BAREKENG: Journal of Mathematics and Its Applications
December 2026 Volume 20 Issue 4 Page 3123-3136
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol20iss4pp3123-3136

A GENERALIZED PICTURE FUZZY DISTANCE MEASURE FOR RESEARCH TOPIC RECOMMENDATION IN MATHEMATICS

Dinni Rahma Oktaviani [1*], **Yolanda Norasia** [2], **Ainun Esti Candra** [3]

[1,2,3]*Mathematics Department, Faculty of Science and Technology, UIN Walisongo Semarang*
Jln. Walisongo 3-5, Tambakaji, Ngaliyan, Semarang, 50185, Indonesia

*Corresponding author's e-mail: *dinni@walisongo.ac..id*

Article Info

Article History:
Received: 29th September 2025
Revised: 27th April 2026
Accepted: 8th June 2026
Available online: 24th August 2026

Keywords:
Fuzzy Distance ;
PFDM;
Picture Fuzzy;
Research Topic.

ABSTRACT

Selecting an appropriate research topic is a critical yet challenging task for mathematics students, often hindered by misalignment between student interests, academic competencies, and supervisor availability. Traditional recommendation systems fail to capture the inherent uncertainty and ambiguity in human preferences, leading to suboptimal topic matches and prolonged study durations. This paper develops a research topic recommendation system specifically designed for mathematics students using an innovative Picture Fuzzy Distance Measure (PFDM). The picture fuzzy approach was selected for its comprehensiveness and detail in decision-making. Unlike traditional binary or even intuitionistic fuzzy models, this approach uniquely and simultaneously accommodates three degrees of consideration: positive membership, neutral membership, and negative membership degrees, thereby enabling more comprehensive representation of the complexity and ambiguity in human preferences. The proposed method integrates three main criteria through this approach: student interest measured through structured questionnaires, academic competence based on transcript analysis, and supervisor suitability based on expertise and availability. The developed PFDM has been proven to satisfy all basic metric axioms—non-negativity, identity, symmetry, and triangle inequality—with values bounded within the [0,1] interval. This paper not only provides a theoretical contribution to the development of fuzzy distance measures but also offers a practical solution for research topic recommendation.

How to cite this article:

D. R. Oktaviani, Y. Norasia, and A. E. Candra, "A GENERALIZED PICTURE FUZZY DISTANCE MEASURE FOR RESEARCH TOPIC RECOMMENDATION IN MATHEMATICS", *BAREKENG: J. Math. & App.*, vol. 20, no. 4, pp. 3123-3126, Dec. 2026.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · Open Access

1. INTRODUCTION

The theory of uncertainty was first introduced by Lotfi A. Zadeh in 1965 through the basic concept of the Fuzzy Set (FS) [1]. A fuzzy set is defined by a membership degree, which allows an element to belong to a set in a graded manner, rather than in a binary fashion (where 0 means non-member and 1 means member). This approach provides flexibility in handling uncertainty and enables its application in various fields such as artificial intelligence [2], decision-making [3], and data analysis [4]. In 1986, K. Atanassov introduced the more complex concept of the Intuitionistic Fuzzy Set (IFS) [5]. IFS involves both a membership degree ($\mu(x)$) and a non-membership degree ($\nu(x)$), allowing for additional information to accommodate uncertainty that cannot be represented by a single membership degree [6]. In IFS, a hesitation or indeterminacy degree is also defined as $\pi(x) = 1 - \mu(x) - \nu(x)$ [5]. This hesitation degree improves the completeness of the representation by capturing the amount of uncertainty or lack of information regarding whether an element belongs to the set or not, which is particularly useful when decision-makers are unable to fully commit to either membership or non-membership.

The development of fuzzy set theory continued, and research by Bui Cong Cuong and Vladik Kreinovich in 2013 introduced the Picture Fuzzy Set (PFS), which adds an explicit degree of neutrality (also called neutral membership or hesitation) [7]. Unlike IFS, where hesitation is derived from membership and non-membership as $\pi(x) = 1 - \mu(x) - \nu(x)$, PFS treats neutral membership as an independent, explicitly defined parameter. PFS provides a new way to represent uncertainty by considering three explicit aspects: positive membership, neutral membership, and negative membership (non-membership), each playing a distinct role [8]. A fourth degree, refusal membership, is then derived as $\xi(x) = 1 - \mu(x) - \eta(x) - \nu(x)$, representing the proportion of information that is withheld or refused. Consequently, PFS can perform more detailed analysis in situations where acceptance, uncertainty, and rejection cannot be considered equal or independent. PFS theory has proven effective in educational contexts, including the evaluation of physical education quality [9] and teaching quality assessment [10], where it captures neutral and non-participating situations. One of the tools in PFS used in decision-making is the distance measure [11], [12], which quantifies the degree of similarity or dissimilarity between two PFSs. A distance measure typically satisfies metric properties such as non-negativity, identity of indiscernibles, symmetry, and the triangle inequality. In decision-making contexts, distance measure enables the comparison of multiple alternatives against an ideal reference by converting the three-dimensional picture fuzzy information into a single scalar value, where a smaller distance indicates a closer match to desired criteria. The choice of distance function directly influences the ranking of alternatives and, consequently, the quality of the final recommendation.

Selecting a research topic is a major problem for mathematics students because it requires many considerations that involve uncertainty [13]. Students need to consider their interests, academic abilities, the availability of supervising lecturers, the availability of references, alignment with the study program's roadmap, and more [14]. Many students struggle to determine a suitable topic due to uncertainty in their interests and the complexity of mathematics itself. PFS is a concept that can be used to provide a solution for this kind of multifaceted uncertainty. PFS is capable of modeling preferences more comprehensively with its three membership degrees: positive, neutral, and negative. This approach is expected to provide more accurate and personalized recommendations.

The recommendation of research topics for mathematics students can be addressed using the distance measure in PFS, known as the Picture Fuzzy Distance Measure (PFDM)[15]. The Picture Fuzzy Distance Measure (PFDM) has broad and vital applications in various fields such as pattern recognition problems [16], multi-criteria decision-making problems [17], medical diagnosis[18], pattern analysis[19], and transportation problems [20]. In decision support systems, PFDM is used to compare alternatives against an ideal condition by simultaneously considering the degrees of agreement, disagreement, and neutrality. For example, in medical diagnosis, PFDM helps measure the distance between the symptoms experienced by a patient and the symptoms of a specific disease, taking into account the physician's confidence, uncertainty, and belief that it is not the disease [21]. PFDM is a quantitative metric for measuring the distance between two PFSs. The calculation of the distance between two PFSs involves all three degrees of the PFS [22]. A larger PFDM value indicates a greater distance between the two PFSs. In the context of recommending research topics to students, the PFDM value represents the suitability of a research topic for a mathematics student, where the ideal topic profile is defined as having maximum positive membership, zero neutral, and zero negative membership $(1,0,0)$. A smaller PFDM value (i.e., a shorter distance to this ideal profile) indicates that the topic is more suitable for the student.

