BAREKENG: Journal of Mathematics and Its Applications
December 2026 Volume 20 Issue 4 Page 2967-2978
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol20iss4pp2967-2978

EFFECT OF NANOPARTICLE SHAPE ON MELTING HEAT TRANSFER IN HYBRID NANOFLUID FLOW AT STAGNATION POINT

Nurul Syuhada Ismail [1*], Norhunaini Mohd Shaipullah [2], Jane Labadin [3], Rusya Iryanti Yahaya [4], Nur Fazliana Rahim [5], Norihan Md Arifin [6]

[1,2] Centre for Pre-University Studies, Universiti Malaysia Sarawak
[3] Faculty of Computer Science and Information Technology, Universiti Malaysia Sarawak
Kota Samarahan, Sarawak, 94300, Malaysia

[4] Institute of Mathematical Research, Universiti Putra Malaysia
[6] Department of Mathematics, Faculty of Science and Institute for Mathematical Research, Universiti Putra Malaysia
UPM Serdang, Selangor, 43400, Malaysia

[5] Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara
Perlis Branch, Arau Campus, Arau, Perlis, 02600, Malaysia

*Corresponding author's e-mail: * insyuhada@unimas.my*

Article Info

Article History:
Received: 20th September 2025
Revised: 2nd April 2026
Accepted: 26th May 2026
Available online: 24th August 2026

Keywords:
Nanofluid;
Nanoparticle Shape;
Melting Heat Transfer.

ABSTRACT

This work examines the influence of nanoparticle geometry on the transfer of heat characteristics during melting in hybrid nanofluid flow at the stagnation point. The hybrid nanofluid, composed of aluminium oxide (Al_2O_3) and copper (Cu) nanoparticles dispersed in a water-based fluid, is analysed under varying conditions of nanoparticle geometry, including spherical, cylindrical, and platelet shapes. A mathematical model is developed by incorporating the melting boundary condition into the fluid flow and heat transfer governing equations. The utilization of similarity variables transforms the governing equations into a series of ordinary differential equations. The MATLAB bvp4c solver solves these equations numerically. The results reveal that nanoparticle shape significantly influences thermal conductivity and flow dynamics, thereby affecting heat transfer efficiency. The platelet-shaped nanoparticles produce the highest local skin friction coefficient, succeeded by cylindrical- and spherical-shaped nanoparticles. Furthermore, the interplay of melting parameters and stagnation point flow dynamics further amplifies heat transfer performance. This study provides critical insights into optimizing nanofluid designs for industrial applications requiring efficient thermal management near the stagnation point.

How to cite this article:

N. S. Ismail, N. M. Shaipullah, J. Labadin, R. I. Yahaya, N. F. Rahim, and N. M. Arifin, "EFFECT OF NANOPARTICLE SHAPE ON MELTING HEAT TRANSFER IN HYBRID NANOFLUID FLOW AT STAGNATION POINT", *BAREKENG: J. Math. & App.*, vol. 20, no. 4, pp. 2967-2978, Dec. 2026.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · Open Access

1. INTRODUCTION

Hybrid nanofluids have attracted significant attention due to their superior thermophysical properties compared with conventional nanofluids. Hybrid nanofluids can enhance their heat transfer performance and increase thermal conductivity by combining two distinct nanoparticles in a base fluid. Prior research indicates that hybrid nanofluids, such as Al_2O_3–Cu or Al_2O_3–CuO dispersed in water, provide superior thermal conductivity and heat transfer efficiency relative to single nanoparticle nanofluids. For example, studies reported that Al_2O_3–Cu hybrid nanofluids can significantly increase the Nusselt number and heat transfer coefficient due to the synergistic effect between the metallic and metal-oxide nanoparticles [1]. The selection of aluminium oxide (Al_2O_3) and copper (Cu) nanoparticles is motivated by their favourable thermophysical properties. Copper nanoparticles have extremely high thermal conductivity, whereas aluminium oxide nanoparticles are commonly utilised because of their availability, chemical stability, and favourable dispersion properties. The combination of these nanoparticles in a hybrid system can therefore enhance heat transfer performance more effectively than single-component nanofluids [2].

The combined effects of various nanoparticle materials, shapes, and concentrations result in hybrid nanofluids comprising two or more nanoparticles that exhibit enhanced heat transfer capabilities. The shape and configuration of nanoparticles greatly affect hybrid nanofluid thermal conductivity and heat transfer efficiency, according to extensive research. The research conducted by Takabi and Salehi [3] emphasises that the thermophysical properties of nanofluids are profoundly affected by the dimensions, geometry, and concentration of nanoparticles, which in turn affect the characteristics of heat transfer and pressure drop. Hwang et al. [4] highlighted that the drag force and pressure drop in nanofluids depend on the shape of the nanoparticles, with smaller, more surface area-efficient shapes yielding superior performance. This aligns with the findings of Ilias et al. [5], who observed that platelet-shaped nanoparticles exhibited better heat transfer properties than other shapes in both laminar and turbulent flow regimes. Furthermore, Benkhedda et al. [6] showed that specific nanoparticle shapes increase hybrid nanofluid thermal conductivity, especially in horizontal pipe flows. According to their research, the highest heat transfer rates were attained by platelet-shaped SiO_2 nanoparticles, highlighting the crucial role that shape plays in thermal performance. Madhura et al. [7] also contributed to this understanding by showing that blade-shaped nanoparticles produced the highest temperatures due to their elevated thermal conductivity, while brick-shaped nanoparticles displayed the opposite trend, indicating the complex interplay between shape and thermal behaviour.

Moreover, the work of Samantaray et al. [8] found that different shapes significantly alter the thermophysical properties of nanofluids, impacting entropy generation and flow characteristics. Their findings were echoed by Rashid et al. [9], who emphasised the importance of nanoparticle shape in fluid flow and heat transfer dynamics within divergent-convergent channels. This suggests that the shape of nanoparticles affects not only thermal conductivity but also the overall flow behaviour in hybrid nanofluids. Furthermore, studies such as those by Gireesha et al. [10] and Vanaki et al. [11] have shown that nanoparticle shape can lead to substantial variations in heat transfer enhancement across different channel configurations. Gireesha et al. [10] specifically noted that shape influences the heat transfer phenomenon in moving porous fins, while Vanaki et al. [11] examined the influence of nanoparticle concentration and shape on heat transfer efficiency in wavy channels. Mahmood et al. [12] emphasised the potential of ternary hybrid nanofluids in enhancing heat flow at stagnation points, suggesting that the interaction between different nanoparticle shapes can improve thermal performance. This is further supported by the study of Aziz et al. [13], which demonstrates that the shape factor of nanoparticles can influence the rate of heat transfer, indicating that specific geometrical configurations may enhance thermal conduction within the fluid.

