compute the synaptic weight of the induced logic.

Logic mining extracts information from real-world datasets by discovering patterns through logical formulation. In the past few years, the 2 Satisfiability Reverse Analysis Method (2SATRA) was employed by Alway et al. [8] to analyse a dataset concerning palm oil commodity prices. 2SATRA is a logic mining model with 2SAT and a reverse analysis mechanism. The trend of palm oil price can be interpreted as it effectively discovers the best-induced logic from the data. The extracted best logic is based on frequency comparison in the training phase. However, the random nature of the attribute selection method leads to limited interpretability of the induced logical rules. After that, Rusdi et al. [9] proposed SHoRA to develop a new objective function of the best logic that considers the true positive and true negative outcomes. Additionally, SHoRA utilises supervised learning for higher-order logic during the pre-processing phase of DHNN. The attributes are filtered by employing correlation analysis. Moreover, there is other work, Manoharam et al. [10] formulated a log-linear approach known as 3-satisfiability-based reverse analysis multi-unit method (G3SATRAμ) which utilized a multi-unit DHNN governed by the best logic extracted from the datasets. However, the SAT structure of G3SATRAμ cannot be applied to analyse data that consists of a high number of attributes because the application of the log-linear method as an attribute selection method can only analyse up to 10 attributes at one time. Consequently, the analysis of relationships among attributes becomes less effective when the number of attributes exceeds ten. Although ShoRA, G3SATRAμ, and 2SATRA have contributed to advancements in logic mining, issues such as bias toward true positives and limitations in handling high-dimensional data continue to exist. Future research should explore alternative attribute selection methods, such as unsupervised attribute selection approaches, to enhance logic mining effectiveness and interpretability.

In this research, logic mining model Permutation Weighted Random 2 Satisfiability is utilized, and the main components of Pr2SATRA are DHNN, logical rule Weighted Random 2 Satisfiability, Election

Algorithm as training algorithm, Jaccard Feature Selection Method (JFSM) as attribute selection method involved in data pre-processing, and reverse analysis method (RA). The proposed logic mining model is applied to the Telecommunication Churn dataset. Accordingly, this study has three main objectives: (i) to apply Weighted Random 2 Satisfiability in Discrete Hopfield Neural Network as neuron representation, (ii) to construct a logic mining model namely Permutation Weighted Random 2 Satisfiability Reverse Analysis for Telecom Churn classification, and (iii) to evaluate the performance of the Permutation Weighted Random 2 Satisfiability Reverse Analysis using multiple metrics such as accuracy, precision, specificity, Matthew's correlation coefficient and F1-Score.

From the perspective of the conventional churn prediction approaches, including logistic regression, support vector machines, and tree-based models, they mainly focus on predictive accuracy and often lack the ability to extract explicit logical rules that explain customer churn behaviour. This creates a research gap in developing churn prediction frameworks that integrate both learning and logical reasoning. To address this gap, the proposed Pr2SATRA approach introduces a logic-mining framework based on Weighted Random 2-Satisfiability implemented in a Discrete Hopfield Neural Network, enabling the extraction of interpretable logical rules while maintaining competitive predictive performance. The content in this paper is organized into a total of four Sections. Section 1 is the introduction. Section 2 shows the methodology of the proposed work in detail. Next, Section 3 presented the experimental setup and findings of the proposed work. Finally, Section 4 summarizes the findings of this research.

2. RESEARCH METHODS

This section discusses the formulation of the logical rule, which is then integrated into DHNN. The second subsection reveals the general concepts of the logical rule in DHNN, and the third subsection describes the methodology of the proposed logic mining, including the phases involved in finding the best induced logic.

2.1 Weighted Random 2 Satisfiability

In this research, the logical rule, namely Weighted Random 2 Satisfiability (r2SAT), is utilized. As described by [4], the non-systematic r2SAT includes several elements: each clause contains a different number of k literals, where k equals 1 or 2, and the quantity of negated literals produced is determined by r. The following Eq. (1) represents the general formulation of r2SAT.

$$\Gamma_{r2SAT} = \wedge_{i=1}^{u} J_i^{(2)} \wedge_{i=1}^{v} J_i^{(1)}, \tag{1}$$

where $u, v \in \mathbb{N}$ denotes the number of clauses for second and first-order clauses, respectively. The formula $m = u + v$ represents the overall number of clauses. Observe that, $J_i^{(k)}$ denotes the logical clause associated with index i at order k as in Eq. (2) below:

$$J_i^{(k)} = \begin{cases} (A_i \vee B_i), & k = 2, \\ (C_i), & k = 1. \end{cases} \tag{2}$$

