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OPTIMIZATION OF DISASTER RELIEF LOGISTICS DISTRIBUTION USING THE FUZZY TRANSPORTATION PROBLEM MODEL

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ABSTRACT

The distribution of disaster relief logistics faces significant challenges due to uncertainty in demand, supply constraints, and accessibility constraints in affected areas. The novelty of this study lies in integrating trapezoidal fuzzy numbers to represent uncertainty in disaster logistics, thereby offering a more realistic model than conventional deterministic models. This study proposes developing a fuzzy-logic-based transportation model to optimize logistics resource allocation. The model was applied to a disaster relief distribution scenario with five source locations and five destination points. The model is solved using the Vogel Approximation Method and optimality test using the Simplex Transportation Method. Next, to determine the distribution route that minimizes costs and distance, a simulation was conducted in MATLAB. The results show that the fuzzy transportation problem model produces more efficient distribution solutions than conventional transportation models, which can be used only for certain data. However, this study is limited to single-objective cost minimization using simulated data. Therefore, future research should consider applying multi-objective optimization to minimize both distribution cost and time simultaneously using real-time geospatial data.

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1. INTRODUCTION

Logistics distribution in disaster situations is a major challenge for governments and humanitarian organizations. Uncertainty about information on victims' needs, infrastructure conditions, and transportation availability adds to the complexity of the distribution process. In emergency situations, logistics distribution is a complex and critical process. The speed and accuracy of distribution are crucial for the safety of victims and the effectiveness of post-disaster response. However, emergencies are often characterized by data uncertainty, including the amount of need, the availability of supplies, and transportation access. This inherent ambiguity necessitates the development of robust models that can effectively manage imprecise information, particularly concerning supply, demand, and transportation costs [1].

Transportation is the movement of people or goods from one place to another by land, sea, or air, with or without the aid of a machine [2]. Transportation issues have become significant and require significant attention. This is not only about the movement of people and goods, but also about limited infrastructure, the availability of facilities, and the efficiency of distribution channels. Hasbiyati et al. have studied various forms of transportation problems using diverse approaches to demonstrate the complexity of real-world problems [3], [4], [5].

Transportation problems, as a fundamental aspect of operational research, traditionally focus on minimizing the cost of shipping goods from source to destination, assuming that the parameters of supply, demand, and transportation costs are certain [6]. Classical transportation models based on linear programming generally require precise data on supply, demand, and shipping costs. However, in disaster contexts, deterministic data is often unavailable or even impossible to obtain accurately. Classical deterministic transportation models are not adaptive enough to handle the dynamics and uncertainties that occur during disasters. For this reason, the fuzzy logic approach offers advantages, as it can handle vague information [7].

This approach provides a more realistic representation of logistical challenges when precise values are difficult to obtain, thereby enabling more robust decision-making [8]. This approach has been used in various logistics optimization studies, but its application to disaster relief distribution that considers simultaneous uncertainty in demand and supply remains limited. Fuzzy transportation models can be used because they allow the use of fuzzy data to represent uncertainty in parameters such as shipping costs, inventory levels, and demand. This is expected to enable more efficient logistics distribution, even under uncertain conditions.

Several studies have examined the solution of transportation problems by developing fuzzy-based models to accommodate uncertainty in cost, supply, and demand parameters. Gurukumaresan, et al. proposed a solution to the fuzzy transportation problem using octagonal fuzzy numbers to represent uncertainty [9]. In addition, Pooja et.al., developed an algorithm to solve fuzzy transportation problem with trapezoidal fuzzy numbers which was proven to reduce the optimal cost in solving fuzzy-based transportation problems [10]. Furthermore, many previous researchers have studied transportation problems using fuzzy-based models and the use of other complementary methods to continue developing science in this field, Dhouib [11] introduce a new heuristic to solve trapezoidal fuzzy transportation problem, Pratihar et al. [12] and Mahdi et al. [13] developed modification to Vogel's Approximation Method (VAM). In a broader context, Lu and Li [14] extended transportation models to minimize carbon emissions. Furthermore, Kane et al. [15] addressed the fuzzy transportation problem by using trapezoidal fuzzy numbers. Although existing studies provide a strong foundation in general supply chain optimization, applying Trapezoidal Fuzzy Numbers combined with VAM and Simplex Transportation Methods specifically to the volatile nature of disaster relief presents an opportunity to enhance decision-making efficiency. This study fills this gap by proposing an integrated model that combines these approaches.

This research aims to develop a fuzzy-based transportation model that can accommodate data uncertainty in the disaster relief logistics distribution process and provide optimal solutions for the distribution of disaster relief logistics under uncertain demand and supply. In this model, important parameters, such as shipping costs, supply capacity, and demand, are represented as fuzzy numbers. The model is solved using Vogel's Approximation Method to find initial basic solutions and the Simplex Transportation Method for the optimality test, enabling the search for optimal solutions even in the presence of fuzzy data. This approach specifically addresses the challenges posed by the unpredictable nature of post-disaster scenarios, where traditional deterministic models often fall short due to their inability to account for the inherent vagueness in real-world data [1].

