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MULTI-OBJECTIVE MIXED-INTEGER PROGRAMMING MODEL WITH BATTERY AND CHARGING CONSTRAINTS FOR ELECTRIC FEEDER BUS NETWORKS

Rini Yanti [1*], Parlindungan Kudadiri [2], Eka Setia Novi [3], Febria Marta Siska [4], Deshinta Arrova Dewi [5], R. Raja Subramanian [6]

[1,2]*Department of Informatics Engineering, Faculty of Engineering and Informatics, Universitas Sains dan Teknologi Indonesia*
[3]*Department of Business Law, Faculty of Economics, Education, and Law, Universitas Sains dan Teknologi Indonesia*
[4]*Department of International Business Management, Faculty Economics, Education, and Law, Universitas Sains dan Teknologi Indonesia*
Jln. Purwodadi Ujung, Pekanbaru, 28299, Indonesia

[5]*Center for Data Science and Sustainable Technologies, INTI International University*
Jln. Persiaran Perdana Bandar Baru Nilai, 71800, Malaysia

[5]*Faculty of Engineering and Technology, Shinawatra University*
Bang Toei, Sam Khok District, Pathum Thani, 12160, Thailand

[6]*Department of Computer Science and Engineering, Kalasalingam Academy of Research and Education*
Krishnankoil, Srivilliputhur, Tamil Nadu 626126, India

*Corresponding author's e-mail: * **riniyanti@usti.ac.id***

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ABSTRACT

The deployment of electric vehicle (EV)–based feeder bus networks is increasingly promoted to support sustainable urban transportation systems. However, their operational planning is challenged by limited battery capacity, charging time requirements, and restricted charging infrastructure, which introduce complex trade-offs between operational efficiency, energy consumption, and service coverage. This study aims to develop a Multi-Objective Mixed-Integer Programming (MOMIP) model that explicitly incorporates battery state-of-charge dynamics and charging station constraints for optimizing electric feeder bus networks. The proposed model simultaneously minimizes operational costs and total energy consumption while maximizing service coverage, enabling a comprehensive evaluation of conflicting operational objectives. The use of MOMIP is justified by the need to capture Pareto-optimal trade-offs among these competing objectives within a unified mathematical formulation. Numerical experiments based on hypothetical operational scenarios demonstrate that the model generates feasible Pareto-optimal solutions, revealing clear trade-offs between cost efficiency, energy usage, and network accessibility. Analysis further indicates that increasing charging capacity significantly enhances system performance, reducing energy consumption by more than 20% and improving service coverage by over 7 percentage points. The proposed model provides a robust decision-support tool for transport planners and contributes to the development of energy-efficient and sustainable electric feeder bus operations.

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1. INTRODUCTION

The rapid growth of urban populations has intensified demand for sustainable, efficient public transportation systems. In recent years, electric vehicle (EV) technology has emerged as a promising alternative to conventional fossil-fuel-based transport due to its potential to reduce greenhouse gas emissions, mitigate air pollution, and lower operational costs [1]. Electric feeder bus networks play a crucial role in bridging local demand nodes with major transit corridors [2]. Therefore, enhancing accessibility and operational efficiency in transportation systems is critical for ensuring the long-term sustainability of urban transit networks. Within this domain, electric feeder bus networks serve as critical connectors between local demand nodes and major transit corridors, playing a pivotal role in enhancing the integration and coverage of urban transit systems [3]. Ensuring both accessibility and operational efficiency in such networks is essential to achieving the long-term sustainability of metropolitan transportation infrastructure [4].

The integration of battery-electric buses into feeder networks, however, introduces complex operational and planning challenges. These include limited driving range, extended charging durations, and the necessity for strategically located charging infrastructure [5]. While existing studies have developed various optimization models for scheduling and routing electric buses, the majority have primarily addressed single-objective formulations, typically focusing on either cost minimization or environmental impact reduction in isolation [6]. Such approaches often overlook the inherent trade-offs between multiple, and sometimes conflicting, operational objectives [7]. Furthermore, a critical shortcoming is the limited incorporation of battery degradation dynamics and detailed charging constraints into network-level planning [8]. Neglecting these factors can lead to operational plans that are either suboptimal or infeasible in real-world deployment scenarios.

To address these gaps, this study proposes a Multi-Objective Mixed-Integer Programming (MOMIP) model that explicitly integrates battery performance characteristics, charging constraints, and state-of-charge dynamics into the planning of electric feeder bus networks [9]. The model simultaneously optimizes multiple objectives, including minimizing operational costs, reducing energy consumption, and maximizing service accessibility. Unlike conventional approaches, the proposed model holistically captures the interactions between route assignment, charging schedules, and battery constraints within a unified mathematical formulation, thereby providing a more realistic and adaptable decision-support tool for transport planners operating in battery-constrained environments.

The research involves a Multi-Objective Mixed-Integer Programming (MOMIP) model for optimizing electric feeder bus networks, which involves several stages. It begins with a literature review to build on existing knowledge and identify gaps in current methods. The problem definition outlines the network components, assumptions, and objectives. The model formulation step creates a mathematical model to address these objectives, incorporating constraints. Model validation ensures feasibility through dimensional analysis and testing against hypothetical scenarios. The discussion interprets the results, compares the model's performance with existing approaches, and addresses limitations. Finally, the conclusion summarizes the findings and suggests future research directions.

2. RESEARCH METHODS

1.1 Problem Definition and Assumptions

The feeder bus network under consideration connects peripheral regions (suburban or rural areas) to central transit hubs (such as bus terminals, metro stations, or railway stations). The feeder buses, operating with electric vehicles, serve to enhance public transportation accessibility while minimizing environmental impact and operational costs. The components of the network include:

1. Feeder bus fleet, that the fleet size is determined based on demand, service frequency, and available charging infrastructure. Each bus in the fleet has an associated battery capacity and range, limiting the distance it can travel before requiring a recharge.
2. Service routes that connect the peripheral areas (origin nodes) to central transit hubs (destination nodes). These routes are designed to ensure maximum coverage and accessibility while minimizing operational costs and energy consumption.

3. Nodes that represent key locations in the network. There are origin nodes (locations in peripheral areas where buses pick up passengers) and destination nodes (central transit hubs where passengers transfer to larger transit systems)
4. Charging stations where the buses can recharge. Charging stations are characterized by their capacity (the maximum number of buses that can charge simultaneously) and charging rate (the rate at which energy is transferred to the bus battery).

To simplify the modeling process, the following assumptions are made.