Each literal $\{A_i, B_i, C_i\}$ are presented in bipolar form $\{1, -1\}$ mapped to true and false. The arrangement of literals in each $J_i^{(k)}$ may consist of either positive (A_i) or negative $(\neg A_i)$. Eq. (3) indicates the total number of literals, denoted by λ in Γ_{r2SAT}.

$$\lambda = 2u + v. \tag{3}$$

The second key feature of Γ_{r2SAT} is that the arrangement of negative literals can be determined based on the proportions desired by r. Simply said, r denotes the ratio of negative literals in Γ_{r2SAT}. The total number of negative literals is denoted as N_v can be calculated as stated in Eq. (4).

$$N_v = r\lambda, \tag{4}$$

where $N_v \in \mathbb{N}$ and the range of $r = [0.1, 0.9]$ with a step size of $\Delta r = 0.1$ is considered. Based on the research by Romli et al. [11], the values of $r = 0$ or $r = 1$ are not considered as the literals produced will solely reflect on either all positive or all negative literals. An optimization of the possible weight of all literals occurs in the logic phase to obtain optimal generation of Γ_{r2SAT} with desired values of r. Eq. (5) shows the possible weight, η_i of the literal.

$$\eta_i = \begin{cases} 1, & if\ (\neg A_i), \\ 0, & if\ (A_i) \end{cases}. \tag{5}$$

The total η_i in Γ_{r2SAT}, denoted as κ can be computed as shown in Eq. (6).

$$\kappa = \sum_{i=1}^{\lambda} \eta_i\,. \tag{6}$$

Note that the number of negative literals will consistently be produced in relation to N_v. Therefore, the logic phase will include a minimization approach by using Artificial Bee Colony (ABC) [12] to ensure the desired fitness of $f_{LP} = 0$, where f_{LP} is the fitness in the logic phase. The minimization of f_{LP} in the logic phase is defined as displayed in Eq. (7).

$$min[f_{LP}] = |N_v - \kappa| \tag{7}$$

An example of Γ_{r2SAT} consists of $r = 0.4$ and $\lambda = 10$ can be formulated as in Eq. (8).

$$\Gamma_{r2SAT} = (A_1 \vee \neg B_1) \wedge (A_2 \vee \neg B_2) \wedge (\neg A_3 \vee B_3) \wedge (A_4 \vee B_4) \wedge (\neg C_1) \wedge (C_2). \tag{8}$$

Note that Eq. (8) is one of the examples of Γ_{r2SAT} before being implemented in DHNN to evaluate the neuron connections.

2.2 Weighted Random 2 Satisfiability in the Discrete Hopfield Neural Network

For this section, *r*2SAT, as the logical representation, is implemented in DHNN and abbreviated as DHNN-*r*2SAT will be discussed. DHNN is one of the ANNs and operates through two primary phases, which are the training phase and the retrieval phase. In the training phase, the Wan Abdullah (WA) method is used to train Γ_{r2SAT} ensuring optimal synaptic weight management [13]. The synaptic weight obtained will be stored in the Content Addressable Memory (CAM) as a form of associative memory, which will then be applied in the retrieval phase of DHNN. During the retrieval phase, the local field is used to compute the production of final neuron states. Eqs. (9) - (10) displayed the formulation of the cost function in the DHNN-*r*2SAT model.

$$E_{\Gamma_{r2SAT}} = \frac{1}{4}\sum_{i=1}^{u}\left(\prod_{j=1}^{2} Q_{ij}\right) + \frac{1}{2}\sum_{i=1}^{v}\left(\prod_{j=1}^{1} Q_{ij}\right), \tag{9}$$

$$Q_{ij} = \begin{cases} (1 - S_{A_i}), & if\ (\neg A_i), \\ (1 + S_{A_i}), & if\ (A_i). \end{cases} \tag{10}$$

In this research, effectively minimizing the cost function or Eq. (9) in DHNN will guarantee optimal synaptic weights. A cost function of zero or $E_{\Gamma_{r2SAT}} = 0$ indicates that every clause within the logical rule is fulfilled. On the contrary, when $E_{\Gamma_{r2SAT}} \neq 0$ shows the number of unsatisfied clauses in Γ_{r2SAT}.