In many engineering applications, melting heat transfer is a crucial phenomenon, especially in systems that involve non-Newtonian fluids and phase change materials (PCMs). The study of melting heat transfer encompasses a wide range of scenarios, including thermal energy storage, manufacturing processes, and environmental applications such as permafrost melting and geothermal energy recovery. One significant aspect of melting heat transfer is its application in thermal energy storage systems, where PCMs are utilised to absorb and release thermal energy during phase transitions. Alshara's [14] investigation focuses on the energy storage capabilities of packed beds of cylindrical capsules filled with n-octadecane as a PCM and highlights the importance of understanding the dimensionless temperature distribution and melt fraction in optimising heat transfer performance. The findings demonstrate that water, utilized as a heat transfer fluid, significantly contributes to the improvement of thermal efficiency in these systems.

For non-Newtonian fluids, Hayat et al. [15] explored the melting heat transfer characteristics in boundary layer flows of second-grade fluids, emphasising the influence of Soret and Dufour effects on thermal performance. Their findings suggest that the unique properties of non-Newtonian fluids can significantly affect the melting process, which is vital for applications such as oil extraction and geothermal energy recovery. Furthermore, the melting effect in stagnation point flows has been investigated by Hayat et al. [16], who focused on the behaviour of Jeffrey nanomaterials under MHD conditions. Their research highlights the complexities of melting heat transfer in non-Newtonian fluids, especially when heat gradients and chemical reactions are present. The findings contribute to a deeper understanding of how melting processes can be controlled and optimised in various engineering applications.

The effects of hybrid nanofluids on stagnation point flow have been studied in various contexts. For instance, research on unsteady separated stagnation-point flow over a moving plate using hybrid nanofluids shows that the addition of dual-type nanoparticles notably influences the hydrodynamic flow and heat transfer properties [17]. Similarly, the use of hybrid nanofluids, particularly those combining copper and alumina nanoparticles, enhances heat transfer performance in stagnation point flows, demonstrating the effectiveness of these materials in practical applications [18]. Furthermore, the effectiveness of hybrid nanofluids in heat transfer procedures is mostly determined by their thermal conductivity and colloidal stability. Babar et al. [19] have reported that hybrid nanofluids demonstrate improved thermal performance compared to mono-nanoparticle suspensions, which is necessary for applications where efficient heat transfer is required. The experimental studies performed by Waini et al. [20] also support these findings, showing that hybrid nanofluids outperform conventional nanofluids in terms of heat transfer rates at stagnation points.

This study aims to advance the field by examining fluid flow at stagnation points and melting heat transfer with the effect of nanoparticle shape in hybrid nanofluids. The increasing demand for effective thermal control in contemporary engineering systems, such as energy storage, electronic cooling, and phase change materials, requires superior heat transfer fluids with enhanced thermal performance. Hybrid nanofluids offer a promising alternative to conventional heat transfer fluids. However, research on how the form of nanoparticles affects heat transfer, especially in melting environments, remains scarce. The current study's originality is in investigating the influence of nanoparticle shape on melting heat transfer. Our study provides a new outlook on this issue, addressing an aspect not previously investigated, and compares our results with those of Waini et al. [21]. Thus, providing a new perspective on how nanoparticle shape influences thermal performance. The investigation into the effect of nanoparticle shape on melting heat transfer in hybrid nanofluid flow at stagnation points is a critical area of research, especially considering the growing use of nanofluids in heat control systems. The heat transfer characteristics and thermal conductivity of nanofluids are greatly affected by the shape and size of the nanoparticles, which is important to optimize their performance in various engineering applications. Subsequent investigations are necessary to clarify the combined impact of nanoparticle shape, concentration, and type on the thermal efficiency of hybrid nanofluids in heat transfer applications.

2. RESEARCH METHODS

This section outlines the mathematical modelling and solution procedure used to investigate the melting heat transfer in hybrid nanofluid flow at a stagnation point. The physical problem is first formulated based on the boundary layer assumptions for a steady two-dimensional flow over a stretching or shrinking surface. Fig. 1 illustrates the flow structure of a steady, two-dimensional flow of a hybrid nanofluid at the stagnation point towards a stretching or shrinking surface, considering the impact of melting heat transfer. This flow problem has real-world applications in industrial operations, which include the reheating or remelting of metal strips, where precise thermal control over deforming surfaces is essential.

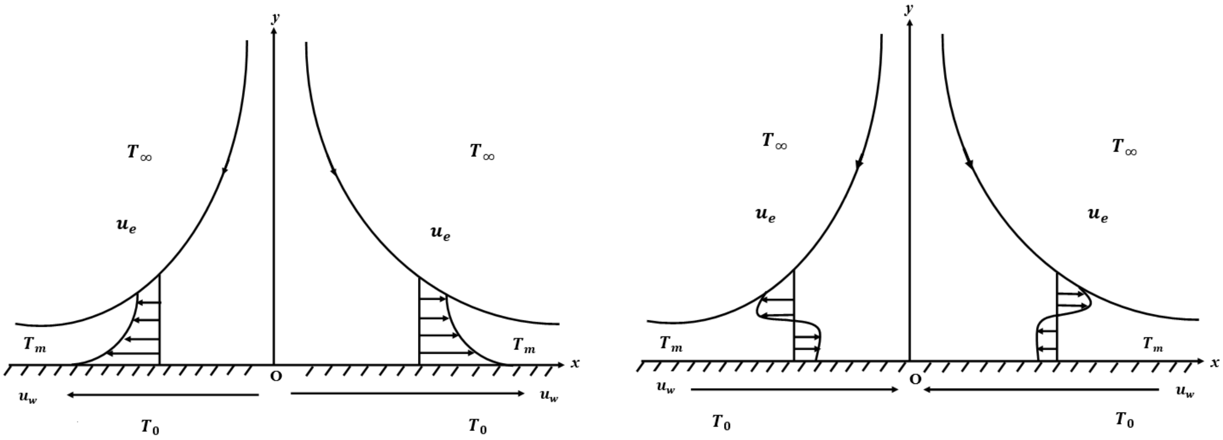


Figure 1. A Sketch of the Physical Model Illustrating the Flow Over (a) A Stretching Sheet and (b) A Shrinking Sheet

The equations $u_e(x) = ax$ and $u_w(x) = bx$, denote the external flow velocity and the surface velocity, subsequently, with a and b being constants. The symbol T_m represents the temperature of the melting surface, while temperatures T_∞ and T_0 indicate the liquid phase's temperature away from the plate and the solid phase's temperature away from the interface, respectively. Therefore, the governing equations for this study are given in [22] and [23].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho C_p)_{hnf}}\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

subject to:

$$\begin{aligned} u &= u_w(x), \quad T = T_m, \quad \text{at} \quad y = 0, \\ u &\to u_e(x), \quad T \to T_\infty, \quad \text{as} \quad y \to \infty, \end{aligned} \tag{4}$$

$$k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0} = \rho_{hnf}[L + c_s(T_m - T_0)]v, \tag{5}$$

where u and v represent the x- and y- components of the velocity, respectively. The latent heat of the fluid (L), the temperature of the hybrid nanofluid (T), and the heat capacity of the solid surface (c_s) appear in all the equations above. Specifically, the melting surface condition originally proposed by Roberts [24] has been widely adopted and validated in subsequent studies [25]-[29], and is represented by Eq. (5). Table 1 specifies the thermophysical properties of nanofluids and hybrid nanofluids, while Table 2 presents the values of A, B, and m for different nanoparticle shapes.