1. The fleet size is assumed to be fixed and known in advance, based on estimated passenger demand and service frequency requirements,
2. each charging station has a fixed charging rate,
3. each electric bus has a fixed battery capacity,
4. the travel time between nodes is assumed to be fixed and deterministic,
5. the passenger demand at each origin node is known and remains constant,
6. charging stations are assumed to be available at fixed locations with known capacities.

2.2 Sets, Parameters, and Decision Variables

In order to make a model formulation, we have to define the following sets, indices, parameters, and decision variables.

1. Sets:

   $\mathcal{N}$: set of all nodes in the network, including origin, destination, and charging stations;

   $\mathcal{R}$: set of all feeder bus routes;

   $\mathcal{F}$: set of all buses in the fleet;

   $\mathcal{C}$: set of all charging stations.

2. Indices:

   $i, j \in \mathcal{N}$: indices representing nodes;

   $r \in \mathcal{R}$: index for routes;

   $f \in \mathcal{F}$: index for buses;

   $c \in \mathcal{C}$: index for charging stations.

3. Parameters:

   c_{ij} : cost to travel from node i to node j;

   d_i : demand at origin node i;

   B : battery capacity of each bus (in kWh);

   r_j : charging rate at charging station j (kW);

   t_{ij} : travel time between nodes i and j;

   α : energy consumption per distance unit (kWh per km);

   S_j : capacity of charging station j (number of buses that can charge simultaneously);

   Q_f : capacity of bus f (maximum number of passengers it can carry);

   T_{max} : maximum total service time for any bus (from the start of the service to the end of its scheduled shifts).

4. Decision Variables:

   $x_{ij}^{f} \in \{0,1\}$: binary decision variable indicating whether bus f travels from node i to node j (1 if yes, 0 otherwise);

$y_{fc} \in \{0,1\}$: binary decision variable indicating whether bus f is assigned to charging station c (1 if yes, 0 otherwise);

z_f : total energy consumed by bus f over its route;

p_i^f : number of passengers served by bus f at node i.

2.3 Model Formulation

2.3.1 Electric Vehicle-Based Feeder Bus Systems

A feeder bus system is a public transport solution designed to connect suburban or remote areas to major transit hubs, such as train stations or bus terminals, using smaller vehicles [10]. It enhances accessibility to public transportation by providing convenient connections to high-capacity transit systems, reducing reliance on private cars, and helping alleviate traffic congestion. Feeder bus systems are cost-effective, sustainable, and improve public transport ridership, especially in less densely populated areas [11] . However, they require careful coordination with larger transit networks, efficient route planning, and consistent fleet management to ensure effectiveness [12].

Electric Vehicle-Based Feeder Bus Systems (EV-FBS) are increasingly recognized as a sustainable and efficient transportation solution for urban mobility. These systems leverage electric vehicles (EVs) to serve as feeder buses in urban transit networks, connecting suburban or remote areas to main transportation hubs, such as subway stations or bus terminals [13]. The transition to electric feeder buses not only supports the global shift towards sustainability but also reduces the environmental footprint of public transportation. The use of electric buses in urban feeder systems has gained significant attention due to their potential to reduce greenhouse gas emissions, lower operational costs, and improve air quality [14]. A key challenge in implementing Electric Vehicle-Based Feeder Bus Systems is integrating EVs into existing transport networks while ensuring they can operate efficiently over long distances with limited battery life. Feeder bus networks, by definition, serve as connectors between the main transportation grid and less accessible regions, thereby requiring precise planning to ensure EVs can operate effectively without running out of battery before reaching charging stations.

The adoption of electric vehicles in feeder bus systems not only reduces reliance on fossil fuels but also enhances the overall energy efficiency of urban mobility [15]. However, integrating these systems presents a number of logistical and technological challenges, such as managing the battery life of electric buses, ensuring optimal routing, and providing sufficient charging infrastructure. The optimal scheduling and routing of electric feeder buses are complex tasks that require accounting for variables such as battery constraints, energy consumption, charging times, and real-time traffic conditions.

One of the most critical elements in the design of Electric Vehicle-Based Feeder Bus Systems is addressing the battery and charging constraints [16]. The limited range of electric buses necessitates the inclusion of charging stations along routes, introducing new operational constraints in terms of time and location. The importance of strategically placing charging stations to minimize bus downtime and optimize energy use. Efficient battery management is essential for maintaining the operational feasibility of these systems, as electric buses may need to recharge multiple times throughout the day.

2.3.2 Mixed-Integer Programming in Public Transport Planning

Mixed-Integer Programming (MIP) is a type of mathematical optimization problem that involves both continuous and integer decision variables. It is widely used in fields such as operations research, logistics, finance, and engineering. The general form of a MIP problem is structured as Eq. (1)

$$Minimize\ f(x) = c^T x, \tag{1}$$

where:

$x : (x_1, x_2, \dots, x_n)$ is the vector of decision variables;

$c : (c_1, c_2, \dots, c_n)$ is a vector of coefficients;

$c^T x$ represents the objective function, which is to be minimized or maximized (depending on the problem).

The decision variables are subject to a set of constraints, which typically include as Eq. (2)

$$\begin{aligned} Ax &= b \quad (Equality\ Constraints) \\ A_{ineq}x &\leq b_{ineq} \quad (Inequality\ Constraints) \\ x_{lower} \leq x &\leq x_{upper} \quad (Variable\ Bounds), \end{aligned} \tag{2}$$

where:

A is an $m \times n$ matrix of coefficients;

b is a vector of length m;

A_{ineq} is an $m_{ineq} \times n$ matrix of coefficients;

b_{ineq} is a vector of length m_{ineq};

x_{lower} and x_{upper} are the lower and upper bounds for the variables x;

$x_i \in \mathbb{R}$ continuous variables);

$x_i \in \mathbb{Z}$ (integer variables); or

$x_i \in \{0,1\}$ (binary variables).

The objective function $f(x) = c^T x$ could represent various operational goals such as cost minimization, profit maximization, or efficiency. Constraints in MIP ensure that the decision variables satisfy certain conditions. The inclusion of both continuous and integer variables makes MIP particularly suited for problems where decisions involve both numerical quantities (e.g., distance, time, or cost) and discrete choices (e.g., selecting a specific route or a particular facility). MIP has emerged as a robust tool for solving various complex problems in public transport, including vehicle routing, scheduling, and resource allocation [17].