In the retrieval phase of DHNN-*r*2SAT, the final neuron states are determined by computing the local field (h_i) of DHNN and then applying an activation function for filtering. The formula of h_i is shown in Eq. (11).

$$h_i(t) = \sum_{j=1, i=j}^{N} W_{ij}^{(2)} S_j + W_i^{(1)}. \tag{11}$$

Note that $W_{ij}^{(2)}$ and $W_i^{(1)}$ depicts the synaptic weight of the DHNN for second and first-order, respectively, involving neurons *i* and *j*. The WA method is applied to obtain the computation of the optimal synaptic weight. The hyperbolic activation function (HTAF) shown in Eq. (12) is applied to aid DHNN in squashing the values of h_i into the bipolar final neuron state.

$$S_i(t) = \begin{cases} 1, & tanh(h_i) \geq 0, \\ -1, & otherwise. \end{cases} \tag{12}$$

It is worth mentioning that the final neuron state is then used in the logic mining process to retrieve the induced logic for the classification related to the dataset. The next section will discuss in detail the proposed logic mining approach.

2.3 Permutation Weighted Random 2 Satisfiability Reverse Analysis

The proposed logic mining, namely Permutation Weighted Random 2 Satisfiability Reverse Analysis (Pr2SATRA), consists of five components. The components are DHNN, logical rule r2SAT, the Election Algorithm (EA) as the training algorithm, Jaccard Feature Selection Method (JFSM) as the attribute selection method, and Reverse Analysis (RA). The proposed logic mining model consists of four main phases, which are the preliminary phase, logic phase, training phase, and retrieval phase. During the preliminary phase, three main steps are conducted for the data preprocessing process, which are data preparation, JFSM, and data splitting. First, the raw entries will be converted to bipolar representation $\{1,-1\}$. The data consists of categorical and non-categorical attributes. The categorical attributes entries are converted into 1 and -1 based on the value assigned, while the entries for non-categorical attributes are changed to 1 and -1 using k-means clustering [14]. After that, JFSM is applied to remove irrelevant attributes that show low similarity to the dependent attribute of the dataset. As supported by Zamri et al. [4], the formula of JFSM can be seen in Eqs. (13) - (17).

$$JI = \frac{\sum_{j=1}^{n} a_j}{\sum_{j=1}^{n} a_j + \sum_{j=1}^{n} b_j + \sum_{j=1}^{n} c_j}, \tag{13}$$

where

$$a_j(e_i) = \begin{cases} 1, & if\ (E_i, P) = (1,1), \\ 0, & otherwise, \end{cases} \tag{14}$$

$$b_j(e_i) = \begin{cases} 1, & if\ (E_i, P) = (1,-1), \\ 0, & otherwise, \end{cases} \tag{15}$$

$$c_j(e_i) = \begin{cases} 1, & if\ (E_i, P) = (-1,1), \\ 0, & otherwise, \end{cases} \tag{16}$$

$$d_j(e_i) = \begin{cases} 1, & if\ (E_i, P) = (-1,-1), \\ 0, & otherwise. \end{cases} \tag{17}$$

Note that JI denotes the Jaccard Index. Meanwhile, e_i, E_i, and P represents raw entries, entries of independent attributes, and the actual outcome of the dataset, respectively. Consider that n represents the overall number of entries. In this study, JFSM is utilized to identify 10 significant attributes using the formula. Higher JI values are preferred to imply a high distribution of entries, which contributes to a positive outcome of the dataset. Since the optimal JI depends on the dataset, the highest value is selected relative to the experimental results rather than using a fixed threshold. After that, the dataset was split into $60:40$ ratio where 60% classified as the training dataset and 40% classified as a testing dataset [15]. In addition, 5-fold cross-validation is considered to ensure a non-overfitting dataset.

After the preliminary phase, the logic phase is performed, which focuses on extracting meaningful logical rules and relationships from the dataset. During the logic phase, the r2SAT logical rule will be produced, and only unique structures of Γ_{r2SAT} are generated. Examining all the unique structures of Γ_{r2SAT} is essential to ensure that Pr2SATRA considers all potential neuron connections when negative links are assigned to different literals. Following the logic phase, the training phase is a crucial step where the model learns from the dataset and the extracted logical rules to make accurate classifications. All the generated Γ_{r2SAT} from the logic phase are the possible super logics. The selection of the super logic, Γ_{r2SAT}^{s} is determined by the highest summation of true positive (TP) and true negative (TN) as shown in Eq. (18) [11].