Table 1. Thermophysical Properties of Hybrid Nanofluid [6], [23]

Characteristics	Hybrid Nanofluid
Dynamic viscosity, μ	Spherical nanoparticle : $\mu_{hnf} = \dfrac{\mu_f}{(1-\varphi)^{2.5}}$ Non - spherical nanoparticle : $\mu_{hnf} = \mu_f(1 + A\varphi + B\varphi^2)$
Heat capacity, ρC_p	$(\rho C_p)_{hnf} = (1-\varphi)(\rho C_\rho)_f + \varphi_{n1}(\rho C_\rho)_{n1} + \varphi_{n2}(\rho C_\rho)_{n2}$
Density, ρ	$\rho_{hnf} = (1-\varphi)\rho_f + \varphi_{n1}\rho_{n1} + \varphi_{n2}\rho_{n2}$

Characteristics	Hybrid Nanofluid
Thermal conductivity, k	$k_{hnf} = k_f \left[\dfrac{\dfrac{\varphi_{n1}k_{n1} + \varphi_{n2}k_{n2}}{\varphi} + (m-1)k_f - (m-1)\varphi\left(k_f - \dfrac{\varphi_{n1}k_{n1} + \varphi_{n2}k_{n2}}{\varphi}\right)}{\dfrac{\varphi_{n1}k_{n1} + \varphi_{n2}k_{n2}}{\varphi} + (m-1)k_f + \varphi\left(k_f - \dfrac{\varphi_{n1}k_{n1} + \varphi_{n2}k_{n2}}{\varphi}\right)} \right]$

Table 2. Values of A, B, and Shape Factor M for Various Nanoparticle Shapes [30], [31]

Nanoparticle shape	A	B	m
Spherical	–	–	3
Cylindrical	13.5	904.4	4.9
Platelet	37.1	612.6	5.7

The nanoparticle volume fraction is represented by φ in Table 1, with the understanding that $\varphi = \varphi_{n1} + \varphi_{n2}$. The shape factor is denoted by m, and A and B are constants that are determined by the shapes of the nanoparticles, which are described in detail in Table 2. Table 3 describes the thermophysical characteristics of nanoparticles and water.

Table 3. Thermophysical Characteristics of Nanoparticles and Water [32]

Thermophysical Characteristics	Al_2O_3	Cu	water
ρ (kg/m^3)	3970	8933	997.1
C_p (J/ kgK)	765	385	4179
k (W/mK)	40	400	0.613
Prandtl number, Pr			6.2

Next, the dimensionless variables are [22], [28]

$$u = axf'(\eta), \qquad v = -\sqrt{a\nu_f}f(\eta), \qquad \theta(\eta) = \frac{T - T_m}{T_\infty - T_m}, \qquad \eta = y\sqrt{\frac{a}{\nu_f}}. \tag{6}$$

Then, by substituting Eq. (6) into Eqs. (2) and (3), the equations reduce to:

$$\frac{\dfrac{\mu_{hnf}}{\mu_f}}{\dfrac{\rho_{hnf}}{\rho_f}} f''' + ff'' - f'^2 + 1 = 0, \tag{7}$$

$$\frac{1}{Pr}\frac{\dfrac{k_{hnf}}{k_f}}{\dfrac{(\rho C_p)_{hnf}}{(\rho C_p)_f}}\theta'' + f\theta' = 0, \tag{8}$$

where the continuity Eq. (1) is unequivocally fulfilled. The boundary conditions are:

$$\begin{gathered} Pr\frac{\rho_{hnf}}{\rho_f}f(0) + M\frac{k_{hnf}}{k_f}\theta'(0) = 0, \\ f'(0) = \lambda, \qquad \theta(0) = 0, \\ f'(\eta) \to 1, \qquad \theta(\eta) \to 1, \text{as } \eta \to \infty. \end{gathered} \tag{9}$$

Then, ν_f is the kinematic viscosity of the fluid, $(\rho C_p)_{hnf}, k_{hnf}, \mu_{hnf}$, and ρ_{hnf} represent the heat capacity, thermal conductivity, dynamic viscosity, and density of the hybrid nanofluid, respectively. The kinematic viscosity of the fluid, density of the hybrid nanofluid, heat capacity, thermal conductivity, and dynamic viscosity are represented by ν_f, ρ_{hnf}, $(\rho C_p)_{hnf}, k_{hnf}$, and μ_{hnf}, accordingly. Next, λ is the shrinking/stretching parameter, where stretched sheets are represented by $\lambda > 0$ and shrinking sheets by $\lambda < 0$, M is the constant melting parameter and Pr is the Prandtl number, defined as

$$\lambda = \frac{b}{a}, \qquad \Pr = \frac{(\mu C_p)_f}{k_f}, \qquad M = \frac{(C_p)_f (T_\infty - T_m)}{L + c_s}. \tag{10}$$

The following definitions provide the local Nusselt number Nu_x and the skin friction coefficient C_f:

$$C_f = \frac{\mu_{hnf}}{\rho_f u_e^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad Nu_x = -\frac{x k_{hnf}}{k_f (T_\infty - T_m)}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{11}$$

Further, substituting Eqs. (6) and (10) into Eq. (11) yields:

$$Re_x^{\frac{1}{2}} C_f = \frac{\mu_{hnf}}{\mu_f} f''(0), \qquad Re_x^{-\frac{1}{2}} Nu_x = -\frac{k_{hnf}}{k_f}\theta'(0), \tag{12}$$

where $Re_x = u_e x/\nu_f$ is referring to the local Reynolds number.