2.3.3 Vehicle Routing

The Vehicle Routing Problem (VRP) is one of the primary applications of MIP in public transport. In the VRP, the objective is to determine the optimal set of routes for a fleet of vehicles that serve a set of locations while minimizing the total distance or time traveled, subject to various constraints [18]. The standard Mixed-Integer Programming formulation for the VRP is stated as Eq. (3)

$$Minimize\ Z = \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} x_{ij}\ , \tag{3}$$

where:

c_{ij} is the travel cost (distance or time) from location i to location j;

x_{ij} is a binary variable that equals 1 if a vehicle travels from location i to location j, and 0 otherwise.

Subject to the following constraints as Eq. (4) as vehicle flow constraint:

$$\begin{aligned} \sum_{j=1}^{N} x_{ij} = 1 \quad \forall i \in \{1, 2, \dots, N\} \\ \sum_{i=1}^{N} x_{ji} = 1 \quad \forall j \in \{1, 2, \dots, N\} \end{aligned}\ , \tag{4}$$

and Eq. (5) as capacity constraint

$$\sum_{i=1}^{N} q_i x_{ij} \leq Q \quad \forall j \in \{1, 2, \dots, N\}, \tag{5}$$

where:

q_i is the demand at location i;

Q is the vehicle capacity.

These formulations ensure that each location is visited exactly once and that the vehicle capacity is not exceeded. The decision variables x_{ij} represent the assignment of vehicles to travel between specific locations, optimizing for travel distance or time.

2.3.4 Fleet Scheduling

Fleet scheduling involves assigning vehicles to routes over a planning horizon, typically while respecting various operational constraints such as crew working hours, maintenance schedules, and vehicle availability. The objective is to minimize operational costs while maximizing service efficiency and ensuring regulatory compliance. A Mixed-Integer Programming formulation for fleet scheduling can be represented as Eq. (6) [19]

$$Minimize\ Z = \sum_{t=1}^{T}\sum_{v=1}^{V}(Operating\ Cost_v(t) \cdot y_{vt}), \tag{6}$$

where:

T is the set of time periods in the scheduling horizon;

V is the set of vehicles;

y_{vt} is a binary decision variable that equals 1 if vehicle v is assigned to time period t, and 0 otherwise.

Objective function in Eq. (6) subject to vehicle assignment constraints represented as Eq. (7)

$$\sum_{v=1}^{V} y_{vt} = 1 \quad \forall t \in \{1, 2, \dots, T\}, \tag{7}$$

and crew and vehicle constraints as (Eq. (8)

$$\sum_{t=1}^{T} y_{vt} \cdot Working\ Hours_v(t) \leq Max\ Working\ Hours_v \quad \forall v \in \{1, 2, \dots, V\}. \tag{8}$$

This constraint ensures that crew working hours comply with legal regulations, and vehicles are scheduled within available operating hours.

2.3.5 Charging Infrastructure for Electric Vehicles

The integration of electric vehicles (EVs) in public transport presents new challenges related to battery life and charging infrastructure [20]. MIP formulations are used to optimize the placement of charging stations, schedule charging times, and allocate vehicles to ensure efficient operation throughout the day. A Multi-Objective Mixed-Integer Programming formulation for EV charging can be expressed as Eq. (9)[21].

$$Minimize\ Z = \sum_{k=1}^{K}\sum_{i=1}^{N}\left(c_{ij} \cdot x_{ij} + \lambda \cdot Charging\ Cost_i \cdot y_i\right), \tag{9}$$

where:

c_{ij} is the travel cost from location i to location j;

x_{ij} is a binary variable that equals 1 if a vehicle travels from location i to location j;

$Charging\ Cost_i$ is the cost of charging at station i;

y_i is a binary decision variable that equals 1 if charging station i is used, and 0 otherwise;

λ is a weighting factor that balances the cost of charging with the operational cost of vehicle routing.

Objective function in Eq. (9) is subject to charging constraints as Eq. (10).

$$\sum_{i=1}^{N} y_i \cdot Charging\ Time_i \leq Available\ Charging\ Time. \tag{10}$$

This ensures that the total charging time required for all vehicles does not exceed the available charging time within a given time window.

2.3.6 Battery and Charging Constraints in Optimization Models

Battery and charging constraints are central to the operation of electric vehicles. These constraints arise from the inherent limitations of battery technology, including energy capacity, charge and discharge rates, charging times, and state-of-charge (SOC) requirements. Furthermore, the availability and capacity of charging stations add another layer of complexity, as these facilities must be efficiently integrated into the vehicle fleet's operational schedule. Battery constraints consist of capacity constraints and State-of-Charge (SOC) Constraints [22]. The limited energy storage capacity of batteries dictates the distance that an EV can travel before recharging is required. This limitation, referred to as the range constraint, is a critical factor in the planning and scheduling of electric vehicles. In optimization models, the battery capacity B is often represented as a parameter that limits the maximum energy a vehicle can consume during a route, as represented in Eq. (11).

$$\sum_{i=1}^{n} d_{ij} \cdot x_{ij} \leq B, \quad \forall j \in \{1, 2, \dots, n\}, \tag{11}$$

where d_{ij} represents the distance between locations i and j,, and x_{ij} is a binary decision variable indicating whether the vehicle travels between locations i and j.

The SOC of a vehicle's battery must be carefully managed to ensure that it is neither overcharged nor depleted beyond a safe threshold. These constraints are particularly important in optimization models for fleet management, where real-time monitoring of battery levels ensures operational reliability. The SOC is modeled as Eq. (12).

$$SOC_i(t+1) = SOC_i(t) - E_i(t) + C_i(t), \tag{12}$$

where:

$SOC_i(t)$ is the state-of-charge at time t;

$E_i(t)$ is the energy consumption of vehicle i at time t;

$C_i(t)$ is the energy replenished by charging at time t.

Charging constraints include charging rate and time constraints, as well as charging station availability. Charging times are often modeled based on the charging rate r, which can vary depending on factors such as the vehicle's battery health and the charging infrastructure's capabilities. A typical charging constraint is represented as Eq. (13).

$$C_{ij}(t) \leq r_{ij}(t) \cdot \Delta t, \tag{13}$$

where:

$C_{ij}(t)$ is the charge delivered to vehicle i at station j during time period t;

$r_{ij}(t)$ is the charging rate at station j for vehicle i at time t;

Δt is the time period during which charging occurs.

The availability and capacity of charging stations are critical constraints. Multiple vehicles might need to recharge at different times, requiring optimization models to determine the optimal allocation of vehicles to available charging stations. This constraint ensures that the total number of vehicles assigned to each station does not exceed the station's capacity. The typical formulation is represented as Eq. (14) as follows.

$$\sum_{i=1}^{n} x_{ij} \leq S_j, \quad \forall j \in \{1, 2, \dots, m\}, \tag{14}$$

where:

x_{ij} is a binary decision variable indicating whether vehicle i is assigned to station j;

S_j is the capacity of charging station j.