$$\Gamma_{r2SAT}^{s} = max[TP, TN]. \tag{18}$$

This approach will aid the proposed logic mining in training the significant structure of Γ_{r2SAT} and enable the network to capture the majority patterns in the training data (Γ_{train}). Once the optimal Γ_{r2SAT}^{b} is generated, EA will optimize the logic, ensuring optimal synaptic weights are obtained. EA is utilized to maximize the satisfied interpretation for Γ_{r2SAT} that minimizes the cost function during the training phase of DHNN [11]. Comparison between the logical outcome of Γ_{r2SAT} and P from the Γ_{train} will be conducted according to Fig. 1.

		P from the Γ_{train}	
		Positive (1)	Negative (-1)
Outcome from Γ^{s}_{r2SAT}	Positive (1)	TP	FN
	Negative (-1)	FP	TN

Figure 1. The Confusion Matrix Classification in the Training Phase of P*r*2SATRA

The retrieval phase is the last phase in Pr2SATRA. During this phase, the local field, as stated in Eq. (11), is calculated to produce the final neuron state. According to the work by Romli et al. [11], the obtained final neuron states will be converted into the format of a SAT logical rule. All the possible induced logic, Γ^{i}_{r2SAT} will be retrieved based on the final neuron state as shown in Eq. (19).

$$S_i^{induced} = \begin{cases} A_i, & S_i = 1, \\ \neg A_i, & S_i = -1. \end{cases} \tag{19}$$

Subsequently, the outcome from the retrieved Γ^{i}_{r2SAT} will be compared with the outcome from testing data (Γ_{test}) as shown in Fig. 2. The best Γ^{i}_{r2SAT} is selected according to the highest accuracy (ACC) achieved from the classification because ACC offers general insights into logic mining by producing optimal outcomes with high TP and TN that align with the actual outcome of the dataset. Fig. 2 illustrates the flowchart of the main phases in Pr2SATRA.

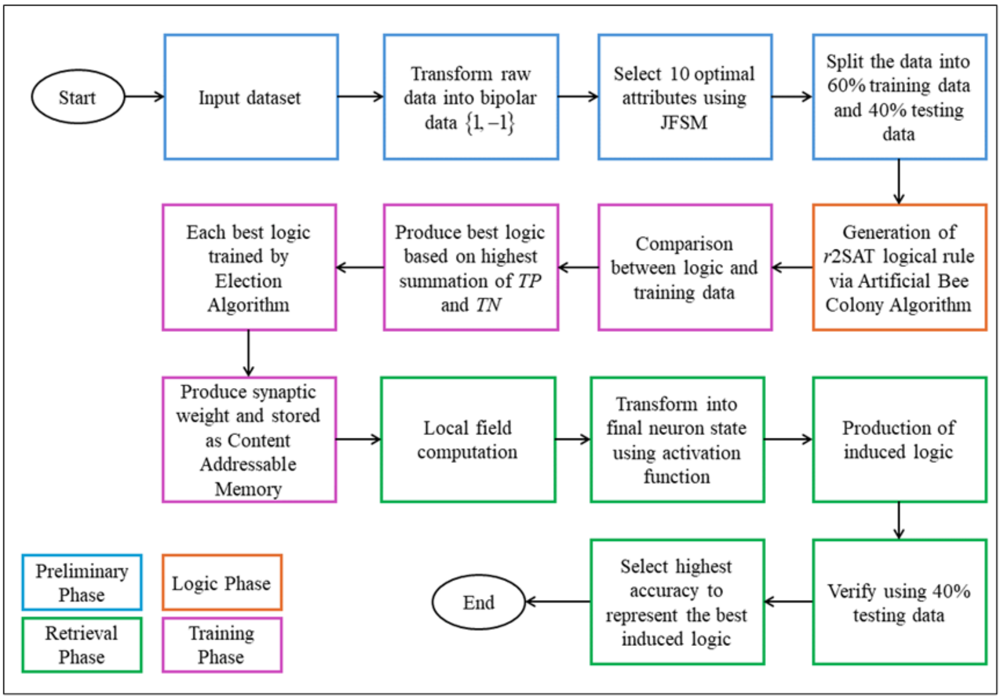

Figure 2. Flowchart of P*r*2SATRA

2.4 Experimental Setup

The experimental settings throughout the research will be described. The simulations were run using Microsoft Dev C++ Version 5.11. A single personal laptop featuring an Intel Core i5 processor is used to execute the simulations. This section focuses on the evaluation of the performance of the induced logic produced by Pr2SATRA. Table 1 displays the list of parameters used in the experiment.