The expected outcome of this research is a mathematical model capable of improving the efficiency of distribution in emergency situations, with more adaptive and realistic performance compared to conventional approaches. A disaster case study will be used to test and evaluate the model's performance. Thus, this research is expected to provide theoretical contributions to the development of fuzzy transportation models and practical benefits in helping relevant agencies design more efficient and responsive logistics strategies to field conditions. This includes addressing the challenge of modeling parameters such as transportation costs, supply, and demand as trapezoidal fuzzy numbers to better reflect real-world variability in humanitarian logistics [6].

2. RESEARCH METHODS

This study applies a fuzzy transportation problem model to optimize the distribution of disaster relief logistics, addressing the limitations of classical deterministic approaches. Before detailing the specific mathematical procedures, the conceptual framework presented in Fig. 1 highlights the fundamental shift in this research. Unlike classical models that assume fixed parameters for supply, demand, and costs, the proposed framework utilizes Trapezoidal Fuzzy Numbers to accommodate real-world uncertainty.

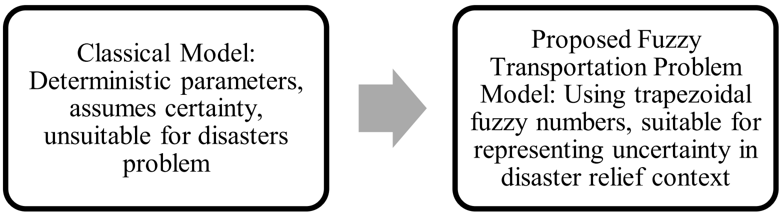

Figure 1. Comparison Conceptual Framework Between Classical Model and The Proposed Model.

To ensure systematic execution of this study within the framework above, a comprehensive research workflow is established. The step-by-step procedure, ranging from the setup of numerical examples to the final optimization using VAM and Simplex methods, is illustrated in Fig. 2.

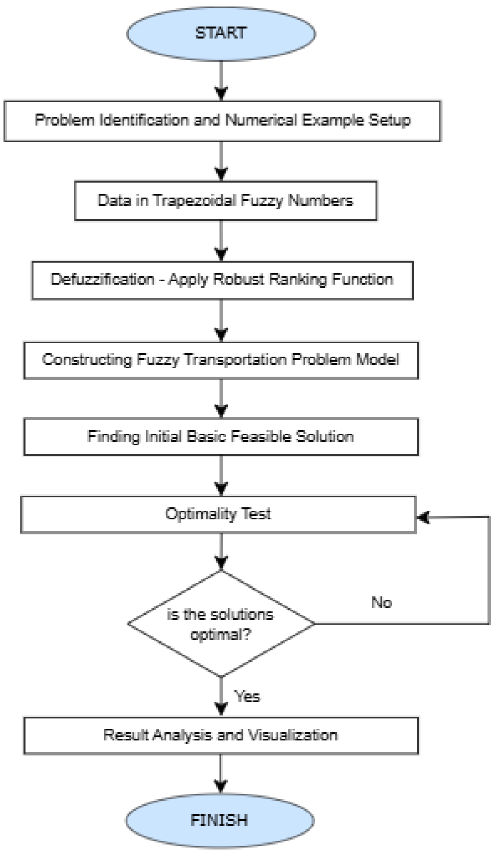

Figure 2. Research Methodology Flowchart

2.1 Transportation Model

A transportation model is a part of an optimization method that is used to determine the distribution of goods from several sources to several destinations at minimum cost [16]. In general, this model can be applied to various fields, such as logistics distribution, supply chain management, and transportation planning, where each source has a limited supply capacity, and each destination has specific demand requirements. In its conventional form, this model requires definite input data for the quantity of supply, demand, and distribution costs [17].

A transportation model is a special form of model in Operations Research that determines the most effective distribution path from a number of sources (supply) to a number of destinations (demand) with optimal total cost. The main problem in transportation problems is how to distribute goods from sources to destinations so that the total distribution cost is minimized while satisfying the constraints of inventory capacity and demand. The main parameters used in transportation models include distribution costs, inventory levels at each source, and demand at each destination.