2.4 Multi-Objective Optimization in Transport Systems

Multi-Objective Optimization (MOO) refers to optimization problems involving more than one objective function that need to be simultaneously optimized. MOO is applicable when the system under consideration has multiple, often conflicting goals. MOO aims to find solutions that strike a balance between these objectives, providing decision-makers with a set of trade-offs rather than a single optimal solution. Multi-objective optimization has broad applications across various fields, including engineering, economics, logistics, transportation, and resource management. In these areas, decision-makers often face the challenge of optimizing multiple competing objectives, each with its own constraints and requirements. MOO is crucial for handling such complex problems, especially when trade-offs are needed to find the most feasible or practical solution. In MOO, solutions are evaluated based on Pareto optimality. A solution is considered Pareto optimal if there is no other solution that improves one objective without worsening another [23]. This leads to a set of solutions known as the Pareto front, which represents the best trade-offs between the different objectives. A decision-maker must choose a solution from this Pareto front, depending on their specific preferences and priorities.

In mathematical terms, a general MOO Problems can be formulated as Eq. (15) as follows [24]

$$Minimize\ f(x) = \{f_1(x), f_2(x), \dots, f_k(x)\}, \tag{15}$$

with

$x = (x_1, x_2, \dots, x_n\ )$ is the vector of decision variables;

$f_i(x)$ represents the i-th objective function (for $i = 1, 2, \dots, k$) that needs to be minimized or maximized.

The objective function on Eq. (15) subject to Eq. (16) as follows

$$\begin{aligned} g_j(x) \le 0, \quad \forall j \in = 1, 2, \dots, m \\ h_i(x) = 0, \quad \forall i = 1, 2, \dots, p \end{aligned} \tag{16}$$

with

$g_j(x)$ represents the inequality constraints;

$h_i(x)$ represents the equality constraints;

$x \in \mathbb{R}^n$ represents the decision variables that can be continuous or integer.

Transport systems are inherently complex, as they involve coordinating multiple elements, including vehicles, infrastructure, passengers, and environmental considerations. MOO helps address these complexities by offering decision-makers a range of solutions that balance the objectives in different ways. The vehicle routing problem (VRP) is one of the most common transportation optimization problems. It involves finding the optimal set of routes for a fleet of vehicles to serve a set of customers or destinations. In multi-objective vehicle routing, the objectives could include minimizing travel distance or time, fuel consumption, emissions, and service time or waiting time [25]. MOO is widely used in public transport scheduling, where the goal is to allocate buses or trains to various routes and time slots. Multiple objectives may include minimizing passenger waiting time, maximizing service frequency, and minimizing operational costs. In this case, MOO helps determine the best balance between service quality and cost efficiency [26].

A Multi-Objective Mixed-Integer Programming (MOMIP) model is a mathematical optimization that simultaneously addresses multiple conflicting objectives while considering decision variables that can be either continuous or discrete (integer or binary). This model is valid if all constraints are satisfied, decision variables are correctly defined, the model is mathematically sound, and solutions are feasible in hypothetical or real-world scenarios. A feasible solution exists if the model lies on the Pareto Front, and the solution is called Pareto optimal.

2.5 Multi-Objective Mixed-Integer Programming Model

The problem aims to optimize three main objectives simultaneously. The objective functions in a Multi-Objective Mixed-Integer Programming (MOMIP) model for electric feeder bus networks are designed to address key performance indicators that reflect both operational and sustainability goals of the transportation system. The following explanation explains why the objective function and its constraints were chosen.

1. Minimize Operational Costs (Objective 1)

   Operational costs are a fundamental concern for any transportation system, and for an electric feeder bus network, they include vehicle operation, energy consumption, and maintenance expenses. The operational costs are directly tied to the distance traveled and the energy consumed by the buses. For electric buses, charging costs also play a critical role, which is why energy consumption (and its associated costs) is an important factor in this objective.

2. Minimize Total Energy Consumption (Objective 2)

   Energy consumption is the most significant environmental impact of transportation systems. For electric buses, the focus is on reducing electricity consumption for travel, which directly affects emissions (if the electricity comes from non-renewable sources)

3. Maximize Service Coverage or Accessibility (Objective 3)

   Service coverage and accessibility are vital for the effectiveness of the feeder bus network. Maximizing coverage means ensuring that buses reach as many origin nodes (in suburban or rural areas) as possible, thereby providing essential connections to central hubs. Accessibility also improves passenger convenience, making public transportation a more attractive option and potentially reducing reliance on private cars, which aligns with sustainability goals.

The constraints in the MOMIP model are critical to ensuring that the feeder bus network operates within practical and operational limits while achieving the objectives.

1. Network and Scheduling

   The network and scheduling constraints ensure that each bus follows a feasible route and adheres to service requirements. It guarantees that the bus network is both operationally effective and logistically feasible, serving all necessary nodes (origin and destination). This constraint ensures that each bus travels between nodes in a valid, scheduled manner, adhering to the feeder system's connectivity requirements.

2. Battery State-of-Charge Dynamics

   This constraint ensures that the battery is monitored dynamically, maintaining a feasible SOC throughout the trip. The SOC constraint accounts for the energy consumed during travel and ensures that the battery is charged at the correct intervals.

3. Charging Time and Station Availability

   The charging time and station availability constraints ensure that buses are charged efficiently without exceeding charging station capacity. Charging stations have limited $(S_j)$and charging rate (r_j), so this constraint ensures that buses do not exceed the station’s charging limits and that they charge in a reasonable time frame to minimize downtime.

4. Fleet Size and Capacity Limits

   The fleet size and capacity limit constraints ensure that the number of buses and their passenger capacity do not exceed the operational limits. It also ensures that each bus operates within its passenger capacity, which is critical for maximizing service and efficiency while balancing demand and fleet size.