Table 1. List of Parameters in P*r*2SATRA

Parameter	Parameter Value
Neuron Combination	100
Number of Learning	100
Number of Trials	100
Number of Selected Attributes	10

Parameter	Parameter Value
Maximum Permutation	100
Ratio of Training Data	60
Ratio of Testing Data	40

3. RESULTS AND DISCUSSION

3.1 Dataset Description

The dataset was collected from the Kaggle website and focuses on Orange Telecommunications Customer Churn. The dataset consists of textual data and represents customer-related attributes and their corresponding churn status. The dataset has no missing values. The dataset consisted of 20 attributes, but only 19 attributes were considered. This happened because one of the attributes, 'State' contains unique values. Table 2 presents details of the attributes with the clustering classification for the bipolar value.

Table 2. Details of the Attributes in the Orange Telecommunications Customer Churn Dataset

Attribute	Name of Attribute	Bipolar Class	Description
1	Account length	(1, -1) = (More than or equal to 101, Less than or equal to 100)	The length of the account
2	Area code	(1, -1) = (more than or equal to 437, less than or equal to 436)	Code of the city
3	International plan	(1, -1) = (Yes, No)	Indicates whether the customer has an international plan or not.
4	Voicemail plan	(1, -1) = (Yes, No)	Indicates whether the customer has subscribed to a voicemail service or not
5	Number of voicemail messages	(1, -1) = (Yes, No)	The number of voicemail messages received by the customer
6	Total day minutes	(1, -1) = (more than or equal to 179.48, less than or equal to 179.47)	The total number of minutes the customer has used for morning calls
7	Total day calls	(1, -1) = (More than or equal to 101, Less than or equal to 100)	The total number of calls made or received during the morning
8	Total day charge	(1, -1) = (More than or equal to 30.51, Less than or equal to 30.50)	The total charges in the morning incurred by the customer
9	Total eve minutes	(1, -1) = (More than or equal to 200.39, Less than or equal to 200.38)	The total number of minutes the customer has used for evening calls
10	Total eve calls	(1, -1) = (More than or equal to 101, Less than or equal to 100)	The total number of calls made or received during the evening
11	Total eve charge	(1, -1) = (More than or equal to 17.03, Less than or equal to 17.02)	The total charges incurred by the customer in the evening
12	Total night minutes	(1, -1) = (More than or equal to 201.17, Less than or equal to 201.16)	The total number of minutes the customer has used for nighttime calls
13	Total night calls	(1, -1) = (More than or equal to 101, Less than or equal to 100)	The total number of calls made or received during the nighttime
14	Total night charge	(1, -1) = (More than or equal to 9.05, Less than or equal to 9.04)	The total charges incurred by the customer during the nighttime
15	Total intl minutes	(1, -1) = (More than or equal to 10.24, Less than or equal to 10.23)	Total number of international calls in minutes
16	Total intl calls	(1, -1) = (More than or equal to 4, Less than or equal to 3)	Total number of international calls made
17	Total intl charge	(1, -1) = (More than or equal to 2.76, Less than or equal to 2.75)	Total charges for international calls

Attribute	Name of Attribute	Bipolar Class	Description
18	Customer service calls	(1, -1) = (More than or equal to 2 times, Less than or equal to 1 time)	Number of customer service calls made by the customer
19	Churn	(1, -1) = (Yes, No)	Indicates whether customers churn or not

3.2 Performance Metrics

For performance metrics evaluation, the top five metrics are used to measure the performance of the proposed model, which are accuracy (ACC), precision (PRE), specificity (SPE), Matthews Correlation Coefficient (MCC), and F1-Score ($F1$). According to Chico and Jurman [16], ACC is the commonly used metric and is defined as the ratio of the number correctly predicted to the overall behaviours. The formula of ACC is provided in Eq. (20) as supported by Chico and Jurman [16].

$$ACC = \frac{TP + TN}{TP + TN + FP + FN}. \tag{20}$$

Note that TP (true positive) represents correctly classified positive outcomes, FN (false negative) represents incorrectly classified positive outcomes, TN (true negative indicates correctly classified negative outcomes, and FP (false positive) denotes incorrectly classified negative outcomes. PRE is employed to estimate the predictive ability of the model [17]. The formulation of PRE is defined in Eq. (21) as follows.