3. RESULTS AND DISCUSSION

The MATLAB bvp4c solver [33] was employed to numerically solve Eqs. (7) to (9). The thickness of the boundary layer, η_∞ and initial guess were carefully adjusted according to the specific parameters used to achieve the desired solutions. To ensure the validity of the computations and the methodology adopted in this study, the results were compared with previously published findings that employed the same method, as shown in Table 4. When certain parameters were omitted, the numerical outcomes closely matched those from earlier studies, thereby reinforcing confidence in the accuracy of the data obtained. The study by [21] was selected for comparison because it considers a similar stagnation-point flow configuration with melting heat transfer, which provides an appropriate benchmark to validate the numerical method and the similarity formulation used in the present study. By reducing certain parameters in our model to match the conditions considered in their work, the governing equations become consistent with those reported by [21], allowing for a direct and meaningful comparison of the numerical results.

Table 4. Comparison of Results Calculated at $M = 0, m = 3$, and $Pr = 6.2$

λ	φ_{n2}	$Re_x^{1/2}C_f$		$Re_x^{-1/2}Nu_x$	
		Waini et al. [21]	Present study	Waini et al. [21]	Present study
-0.5	0.01	1.573697	1.57370	-0.589813	-0.58981
	0.05	1.885501	1.88550	-0.706314	-0.70631
	0.1	2.286512	2.28651	-0.838510	-0.83851
0	0.01	1.296890	1.29689	-1.157073	-1.15707
	0.05	1.553850	1.55385	-1.269379	-1.26938
	0.1	1.884324	1.88432	-1.404327	-1.40433
0.5	0.01	0.750507	0.75051	-1.623251	-1.62325
	0.05	0.899208	0.89921	-1.733859	-1.73386
	0.1	1.090453	1.09045	-1.872386	-1.87239

Next, Table 5 presents the local $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ values for various nanoparticle shapes and different values of M. This study takes into account both the stretching and the shrinking sheet cases. The results indicate that the negative values of $Re_x^{-1/2}Nu_x$ in both cases signify that the direction of heat transfer is from the hybrid nanofluid to the sheet, thereby facilitating melting.

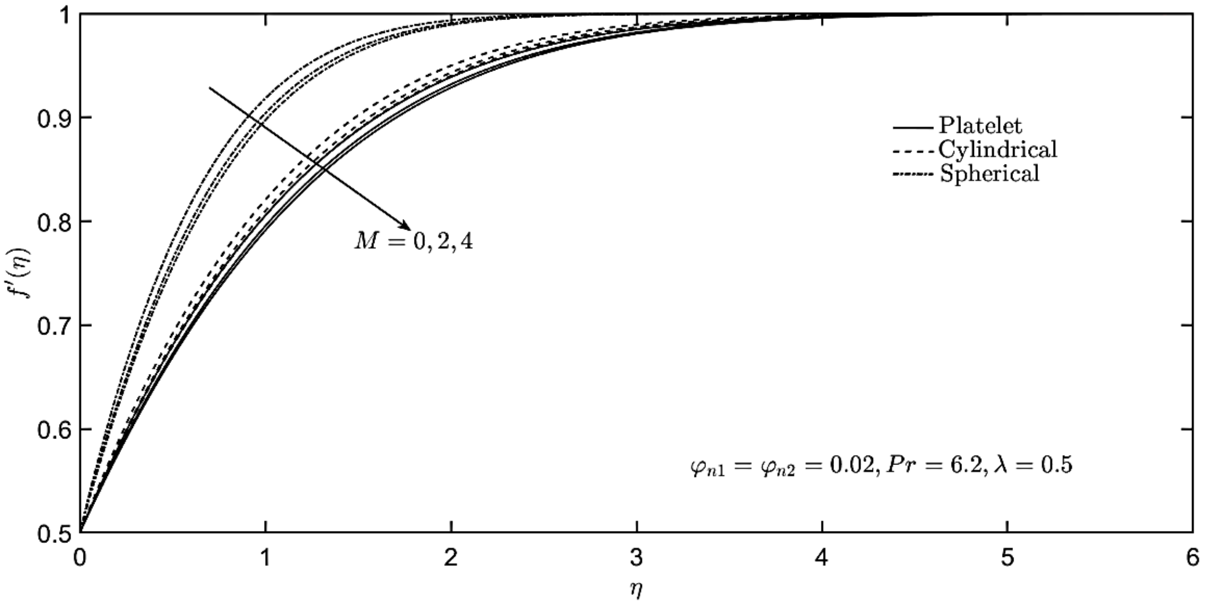
Table 5. Values of $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ when $Pr = 6.2$ and $\varphi_{n1} = \varphi_{n2} = 0.02$

λ	M	m	$Re_x^{1/2}C_f$	$Re_x^{-1/2}Nu_x$
0.5	0	5.7 (Platelet)	1.46568	-1.71126
		4.9 (Cylindrical)	1.36100	-1.69577
		3 (Spherical)	0.82870	-1.69887
	2	5.7	1.38567	-0.90968
		4.9	1.28182	-0.90146
		3	0.75066	-0.90208
	4	5.7	1.35246	-0.64967
		4.9	1.24904	-0.64380

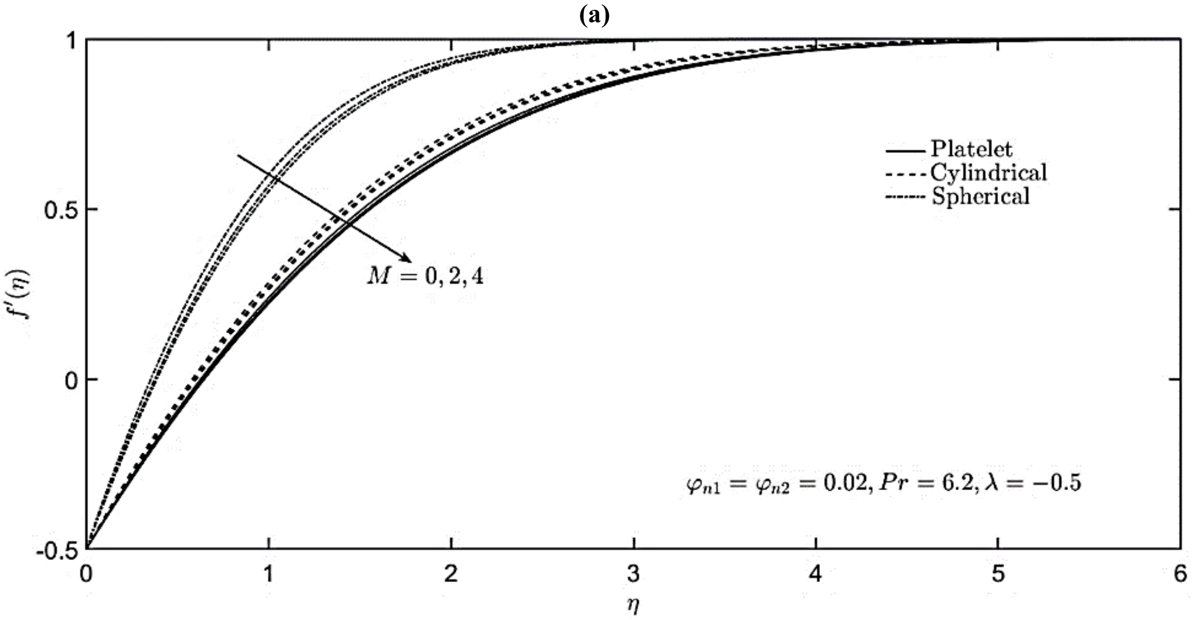
λ	M	m	$Re_x^{1/2}C_f$	$Re_x^{-1/2}Nu_x$
		3	0.71919	-0.64386
-0.5	0	5.7	3.07349	-0.36567
		4.9	2.85387	-0.39651
		3	1.73766	-0.65480
	2	5.7	3.00402	-0.23676
		4.9	2.77978	-0.25271
		3	1.62697	-0.38182
	4	5.7	2.96666	-0.18270
		4.9	2.74072	-0.19376
		3	1.57534	-0.28261