Mathematical representations of MOMIP model for electric feeder bus networks with battery and charging constraints can be stated as Eq. (17) and Eq. (18) as follows

$$\begin{aligned} Minimize\ Z_1 &= \sum_{f\in F}\sum_{i,j\in N} c_{ij}x_{ij}^{f} + \sum_{f\in F} z_f \\ Minimize\ Z_2 &= \sum_{f\in F} z_f \\ Maximize\ Z_3 &= \sum_{i\in N}\sum_{f\in F} p_i^{f} \end{aligned} \quad . \tag{17}$$

Subject to

$$\sum_{j\in N} x_{ij}^{f} = 1 \quad \forall f \in F, \forall i \in N$$
$$SOC_f(t+1) = SOC_f(t) - \alpha \cdot d_f(t) + \sum_{c\in C} y_{fc} \cdot r_c \cdot \Delta t$$
$$\sum_{f\in F} y_{fc} \leq S_j \quad \forall c \in C \qquad . \qquad (18)$$
$$\sum_{i\in N} p_i^f \leq Q_f \quad \forall f \in F$$

2.6. Model Validation Procedure

Validity ensures that all model constraints produce solutions that can actually be implemented. A valid model provides results that accurately reflect the system being studied, enabling transport planners to make data-driven decisions that balance cost, energy consumption, and service quality. Checking validity confirms that equations, units, and variable definitions are consistent, preventing errors that could distort optimization outcomes.

2.6.1 Dimensional Analysis Checking

Dimensional analysis is a method used to ensure the consistency of units and measurements within the model. This process involves checking that all terms in the model's objective functions, constraints, and variables have consistent units so that the model's formulations are physically meaningful and mathematically valid.

1. Units for each parameter and variable.

   Operational costs are represented in monetary units (dollars). Travel time is in hours, and the cost per hour of travel is in dollars/hour; the operational cost should be in dollars. Energy is usually measured in kWh (kilowatt-hours), as electric buses consume energy to travel certain distances. If the energy consumption per bus per distance is denoted by α (kWh/km), the energy consumed for a bus traveling a distance d (in km) should be $\alpha \cdot d$, and this product should result in kWh.

2. Units in Objective Functions

   In operational costs, the objective function involves terms travel costs c_{ij} per km has units of dollar/km and energy consumption z_f has units of kWh, which needs to be converted into dollars using the cost of electricity.

3. Constraints on dimensional consistency

   Battery capacity B is in kWh, and energy consumption z_f is also in kWh, battery usage is mathematically consistent with the battery's energy consumption rate during travel. Travel time t_{ij} (in hours) multiplied by the number of vehicles should not exceed the available operational time for a bus.

2.6.2 Constraint Satisfaction Checking

To check constraint satisfaction of the model, we need to systematically evaluate whether all defined constraints are met by the solutions generated by the model.

1. Network and Scheduling Constraints

   These ensure that buses are correctly assigned to routes and scheduled properly. Each bus must serve only assigned routes and no route is left unattended unless explicitly allowed. The buses meet the minimum and maximum frequency requirements per route and adhere to time windows. For each bus f and route r

$$\sum_{f\in F} x_{fr}(t) \geq mimimum\ frequency\ requirment, \forall r \in R, \forall t,$$

   with $x_{fr}(t) \in \{0,1\}$ indicates if bus f serves route r at time t.

2. Battery State-of-Charge (SoC) Dynamics

   These constraints ensure that buses never run below minimum battery levels and can complete assigned trips. For Battery consumption, calculate SoC after each trip using

$$SOC_f(t+1) = SOC_f(t) - \frac{E_{trip}}{C_{battery}},$$

   where E_{trip} is the energy consumed for the trip, and $C_{battery}$ is the battery capacity. And

$$SOC_f(t) \geq SOC_{min}, \qquad \forall f, \forall t,$$

   ensuring no bus runs out of energy mid-trip.

3. Charging Time and Station Availability

   These constraints ensure that charging operations are feasible and do not exceed station capacity. Each bus must respect station-specific charging rates

$$SOC_f(t+1) = SOC_f(t) + r_{charge} \cdot \Delta t,$$

   and the number of buses charging does not exceed the number of charging ports $\mathbb{C}$

$$\sum_{f \in \mathbb{F}} y_{f,c}(t) \leq \mathbb{C}, \forall c \in \mathbb{C},$$

   where $y_{f,c}(t) \in \{0,1\}$ if bus f is charging at station c at time t.

4. Fleet Size and Capacity Limits

   These constraints ensure operational feasibility in terms of fleet availability and passenger capacity. Total buses assigned must not exceed fleet size

$$\sum_{f \in \mathbb{F}} \sum_{r \in \mathbb{R}} x_{fr}(t) \leq N_{fleet}, \forall t,$$

   and bus capacity must meet passenger demand

$$\sum_{r \in \mathbb{R}} d_r(t) \leq \sum_{f \in \mathbb{F}} Cap_f \cdot x_{fr}(t), \ \forall t,$$

   where $d_r(t)$ is the passenger demand on route r at time t, and Cap_f is bus f's seating capacity.

The performance of the proposed MOMIP model is evaluated through numerical experiments, as presented in the following section.

3. RESULTS AND DISCUSSION

This section presents the numerical results obtained from solving the proposed multi-objective optimization model.

3.1 Numerical Simulation

The test network consists of 10 feeder routes served by a fleet of 12 electric buses, operating over a single daily planning horizon. Each bus is equipped with 300 kWh battery, with a minimum allowable state-of-charge (SoC) of 20%. The average energy consumption rate is set to 1.2 kWh/km, and charging is permitted only at terminal nodes equipped with 60 kW chargers. Three charging capacity scenarios are considered, with 1 to 3 chargers per terminal, to examine the impact of infrastructure availability. Operational costs include a fixed daily cost of USD 180 per bus, a variable operating cost of USD 1.1 per km, and an electricity price of USD 0.12 per kWh. Service coverage is defined as the percentage of feeder demand satisfied by the scheduled operations. Three representative solutions are selected to illustrate the extreme and balanced outcomes of the optimization process, as summarized in Table 1.

Table 1. Pareto Optimal Solutions

Type of Solution	Operational Cost (USD/day)	Energy Consumption (kWh/day)	Energy Cost (USD/day)	Service Coverage (%)
Cost-oriented (S1)	4.76	4.12	494	81.5
Balanced (S2)	5.08	3.46	415	88.9
Coverage-oriented (S3)	5.52	2.95	354	93.8

The cost-oriented solution (S1) in Table 1 achieves the lowest daily operational cost of USD 4,760, but results in the highest energy consumption (4,120 kWh/day) and the lowest service coverage (81.5%). In contrast, the coverage-oriented solution (S3) prioritizes service quality, achieving a coverage level of 93.8%. This improvement is associated with an increase in operational cost to USD 5,520 per day. Notably, despite higher cost, energy consumption is reduced to 2,950 kWh/day, indicating that optimized routing and charging schedules can improve energy efficiency even under expanded service coverage. The balanced solution (S2) represents a compromise among the three objectives. With a daily operational cost of USD 5,080, this solution reduces energy consumption by approximately 16.0% compared to the cost-oriented solution, while increasing service coverage by 7.4 percentage points. This solution demonstrates an effective trade-off between economic efficiency and service performance.