Table 1. Public Transport

	Goal 1	**Goal 2**	**Goal 3**	**...**	**Objective n**	**Supply**
Source 1	c_{11}	c_{12}	c_{13}		c_{1n}	s_1
Source 2	c_{21}	c_{22}	c_{23}		c_{2n}	s_3
Source 3	c_{31}	c_{32}	c_{33}		c_{3n}	s_3
⋮	⋮	⋮	⋮		⋮	⋮
Source m	c_{m1}	c_{m2}	c_{m3}		c_{mn}	s_m
Demand	d_1	d_2	d_3		d_n	

The general form of the transportation model from Table 1, by minimizing total distribution costs, can be written mathematically as follows:

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}, \tag{1}$$

constraint: $\sum_{j=1}^{n} x_{ij} = s_i,\ i = 1,2,3,\cdots,m$ (inventory constraints);

$\sum_{i=1}^{m} x_{ij} = d_j,\ j = 1,2,3,\cdots,n$ (demand constraints);

$x_{ij} \geq 0$;

with: c_{ij} := the shipping cost per unit of goods from source i to destination j;

s_i := the supply from source i;

d_j := the demand at destination j;

i := $1,2,3,\dots,m$;

j := $1,2,3,\dots,n$.

If the total supply is equal to the total demand, then the model is called a balanced transportation problem, which can be written systematically as follows:

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j. \tag{2}$$

A transportation model in its standard form is a linear minimum-cost network flow problem under deterministic conditions, that is, when all parameters, such as costs, demand values, and inventory quantities, are certain [18]. However, in practice, unavoidable uncertainties often arise due to unforeseen circumstances, such as distribution delays that exceed estimates or sudden changes in supply needs or availability. Therefore, a mathematical model is needed to handle these uncertainties and obtain optimal solutions. Transportation problems with uncertain parameters are then formulated as a fuzzy transportation model, in which transportation costs, inventory levels, and demand are represented as fuzzy numbers.

2.2 Fuzzy Logic

Fuzzy logic is a mathematical framework developed to address the uncertainty and ambiguity that often arise in real-world problems. Unlike binary logic, which only recognizes true (1) or false (0), fuzzy logic allows for a continuous degree of membership in the range of 0 to 1. This concept was first introduced by Lotfi A. Zadeh in 1965 [7] and has since been widely applied across fields such as decision-making, control systems, optimization, and modeling with qualitative and fuzzy data. In this study, uncertainty is represented using trapezoidal fuzzy numbers, as this shape is simple yet can more flexibly represent a range of uncertain values.

Several definitions of fuzzy numbers, especially trapezoidal fuzzy numbers used in this study, are presented below.

Definition 1. [18] *Suppose R is a set of real numbers, then a fuzzy set A in R is defined as a set of ordered pair $A = \{(x, \mu A(x)) | x \in R\}$ with $\mu A(x)$ is the membership function for a fuzzy set being between 0 and 1.*

Definition 2. [19] *Trapezoidal fuzzy numbers $\tilde{A}$ is a fuzzy number (a, b, c, d) that has a membership function $\mu\tilde{A}(x)$ as follows:*

$$\mu\tilde{A}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & x > d \end{cases} . \quad (3)$$

The membership function of trapezoidal fuzzy numbers is linear and trapezoidal, which contains linguistic assessment uncertainty, as in Fig. 3.

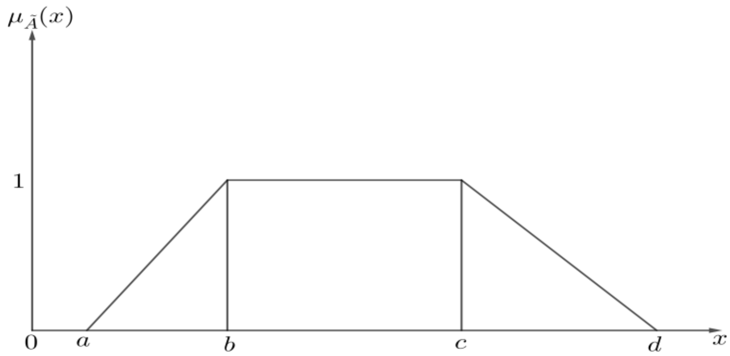

Figure 3. Trapezoidal Curve

2.3 Vogel Approach Method

To determine the optimal solution to a transportation problem, the first step is to determine the initial baseline solution. The initial baseline solution can be determined using various methods, including the Vogel Approximation Method. Taha [16] explains the steps of the Vogel Approximation Method, namely:

1. For each row or column, determine the difference by subtracting the smallest cost element in the row or column.
2. Then allocate inventory to meet demand as much as possible at the lowest cost variable in the selected row or column. Next, readjust inventory and demand, and mark the rows or columns that have been fulfilled. If there are rows and columns that are fulfilled simultaneously, select one to mark.
   a. If the row or column has zero inventory or demand, do not mark it.

b. If the row or column with supply and demand has not been marked, determine the base variable in the row or column with the smallest cost.

c. If all unmarked and columns have zero supply and demand, determine the zero-valued basis variable by selecting the smallest cost and stopping.

2.4 Simplex Transportation Method

The initial baseline solution obtained by solving a transportation problem using the Vogel Approximation Method is not necessarily optimal. Therefore, further methods are needed to test the optimality of the solution. One commonly used approach is the Simplex Transportation Method, which improves the initial solution until it reaches optimal conditions [20].