Therefore, the Picture Fuzzy Distance Measure (PFDM) becomes an essential tool for calculating how well a topic aligns with a student's preferences. Furthermore, PFDM allows for a more detailed comparison by considering all membership degrees in the PFS. Thus, PFDM can enhance the accuracy of recommendations by ensuring that the selected topic truly matches the student's interests and needs. To integrate student preferences with the characteristics of research topics, this paper proposes the implementation of PFDM in a recommendation system for mathematics student research topics, aiming for a better fit for the student. Before implementing PFDM, it is first necessary to understand the properties/characteristics of the PFDM. The main contribution of this paper is the development of a novel Picture Fuzzy Distance Measure (PFDM) that not only satisfies all metric axioms but also demonstrates superior sensitivity to changes in the neutral membership degree compared to existing distance measures. Additionally, this study provides the first application of PFDM in the domain of research topic recommendation for mathematics students, integrating three key criteria: student interest, academic competence, and supervisor suitability. The new findings indicate that the proposed PFDM-based recommendation system produces more accurate and personalized topic suggestions, as validated through experimental comparisons with fuzzy and intuitionistic fuzzy approaches. These contributions advance both the theoretical development of picture fuzzy distance measures and the practical implementation of intelligent recommendation systems in higher education.

2. RESEARCH METHOD

This developmental and applied research constructs a novel decision-support framework for research topic recommendation, presented in two complementary sections. First, the theoretical foundation by proving the proposed Picture Fuzzy Distance Measure (PFDM) satisfies all basic metric axioms—non-negativity, identity, symmetry, and triangle inequality—while also demonstrating robust mathematical properties, including value bounds within the [0, 1] interval. Second, the PFDM is implemented in a practical recommendation system for mathematics students using multi-attribute considerations, which comprise interest data from structured questionnaires, academic ability derived from transcripts, and supervisor suitability based on lecturer expertise and availability.

To prove the proposed Picture Fuzzy Distance Measure (PFDM), first, we define some of the basic concepts of Picture Fuzzy Set, such as the Definition Picture Fuzzy Set, Picture Fuzzy Number, Relation, and Operation.

Definition 1. [7] Let X be a non-empty set. Picture Fuzzy Set (PFS) A in X is denoted as

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) | x \in X\},$$

where $\mu_A(x) \in [0,1]$ is called the positive membership degree of x in A,

$\eta_A(x) \in [0,1]$ is called the neutral membership degree of x in A,

$\nu_A(x) \in [0,1]$ is called the negative membership degree of x in A,

satisfying the condition $0 \leq \mu_A(x), \eta_A(x), \nu_A(x) \leq 1$ and $0 \leq \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

The refusal membership degree of x in A is defined as $\xi_A(x) = 1 - (\mu_A(x) + \eta_A(x) + \nu_A(x))$.

Definition 2. [7] Let A be a Picture Fuzzy Set (PFS) defined on a singleton universe $X = \{x\}$. Then, A can be represented as a triplet $\alpha = (\mu, \eta, \nu)$,

where: $\mu = \mu_A(x) \in [0,1]$ is the degree of positive membership,

$\eta = \eta_A(x) \in [0,1]$ is the degree of neutral membership,

$\nu = \nu_A(x) \in [0,1]$ is the degree of negative membership,

with the condition $0 \leq \mu + \eta + \nu \leq 1$. This triplet is called a Picture Fuzzy Number (PFN). The refusal degree is derived as $\xi = 1 - \mu - \eta - \nu$.

Definition 3. [7] Let $A, B \in PFS(X)$, then

1. $A \subseteq B$, if $\mu_A(x) \leq \mu_B(x), \eta_A(x) \leq \eta_A(x)$ and $\nu_A(x) \geq \nu_B(x)$, for each $x \in X$.
2. $A = B$, if $\mu_A(x) = \mu_B(x), \eta_A(x) = \eta_A(x)$ and $\nu_A(x) = \nu_B(x)$, for each $x \in X$.

3. $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\eta_A(x), \eta_A(x)\}, \min\{\nu_A(x), \nu_B(x)\} \mid x \in X\}$.
4. $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \min\{\eta_A(x), \eta_A(x)\}, \max\{\nu_A(x), \nu_B(x)\} \mid x \in X\}$.
5. $A^c = \{(x, \nu_A(x), \eta_A(x), \mu_A(x)) \mid x \in X\}$.
6. $A \times B = \{((x,y), \mu_{A\times B}(x,y), \eta_{A\times B}(x,y), \nu_{A\times B}(x,y)) \mid x \in A, y \in B\}$

   with $\mu_{A\times B}(x,y) = \min\{\mu_A(x), \mu_B(y)\}$,
   $\eta_{A\times B}(x,y) = \min\{\eta_A(x), \eta_B(y)\}$,
   $\nu_{A\times B}(x,y) = \max\{\nu_A(x), \nu_B(y)\}$.

Definition 4. [7] Let $A, B, C \in PFS(X)$ with $PFS(X)$ is set of Picture Fuzzy Set (PFS) of X. A function $d: PFS(X) \to \mathbb{R}$ is defined as Picture Fuzzy Distance Measure (PFDM) if and only if it satisfies for every $A, B, C \in PFS(X)$:

[D1] Non-Negativity: $d(A,B) \geq 0$

[D2] Identity of Indiscernibles: $d(A,B) = 0$ if and only if $A = B$

[D3] Symmetry: $d(A,B) = d(B,A)$

[D4] Triangle Inequality: $d(A,C) \leq d(A,B) + d(B,C)$

The commonly used PFDMs are the Hamming PFDM and the Euclidean PFDM [11], where d_H denotes the Hamming PFDM and d_E denotes the Euclidean PFDM (quadratic distance that emphasizes larger differences). Their formulas are as follows:

$$d_H(A,B) = \frac{1}{3n}\sum_{i=1}^{n}\left(|\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\right)$$

$$d_E(A,B) = \left(\frac{1}{3n}\sum_{i=1}^{n}\left(|\mu_A(x_i) - \mu_B(x_i)|^2 + |\eta_A(x_i) - \eta_B(x_i)|^2 + |\nu_A(x_i) - \nu_B(x_i)|^2\right)\right)^{\frac{1}{2}}$$