3.2 Pareto Front Analysis

The complete set of non-dominated solutions obtained from the solver is illustrated through the Pareto front shown in Fig. 1.

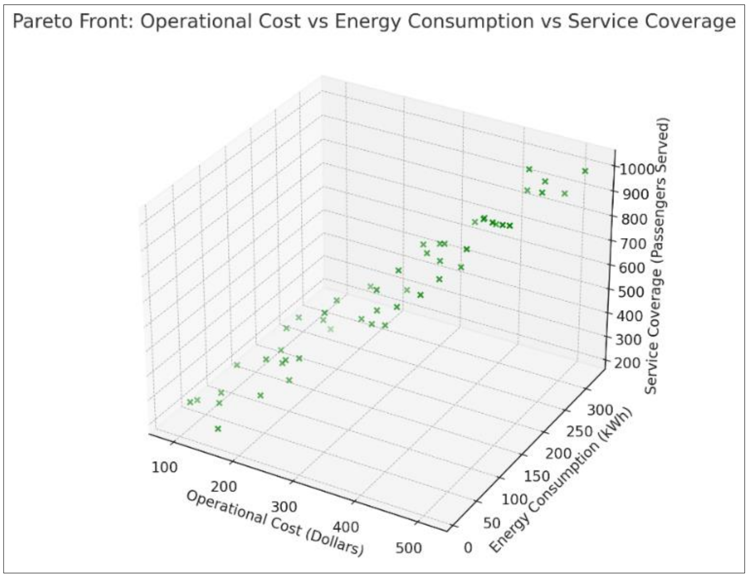

Figure 1. Pareto Front Feasible Solution for Three Objectives Simultaneously

Fig. 1 shows that each point in the scatter plot corresponds to a potential solution, with the goal of minimizing operational costs and energy consumption while maximizing service coverage. The Pareto-optimal solutions lie on the trade-off surface, illustrating the set of optimal solutions that offer different balances between these competing objectives. From the extreme Pareto solutions, the solver results indicate that energy consumption can be reduced by approximately 28.4% and service coverage increased by 12.3 percentage points when moving from the cost-oriented to the coverage-oriented solution. These results quantitatively demonstrate the conflict between the objectives and highlight the importance of multi-objective optimization in electric feeder bus operations.

3.3 Impact of Charging Infrastructure Capacity

To evaluate the sensitivity of the optimization results to charging infrastructure availability, additional simulations were conducted under different charging capacity scenarios. The results are summarized in Table 2.

Table 2. Impact of Charging Capacity on Optimization Results

Charging Conditions	Chargers per Terminal	Operational Cost (USD/day)	Energy Consumption (kWh/day)	Service Coverage (%)
Limited capacity	1	5.43	3.98	84.2
Moderate capacity	2	5.08	3.46	88.9
Enhanced capacity	3	4.94	3.12	91.6

The results in Table 2 show that increasing charging capacity leads to a consistent reduction in energy consumption and a significant improvement in service coverage. When the number of chargers per terminal increases from 1 to 3, energy consumption decreases by approximately 21.6%, while service coverage improves by 7.4 percentage points. At the same time, operational costs slightly decrease due to reduced waiting times and more efficient vehicle scheduling.

3.4 Discussion

The numerical results confirm that the proposed multi-objective mixed-integer programming model effectively captures the trade-offs among operational cost, energy consumption, and service coverage in electric feeder bus operations. The Pareto-optimal solutions indicate that improvements in energy efficiency and service coverage come at higher operational costs, underscoring the need for a multi-objective perspective. The balanced solution demonstrates that moderate cost increases can yield substantial reductions in energy consumption and meaningful service improvements, providing practical insights for transport operators. The availability of charging infrastructure has a significant impact on system performance. Scenario-based results reveal that increasing charging capacity improves operational flexibility, reduces total energy consumption, and enhances service coverage without proportionally increasing operational cost. This indicates that charging constraints are a critical bottleneck in electric feeder bus systems and should be jointly considered with operational planning rather than treated as exogenous factors.

Recent studies have explored multi-objective optimization within the context of Discrete Hopfield Neural Networks (DHNN) and logic mining, particularly for satisfiability problems and supervised learning tasks. From a methodological perspective, the proposed model differs from existing multi-objective models in Table 3.

Table 3. Comparison of Multi-Objective Perspectives

Aspect	Proposed Model	DHNN + Modified NGA [27]	DHNN + Election Algorithm [28]	Multi-unit DHNN [29]
Problem domain	Electric feeder bus network optimization	Logic mining & satisfiability learning	Logical rule optimization	Supervised logic-based learning
Multi-objective formulation	Explicit mathematical objectives (cost, energy, coverage)	Fitness-based objectives	Fitness-based objectives	Performance-based objectives
Decision variables	Discrete & continuous (routing, charging, fleet)	Neural states & logic variables	Neural states & logical clauses	Neural weights & logic units
Constraint handling	Explicit (battery SoC, charging capacity)	Implicit via fitness	Implicit via fitness	Implicit
Pareto-optimal solutions	Explicit Pareto front	Implicit trade-offs	Implicit trade-offs	Not explicitly Pareto-based
Suitability for transport planning	High	Low	Low	Very limited

Based on Table 3, the proposed MOMIP framework complements existing DHNN-based multi-objective methods by addressing a distinct class of multi-objective problems characterized by operational feasibility, infrastructure dependency, and decision interpretability.

This study is limited by the use of hypothetical operational data and a static planning horizon. Future research will focus on large-scale real-world datasets, dynamic and stochastic extensions, and hybrid integration with data-driven or learning-based approaches.

4. CONCLUSION

This study proposes a Multi-Objective Mixed-Integer Programming model for electric feeder bus networks, integrating battery dynamics, charging constraints, and fleet limitations as Eq. (18). The model effectively captures trade-offs between operational costs, energy consumption, and service coverage as Eq. (17), producing feasible Pareto-optimal solutions that provide actionable insights for urban transport planning. Numerical results demonstrate that the proposed model generates well-distributed Pareto-optimal solutions, revealing clear trade-offs among operational cost, energy consumption, and service coverage. Compared to cost-oriented solutions, coverage-oriented strategies reduce daily energy consumption by up to 28.4% while improving service coverage by approximately 12.3 percentage points, at the expense of higher operational costs. The balanced Pareto solution shows that a moderate increase in daily cost (approximately 6–7%) can achieve a substantial reduction in energy consumption (about 16%) while increasing service coverage to nearly 89%. Scenario-based analysis further indicates that increasing charging capacity from one to three chargers per terminal reduces energy consumption by more than 20% and improves service coverage by over 7 percentage points. These quantitative findings confirm that the proposed MOMIP framework provides an effective and interpretable decision-support tool for electric feeder bus planning, offering advantages over learning-oriented multi-objective approaches by explicitly modeling operational and infrastructure constraints.