Let u_i be the dual supply variable and v_j the dual demand variable with $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$. The dual form of Winston's transportation problem states that if

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \,. \tag{4}$$

Then it is said to be balanced [21]. In a balanced transportation problem, the dual form can be written as:

$$\min z' = s_1u_1 + s_2u_2 + \cdots + s_mu_m + +d_1v_1 + d_2v_2 + \cdots + d_nv_n, \tag{5}$$

constraints

$$\begin{aligned} u_1 + v_1 &\le c_{11}, \\ u_1 + v_2 &\le c_{12}, \\ &\vdots \\ u_1 + v_n &\le c_{1n}, \\ u_2 + v_1 &\le c_{21}, \\ u_2 + v_2 &\le c_{22}, \\ &\vdots \\ u_2 + v_n &\le c_{2n}, \\ &\vdots \\ u_m + v_1 &\le c_{m1}, \\ u_m + v_2 &\le c_{m2}, \\ &\vdots \\ u_m + v_n &\le c_{mn}, \end{aligned} \tag{6}$$

where u_i and v_j free of signs; and $i := 1,2,3,\ldots,m$, $j := 1,2,3,\ldots,n$.

If written in sigma notation, it is obtained

$$\min z' = \sum_{i=1}^{m} s_iu_i = \sum_{j=1}^{n} d_jv_j, \tag{7}$$

constraint

$$u_i + v_j \le c_{ij}, i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n, \tag{8}$$

u_i and v_j free of signs.

The steps of the simplex method for transportation are as follows:

1. If the problem is unbalanced, balance it first using the Eq. (1).
2. Use one of the methods to find the initial basic feasible solution.
3. Use the fact that $u_1 = 0$ for all basic variables to find the initial basic feasible solution $[u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n]$ with the Eq. (9)

$$u_i + v_j = c_{ij}. \tag{9}$$

4. Determine the possible costs for each non-basic variable using the equation

$$\bar{c}_{ij} = u_i + v_j - c_{ij} \le 0. \tag{10}$$

with $\bar{c}_{ij}$ is the possible cost.

5. There are several cases, namely:
   a. If $\bar{c}_{ij}$ is negative, then the solution obtained previously is the optimal solution.
   b. If $\bar{c}_{ij}$ is positive, the solution obtained is not yet optimal. Improve the solution by finding the variables that will enter the basis (input variables) and variables that will leave the basis (output variables). The input variable is the variable that corresponds to the most positive value of $\bar{c}_{ij}$. To find the input and output variables, construct a closed loop that begins and ends at the incoming variable. The loop consists of successive vertical and horizontal segments with an unspecified direction, meaning it can be clockwise or counterclockwise. The output variable is determined by selecting the smallest basis variable in the constructed loop.
   c. Next, the input variables are filled in as many times as the output variables. For example, x_{ij} is an input variable. When x_{ij} is increased by 1 unit to maintain the solution, the base variables within the loop are adjusted according to the number of permits and supplies, i.e. by increasing and decreasing the base variable in the loop. This process is summarized in the table with (+) and (-) sign in the corresponding cell. The changes obtained will maintain the supply and demand constraints so that they remain satisfied. Then return to Step (3).

2.5 Robust Ranking Function

In real-world scenarios, objective coefficients are often imprecise and are treated as fuzzy numbers [22]. In this study, the Robust Ranking function is selected for defuzzification because it explicitly incorporates α-cuts, allowing the model to capture the full spread of uncertainty rather than relying solely on the geometric center. Furthermore, due to its simplicity and accuracy [23], Robust Ranking function is utilized in this case used to convert trapezoidal fuzzy numbers into crisp numbers. Crisp numbers are numbers whose membership in explicitly stated. A variable in a crisp number has two possible membership values (degree of membership), namely x is member of A represented by 1 or x non-member of A represented by 0. Then to convert fuzzy numbers into crisp numbers, the robust ranking function can be used.

Definition 3. [2] *Fuzzy numbers do not have a natural order, but in the decision-making process, fuzzy numbers must be systematic. If $A = (a, b, c, d)$ are fuzzy numbers, then robust ranking is defined as follow:*

$$R(A) = \int_0^1 0.5\ (A_\alpha^L, A_\alpha^U) d\alpha, \tag{11}$$

with

$$(A_\alpha^L, A_\alpha^U) = \big((b-a)\alpha + a, d - (d-c)\alpha\big), \tag{12}$$

where (A_α^L, A_α^U) represents the α-cut interval of the fuzzy number with the lower bound $(A_\alpha^L) = (b - a)\alpha + a$ and upper bound $(A_\alpha^U) = d - (d - c)\alpha$. Furthermore, $R(A)$ is called the robust ranking index to represent the value of fuzzy numbers to crisp numbers. Based on Definition 3, the ranking value of trapezoidal fuzzy numbers is obtained by substituting the Eq. (12) into the Eq. (11) that is:

$$\begin{aligned}
R(A) &= \int_0^1 0.5\ (A_\alpha^L, A_\alpha^U)\ d\alpha,\\
&= \int_0^1 0.5\ \big((b - a)\alpha + a, d - (d - c)\alpha\big)\ d\alpha,\\
&= \int_0^1 0.5\ \big((b\alpha - a\alpha) + a, d - (d\alpha - c\alpha)\big)\ d\alpha,\\
&= 0.5 \int_0^1 \big((b - a - d + c)\alpha + (a + d)\big)\ d\alpha,\\
&= 0.5\ \left[\left(\frac{1}{2}\right)(b - a - d + c)\alpha^2 + (a + d)\alpha\right]_0^1\\
&= 0.5\ \left[\left(\left(\frac{1}{2}\right)b - \left(\frac{1}{2}\right)a - \left(\frac{1}{2}\right)d + \left(\frac{1}{2}\right)c\right)\alpha^2 + (a + d)\alpha\right]_0^1
\end{aligned}$$

$$= 0.5\left(\left(\left(\frac{1}{2}\right)b-\left(\frac{1}{2}\right)a-\left(\frac{1}{2}\right)d+\left(\frac{1}{2}\right)c\right)+(a+d)\right)$$

$$= 0.5\left(\left(\frac{1}{2}\right)(b+a+c+d)\right)$$

$$R(A) = \left(\frac{a+b+c+d}{4}\right) \tag{13}$$

3. RESULTS AND DISCUSSION

This section explains the process of constructing a fuzzy transportation model, its application in disaster logistics distribution, and the determination of initial and optimal solutions.

3.1 Fuzzy Transportation Model

The fuzzy transportation model represents supply, demand, and transportation costs in fuzzy numbers. In this study, trapezoidal fuzzy numbers are used, referred to as the trapezoidal fuzzy transportation model. Membership functions are used to transform estimated data into fuzzy parameters. The general form of the trapezoidal fuzzy transportation problem is written as follows:

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}x_{ij}, \tag{14}$$

constraint: $\sum_{j=1}^{n} x_{ij} = \tilde{s}_i,\ i = 1,2,3,\cdots,m$ (inventory constraints);

(15)

$\sum_{i=1}^{m} x_{ij} = \tilde{d}_j,\ j = 1,2,3,\cdots,n$ (demand constraints);

$x_{ij} \geq 0;$

where $\tilde{c}$ is a cost in fuzzy form, $\tilde{s}$ in inventory in fuzzy form, and $\tilde{d}$ is demand in fuzzy form.

3.2 Application of the transportation model in disaster logistic distribution

To validate the feasibility and effectiveness of the proposed Fuzzy Transportation Model, a numerical experiment is conducted. The data presented in Table 2 represents a hypothetical disaster relief scenario designed to simulate the uncertainty inherent in logistics distribution. The supply and demand parameters are generated as trapezoidal fuzzy numbers to reflect the ambiguity inherent in real-world data, where exact values are often unavailable. The fuzzy transportation model with five sources and five destinations is presented in Table 2 below, with costs, inventories, and demands expressed as trapezoidal fuzzy numbers.

Table 2. Transportation in Trapezoidal Fuzzy Numbers

	Destination 1	**Destination 2**	**Destination 3**	**Destination 4**	**Destination 5**	**Supply**
Source 1	(0, 0, 0, 0)	(3, 5, 7, 9)	(5, 8, 10, 13)	(2, 3, 3, 4)	(5, 7, 19, 21)	(170, 190, 210, 230)
Source 2	(5, 9, 13, 17)	(0, 0, 0, 0)	(2, 4, 6, 8)	(1, 3, 5, 7)	(5, 7, 7, 17)	(136, 152, 168, 184)
Source 3	(1, 2, 6, 7)	(0, 2, 2, 4)	(0, 0, 0, 0)	(1, 7, 9, 11)	(4, 8, 16, 20)	(102, 114, 126, 138)
Source 4	(1, 2, 4, 5)	(3, 6, 10, 13)	(3, 8, 12, 13)	(0, 0, 0, 0)	(1, 2, 3, 6)	(102, 114, 126, 138)
Source 5	(2, 4, 9, 13)	(4, 7, 11, 14)	(5, 6, 8, 13)	(4, 5, 5, 10)	(0, 0, 0, 0)	(102, 114, 126, 138)
Demand	(102, 114, 126, 138)	(102, 114, 126, 138)	(152, 160, 176, 192)	(136, 152, 168, 184)	(128, 142, 158, 172)	