3. RESULTS AND DISCUSSION

The results and discussion in this study are presented in two main subsections to comprehensively address the research questions. The first subsection discusses the characteristics and mathematical properties of the proposed Picture Fuzzy Distance Measure (PFDM). The second subsection examines the practical implementation of PFDM in a research topic recommendation system for mathematics students based on three main criteria. This division enables an in-depth analysis of both the theoretical and applicative aspects of PFDM. Although several PFDMs have been previously proposed, most of them are limited to specific instnatiations such as the Hamming distance ($p = 1$) and the Euclidean distance ($p = 2$). The PFDM was proposed to generalize these by introducing a flexible parameter p, where p is any positive integer. This generalization allows the distance measure to be adjusted according to the preference or sensitivity level required in a given decision-making context. The proposed PFDM satisfies all metric axioms and is bounded within [0,1]. Furthermore, empirical validation demonstrates that different p values yield different ranking outcomes. Based on experimental validation using respondent data, it was found that $p = 3$ produces the closest alignment with user satisfaction compared to Hamming ($p = 1$) and Euclidean ($p = 2$). Thus, the main contribution of the proposed PFDM lies not merely in proving its mathematical properties, but in providing a tunable distance measure that can be adapted to specific application needs, with $p = 3$ offering the most accurate recommendation results for the case of the mathematics student research topics. Through this approach, it can be demonstrated how the mathematical properties of the PFDM support its effectiveness in generating accurate and personalized recommendations and in addressing the research questions.

3.1 Generalized Picture Fuzzy Distance Measure (PFDM)

Theorem 1. *Let $A, B \in PFS(X)$ with $PFS(X)$ is a set of Picture Fuzzy Set (PFS) of X, $p \in \mathbb{N}$. Given*

$$d_p(A,B) = \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p)\right)^{\frac{1}{p}}$$

is Picture Fuzzy Distance Measure (PFDM).

With:

$\mu_A(x_i)$: *positive membership* (x_i) *of PFS A;*

$\eta_A(x_i)$: *neutral membership* (x_i) *of PFS A;*

$\nu_A(x_i)$: *negative membership* (x_i) *of PFS A.*

Proof.

1. Non-Negativity: $d_p(A,B) \geq 0$

   Let $x_i \in X$, from the absolute properties we have $|\mu_A(x_i) - \mu_B(x_i)|$, $|\eta_A(x_i) - \eta_B(x_i)|$, $|\nu_A(x_i) - \nu_B(x_i)| \geq 0$ then $|\mu_A(x_i) - \mu_B(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p \geq 0$. Thus, $d_p(A,B) \geq 0$

2. Identity of Indiscernibles: $d_p(A,B) = 0$ if and only if $A = B$

   a. $d_p(A,B) = 0 \implies A = B$

      Let $d_p(A,B) = 0$ then

      $$\left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p)\right)^{\frac{1}{p}} = 0.$$

      Based on the absolute properties we have $\forall x_i \in X$, $|\mu_A(x_i) - \mu_B(x_i)| = |\eta_A(x_i) - \eta_B(x_i)| = |\nu_A(x_i) - \nu_B(x_i)| = 0$, then $\mu_A(x_i) = \mu_B(x_i)$, $\eta_A(x_i) = \eta_B(x_i)$, and $\nu_A(x_i) = \nu_B(x_i)$. Since the positive, neutral, and negative membership degrees of PFS A and B are equal for all elements, it follows that $A = B$.

   b. $A = B \implies d_p(A,B) = 0$

      Let $A = B$ then we have $\mu_A(x_i) = \mu_B(x_i)$, $\eta_A(x_i) = \eta_B(x_i)$ and $\nu_A(x_i) = \nu_B(x_i) \forall x_i \in X$. So, $\forall x_i \in X$, $|\mu_A(x_i) - \mu_B(x_i)| = |\eta_A(x_i) - \eta_B(x_i)| = |\nu_A(x_i) - \nu_B(x_i)| = 0$ then $d_p(A,B) = 0$.

3. Symmetry: $d_p(A,B) = d_p(B,A)$

   $$\begin{aligned} d_p(A,B) &= \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p)\right)^{\frac{1}{p}} \\ &= \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_B(x_i) - \mu_A(x_i)|^p + |\eta_B(x_i) - \eta_A(x_i)|^p + |\nu_B(x_i) - \nu_A(x_i)|^p)\right)^{\frac{1}{p}} \\ &= d_p(B,A) \end{aligned}$$

4. Triangle Inequality: $d_p(A,C) \leq d_p(A,B) + d_p(B,C)$

   $$d_p(A,C) = \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_C(x_i)|^p + |\eta_A(x_i) - \eta_C(x_i)|^p + |\nu_A(x_i) - \nu_C(x_i)|^p)\right)^{\frac{1}{p}}$$

$$= \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i) + \mu_B(x_i) - \mu_C(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i) + \eta_B(x_i) - \eta_C(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i) + \nu_B(x_i) - \nu_C(x_i)|^p)\right)^{\frac{1}{p}}$$

$$\leq \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)|^p + |\mu_B(x_i) - \mu_C(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i)|^p + |\eta_B(x_i) - \eta_C(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p + |\nu_B(x_i) - \nu_C(x_i)|^p)\right)^{\frac{1}{p}}$$

$$\leq \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p)\right)^{\frac{1}{p}} + \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_B(x_i) - \mu_C(x_i)|^p + |\eta_B(x_i) - \eta_C(x_i)|^p + |\nu_B(x_i) - \nu_C(x_i)|^p)\right)^{\frac{1}{p}}$$

$$= d_p(A,B) + d_p(B,C). \blacksquare$$

Theorem 2. *Let $A, B \in PFS(X)$ with $PFS(X)$ is a set of Picture Fuzzy Set (PFS) of $X, p \in \mathbb{N}$ then*

$$0 \leq d_p(A,B) \leq 1.$$

Proof. Since $d_p(A,B)$ is PFDM then $d_p(A,B) \geq 0$. All membership degrees lie in the interval [0,1], $0 \leq \mu_A(x_i), \eta_A(x_i), \nu_A(x_i) \leq 1$. The same holds $\mu_B(x_i), \eta_B(x_i), \nu_B(x_i)$. Therefore, the maximum possible absolute difference for any membership degree is 1. Hence, for each i and $p \in \mathbb{N}$:

$$|\mu_A(x_i) - \mu_B(x_i)|^p \leq 1, |\eta_A(x_i) - \eta_B(x_i)|^p \leq 1, |\nu_A(x_i) - \nu_B(x_i)|^p \leq 1.$$

Summing these three terms for a given i,

$$|\mu_A(x_i) - \mu_B(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p \leq 3.$$

Now sum over all $i = 1,2, \dots, n$:

$$\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p \leq 3n.$$

Divide both sides by 3:

$$\frac{1}{3n}\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p \leq 1.$$

Finally, take the p-th root of both sides. Since the p-th root function is monotonically increasing for non-negative arguments:

$$d_p(A,B) = \left(\frac{1}{3n}\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|^p + \left|\eta_A(x_i) - \eta_B(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p\right)^{\frac{1}{p}} \leq 1^{\frac{1}{p}} = 1$$

Thus, $d_p(A,B) \leq 1$.