Author Contributions

Rini Yanti: Conceptualization, Formal Analysis, Investigation, Methodology, Validation, Writing-Original Draft. Parlindungan Kudadiri: Data Curation, Resources, Software, Visualization, Writing - Review and Editing. Eka Setia Novi: Data Curation, Project Administration, Writing - Review and Editing. Febria Marta Siska: Data Curation, Project Administration, Writing - Review and Editing. Deshinta Arrova Dewi: Funding Acquisition, Supervision, Writing - Review and Editing. R. Raja Subramanian: Supervision, Writing - Review and Editing. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare that no potential conflicts of interest exist in relation to the research, authorship, or publication of this article.

Declaration of Generative AI and AI-assisted technologies

Generative AI tools (e.g., ChatGPT) were used solely for language refinement (grammar, spelling, and clarity). The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. The authors reviewed and approved all final text.

REFERENCES

[1] I. Veza, M. Z. Asy'ari, M. Idris, V. Epin, I. M. Rizwanul Fattah, and M. Spraggon, "ELECTRIC VEHICLE (EV) AND DRIVING TOWARDS SUSTAINABILITY: COMPARISON BETWEEN EV, HEV, PHEV, AND ICE VEHICLES TO ACHIEVE NET ZERO EMISSIONS BY 2050 FROM EV," *Alexandria Engineering Journal*, vol. 82, pp. 459–467, Nov. 2023, doi: https://doi.org/10.1016/j.aej.2023.10.020

[2] L. Shams Ashkezari, H. J. Kaleybar, and M. Brenna, "ELECTRIC BUS CHARGING INFRASTRUCTURES: TECHNOLOGIES, STANDARDS, AND CONFIGURATIONS," *IEEE Access*, vol. 12, pp. 80505–80528, 2024, doi: https://doi.org/10.1109/ACCESS.2024.3410880

[3] V. Smoliarov, "ENHANCING THE EFFICIENCY OF MANAGEMENT OF TRANSPORT ENTERPRISES: MODERN TRENDS AND CHALLENGES," *Journal of Economics, Innovative Management, and Entrepreneurshir*, vol. 2, no. 3, pp. 81–91, 2024, doi: https://doi.org/10.59652/jeime.v2i3.305

[4] N. Panossian, M. Muratori, B. Palmintier, A. Meintz, T. Lipman, and K. Moffat, "CHALLENGES AND OPPORTUNITIES OF INTEGRATING ELECTRIC VEHICLES IN ELECTRICITY DISTRIBUTION SYSTEMS," Jun. 01, 2022, *Springer Nature*, doi: https://doi.org/10.1007/s40518-022-00201-2

[5] M. Rogge, E. van der Hurk, A. Larsen, and D. U. Sauer, "ELECTRIC BUS FLEET SIZE AND MIX PROBLEM WITH OPTIMIZATION OF CHARGING INFRASTRUCTURE," *Appl Energy*, vol. 211, pp. 282–295, Feb. 2018, doi: https://doi.org/10.1016/j.apenergy.2017.11.051

[6] M. Rupp, C. Rieke, N. Handschuh, and I. Kuperjans, "ECONOMIC AND ECOLOGICAL OPTIMIZATION OF ELECTRIC BUS CHARGING CONSIDERING VARIABLE ELECTRICITY PRICES AND CO2EQ INTENSITIES," *Transp Res D Transp Environ*, vol. 81, Apr. 2020, doi: https://doi.org/10.1016/j.trd.2020.102293

[7] J. J. Lin and H. I. Wong, "OPTIMIZATION OF A FEEDER-BUS ROUTE DESIGN BY USING A MULTIOBJECTIVE PROGRAMMING APPROACH," *Transportation Planning and Technology*, vol. 37, no. 5, pp. 430–449, 2014, doi: https://doi.org/10.1080/03081060.2014.912418

[8] Z. Gao *et al.*, "BATTERY CAPACITY AND RECHARGING NEEDS FOR ELECTRIC BUSES IN CITY TRANSIT SERVICE," 2017, doi: https://doi.org/10.1016/j.energy.2017.01.101

[9] H. Fatorachian and H. Kazemi, "SUSTAINABLE OPTIMIZATION STRATEGIES FOR ON-DEMAND TRANSPORTATION SYSTEMS: ENHANCING EFFICIENCY AND REDUCING ENERGY USE," 2025, *Taylor and Francis Ltd,* doi: https://doi.org/10.1080/27658511.2025.2464388

[10] G. K. Kuah and J. Perl, "THE FEEDER BUS NETWORK DESIGN PROBLEM," *J. Operational Research Society*, vol. 40, no. 8, pp. 751–767, 1989, doi: https://doi.org/10.1057/jors.1989.127

[11] T. Sangveraphunsiri, M. J. Cassidy, and C. F. Daganzo, "JITNEY-LITE: A FLEXIBLE-ROUTE FEEDER SERVICE FOR DEVELOPING COUNTRIES," *Transportation Research Part B: Methodological*, vol. 156, pp. 1–13, Feb. 2022, doi: https://doi.org/10.1016/j.trb.2021.12.015

[12] T. Rongen, T. Tillema, S. Lenferink, and J. Arts, "HOW TO ENSURE SUFFICIENT ACCESS TO PUBLIC TRANSPORT IN RURAL AREAS? A COMPARATIVE ANALYSIS OF INSTITUTIONAL DESIGNS FOR MULTIMODAL INTEGRATION," *Travel Behav Soc*, vol. 41, Oct. 2025, doi: https://doi.org/10.1016/j.tbs.2025.101096

[13] H. Badia and E. Jenelius, "DESIGN AND OPERATION OF FEEDER SYSTEMS IN THE ERA OF AUTOMATED AND ELECTRIC BUSES," *Transp Res Part A Policy Pract*, vol. 152, pp. 146–172, Oct. 2021, doi: https://doi.org/10.1016/j.tra.2021.07.015