The mathematical model of the fuzzy transportation problem in Table 2 can be written as follows:

$$\begin{aligned}\min Z = {} & (0,0,0,0)x_{11} + (3,5,7,9)x_{12} + (5,8,10,13)x_{13} + (2,3,3,4)x_{14} + \\ & (5,7,19,21)x_{15} + (5,9,13,17)x_{21} + (0,0,0,0)x_{22} + (2,4,6,8)x_{23} + \\ & (1,3,5,7)x_{24} + (5,7,7,17)x_{25} + (1,2,6,7)x_{31} + (0,2,2,4)x_{32} + \\ & (0,0,0,0)x_{33} + (1,7,9,11)x_{34} + (4,8,16,20)x_{35} + (1,2,4,5)x_{41} + \\ & (3,6,10,13)x_{42} + (3,8,12,13)x_{43} + (0,0,0,0)x_{44} + (1,2,3,6)x_{45} + \\ & (2,4,9,13)x_{51} + (4,7,11,14)x_{52} + (5,6,8,13)x_{53} + (4,5,5,10)x_{54} + \\ & (0,0,0,0)x_{55},\end{aligned} \tag{16}$$

constraints:

$$\begin{gathered} x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = (170,190,210,230), \\ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = (136,152,168,184), \\ x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = (102,114,126,138), \\ x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = (102,114,126,138), \\ x_{51} + x_{52} + x_{53} + x_{54} + x_{55} = (102,114,126,138), \\ x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = (102,114,126,138), \\ x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = (102,114,126,138), \\ x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = (152,160,176,192), \\ x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = (136,152,168,184), \\ x_{15} + x_{25} + x_{35} + x_{45} + x_{55} = (128,142,158,172), \\ x_{ij} \geq 0 \text{ with } i = 1,2,3,4,5 \text{ and } j = 1,2,3,4,5, \\ \tilde{c}_{ii} = 0, i = 1,2,3,4,5, \end{gathered} \tag{17}$$

where Z represents the total fuzzy transportation cost, x_{ij} is the decision variable representing the quatity of unit transported from source i to destination j.

Next, the optimal solution of the model in Eq. (16) will be determined, namely, with the following steps:

1. Check whether the model is balanced, namely by checking whether the amount of supply is equal to the amount of demand.
2. Determining the initial basic solution using the Vogel Approximation Method.
3. Determining the optimal solution (optimality test) using the Simplex Transportation Method.

To determine whether the model is balanced, first change supply and demand, which are in the form of fuzzy numbers, into firm numbers using the robust ranking function, as follows:

$$R(\tilde{A}) = \left(\frac{a+b+c+d}{4}\right),$$

for the stock

$$R(\tilde{s}_1) = \left(\frac{170+190+210+230}{4}\right) = 200,$$

for request

$$R(\tilde{d}_1) = \left(\frac{102+114+126+138}{4}\right) = 120.$$

In the same way, for all supplies and requests in Table 2, the quantities are crisp numbers. Next, the total supplies and the total requests are calculated as follows:

The amount of inventory can be write

$$\sum_{i=1}^{5} s_i = 200 + 160 + 120 + 120 + 120 = 720,$$

and the number of requests

$$\sum_{i=1}^{5} d_j = 120 + 120 + 170 + 160 + 150 = 720,$$

because supply and demand have the same value, namely

$$\sum_{i=1}^{5} s_i = \sum_{i=1}^{5} d_j = 720.$$

So the transportation problem is said to be balanced. The change in transportation cost, from fuzzy numbers to crisp numbers, is shown in Table 3. This table presents the defuzzified values derived from the trapezoidal fuzzy numbers in Table 2. These crisp values are calculated using the Robust Ranking Function defined in Eq. (13). This transformation is necessary to convert the uncertain cost, supply, and demand parameters into deterministic values, enabling the problem to be solved using standard transportation algorithms such as the Vogel Approximation Method (VAM).

Table 3. Conversion of Fuzzy Transportation Cost to Crisp Value Using the Robust Ranking Functions

	Destination 1	**Destination 2**	**Destination 3**	**Destination 4**	**Destination 5**	**Supply**
Source 1	0	6	9	3	13	200
Source 2	11	0	5	4	9	160
Source 3	4	2	0	7	12	120
Source 4	3	8	9	0	3	120
Source 5	7	9	8	6	0	120
Demand	120	120	170	160	150	

To illustrate the allocation mechanism of the Vogel Approximation Method (VAM) presented in Table 4, the penalty cost is calculated for each row and column by finding the difference between the two lowest unit costs. For example, in the first iteration for Source 1, the two lowest transportation costs are 3 and 0. The penalty is thus calculated as $3 - 0 = 3$. This process is repeated for all rows and columns. The allocation is then prioritized for the cell with the lowest cost in the row or column that possesses the highest penalty value. Then, by using this method, an initial basic solution to the transportation problem is obtained namely $x_{11} = 120, x_{13} = 10, x_{14} = 70, x_{22} = 120, x_{23} = 40, x_{33} = 120, x_{44} = 90, x_{45} = 30,$ and $x_{55} = 120$ with the total transportation costs $z = (0 \times 120) + (9 \times 10) + (3 \times 70) + (0 \times 120) + (5 \times 40) + (0 \times 120) + (0 \times 90) + (3 \times 30) + (0 \times 120) = 590$. The results of this calculation are shown in Table 4. The values in regular font represent the unit transportation cost, while the values in bold parentheses represent the allocated quantity.