Based on the data in Table 5, the stretching case ($\lambda = 0.5$) exhibits a reduced coefficient of local skin friction $Re_x^{1/2}C_f$ but higher magnitudes of the local Nusselt number $Re_x^{-1/2}Nu_x$ compared to the shrinking case ($\lambda = -0.5$). This indicates that, relative to the shrinking sheet, the stretching sheet facilitates a higher heat transfer rate while inducing less wall shear stress. In both stretching and shrinking scenarios, platelet-shaped nanoparticles produce the highest local $Re_x^{1/2}C_f$, followed by cylindrical and spherical nanoparticles. Meanwhile, for the stretching case specifically, platelet-shaped nanoparticles also yield the highest magnitude of the local $Re_x^{-1/2}Nu_x$, followed by spherical- and cylindrical-shapes. Generally, platelet-shaped nanoparticles have a large surface area-to-volume ratio, which enhances their thermal conductivity compared to cylindrical- and spherical-shaped nanoparticles. However, their large surface area also increases resistance to motion, resulting in a higher local $Re_x^{1/2}C_f$. Next, the shrinking case shows that spherical-shaped nanoparticles result in the highest magnitude of the local $Re_x^{-1/2}Nu_x$, while the platelet shape yields the lowest. Since the shrinking sheet has a higher local $Re_x^{1/2}C_f$ than the stretching sheet, the use of platelet-shaped nanoparticles introduces greater resistance, which reduces heat transfer performance. Spherical-shaped nanoparticles, with their smoother flow characteristics, exhibit better heat transfer performance in the shrinking case, achieving a higher magnitude of the local $Re_x^{-1/2}Nu_x$. Thus, for optimal heat transfer performance, platelet-shaped nanoparticles are recommended for stretching sheet configurations, whereas spherical nanoparticles are better suited for shrinking sheet conditions.

Fig. 2 shows how the melting parameter M affects velocity profiles across the stretching and shrinking scenarios. As shown in Figs. 2 (a) and (b), regardless of the shape of the nanoparticles, an increase in the M results in a decrease in the hybrid nanofluid's velocity profile. This observation indicates that intense melting enhances the momentum boundary layer thickness. From a physical perspective, the melting process introduces a blowing-type boundary condition at the stretching or shrinking surface. Accordingly, an increase in the parameter M, associated with enhanced melting intensity, results in a thickening of the boundary layer. Consequently, the velocity gradient diminishes and causes the local $Re_x^{1/2}C_f$ to decrease, as indicated by the data in Table 4.



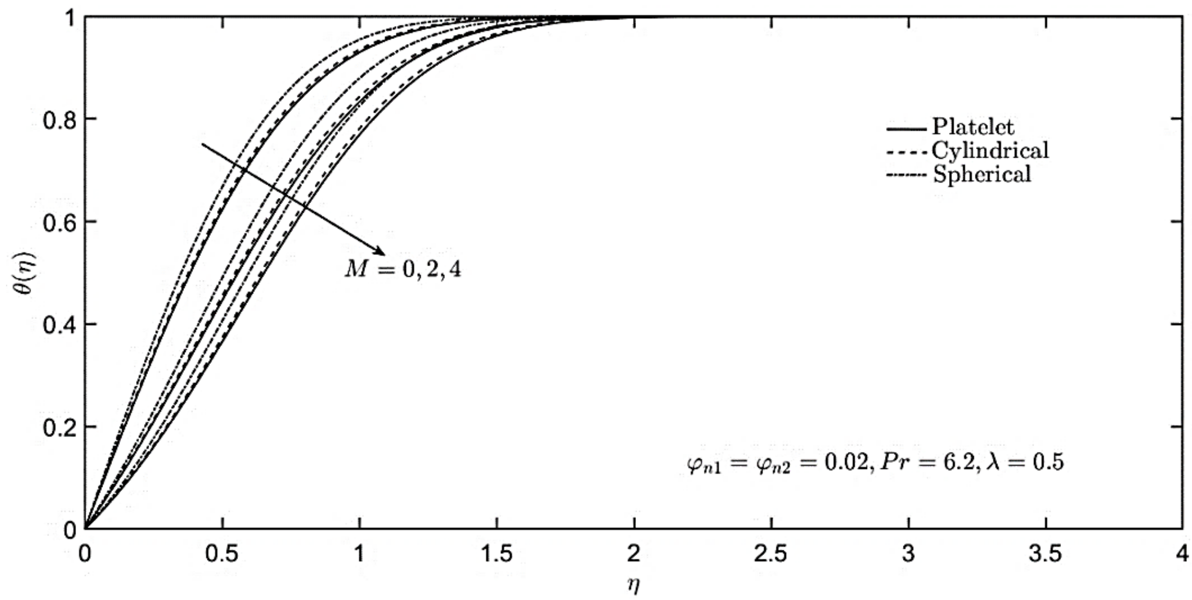
(a)