[14] H. Fatorachian and H. Kazemi, "SUSTAINABLE OPTIMIZATION STRATEGIES FOR ON-DEMAND TRANSPORTATION SYSTEMS: ENHANCING EFFICIENCY AND REDUCING ENERGY USE," *Sustainable Environment*, vol. 11, no. 1, Dec. 2025, doi: https://doi.org/10.1080/27658511.2025.2464388

[15] A. Poullikkas, "SUSTAINABLE OPTIONS FOR ELECTRIC VEHICLE TECHNOLOGIES," 2015, *Elsevier Ltd*. doi: https://doi.org/10.1016/j.rser.2014.09.016

[16] S. Mishra, S. Rodriguez, M. Morales, and N. M. Amato, "BATTERY CONSTRAINED COVERAGE," in *IEEE International Conference on Automation Science and Engineering (CASE)*, IEEE, Aug. 2016. doi: https://doi.org/10.1109/COASE.2016.7743470

[17] M. Tóth, T. Hajba, and A. Horváth, "MILP MODELS OF A PATIENT TRANSPORTATION PROBLEM," *Cent Eur J Oper Res*, vol. 32, no. 4, pp. 903–922, Dec. 2024, doi: https://doi.org/10.1007/s10100-023-00902-z

[18] G. Kim, Y. S. Ong, C. K. Heng, P. S. Tan, and N. A. Zhang, "CITY VEHICLE ROUTING PROBLEM (CITY VRP): A REVIEW," *IEEE Transactions on Intelligent Transportation Systems*, vol. 16, no. 4, pp. 1654–1666, Aug. 2015, doi: https://doi.org/10.1109/TITS.2015.2395536

[19] N. Bodenschatz, M. Eider, and A. Berl, "MIXED INTEGER LINIER PROGRAMMING MODEL FOR THE CHARGING SCHEDULING OF ELECTRIC VEHICLE FLEETS," in *10th International Conference on Advanced Computer Information Technologies (ACIT)*, IEEE, Sep. 2020, pp. 741–746, doi: https://doi.org/10.1109/ACIT49673.2020.9208875

[20] B. R. Ke, Y. H. Lin, H. Z. Chen, and S. C. Fang, "BATTERY CHARGING AND DISCHARGING SCHEDULING WITH DEMAND RESPONSE FOR AN ELECTRIC BUS PUBLIC TRANSPORTATION SYSTEM," *Sustainable Energy Technologies and Assessments*, vol. 40, Aug. 2020, doi: https://doi.org/10.1016/j.seta.2020.100741

[21] J. F. Franco, M. J. Rider, and R. Romero, "A MIXED-INTEGER LINEAR PROGRAMMING MODEL FOR THE ELECTRIC VEHICLE CHARGING COORDINATION PROBLEM IN UNBALANCED ELECTRICAL DISTRIBUTION

SYSTEMS," *IEEE Trans Smart Grid*, vol. 6, no. 5, pp. 2200–2210, Sep. 2015, doi: https://doi.org/10.1109/TSG.2015.2394489

[22] F. Sun, R. Xiong, and H. He, "ESTIMATION OF STATE-OF-CHARGE AND STATE-OF-POWER CAPABILITY OF LITHIUM-ION BATTERY CONSIDERING VARYING HEALTH CONDITIONS," *J Power Sources*, vol. 259, pp. 166–176, Aug. 2014, doi: https://doi.org/10.1016/j.jpowsour.2014.02.095

[23] J. Wang *et al.*, "MULTI-OBJECTIVE OPTIMIZATION ALGORITHM FOR GROUPING DECISION VARIABLES BASED ON EXTREME POINT PARETO FRONTIER," *Computers, Materials and Continua*, vol. 79, no. 1, pp. 1281–1299, 2024, doi: https://doi.org/10.32604/cmc.2024.048495

[24] K. Sriprateep *et al.*, "MULTI-OBJECTIVE OPTIMIZATION OF RESILIENT, SUSTAINABLE, AND SAFE URBAN BUS ROUTES FOR TOURISM PROMOTION USING A HYBRID REINFORCEMENT LEARNING ALGORITHM," *Mathematics*, vol. 12, no. 14, Jul. 2024, doi: https://doi.org/10.3390/math12142283

[25] Z. Zhang, H. Qin, and Y. Li, "MULTI-OBJECTIVE OPTIMIZATION FOR THE VEHICLE ROUTING PROBLEM WITH OUTSOURCING AND PROFIT BALANCING," *IEEE Transactions on Intelligent Transportation Systems*, vol. 21, no. 5, pp. 1987–2001, May 2020, doi: https://doi.org/10.1109/TITS.2019.2910274

[26] T. Hai *et al.*, "TECHNO-ECONOMIC-ENVIRO EVALUATION OF A PV/BIOGAS/DIESEL/BATTERY HYBRID SYSTEM TO SUPPORT THE POWER NETWORK FOR THE WELFARE OF RURAL COMMUNITIES," *International Journal of Low-Carbon Technologies*, vol. 20, pp. 659–670, 2025, doi: https://doi.org/10.1093/ijlct/ctae256

[27] N. E. Zamri *et al.*, "A MODIFIED REVERSE-BASED ANALYSIS LOGIC MINING MODEL WITH WEIGHTED RANDOM 2 SATISFIABILITY LOGIC IN DISCRETE HOPFIELD NEURAL NETWORK AND MULTI-OBJECTIVE TRAINING OF MODIFIED NICHED GENETIC ALGORITHM," *Expert Syst Appl*, vol. 240, Apr. 2024, doi: https://doi.org/10.1016/j.eswa.2023.122307

[28] M. Aqmar, F. Roslan, N. E. Zamri, M. Asyraf Mansor, M. Shareduwan, and M. Kasihmuddin, "MAJOR 3 SATISFIABILITY LOGIC IN DISCRETE HOPFIELD NEURAL NETWORK INTEGRATED WITH MULTI-OBJECTIVE ELECTION ALGORITHM," *AIMS Mathematics*, 2023, doi: https://doi.org/10.3934/math.20231145

[29] N. A. Rusdi, M. S. M. Kasihmuddin, N. A. Romli, G. Manoharam, and M. A. Mansor, "MULTI-UNIT DISCRETE HOPFIELD NEURAL NETWORK FOR HIGHER ORDER SUPERVISED LEARNING THROUGH LOGIC MINING: OPTIMAL PERFORMANCE DESIGN AND ATTRIBUTE SELECTION," *Journal of King Saud University - Computer and Information Sciences*, vol. 35, no. 5, May 2023, doi: https://doi.org/10.1016/j.jksuci.2023.101554