Table 4. Transportation Cost Allocation Using the Vogel Approach Method

	Destination 1	**Destination 2**	**Destination 3**	**Destination 4**	**Destination 5**	**Supply**
Source 1	0 **(120)**	6	9 **(10)**	3 **(70)**	13	200
Source 2	11	0 **(120)**	5 **(40)**	4	9	160
Source 3	4	2	0 **(120)**	7	12	120
Source 4	3	8	9	0 **(90)**	3 **(30)**	120
Source 5	7	9	8	6	0 **(120)**	120
Demand	120	120	170	160	150	

Next, the initial basic solution obtained will be tested for optimality using the Simplex Transportation Method. The values of u_i and v_j are determined through each basic variable with Eq. (9), which is shown in Table 5.

Table 5. Basic Variable

Basic Variable	$(\boldsymbol{u},\boldsymbol{v})$**-Equation**	**Solution**
x_{11}	$u_1+v_1=0$	Set $u_1=0\Rightarrow v_1=0$
x_{13}	$u_1+v_3=9$	$u_1=0\Rightarrow v_3=9$
x_{14}	$u_1+v_4=3$	$u_1=0\Rightarrow v_4=3$
x_{23}	$u_2+v_3=5$	$v_3=9\Rightarrow u_2=-4$
x_{22}	$u_2+v_2=0$	$u_2=-4\Rightarrow v_2=4$
x_{33}	$u_3+v_3=0$	$v_3=9\Rightarrow u_3=-9$
x_{44}	$u_4+v_4=0$	$v_4=3\Rightarrow u_4=-3$
x_{45}	$u_4+v_5=3$	$u_4=-3\Rightarrow v_5=6$
x_{55}	$u_5+v_5=0$	$v_5=6\Rightarrow u_5=-6$

Next, each non-basic variable $\tilde{c}_{ij}$ is determined by the Eq. (10), namely $\bar{c}_{12}=-2$, $\bar{c}_{15}=-13$, $\bar{c}_{21}=-15$, $\bar{c}_{24}=-5$, $\bar{c}_{25}=-13$, $\bar{c}_{31}=-13$, $\bar{c}_{32}=-7$, $\bar{c}_{34}=-13$, $\bar{c}_{35}=-21$, $\bar{c}_{41}=-6$, $\bar{c}_{42}=-7$, $\bar{c}_{43}=-3$, $\bar{c}_{51}=-13$, $\bar{c}_{52}=-11$, $\bar{c}_{53}=-5$, and $\bar{c}_{54}=-3$. It can be seen that all non-basic variables have values less than zero or are negative. The optimality conditions have been met, so that the optimal solution to the transportation problem is obtained, namely $x_{13}=10$, $x_{14}=70$, $x_{23}=40$, and $x_{45}=30$ with the minimum total cost of:

$$\begin{aligned}\min Z &= \tilde{c}_{13}x_{13}+\tilde{c}_{14}x_{14}+\tilde{c}_{23}x_{23}+\tilde{c}_{45}x_{45}\\ &= (5,8,10,13)(10)+(2,3,3,4)(70)+(2,4,6,8)(40)+(1,2,3,6)(30)\\ &= (50,80,100,130)+(140,210,210,280)+(80,160,240,320)+(30,60,90,180)\\ &= (50+140+80+30,80+210+160+60,100+210+240+90,130+280+320+180)\\ &= (300,510,640,910)\end{aligned}$$

3.3 Simulation of the Fuzzy Transportation Model in Disaster Logistics Distribution

To validate the optimal solution obtained from the manual Fuzzy Transportation Problem Model, a computational validation was conducted using MATLAB. This validation utilizes the Linear Programming solver to ensure the mathematical precision of the global optimum. The validation process involves two main stages:

1. Optimization: The fuzzified parameters are converted into a linear programming matrix structure. The solver employs the Dual-Simplex Algorithm to minimize the objective function Z with supply and demand constraints.
2. Visualization: The resulting optimal allocation matrix is visualized using a graph-theoretic approach. The directed graph function in MATLAB plots the nodes and generates directed edges with positive allocation values. In the simulation, there are five source locations and five destination locations. Each location is associated with an inter-location cost, which represents the resources (cost, labor, or time) required.