From the arguments above, it follows that $0 \leq d_p(A,B) \leq 1$ for all $A, B \in PFS(X)$, and $p \in \mathbb{N}$. $\blacksquare$

Theorem 3. *Let $A, B, C \in PFS(X)$ with $A \subseteq B \subseteq C$ then $d_p(A,B) \leq d_p(A,C)$ and $d_p(B,C) \leq d_p(A,C)$.*

Proof. Let $A, B, C \in PFS(X)$ with $A \subseteq B \subseteq C$ then $\mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i), \eta_A(x_i) \leq \eta_B(x_i) \leq \eta_C(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i) \geq \nu_C(x_i)$. Consequently,

$$d_p(A,B) = \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)|^p + |\eta_A(x_i) - \eta_B(x_i)|^p + |\nu_A(x_i) - \nu_B(x_i)|^p)\right)^{\frac{1}{p}}$$

$$\leq \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_C(x_i)|^p + |\eta_A(x_i) - \eta_C(x_i)|^p + |\nu_A(x_i) - \nu_C(x_i)|^p)\right)^{\frac{1}{p}}$$

$$= d_p(A,C)$$

and

$$d_p(B,C) = \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_B(x_i) - \mu_C(x_i)|^p + |\eta_B(x_i) - \eta_C(x_i)|^p + |\nu_B(x_i) - \nu_C(x_i)|^p)\right)^{\frac{1}{p}}$$

$$\leq \left(\frac{1}{3n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_C(x_i)|^p + |\eta_A(x_i) - \eta_C(x_i)|^p + |\nu_A(x_i) - \nu_C(x_i)|^p)\right)^{\frac{1}{p}}$$

$$= d_p(A,C). \blacksquare$$

Corollary 4. *Let $A, B, C \in PFS(X)$ with $A \subseteq B \subseteq C$ then* $\frac{d_p(A,B)+d_p(B,C)}{2} \leq d_p(A,C)$

Proof. Based on Theorem 3 and metric properties.

3.2 Implementation Generalized Picture Fuzzy Distance Measure (PFDM)

3.2.1 Definition of the Research Topic Universe

The domain of potential research topics is defined based on the core concentrations within the Mathematics study program. The set of alternatives $T = \{T_1, T_2, T_3, T_4, T_5, T_6\}$ comprises the following six major fields: T_1 = Algebra, T_2 = Analysis, T_3 = Statistics, T_4 = Computational Mathematics, T_5 = Applied Mathematics, and T_6 = Financial Mathematics.

3.2.2 Criteria Definition and Data Collection as Picture Fuzzy Numbers (PFNs)

The suitability of a topic for a student is evaluated based on three primary criteria. All data will be converted into Picture Fuzzy Numbers (PFNs) to effectively handle the inherent uncertainty and hesitancy in human preferences. Let a PFN for a criterion C_i be denoted as (μ_i, η_i, ν_i), where μ_i is the positive membership (agreement), η_i is the neutral membership (hesitancy), and ν_i is the negative membership (disagreement). The three primary criteria are student interest, student competence, and supervisor suitability.

Student interest data ($C1$) is gathered through a questionnaire comprising 24 statements, each rated on a 5-point Likert scale. The statements are designed to probe the student's interest in the six defined mathematical fields.

Table 1. Linguistic Terms for Picture Fuzzy Number (PFN)

Linguistic Term	PFN (μ, η, ν)
Very Suitable	(0.85, 0.10, 0.05)
Suitable	(0.55, 0.30, 0.15)
Neutral	(0.25, 0.50, 0.25)
Unsuitable	(0.15, 0.30, 0.55)
Very Unsuitable	(0.05, 0.10, 0.85)

Each of the 24 statements features response options ranging from very suitable, suitable, neutral, unsuitable, and very unsuitable, which are subsequently converted into Picture Fuzzy Numbers (PFNs) according to Table 1. Among these 24 statements, four are dedicated to representing interest in each respective topic. The PFN for each topic is computed by aggregating the four corresponding items using the arithmetic mean (average), i.e., each component (μ, η, ν) of the resulting PFN is the average of the respective components from the four items. Accordingly, the PFN values representing student interest, denoted as $C1$, are derived and presented in Table 2.

Table 2. Picture Fuzzy Number of Student Interest (PFN C1)

No	T_1			T_2			T_3			T_4			T_5			T_6		
	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν
1	0.8	0.2	0.1	0.5	0.3	0.2	0.5	0.2	0.3	0.7	0.2	0.1	0.7	0.2	0.1	0.2	0.5	0.3
2	0.2	0.5	0.3	0.2	0.4	0.4	0.4	0.4	0.2	0.3	0.4	0.4	0.3	0.3	0.5	0.3	0.4	0.4
3	0.1	0.2	0.8	0.1	0.2	0.8	0.2	0.2	0.7	0.3	0.4	0.4	0.2	0.3	0.6	0.7	0.1	0.3
4	0.2	0.3	0.6	0.3	0.3	0.5	0.4	0.3	0.4	0.3	0.5	0.3	0.3	0.4	0.3	0.8	0.2	0.1
5	0.3	0.5	0.2	0.2	0.4	0.4	0.3	0.5	0.3	0.3	0.5	0.2	0.3	0.4	0.3	0.3	0.5	0.3
6	0.3	0.5	0.3	0.3	0.5	0.3	0.4	0.4	0.2	0.5	0.4	0.2	0.4	0.4	0.2	0.4	0.4	0.2
7	0.2	0.3	0.6	0.3	0.3	0.5	0.6	0.3	0.2	0.2	0.2	0.6	0.4	0.4	0.3	0.6	0.3	0.1
8	0.3	0.5	0.3	0.3	0.5	0.3	0.3	0.5	0.3	0.3	0.5	0.3	0.3	0.5	0.3	0.3	0.5	0.3
9	0.3	0.5	0.3	0.3	0.5	0.3	0.3	0.5	0.3	0.3	0.5	0.3	0.3	0.5	0.3	0.3	0.5	0.3
10	0.2	0.4	0.4	0.2	0.4	0.4	0.2	0.5	0.3	0.3	0.5	0.3	0.2	0.4	0.4	0.5	0.4	0.2
11	0.4	0.4	0.2	0.3	0.5	0.2	0.4	0.4	0.2	0.4	0.4	0.2	0.5	0.4	0.2	0.6	0.3	0.2
12	0.2	0.5	0.3	0.6	0.3	0.2	0.3	0.5	0.2	0.3	0.5	0.2	0.8	0.2	0.1	0.6	0.3	0.2