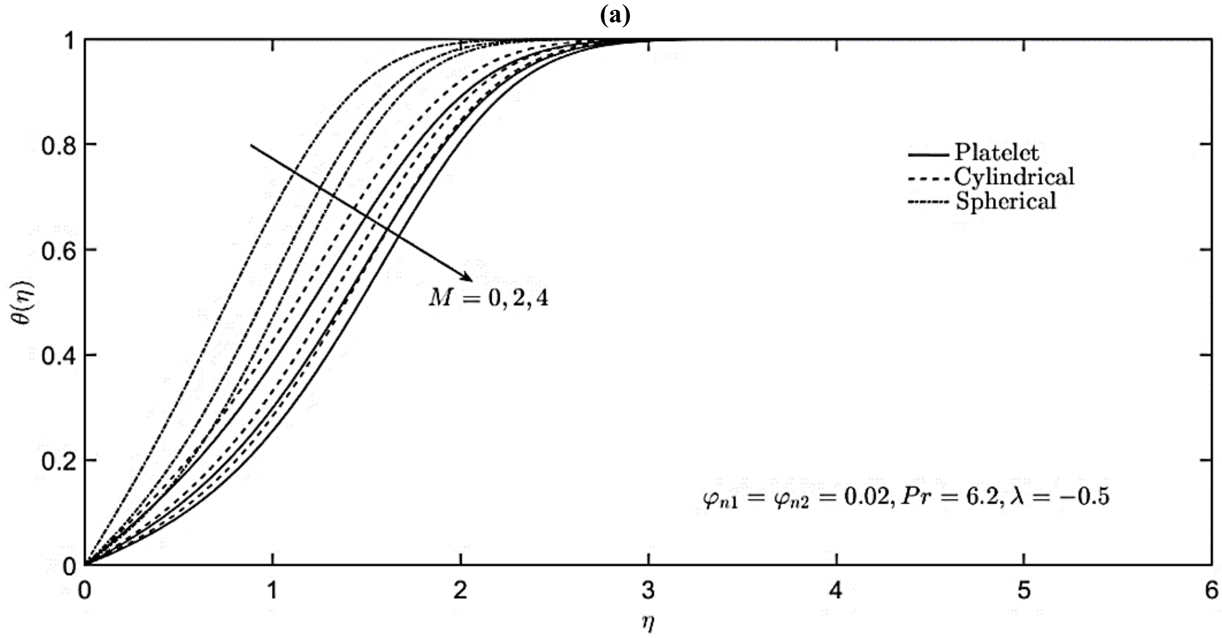
(b)

Figure 2. Velocity Profiles For (a) Stretching And (b) Shrinking Cases with Various Melting Parameters (M) Values

Similarly, Figs. 3 (a) and (b) show that an increase in the melting parameter M reduces the temperature profile in both stretching and shrinking cases. The increasing melting intensity absorbs greater heat from the hybrid nanofluid. Furthermore, an increase in the melting parameter leads to a reduction in fluid temperature, owing to the enhanced temperature gradient between the ambient environment and the melting surface. As a result, the hybrid nanofluid's temperature profile drops as the thermal boundary thickness rises. This reduces the temperature gradient on the surface of the sheet that is shrinking or stretching, resulting in a reduced magnitude of the local Nusselt number $Re_x^{-1/2}Nu_x$, as stated in Table 4.



(a)



(b)

Figure 3. Temperature Profiles for (a) Stretching and (b) Shrinking Cases with Various Melting Parameter (M) Values

4. CONCLUSION

The study demonstrates that the melting parameter significantly influences both the local skin friction coefficient, $Re_x^{1/2}C_f$ and local Nusselt number, $Re_x^{-1/2}Nu_x$ in the context of stretching and shrinking sheets. Negative local $Re_x^{-1/2}Nu_x$ values in both instances demonstrate that the sheet is melted more rapidly due to heat transfer from the hybrid nanofluid. The geometry of nanoparticles significantly affects the thermal transport and shear stress characteristics of the fluid. In the stretching case, platelet-shaped nanoparticles produce the largest local Nusselt number, succeeded by spherical and cylindrical shapes, due to their larger surface area enhancing thermal conductivity. However, platelet-shaped nanoparticles also experience greater resistance, resulting in a higher local skin friction coefficient. In the case of surface shrinking, spherical nanoparticles demonstrate the greatest local Nusselt number values, while platelet-shaped nanoparticles show the lowest. This is attributed to the increased resistance associated with platelet shapes, which reduces heat transfer performance in shrinking flows.

Furthermore, increasing values of the melting parameter result in a reduction to the velocity and temperature distributions when considering stretching and shrinking scenarios. The velocity and temperature gradients near the surface are reduced as a result of the increased thickness of the momentum and thermal boundary layers, which is a result of the enhanced melting effect. In the stretching case, nanoparticles with a platelet geometry exhibit superior thermal performance compared to other shapes, while spherical-shaped nanoparticles are more suitable for the shrinking case, providing better heat transfer performance with reduced resistance. The scope of this study may be broadened in future research to include the effect of varying melting parameters on different nanoparticle materials such as metals, ceramics, or carbon-based materials. Investigating these materials could lead to identifying optimal combinations for different industrial applications. In addition, future research could employ multi-objective optimization algorithms to determine the precise Pareto-optimal configurations for specific industrial cooling systems.

Author Contributions

Nurul Syuhada Ismail: Problem Formulation, Methodology, Writing-Original Draft, Validation. Norhunaini Mohd Shaipullah: Writing - Review and Editing. Jane Labadin: Formal Analysis. Rusya Iryanti Yahaya: Software, Data Curation. Nur Fazliana Rahim: Writing-Review and Editing. Norihan Md Arifin: Resources, Supervision. All authors discussed the results and contributed to the final manuscript.

Funding Statement

The study is a component of the research conducted under Universiti Malaysia Sarawak's PILOT grant (UNI/C09/PILOT/85088/2022).

Acknowledgment

The study is a component of the research conducted under Universiti Malaysia Sarawak's PILOT grant (UNI/C09/PILOT/85088/2022). The authors gratefully acknowledge the support of the Centre of Pre-University Universiti Malaysia Sarawak, Perlis Branch of Universiti Teknologi Mara (UiTM), and Universiti Putra Malaysia.

Declarations

Regarding this work, the authors disclose no conflicts of interest.

Declaration of Generative AI and AI-assisted technologies

Generative AI tools (e.g., ChatGPT) were used solely for language refinement, including grammar, spelling, and clarity. The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. All final text was reviewed and approved by the authors.

REFERENCES

[1] I. J. K. Wong and N. T. A. Tiong, "SIMULATION APPROACH ON TURBULENT THERMAL PERFORMANCE FACTOR OF AL2O3-CU/WATER HYBRID NANOFLUID IN CIRCULAR AND NON-CIRCULAR DUCTS," *SN Appl. Sci.*, vol. 3, no. 3, pp. 1–15, 2021. doi: https://doi.org/10.1007/s42452-021-04317-w.

[2] T. O. Scott, D. R. E. Ewim, and A. C. Eloka-Eboka, "HYBRID NANOFLUIDS FLOW AND HEAT TRANSFER IN CAVITIES: A TECHNOLOGICAL REVIEW," *International Journal of Low-Carbon Technologies*, vol. 17, no. September, pp. 1104–1123, 2022. doi: https://doi.org/10.1093/ijlct/ctac093.