$$PRE = \frac{TP}{TP + FP}. \tag{21}$$

Subsequently, SPE measures the capability of the model to predict the results. SPE refers to the proportion of negative samples that are correctly classified[18]. The following Eq. (22) represents the formulation of SPE.

$$SPE = \frac{TN}{TN + FP}. \tag{22}$$

On the other hand, MCC is a reliable evaluation metric that remains effective even when dealing with imbalanced [16]. The formulation of MCC is presented as in Eq. (23).

$$MCC = \frac{TP \cdot TN - FP \cdot FN}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}}. \tag{23}$$

In addition, $F1$ is also useful for measuring the performance of classification models when handling imbalanced data. Eq. (24) represents the formulation of F1 as follows.

$$F1 = \frac{TP}{TP + \frac{1}{2}(FP + FN)}. \tag{24}$$

It is worth mentioning that the selected metrics are utilized to evaluate the quality of the retrieved induced logic generated by the proposed logic mining model.

3.3 Findings and Analysis

This section will be discussed from three perspectives. First, the result from JFSM computation is explained, and the heatmap obtained for all folds is shown in this section. The second subsection describes the possible cases from the best induced logic obtained. Third, several recommendations based on induced logic are discussed.

3.3.1 Results of Logic Mining

The proposed Pr2SATRA utilized an unsupervised attribute selection method, which is JFSM, by computing the Jaccard Index value for the distance between independent and dependent attributes. The formula in Eq. (13) is applied to calculate the Jaccard Index value. The concept of the attribute selection method is to remove irrelevant attributes. Jaccard Similarity Index is applied to measure the relationship between independent and dependent attributes. JFSM is categorized as an unsupervised attribute selection method in the sense that the selection process is fully automated and independent of any human decisionmaking or label-driven optimization. The inclusion of the churn variable in the formulation serves only for the evaluation of the similarity index, rather than supervising the attribute selection process. Then, the top ten significant attributes with high Jaccard Index values are selected, which depict a high distribution of entries contributing to the positive class of the dataset. Table 3 shows the selected top ten significant attributes with the Jaccard Index value.

Table 3. Jaccard Index Value for the Selected Attributes

Attribute	Attribute Label	Name of Attribute	Jaccard Index Values
3	A_1	International Plan	0.2185
6	B_1	Total day minutes	0.1544
7	A_2	Total day calls	0.1406
8	B_2	Total day charge	0.1544
9	A_3	Total eve minutes	0.1465
11	B_3	Total eve charge	0.1465
14	A_4	Total night charge	0.1369
15	B_4	Total intl minutes	0.1481
17	C_1	Total intl charge	0.1481
18	C_2	Customer service calls	0.1563

All the selected attributes were implemented within Pr2SATRA to retrieve the induced logic. Then, the retrieved induced logic is compared with the testing data in order to evaluate the confusion matrix classification, as shown in Fig. 2. The retrieved induced logic is compared with a 5-fold cross-validation to tackle the class imbalance of the dataset. The classification performance for the confusion matrix is shown in Table 4 as follows.

Table 4. The Results of the Confusion Matrix Classification for Each Fold. The Bold Values Indicate the Best Fold for the Respective Confusion Matrix

Confusion Matrix	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
TP	8	**6**	6	6	8
TN	923	**924**	908	894	881
FP	10	**6**	5	9	11
FN	125	**130**	147	157	166

Based on the confusion matrix classification in Table 4, the values of *TP* and *TN* correspond to the number of correct predictions for positive and negative classes, respectively. Observation shows that the best fold is fold 2. This is because the value of *TN* is highest compared to *TP*, *FP*, and *FN* for fold 2, which shows that the model is able to capture the distribution of negative literals. Further explanation of the reason for choosing fold 2 will be discussed in the next subsection.

3.3.2 Discussion of Best Induced Logic

In this subsection, the analysis for the Telecommunications Churn Dataset will be explained from the best induced logic obtained in Pr2SATRA. After applying JFSM in the preliminary phase, only ten significant attributes were selected. Table 5 displays the results obtained based on the induced logic retrieved by Pr2SATRA of all the metrics in each fold. Notably, Attribute 19, labelled as Γ_d is the dependent attribute, while the other attributes are the independent attributes.