Academic competence $(C2)$ is derived from the student's official transcript, focusing on grades from relevant courses aligned with the six research topics. The average value of students' Grade Point Average (GPA) for each topic is then translated into Picture Fuzzy Numbers (PFNs) using the following membership function of PFS students' GPA:

$$\mu_{GPA}(x) = \begin{cases} \dfrac{x-1}{3}, & 1 < x \leq 4 \\ 0, & 0 < x \leq 1 \end{cases},$$

$$\eta_{GPA}(x) = \frac{|x-4|}{4}, x \in [0,4],$$

$$\eta_{GPA}(x) = \begin{cases} 0, & 3 \leq x \leq 4 \\ \dfrac{3-x}{3}, & 0 \leq x < 3 \end{cases}.$$

The conversion of student GPAs for each topic, utilizing the membership function for the Picture Fuzzy Set (PFS) of student GPA, yielded the PFN $C2$ as presented in Table 3.

Table 3. Picture Fuzzy Number of Student Grade Point Average (PFN C2)

No	T_1			T_2			T_3			T_4			T_5			T_6		
	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν
1	0.7	0.3	0.0	0.4	0.6	0.0	0.3	0.7	0.0	0.3	0.7	0.0	0.7	0.4	0.0	0.4	0.7	0.0
2	0.6	0.1	0.0	0.5	0.0	0.0	0.8	0.0	0.0	0.6	0.0	0.0	0.5	0.2	0.0	0.9	0.1	0.0
3	0.5	0.5	0.0	0.3	0.7	0.0	0.4	0.7	0.0	0.5	0.5	0.0	0.5	0.4	0.1	0.8	0.3	0.0
4	0.8	0.2	0.0	0.8	0.2	0.0	0.9	0.2	0.0	0.8	0.2	0.0	0.9	0.1	0.1	0.9	0.1	0.0
5	0.3	0.6	0.1	0.1	0.8	0.1	0.3	0.7	0.1	0.3	0.7	0.1	0.4	0.6	0.1	0.4	0.6	0.1
6	0.5	0.5	0.0	0.0	1.0	0.0	0.3	0.7	0.0	0.4	0.7	0.0	0.7	0.2	0.1	0.4	0.6	0.0
7	0.5	0.3	0.2	0.1	0.7	0.2	0.5	0.5	0.1	0.8	0.2	0.1	0.4	0.4	0.2	0.9	0.2	0.0
8	0.6	0.3	0.0	0.2	0.4	0.0	0.4	0.4	0.0	0.4	0.4	0.0	0.8	0.2	0.0	0.4	0.4	0.0

No	T_1			T_2			T_3			T_4			T_5			T_6		
	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν
9	0.3	0.5	0.0	0.3	0.5	0.0	0.3	0.5	0.0	0.3	0.6	0.0	0.3	0.5	0.0	0.3	0.5	0.0
10	0.1	0.4	0.1	0.0	0.4	0.0	0.1	0.4	0.1	0.1	0.5	0.1	0.1	0.5	0.1	0.2	0.4	0.1
11	0.5	0.2	0.0	0.4	0.3	0.0	0.5	0.2	0.0	0.2	0.5	0.0	0.7	0.1	0.0	0.4	0.4	0.0
12	0.4	0.5	0.0	0.2	0.8	0.0	0.3	0.8	0.0	0.3	0.6	0.2	0.7	0.3	0.1	0.5	0.5	0.0

Data on supervisor suitability ($C3$) is collected directly via a specialized questionnaire where students provide their perceptions. This questionnaire is designed to yield PFNs directly, asking students to distribute their level of agreement (μ), neutrality (η), and disagreement (ν) regarding a potential supervisor's expertise and mentoring style for a given topic, with the constraint $\mu + \eta + \nu \leq 1$. If the sum is less than 1, the remaining value $\xi = 1 - (\mu + \eta + \nu)$ represents the refusal degree, indicating the portion of uncertainty or unwillingness to respond. In this study, refusal agreements are treated as excluded from distance calculation. The PFN values based on supervisor suitability are presented in Table 4.

Table 4. Picture Fuzzy Number of Supervisor Suitability (PFN $C3$)

No	T_1			T_2			T_3			T_4			T_5			T_6		
	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν	μ	η	ν
1	1.0	0.0	0.0	0.8	0.2	0.0	0.8	0.2	0.0	1.0	0.0	0.0	0.7	0.3	0.0	0.7	0.3	0.0
2	0.9	0.1	0.0	0.7	0.3	0.0	0.9	0.2	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.8	0.2	0.0
3	0.6	0.3	0.0	0.7	0.3	0.0	0.7	0.3	0.0	0.7	0.3	0.0	0.7	0.3	0.0	0.7	0.3	0.0
4	0.9	0.1	0.0	1.0	0.0	0.0	0.8	0.2	0.0	0.9	0.1	0.0	1.0	0.0	0.0	0.8	0.2	0.0
5	0.4	0.4	0.1	0.3	0.5	0.2	0.4	0.4	0.1	0.7	0.3	0.0	0.7	0.3	0.0	0.3	0.5	0.2
6	0.6	0.3	0.0	0.7	0.3	0.0	0.6	0.3	0.0	0.8	0.2	0.0	0.7	0.3	0.0	0.4	0.4	0.1
7	1.0	0.0	0.0	1.0	0.0	0.0	0.9	0.1	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.9	0.1	0.0
8	0.8	0.2	0.0	0.7	0.3	0.0	0.7	0.3	0.0	0.9	0.2	0.0	1.0	0.0	0.0	0.8	0.2	0.0
9	0.6	0.4	0.1	0.7	0.3	0.0	0.7	0.3	0.0	0.7	0.3	0.0	0.7	0.3	0.0	0.7	0.3	0.0
10	0.8	0.2	0.0	0.7	0.3	0.0	0.8	0.2	0.0	0.8	0.2	0.0	0.7	0.3	0.0	0.5	0.4	0.1
11	0.3	0.5	0.2	0.3	0.5	0.2	0.3	0.5	0.2	0.6	0.4	0.1	0.7	0.3	0.0	0.0	0.3	0.6
12	0.9	0.1	0.0	1.0	0.0	0.0	0.8	0.2	0.0	0.9	0.1	0.0	1.0	0.0	0.0	0.9	0.1	0.0

3.2.3 Application of the Picture Fuzzy Distance Measure for Recommendation

For each student and each potential research topic T_i the following process is executed:

Step 1. Construct the Ideal PFS

An ideal profile I_j for topic T_j is defined as the PFS with maximum positive membership, zero neutral membership, and zero negative membership across all three criteria, i.e.,

$$I_j = \langle (1,0,0), (1,0,0), (1,0,0) \rangle.$$

Step 2. Construct the Student PFS

The student's profile S_j for topic T_j is constructed from the collected PFNs:

$$S_j = \langle PFNC1, PFNC2, PFNC3 \rangle.$$

Step 3. Calculate Distance (Suitability Score)

The proposed Generalized PFDM from Theorem 1 with $p = 3$ is used to calculate the distance $d_3(S_j, I_j)$ between the student's profile and the ideal profile for each topic. The formula is defined as follows:

$$d_3(S_j, I_j) = \left(\frac{1}{3n}\sum_{i=1}^{n}\left(\left|\mu_{S_j}(x_i) - \mu_{I_j}(x_i)\right|^3 + |\eta_{S_j}(x_i) - \eta_{I_j}(x_i)|^3 + |\nu_{S_j}(x_i) - \nu_{I_j}(x_i)|^3\right)\right)^{\frac{1}{3}},$$

where $n = 3$ (number of criteria: $C1 =$ student interest, $C2 =$ academic competence, $C3 =$ supervisor suitability), S_j is the student's PFS for topic T_j, I_j is the ideal PFS for topic T_j defined as (1,0,0) for each criterion.

Since the ideal profile I_j has $\mu = 1, \eta = 0, \nu = 0$ for all criteria, the formula simplifies to:

$$d_3(S_j, I_j) = \left(\frac{1}{9}\sum_{i=1}^{3}\left(\left|\mu_{S_j}(x_i) - 1\right|^3 + |\eta_{S_j}(x_i)|^3 + |\nu_{S_j}(x_i)|^3\right)\right)^{\frac{1}{3}}.$$

The results of the distance calculation between the students' PFS and the ideal PFS are presented in Table 5.

Table 5. PFDM d_3 Values between Student Profile and Ideal Profile for Each Topic, along with the Recommended Topic

No	T_1	T_2	T_3	T_4	T_5	T_6	Topic Recommendation
1	**0.218472**	0.384043	0.460009	0.431286	0.261597	0.513111	T_1
2	0.422399	0.449794	**0.325498**	0.391666	0.420748	0.38086	T_3
3	0.57135	0.615936	0.545516	0.439885	0.484223	**0.243611**	T_6
4	0.450182	0.395316	0.345062	0.400914	0.36516	**0.147449**	T_6
5	0.529691	0.632745	0.544285	0.496789	**0.46926**	0.546682	T_5
6	0.469509	0.66246	0.492841	0.438859	**0.350858**	0.479292	T_5
7	0.470256	0.555001	0.358141	0.436304	0.400477	**0.205864**	T_6
8	0.413951	0.503458	0.463647	0.451458	**0.402137**	0.455225	T_5
9	0.502202	0.497948	0.497192	0.497953	0.505624	**0.493927**	T_6
10	0.539833	0.571361	0.554187	0.554029	0.552165	**0.469125**	T_6
11	0.459292	0.50208	0.45328	0.485372	**0.315359**	0.559822	T_5
12	0.465647	0.521834	0.520001	0.489062	**0.211533**	0.346667	T_5

Step 4. Ranking and Recommendation

The topics are ranked in ascending order based on their calculated distance. A smaller distance indicates a higher degree of similarity to the ideal profile, and thus a stronger recommendation for that student. The top-ranked topics are presented as personalized recommendations. The primary research topic recommendation for each student corresponds to the smallest d_3 PFDM value, which is indicated in bold in Table 5, and the corresponding recommended topic are presented in the rightmost column (Topic Recommendation).

Step 5. Comparison of Recommendation Results for Different p values

To determine the optimal parameter, the recommendation outcomes for $p = 1,2,3,4,5$ were compared using the same student data. The ranking order of the six research topics for each student was determined based on their PFDM values across different p parameters. As shown in Table 6, the ranking sequence for each student is presented for $p = 1,2,3,4,5$ where the topics are ordered from the most suitable (left) to the least suitable (right).

Table 6. Ranking Order of Research Topics for Each Student Across Different p Values

No	$p = 1$ (Hamming)	$p = 2$ (Euclidean)	$p = 3$	$p = 4$	$p = 5$
1	$T_1, T_5, T_4, T_3, T_2, T_6$	$T_1, T_5, T_2, T_4, T_3, T_6$	$T_1, T_5, T_2, T_4, T_3, T_6$	$T_1, T_5, T_2, T_4, T_3, T_6$	$T_1, T_5, T_2, T_4, T_3, T_6$
2	$T_3, T_6, T_4, T_5, T_1, T_2$	$T_3, T_6, T_4, T_1, T_5, T_2$	$T_3, T_6, T_4, T_5, T_1, T_2$	$T_3, T_6, T_4, T_5, T_1, T_2$	$T_3, T_6, T_4, T_5, T_1, T_2$
3	$T_6, T_5, T_4, T_3, T_1, T_2$	$T_6, T_4, T_5, T_3, T_1, T_2$	$T_6, T_4, T_5, T_3, T_1, T_2$	$T_6, T_4, T_5, T_3, T_1, T_2$	$T_6, T_4, T_5, T_3, T_1, T_2$
4	$T_6, T_5, T_4, T_3, T_2, T_1$	$T_6, T_3, T_5, T_2, T_4, T_1$	$T_6, T_3, T_5, T_2, T_4, T_1$	$T_6, T_3, T_5, T_2, T_4, T_1$	$T_6, T_3, T_5, T_2, T_4, T_1$
5	$T_5, T_4, T_1, T_3, T_6, T_2$	$T_5, T_4, T_1, T_3, T_6, T_2$	$T_5, T_4, T_1, T_3, T_6, T_2$	$T_5, T_4, T_1, T_3, T_6, T_2$	$T_5, T_4, T_1, T_6, T_3, T_2$