[3] B. Takabi and S. Salehi, "AUGMENTATION OF THE HEAT TRANSFER PERFORMANCE OF A SINUSOIDAL CORRUGATED ENCLOSURE BY EMPLOYING HYBRID NANOFLUID," *Advances in Mechanical Engineering*, vol. 6, p. 147059, 2014. doi: https://doi.org/10.1155/2014/147059.

[4] K. S. Hwang, I. K. Baek, H. K. Shin, and S. P. Jang, "FLOW CHARACTERISTICS OF NANOFLUIDS ACCORDING TO NANOPARTICLES SHAPE," *Applied Mechanics and Materials*, vol. 110–116, pp. 3728–3736, Oct. 2011. doi: https://doi.org/10.4028/www.scientific.net/AMM.110-116.3728.

[5] M. R. Ilias, N. N. Rosli, S. S. Ishak, V. D. David, S. Shafie, and M. N. Abd Rahman, "RADIATION AND NANOPARTICLES SHAPE EFFECT ON ALIGNED MHD JEFFREY HYBRID NANOFLUIDS FLOW AND HEAT

TRANSFER OVER A STRETCHING INCLINED PLATE," *Journal of Advanced Research in Applied Mechanics*, vol. 109, no. 1, pp. 84–102, Oct. 2023. doi: https://doi.org/10.37934/aram.109.1.84102.

[6] M. Benkhedda, T. Boufendi, T. Tayebi, and A. J. Chamkha, "CONVECTIVE HEAT TRANSFER PERFORMANCE OF HYBRID NANOFLUID IN A HORIZONTAL PIPE CONSIDERING NANOPARTICLES SHAPES EFFECT," *J. Therm. Anal. Calorim.*, vol. 140, no. 1, pp. 411–425, Apr. 2020. doi: https://doi.org/10.1007/s10973-019-08836-y

[7] K. R. Madhura, B. Atiwali, and S. S. Iyengar, "INFLUENCE OF NANOPARTICLE SHAPES ON NATURAL CONVECTION FLOW WITH HEAT AND MASS TRANSFER RATES OF NANOFLUIDS WITH FRACTIONAL DERIVATIVE," *Math. Methods Appl. Sci.*, vol. 46, no. 7, pp. 8089–8105, May 2023. doi: https://doi.org/10.1002/mma.7404.

[8] S. S. Samantaray, S. Shaw, A. Misra, M. K. Nayak, and J. Prakash, "DARCY–FORCHHEIMER UP/DOWNFLOW OF ENTROPY OPTIMIZED RADIATIVE NANOFLUIDS WITH SECOND-ORDER SLIP, NONUNIFORM SOURCE/SINK, AND SHAPE EFFECTS," *Heat Transfer*, vol. 51, no. 2, pp. 2318–2342, Mar. 2022. doi: https://doi.org/10.1002/htj.22403.

[9] U. Rashid, A. Iqbal, H. Liang, W. Khan, and M. W. Ashraf, "DYNAMICS OF WATER CONVEYING ZINC OXIDE THROUGH DIVERGENT-CONVERGENT CHANNELS WITH THE EFFECT OF NANOPARTICLES SHAPE WHEN JOULE DISSIPATION ARE SIGNIFICANT," *PLoS One*, vol. 16, no. 1, p. e0245208, Jan. 2021. doi: https://doi.org/10.1371/journal.pone.0245208.

[10] B. Gireesha, G. Sowmya, and R. S. R. Gorla, "NANOPARTICLE SHAPE EFFECT ON THE THERMAL BEHAVIOUR OF MOVING LONGITUDINAL POROUS FIN," *Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanomaterials, Nanoengineering and Nanosystems*, vol. 234, no. 3–4, pp. 115–121, Jul. 2020. doi: https://doi.org/10.1177/2397791420915139.

[11] Sh. M. Vanaki, H. A. Mohammed, A. Abdollahi, and M. A. Wahid, "EFFECT OF NANOPARTICLE SHAPES ON THE HEAT TRANSFER ENHANCEMENT IN A WAVY CHANNEL WITH DIFFERENT PHASE SHIFTS," *J. Mol. Liq.*, vol. 196, pp. 32–42, Aug. 2014. doi: https://doi.org/10.1016/j.molliq.2014.03.001.

[12] Z. Mahmood, Z. Iqbal, M. A. Alyami, B. Alqahtani, M. F. Yassen, and U. Khan, "INFLUENCE OF SUCTION AND HEAT SOURCE ON MHD STAGNATION POINT FLOW OF TERNARY HYBRID NANOFLUID OVER CONVECTIVELY HEATED STRETCHING/SHRINKING CYLINDER," *Advances in Mechanical Engineering*, vol. 14, no. 9, Sep. 2022. doi: https://doi.org/10.1177/16878132221126278.

[13] A. Aziz, W. Jamshed, Y. Ali, and M. Shams, "HEAT TRANSFER AND ENTROPY ANALYSIS OF MAXWELL HYBRID NANOFLUID INCLUDING EFFECTS OF INCLINED MAGNETIC FIELD, JOULE HEATING AND THERMAL RADIATION," *Discrete & Continuous Dynamical Systems - S*, vol. 13, no. 10, pp. 2667–2690, 2020. doi: https://doi.org/10.3934/dcdss.2020142.

[14] A. K. Alshara and M. Kh. Kadhim, "NUMERICAL INVESTIGATION OF ENERGY STORAGE IN PACKED BED OF CYLINDRICAL CAPSULES OF PCM," *Engineering and Technology Journal*, vol. 32, no. 2, pp. 494–510, Feb. 2014. doi: https://doi.org/10.30684/etj.32.2A.15.

[15] T. Hayat, M. Hussain, M. Awais, and S. Obaidat, "MELTING HEAT TRANSFER IN A BOUNDARY LAYER FLOW OF A SECOND GRADE FLUID UNDER SORET AND DUFOUR EFFECTS," *Int. J. Numer. Methods Heat Fluid Flow*, vol. 23, no. 7, pp. 1155–1168, Sep. 2013. doi: https://doi.org/10.1108/HFF-09-2011-0182.

[16] T. Hayat, K. Muhammad, and A. Alsaedi, "MELTING EFFECT IN MHD STAGNATION POINT FLOW OF JEFFREY NANOMATERIAL," *Phys. Scr.*, vol. 94, no. 11, p. 115702, Nov. 2019. doi: 10.1088/1402-4896/ab210e.

[17] N. A. Zainal, R. Nazar, K. Naganthran, and I. Pop, "UNSTEADY SEPARATED STAGNATION-POINT FLOW PAST A MOVING PLATE WITH SUCTION EFFECT IN HYBRID NANOFLUID," *Mathematics*, vol. 10, no. 11, p. 1933, Jun. 2022. doi: https://doi.org/10.3390/math10111933.