Table 5. The Results of Each Metric for All the Folds. The Bold Values Indicate the Best Fold for the Respective Metric

Metrics	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Average
ACC	**0.8734**	0.8687	0.8574	0.8443	0.8340	0.8555
PRE	0.4444	0.4615	**0.5455**	0.4000	0.4211	0.4545

Metrics	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Average
SPE	0.9893	0.9925	**0.9945**	0.9900	0.9877	0.9908
MCC	**0.1268**	0.1093	0.1171	0.0820	0.0940	0.1058
*F*1	**0.1060**	0.0790	0.0732	0.0674	0.0829	0.0817

As shown in Table 5, the proposed approach achieves consistently high accuracy and specificity across all folds, indicating strong performance in correctly identifying customers who do not churn. However, the relatively low *F1* and *MCC* reveal limited effectiveness in detecting churn customers. This performance imbalance is largely attributed to the class imbalance inherent in churn datasets, where customers who do not churn dominate the data distribution. Consequently, while the model demonstrates robustness in overall classification, its ability to capture the minority churn class remains constrained. Nevertheless, the main objective of the proposed approach is interpretable induced logic extraction rather than maximizing churn recall, thereby providing valuable insights into customer behaviour patterns.

Subsequently, Eqs. (25) – (29) represent the retrieved best Γ_{r2SAT}^{i} for all 5-fold cross validations.

$$\Gamma_{induced}^{Fold1} = (A_1 \vee \neg B_1) \wedge (\neg A_2 \vee B_2) \wedge (\neg A_3 \vee \neg B_3) \wedge (A_4 \vee \neg B_4) \wedge (C_1) \wedge (C_2), \tag{25}$$

$$\Gamma_{induced}^{Fold2} = (A_1 \vee B_1) \wedge (A_2 \vee \neg B_2) \wedge (A_3 \vee B_3) \wedge (A_4 \vee \neg B_4) \wedge (C_1) \wedge (C_2), \tag{26}$$

$$\Gamma_{induced}^{Fold3} = (A_1 \vee B_1) \wedge (A_2 \vee \neg B_2) \wedge (A_3 \vee B_3) \wedge (A_4 \vee \neg B_4) \wedge (C_1) \wedge (C_2), \tag{27}$$

$$\Gamma_{induced}^{Fold4} = (A_1 \vee B_1) \wedge (A_2 \vee \neg B_2) \wedge (A_3 \vee B_3) \wedge (A_4 \vee \neg B_4) \wedge (C_1) \wedge (C_2), \tag{28}$$

$$\Gamma_{induced}^{Fold5} = (A_1 \vee \neg B_1) \wedge (A_2 \vee B_2) \wedge (A_3 \vee B_3) \wedge (\neg A_4 \vee \neg B_4) \wedge (C_1) \wedge (C_2). \tag{29}$$

Based on the best induced logic for all the folds, Eq. (26) is chosen to represent the knowledge classification for the dataset because the induced logic is repeated, which depicts the consistency of the induced logic representing the patterns of the dataset. The possible entries that could lead to unsatisfied or produce a negative outcome are $(-1,-1,1,-1,-1,-1,-1,1,1,-1)$. From Table 4, P*r*2SATRA obtained a high *SPE* value. This shows that P*r*2SATRA can generate the best induced logic that minimizes *FP*. Then, the retrieved induced logic mainly represents the attributes that contribute to a negative outcome, which indicates that the customer does not churn. Therefore, several cases based on Eq. (26) that are related to the dataset are discussed as follows:

Case 1: Based on A_2, B_2, and C_2 where A_2 denotes total day calls, B_2 denotes the total day charge and C_2 denotes customer service calls.

THE customers are less likely to churn $\left(\Gamma_{induced}^{Fold2} = -1\right)$ when the number of customer service calls is less than two $(C_2 = -1)$ because the customers rarely encounter any issues with the telecom service. This low frequency of service calls suggests a high level of customer satisfaction. Despite the high number of total daytime calls $(A_2 = 1)$, the total day charge remains low $(B_2 = -1)$. From a customer perspective, this approach provides the benefits of reducing costs. This scenario demonstrates that a seamless, trouble-free experience combined with cost-efficient service encourages customer retention. A previous study shows that enhancing service quality, perceived value, and ensuring customer satisfaction are key factors in fostering loyalty within the telecommunications industry [19]. This underscores the importance of maintaining an efficient and customer-friendly service model.

Case 2: Based on A_1, A_4, B_4, and C_1 where A_1 denotes international plan, A_4 denotes total night charge, B_4 represents total international minutes, and C_1 represents the total international charge.