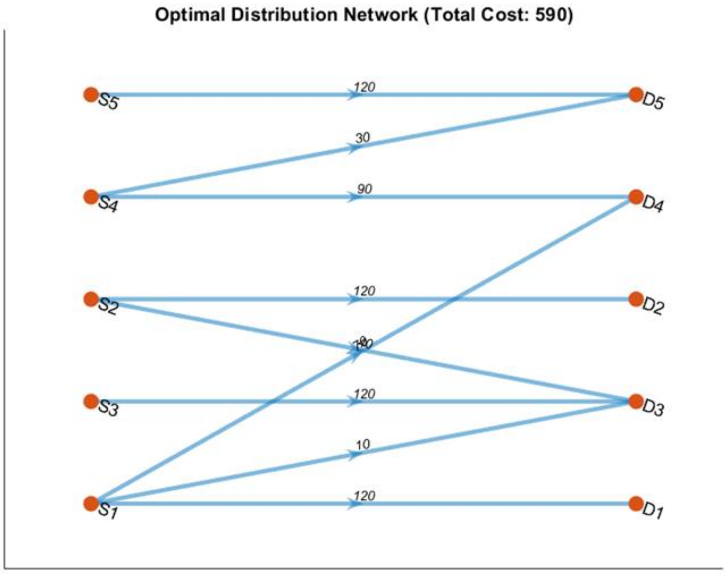

Figure 4. Optimal Distribution Network

The simulation results obtained using MATLAB software show the route with the minimum cost in Fig. 4. In this simulation, the notations $S1, S2, S3, S4,$ and $S5$ are used to represent Source 1 to Source 5, and $D1, D2, D3, D4,$ and $D5$ are used to represent Destination 1 to Destination 5. These points are interconnected by directed blue edges, which represent the active distribution routes selected by the optimization model. This visual representation confirms that the logistics flow effectively satisfies the demand at all destination points without exceeding the capacity of any source, providing a clear, intuitive map of the proposed emergency distribution plan.

Quantitatively, the proposed model demonstrates efficient performance. As shown in the computational analysis, the proposed Fuzzy Transportation Problem Model approach yielded a minimum total cost of 590 monetary units. In contrast, the conventional North West Corner (NWC) method produced a significantly higher baseline cost of 1660. It is important to note that while the NWC solution could theoretically be improved through subsequent optimization iterations starting from such a high-cost initial solution, it requires a significantly larger number of computational steps to reach the global optimum. This proves that the proposed method is not only cost-effective but also computationally efficient for time-sensitive disaster response planning.

Unlike deterministic models that rely on fixed numbers, this model uses fuzzy logic and the Robust Ranking technique to handle real-world uncertainty. By evaluating the spread of trapezoidal fuzzy numbers that capture both best- and worst-case scenarios, the algorithm acts as a filter, naturally avoiding routes with high volatility (wide intervals) even if they seem cheap initially. This creates a built-in safety buffer. The resulting Total Minimum Cost of 590 represents a plan that is both efficient and resilient against unexpected disruptions. Practically, this means humanitarian agencies can stretch their limited budgets further, using the saved funds to provide more aid where it is needed most. This demonstrates that integrating trapezoidal fuzzy numbers into decision-making directly improves the responsiveness and quality of emergency operations.

4. CONCLUSION

Optimization results show that the Fuzzy Transportation Model is capable of adaptively allocating resources, prioritizing more critical areas without violating available supply limits. Unlike deterministic models, which only operate on certain (crisp) values, the fuzzy model can accommodate uncertainty in both demand and supply, resulting in a more realistic solution for field conditions. Based on the numerical example and simulation, the proposed model can reduce the total cost relative to the conventional baseline (NWC) from 1660 to 590. This research demonstrates that the Fuzzy Transportation Problem Model approach is efficient in handling data variability and information uncertainty that often arise in disaster emergencies. This result highlights that the proposed fuzzy approach significantly reduces the computational burden compared to traditional methods. This cost efficiency has significant policy relevance, as the saved funds can be reallocated to expand disaster logistics distribution. Overall, this model has the potential to serve as a decision support tool for humanitarian agencies and governments in planning distribution quickly, accurately, and adaptively. However, this study has several limitations. The model primarily focuses on minimizing transportation costs as a single objective and utilizes simulated data to test the algorithm's feasibility, which may not fully capture the dynamic constraints of real-world disaster zones, such as sudden road blockages. Therefore, further research is recommended to expand the model into a multi-objective optimization problem that considers travel time and vehicle routing constraints, and to validate the model with real-time empirical data from disaster management agencies to enhance its practical applicability.

Author Contributions

Ihda Hasbiyati: Conceptualization, Formal Analysis, Funding Acquisition, Writing – Review and Editing, Validation. Hasriati: Investigation, Resources. Harison: Data curation, Resource. Maimunah: Visualization, Resource. Aziza Masli: Visualization, Writing – Review and Editing, Software, Editing. Ahriyati: Project Administration, Writing – Review and Editing. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare that no conflicts of interest exist in this study.

Declaration of Generative AI and AI-assisted technologies

The authors used generative AI (ChatGPT) only to assist with language polishing and formatting consistency (e.g., improving wording and ensuring uniform terminology). No AI was used to generate research content, perform analyses, or create/modify figures and tables. The authors reviewed the manuscript in full and remain responsible for its content.

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