No	$p = 1$ (Hamming)	$p = 2$ (Euclidean)	$p = 3$	$p = 4$	$p = 5$
6	$T_5, T_4, T_1, T_3, T_6, T_2$	$T_5, T_4, T_1, T_6, T_3, T_2$	$T_5, T_4, T_1, T_6, T_3, T_2$	$T_5, T_4, T_1, T_6, T_3, T_2$	$T_5, T_4, T_6, T_1, T_3, T_2$
7	$T_6, T_4, T_3, T_5, T_1, T_2$	$T_6, T_3, T_4, T_5, T_1, T_2$	$T_6, T_3, T_5, T_4, T_1, T_2$	$T_6, T_3, T_5, T_4, T_1, T_2$	$T_6, T_3, T_5, T_4, T_1, T_2$
8	$T_5, T_1, T_4, T_6, T_3, T_2$	$T_5, T_1, T_4, T_6, T_3, T_2$	$T_5, T_1, T_4, T_6, T_3, T_2$	$T_5, T_1, T_4, T_6, T_3, T_2$	$T_5, T_1, T_4, T_6, T_3, T_2$
9	$T_4, T_6, T_2, T_5, T_3, T_1$	$T_6, T_4, T_2, T_3, T_5, T_1$	$T_6, T_3, T_5, T_4, T_1, T_2$	$T_6, T_3, T_1, T_2, T_4, T_5$	$T_1, T_6, T_3, T_2, T_4, T_5$
10	$T_6, T_3, T_4, T_1, T_2, T_5$	$T_6, T_1, T_3, T_4, T_5, T_2$	$T_6, T_1, T_5, T_4, T_3, T_2$	$T_6, T_1, T_5, T_4, T_3, T_2$	$T_6, T_1, T_5, T_4, T_3, T_2$
11	$T_5, T_3, T_1, T_4, T_2, T_6$	$T_5, T_3, T_1, T_4, T_2, T_6$	$T_5, T_3, T_1, T_4, T_2, T_6$	$T_5, T_3, T_1, T_4, T_2, T_6$	$T_5, T_3, T_1, T_2, T_4, T_6$
12	$T_5, T_6, T_2, T_1, T_4, T_3$	$T_5, T_6, T_1, T_4, T_2, T_3$	$T_5, T_6, T_1, T_4, T_3, T_2$	$T_5, T_6, T_1, T_4, T_3, T_2$	$T_5, T_6, T_1, T_4, T_3, T_2$

Step 6. Validation with the Student Satisfaction Questionnaire

To evaluate which p value yields the most satisfactory recommendations, a post-implementation questionnaire was administered to the 12 participating students. Students rated the suitability of the recommended topics for each p value on a 5-point Likert scale (1 = very unsuitable, 5 = very suitable). The average satisfaction scores are presented in Table 7.

Table 7. Average Student Satisfaction Scores for Different p Values

p Value	Average Satisfaction Score (out of 5)
$p = 1$ (Hamming)	3.75
$p = 2$ (Euclidean)	4.17
$p = 3$	4.83
$p = 4$	4.58
$p = 5$	4.42

The results indicate that $p = 3$ yielded the highest average satisfaction score of 4.83 out of 5, with 10 out of 12 students rating the recommendation as "suitable" or "very suitable". Therefore, $p = 3$ was selected as the optimal parameter for the proposed PFDM-based recommendation system.

3.2.4 Interpretation and Practical Implications

The proposed Picture Fuzzy Distance Measure (PFDM) generates varied recommendations, showing that students' interest alone is insufficient when competence or supervisor suitability is low. The analysis topic (T_2) was never ranked first due to consistently low scores across all three criteria, though it often appeared in subsequent positions. These findings suggest a student's preference for applied mathematics over foundational topics. Accordingly, to balance supervisory workload, a strategy based on top-2 rankings or applying a similarity threshold (e.g., $d_3 \leq 0.3$) is proposed, aligning with existing student-supervisor allocation models that emphasize workload balance [23], [24].

4. CONCLUSION

This study has proven that the proposed Picture Fuzzy Distance Measure (PFDM) satisfies all metric axioms and generalizes the Hamming and Euclidean distances through a flexible parameter $p \in \mathbb{N}$. Empirical validation with 12 fifth-semester mathematics students from UIN Walisongo Semarang showed that $p = 3$ yields the closest alignment with user satisfaction. The PFDM-based recommendation system successfully generates personalized research topic suggestions by integrating student interest, academic competence, and supervisor suitability. However, the findings are limited by the small sample size, single-institution data, subjective PFN inputs, and the absence of comparison with existing methods. Future work should validate the approach with larger, more diverse samples, benchmark against other fuzzy distance measures, and incorporate machine learning for adaptive weighting.

Author Contributions

Dinni Rahma Oktaviani: Conceptualization, methodology, writing-original draft, and validation. Yolanda Norasia: Data curation, resources, draft preparation. Ainun Esti Candra: Writing—review and editing. All authors discussed the results and contributed to the final manuscript.

Funding Statement

This paper received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Acknowledgment

Our Gratitude to the Mathematics Students of UIN Walisongo who have been willing to fill out the questionnaire.

Declarations

The authors declare no conflicts of interest to report study.

Declaration of Generative AI and AI-assisted Technologies

Generative AI tools (e.g., ChatGPT) were used solely for language refinement, including grammar, spelling, and clarity. The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. All final text was reviewed and approved by the authors.

REFERENCES

[1] L. A. Zadeh, “FUZZY SETS,” *Information and Control*, vol. 8, no. 3, pp. 338-353, June 1965. doi: https://doi.org/10.1016/S0019-9958(65)90241-X.

[2] C. Kahraman, S. Cevik, O. Basar, and O. S. Cebi, "ROLE OF FUZZY SETS ON ARTIFICIAL INTELLIGENCE METHODS: A LITERATURE REVIEW," *Transactions on Fuzzy Sets and Systems*, vol. 2, no. 1, pp. 158–178, Mar. 2023.

[3] P. Dutta and A. Konwar, "QUINTIC FUZZY SETS: A NEW CLASS OF FUZZY SETS FOR SOLVING MULTI-CRITERIA DECISION-MAKING PROBLEMS UNDER UNCERTAINTY," *Decision Analytics Journal*, vol. 11, p. 100449, June 2024. doi: https://doi.org/10.1016/j.dajour.2024.100449.

[4] I. Couso, C. Borgelt, E. Hullermeier, and R. Kruse, "FUZZY SETS IN DATA ANALYSIS: FROM STATISTICAL FOUNDATIONS TO MACHINE LEARNING," *IEEE Computational Intelligence Magazine*, vol. 14, no. 1, pp. 31–44, Feb. 2019. doi: https://doi.org/10.1109/MCI.2018.2881642.

[5] K. T. Atanassov, "INTUITIONISTIC FUZZY SETS," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, Aug. 1986. doi: https://doi.org/10.1016/S0165-0114(86)80034-3.

[6] D. Rahma Oktaviani, M. Habiburrohman, I. Kamilia Amalina, and A. History, "APPLICATION OF INTUITIONISTIC FUZZY SETS IN DETERMINING THE MAJOR IN SENIOR HIGH SCHOOL," *Desimal: Jurnal Matematika*, vol. 5, no. 3, pp. 247–254, Oct. 2022. doi: https://doi.org/10.24042/djm.v5i3.13018.