Some of the customers do not churn $\left(\Gamma_{induced}^{Fold2} = -1\right)$ even though the total international charge is high $(C_1 = 1)$ due to a lack of subscription to an international plan $(A_1 = -1)$. Despite not having an international plan, these customers often make international calls, resulting in high total international minutes $(B_4 = 1)$. This behaviour indicates that these customers prioritize the quality and convenience of their international communication over potential cost savings. Additionally, they display low total night charges $(A_4 = -1)$, suggesting a preference for cost-efficient usage during off-peak hours. Research suggests that customer loyalty can be influenced by the quality of products provided by service providers [20]. Additionally, factors such as network coverage and reception quality have been identified as potential reasons for customers to switch to competitors with wider coverage and superior reception quality. This highlights the importance of high-quality service to retain these customers.

Case 3: Based on B_1, A_3 and B_3 where B_1 represents total day minutes, A_3 represents total eve minutes and B_3 represents the total eve charge.

High total day minutes $(B_1 = 1)$ indicate that a customer relies heavily on the telecom services during daytime hours, which often align with work or business activities. For such customers, the reliability and quality of service during these peak hours are crucial. If the telecom provider delivers consistently high-quality service without interruptions, it builds trust and satisfaction. This reliability during critical usage periods makes the customer less likely to consider switching to another provider $\left(\Gamma_{induced}^{Fold2} = -1\right)$. In contrast, low total evening minutes $(A_3 = -1)$ suggest that the customer's need for telecom services significantly decreases during the evening. These customers might have lifestyles or work schedules that do not require heavy evening usage, meaning their needs are already being met by their current plan. Additionally, the low evening charges $(B_3 = -1)$ reduce the risk of receiving unexpected high bills. This predictability increases customer satisfaction, which can greatly strengthen customer trust. Trust is a crucial asset for a company in a competitive industry [21].

3.3.3 Recommendations Based on Induced Logic

The following recommendations are derived directly from the cases explained in the above subsection, where each action corresponds to specific behavioural patterns identified in the churn classification process. For Case 1, the induced logic indicates that long-term customers with frequent service interactions are less likely to churn when engagement and support accessibility are improved. Therefore, the telecom company is recommended to implement loyalty programs that reward long-term customers through discounts or additional benefits to enhance retention. In addition, promoting self-service options via online platforms or mobile applications, such as FAQs, chatbots, and tutorials, can reduce service friction and improve customer experience.

For Case 2, the induced logic reveals that churn behaviour is associated with international call usage patterns and billing uncertainty. Accordingly, the telecom company should introduce flexible international plans, such as unlimited international calling to selected countries at a fixed monthly fee for high-volume users, or pay-per-use plans without long-term commitments for occasional callers. These options provide cost transparency and better align with customer usage needs.

For Case 3, the induced logic suggests that customers with concentrated evening usage are sensitive to usage limits and cost overruns. To address this, the telecom company may encourage off-peak usage through targeted promotions and discounts during evening hours. Additionally, implementing usage alerts and monitoring tools can help customers manage their consumption and avoid unexpected charges, thereby improving overall satisfaction.

4. CONCLUSION

The main conclusions of this study are summarized as follows:

1 The proposed Pr2SATRA logic mining model achieves strong overall classification performance, particularly in terms of accuracy and specificity, enabling reliable identification of non-churn customers and reduction of false positives.
2 Pr2SATRA generates interpretable logical rules, enhancing transparency and supporting knowledge discovery in decision support applications such as telecommunications and potentially healthcare.
3 The model shows limited effectiveness in churn detection, as indicated by lower F1-score and MCC values, mainly due to class imbalance in churn datasets.
4 Future research should focus on addressing class imbalance and extending the framework to other logical structures and larger datasets to improve robustness and scalability.

Author Contributions

Nurul Atiqah Romli: Conceptualization, Writing-Review and Editing. Nurul Ain Najwa Mohamad Jamil: Data Curation, Methodology, Writing-Original Draft. Nur Ezlin Zamri: Formal Analysis, Resources, Validation. Mohd Shareduwan Mohd Kasihmuddin: Supervision, Visualization. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare no competing interests.

Declaration of Generative AI and AI-Assisted Technologies

The authors declare that no generative AI or AI-assisted technologies were used in the preparation of this manuscript, including writing, editing, data analysis, or the creation of tables and figures.

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