[18] N. S. Khashi'ie, N. M. Arifin, I. Pop, and N. S. Wahid, "EFFECT OF SUCTION ON THE STAGNATION POINT FLOW OF HYBRID NANOFLUID TOWARD A PERMEABLE AND VERTICAL RIGA PLATE," *Heat Transfer*, vol. 50, no. 2, pp. 1895–1910, Mar. 2021. doi: https://doi.org/10.1002/htj.21961.

[19] H. Babar, M. Sajid, and H. Ali, "VISCOSITY OF HYBRID NANOFLUIDS: A CRITICAL REVIEW," *Thermal Science*, vol. 23, no. 3 Part B, pp. 1713–1754, 2019. doi: https://doi.org/10.2298/TSCI181128015B.

[20] I. Waini, A. Ishak, and I. Pop, "HYBRID NANOFLUID FLOW TOWARDS A STAGNATION POINT ON A STRETCHING/SHRINKING CYLINDER," *Sci. Rep.*, vol. 10, no. 1, p. 9296, Jun. 2020. doi: https://doi.org/10.1038/s41598-020-66126-2.

[21] I. Waini, A. Ishak, and I. Pop, "MELTING HEAT TRANSFER OF A HYBRID NANOFLUID FLOW TOWARDS A STAGNATION POINT REGION WITH SECOND-ORDER SLIP," *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, vol. 235, no. 2, pp. 405–415, Apr. 2021. doi: https://doi.org/10.1177/0954408920961213.

[22] N. Bachok, A. Ishak, and I. Pop, "MELTING HEAT TRANSFER IN BOUNDARY LAYER STAGNATION-POINT FLOW TOWARDS A STRETCHING/SHRINKING SHEET," *Phys. Lett. A*, vol. 374, no. 40, pp. 4075–4079, Sep. 2010. doi: https://doi.org/10.1016/j.physleta.2010.08.032.

[23] S. P. A. Devi and S. S. U. Devi, "NUMERICAL INVESTIGATION OF HYDROMAGNETIC HYBRID CU – AL_2O_3 /WATER NANOFLUID FLOW OVER A PERMEABLE STRETCHING SHEET WITH SUCTION," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 17, no. 5, pp. 249–257, Aug. 2016. doi: https://doi.org/10.1515/ijnsns-2016-0037.

[24] L. Roberts, "ON THE MELTING OF A SEMI-INFINITE BODY OF ICE PLACED IN A HOT STREAM OF AIR," *J. Fluid Mech.*, vol. 4, no. 5, pp. 505–528, Sep. 1958. doi: https://doi.org/10.1017/S002211205800063X.

[25] M. Epstein and D. H. Cho, "MELTING HEAT TRANSFER IN STEADY LAMINAR FLOW OVER A FLAT PLATE," *J. Heat Transfer*, vol. 98, no. 3, pp. 531–533, Aug. 1976. doi: https://doi.org/10.1115/1.3450595.

[26] S. Ahmad and I. Pop, "MELTING EFFECT ON MIXED CONVECTION BOUNDARY LAYER FLOW ABOUT A VERTICAL SURFACE EMBEDDED IN A POROUS MEDIUM: OPPOSING FLOWS CASE," *Transp. Porous Media*, vol. 102, no. 3, pp. 317–323, Apr. 2014. doi: https://doi.org/10.1007/s11242-014-0291-x.

[27] J. H. Merkin, I. Pop, and S. Ahmad, “NOTE ON THE MELTING EFFECT ON MIXED CONVECTION BOUNDARY-LAYER FLOW OVER A VERTICAL FLAT SURFACE EMBEDDED IN A POROUS MEDIUM,” *Int. J. Heat Mass Transf.*, vol. 84, pp. 786–790, May 2015. doi: https://doi.org/10.1016/j.ijheatmasstransfer.2014.12.040.

[28] F. Mabood, A. Shafiq, T. Hayat, and S. Abelman, “Radiation effects on stagnation point flow with melting heat transfer and second order slip,” *Results Phys.*, vol. 7, pp. 31–42, 2017. doi: https://doi.org/10.1016/j.rinp.2016.11.051.

[29] M. Javed, M. Farooq, A. Anjum, and S. Ahmad, “INSIGHT OF THERMALLY STRATIFIED JEFFREY FLUID FLOW INSIDE POROUS MEDIUM SUBJECT TO CHEMICAL SPECIES AND MELTING HEAT TRANSFER,” *Advances in Mechanical Engineering*, vol. 11, no. 9, Sep. 2019. doi: https://doi.org/10.1177/1687814019876187.

[30] M. Benkhedda, T. Boufendi, T. Tayebi, and A. J. Chamkha, “CONVECTIVE HEAT TRANSFER PERFORMANCE OF HYBRID NANOFLUID IN A HORIZONTAL PIPE CONSIDERING NANOPARTICLES SHAPES EFFECT,” *J. Therm. Anal. Calorim.*, vol. 140, no. 1, pp. 411–425, Apr. 2020. doi: https://doi.org/10.1007/s10973-019-08836-y.

[31] A. Bhattad and J. Sarkar, “EFFECTS OF NANOPARTICLE SHAPE AND SIZE ON THE THERMOHYDRAULIC PERFORMANCE OF PLATE EVAPORATOR USING HYBRID NANOFLUIDS,” *J. Therm. Anal. Calorim.*, vol. 143, no. 1, pp. 767–779, Jan. 2021. doi: https://doi.org/10.1007/s10973-019-09146-z.

[32] H. F. Oztop and E. Abu-Nada, “NUMERICAL STUDY OF NATURAL CONVECTION IN PARTIALLY HEATED RECTANGULAR ENCLOSURES FILLED WITH NANOFLUIDS,” *Int. J. Heat Fluid Flow*, vol. 29, no. 5, pp. 1326–1336, Oct. 2008. doi: https://doi.org/10.1016/j.ijheatfluidflow.2008.04.009

[33] Inc. The MathWorks, “BVP4C – SOLVE BOUNDARY VALUE PROBLEM — FOURTH-ORDER METHOD.” Accessed: Oct. 02, 2025. [Online]. Available: https://www.mathworks.com/help/matlab/ref/bvp4c.html

[34] I. Waini, A. Ishak, and I. Pop, “MELTING HEAT TRANSFER OF A HYBRID NANOFLUID FLOW TOWARDS A STAGNATION POINT REGION WITH SECOND-ORDER SLIP,” *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, vol. 235, no. 2, pp. 405–415, 2020. doi: https://doi.org/10.1177/0954408920961213