[7] BB. C. Cuong and V. Kreinovich, "PICTURE FUZZY SETS - A NEW CONCEPT FOR COMPUTATIONAL INTELLIGENCE PROBLEMS," in *Proc. Third World Congress on Information and Communication Technologies (WICT’2013)*, Hanoi, Vietnam, pp 1-6, Dec. 15-18, 2013. doi: https://doi.org/10.1109/WICT.2013.7113099

[8] D. Rahma Oktaviani, "SUATU KAJIAN PICTURE FUZZY SUBGRUP," *Jurnal Riset dan Aplikasi Matematika*, vol. 7, no. 1, pp. 66–79, Apr. 2023. doi: https://doi.org/10.26740/jram.v7n1.p66-79.

[9] R. Xue, "EVALUATING PHYSICAL EDUCATION QUALITY IN HIGHER EDUCATION USING A PICTURE FUZZY DECISION FRAMEWORK WITH MUIRHEAD MEAN OPERATOR AND MULTIMOORA METHOD," *IEEE Access*, vol. 13, pp. 18277–18293, Jan. 2025. doi: https://doi.org/10.1109/ACCESS.2025.3532949.

[10] Z. Chen, W. Liang, and S. Luo, "A NOVEL INTEGRATED PICTURE FUZZY MACONT METHOD AND ITS APPLICATION IN TEACHING QUALITY EVALUATION IN HIGHER EDUCATION," *IEEE Access*, vol. 12, pp. 88345–88356, June 2024. doi: https://doi.org/10.1109/ACCESS.2024.3418035.

[11] R. Zhao, M. Luo, and S. Li, "A DYNAMIC DISTANCE MEASURE OF PICTURE FUZZY SETS AND ITS APPLICATION," *Symmetry*, vol. 13, no. 3, p. 436, Mar. 2021. doi: https://doi.org/10.3390/sym13030436.

[12] A. K. Adak, M. K. Gunjan, and A. M. Haghighi, "DISTANCE MEASURE OF PICTURE FUZZY SETS AND ITS APPLICATION IN MULTI-CRITERIA DECISION MAKING PROBLEM," *Journal of Fuzzy Extension and Applications*, vol. 5, no. 4, pp. 594–604, Dec. 2024.

[13] J. Bahasa et al., "PENERAPAN METODE INKUIRI DALAM PEMILIHAN TOPIK TULISAN AKADEMIK," *Jurnal Pendidikan dan Pembelajaran*, vol. 18, no. 1, pp. 45–56, Feb. 2023.

[14] R. Untari, N. Alawiyah, L. Permatasari, F. Sulistiyarini, and S. Q. Melati, "FAKTOR-FAKTOR PENGHAMBAT MAHASISWA DALAM MENYUSUN SKRIPSI," *Academia: Journal of Multidisciplinary Studies*, vol. 6, no. 2, pp. 189–204, Dec. 2022. doi: https://doi.org/10.22515/academica.v6i2.5712

[15] J. Yuan, Z. Chen, and M. Wu, "A NOVEL DISTANCE MEASURE AND CRADIS METHOD IN PICTURE FUZZY ENVIRONMENT," *International Journal of Computational Intelligence Systems*, vol. 16, no. 1, p. 150, Dec. 2023. doi: https://doi.org/10.1007/s44196-023-00354-y.

[16] A. H. Ganie, "A PICTURE FUZZY DISTANCE MEASURE AND ITS APPLICATION TO PATTERN RECOGNITION PROBLEMS," *Iranian Journal of Fuzzy Systems*, vol. 20, no. 1, pp. 71–85, Jan. 2023, doi: https://doi.org/10.22111/ijfs.2023.7347.

[17] K. Naeem and S. Memiş, "PICTURE FUZZY SOFT SIGMA-ALGEBRA AND PICTURE FUZZY SOFT MEASURE AND THEIR APPLICATIONS TO MULTI-CRITERIA DECISION-MAKING," *Granular Computing*, vol. 8, no. 2, pp. 397–410, Mar. 2023. doi: https://doi.org/10.1007/s41066-022-00333-2.

[18] G. E. Mostafa, S. Boulaaras, and R. Jan, "NOVEL PICTURE FUZZY DISTANCE MEASURE ρ∗ AND ITS APPLICATION IN MEDICAL DIAGNOSIS," *Partial Differential Equations in Applied Mathematics*, vol. 10, p. 100679, June 2024. doi: https://doi.org/10.1016/j.padiff.2024.100679.

[19] S. Singh and A. H. Ganie, "APPLICATIONS OF A PICTURE FUZZY CORRELATION COEFFICIENT IN PATTERN ANALYSIS AND DECISION-MAKING," *Granular Computing*, vol. 7, no. 2, pp. 353–367, Apr. 2022. doi: https://doi.org/10.1007/s41066-021-00269-z.

[20] S. S. Geetha and K. Selvakumari, "A PICTURE FUZZY APPROACH TO SOLVING TRANSPORTATION PROBLEM," *European Journal of Molecular & Clinical Medicine*, vol. 7, no. 2, pp. 4982–4990, 2020.

[21] S. Singh and S. Kumar, "AN INNOVATIVE DISTANCE MEASURE FOR PICTURE FUZZY SETS AND APPLICATIONS IN MEDICAL DIAGNOSIS AND PATTERN ANALYSIS," *International Journal of Fuzzy Systems*, 2025. [Online]. Available: https://internationalpubls.com (accessed Apr. 7, 2025)

[22] A. R. Mishra, S.-M. Chen, and P. Rani, "MULTI-ATTRIBUTE DECISION-MAKING BASED ON PICTURE FUZZY DISTANCE MEASURE-BASED RELATIVE CLOSENESS COEFFICIENTS AND MODIFIED COMBINED COMPROMISE SOLUTION METHOD," *Information Sciences*, vol. 664, p. 120325, Apr. 2024. doi: https://doi.org/10.1016/j.ins.2024.120325.

[23] V. Sanchez-Anguix, R. Chalumuri, R. Aydoğan, and V. Julian, "A NEAR PARETO OPTIMAL APPROACH TO STUDENT-SUPERVISOR ALLOCATION WITH TWO SIDED PREFERENCES AND WORKLOAD BALANCE," *Applied Soft Computing*, vol. 107, p. 107485, Aug. 2021. doi: https://doi.org/10.1016/j.asoc.2021.107485.

[24] G. Tabansız-Göç, A. Sebatlı-Sağlam, and F. Çavdur, "ANALYZING THE EFFECTS OF DIFFERENT PROBLEM SETTINGS ON THE SATISFACTION LEVELS OF STUDENTS AND SUPERVISORS IN THE STUDENT-SUPERVISOR ALLOCATION PROBLEM," *Journal of the Faculty of Engineering and Architecture of Gazi University*, vol. 40, no. 2, pp. 1131–1146, Feb. 2025. doi: https://doi.org/10.17341/gazimmfd.1430498.