BAREKENG: Journal of Mathematics and Its Applications
December 2026 Volume 20 Issue 4 Page 3137-3150
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol20iss4pp3137-3150

GAME THEORY AND GENETIC ALGORITHM PERSPECTIVE FOR SOLVING MULTI-ECHELON INVENTORY MODELS

Rubono Setiawan [1*], **Muhammad Aditya Tri Ariyanto** [2]

[1,2] *Mathematics Education Department, Universitas Sebelas Maret*
Jln. Ir. Sutami No. 36 A Kentingan, Surakarta, 57126, Indonesia

*Corresponding author's e-mail: ** ***rubono.matematika@staff.uns.ac.id***

Article Info

Article History:
Received: 30th September 2025
Revised: 2nd April 2026
Accepted: 29th May 2026
Available online: 1st July 2026

Keywords:
Game Theory;
Genetic Algorithm;
Inventory;
Nash Equilibrium;
Pareto Optimal.

ABSTRACT

The inventory model provides a mathematical foundation for estimating total costs in managing product flow within a supply chain. As modern supply chains involve multiple decision makers with potentially conflicting objectives, multi-echelon inventory systems require both cost optimization and strategic coordination. Conventional optimization methods are often insufficient to capture these strategic interactions. To address this challenge, this study develops a hybrid model integrating game-theoretic analysis with a Genetic Algorithm (GA) to optimize decision-making in multi-echelon inventory systems. The main contribution of this study lies in integrating game-theoretic equilibrium concepts with a genetic algorithm framework to provide a unified analytical–computational approach for two-echelon inventory decision-making. Game theory represents the strategic behavior among supply chain actors through equilibrium concepts such as Nash and Stackelberg equilibria, while the GA efficiently explores high-dimensional decision spaces to obtain near-optimal solutions. The integration of game-theoretic equilibrium concepts with a genetic algorithm framework allows the resulting solutions to satisfy equilibrium conditions while remaining computationally tractable. This hybrid framework provides a more realistic representation of supply chain decision-making under competitive or cooperative settings. The results demonstrate that the proposed approach enhances the performance of classical game-theoretic inventory models, offering a robust tool for optimizing inventory strategies in complex supply chain environments. The results demonstrate improvements in total inventory cost and coordination efficiency across the vendor–buyer system under different equilibrium schemes. However, this study is limited to a two-echelon deterministic inventory structure with numerical illustrations

How to cite this article:

R. Setiawan and M. A. T. Ariyanto, "GAME THEORY AND GENETIC ALGORITHM PERSPECTIVE FOR SOLVING MULTI-ECHELON INVENTORY MODELS", *BAREKENG: J. Math. & App.*, vol. 20, no. 4, pp. 3137-3150, Dec. 2026.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · Open Access

1. INTRODUCTION

Mathematical inventory models consist of cost functions required by the parties involved in an industrial supply chain system. These cost functions depend on decision variables and parameters that are identified in the system under certain assumptions. As in other mathematical models, the broader and more detailed the assumptions considered, the better the formula represents the actual system, yet it also increases the complexity of the model. Once the mathematical model is established, optimization methods are applied to determine the optimal values of the decision variables. Here, the optimum refers to the minimum value, with the expectation that the total cost incurred will be as low as possible. These optimal values are often referred to as the Economic Order Quantity (EOQ). Initially, inventory models focused only on a single echelon. Later, Goyal [1] introduced a two-echelon inventory model involving vendor and buyer, as well as the concept of integration in inventory systems. The integration concept refers to collaboration between vendor and buyer aimed at achieving better long-term joint benefits. This integrated inventory idea became increasingly popular and has been widely applied in multi-echelon inventory analyses by many researchers. This approach emphasizes that coordinated decision-making can significantly reduce total system costs compared to independent optimization by each party.

In practice, not all parties adopt an integrated approach in determining costs and economic strategies. In inventory systems where one party acts monopolistically as a leader, the system's optimal decision depends on the leader's decision while considering the follower's response. To accommodate such strategic interactions, game theory has been extensively applied in inventory analysis. The equilibrium concept in game theory provides a framework to analyze the optimal decisions of each party within the system. Several studies have highlighted this approach, such as [2] - [7]. Advanced results in the application of game theory in inventory problems and supply chain problems are also provided in several results, such as [8] - [18]. This article discusses the use of equilibrium concepts in game theory based on prior studies and recent developments in inventory modeling. However, most existing studies remain limited in jointly addressing imperfect quality, shortages, and leader–follower decision structures within a two-echelon inventory system, which motivates the proposed model.

As the complexity of multi-compartment inventory models increases, analytical solutions become difficult to derive, especially when the number of decision variables grows or the cost structure is nonlinear. To overcome this, heuristic and metaheuristic approaches have been increasingly employed. One effective method is the Genetic Algorithm (GA), an evolutionary search technique capable of exploring high-dimensional solution spaces and avoiding local optima. The use of computational methods and genetic algorithms in inventory-related topics has also been explored, as can be seen in several references such as [19], [20], and [21]. In previous studies, the integration between game theory concepts and genetic algorithms in the analysis of inventory and supply chain models has not been conducted. The integration of GA with game theory provides a hybrid framework that not only captures the strategic interactions among supply chain parties but also offers a computational tool to obtain near-optimal solutions efficiently. Hence, the combination of these two approaches is expected to yield more realistic and applicable decision-making in complex multi-echelon inventory systems.

Despite extensive studies on game-theoretic and metaheuristic-based inventory models, existing works often address these approaches separately or under simplified cost structures. In particular, the integration of game-theoretic equilibrium concepts with a genetic algorithm to handle nonlinear cost structures arising from imperfect quality items, shortages, and leader–follower decision-making in a two-echelon inventory system remains limited. This study fills this gap by proposing an integrated analytical–computational framework that combines equilibrium analysis with a genetic algorithm to address these complexities.

Based on this background, the main research question addressed in this paper is: How can the equilibrium concept of game theory be combined with a genetic algorithm to obtain optimal or near-optimal solutions in complex multi-compartment inventory models?. Therefore, the objective of this study is to explain how the integration of game theory concepts and genetic algorithms can be applied in analyzing multi-compartment inventory models.

2. RESEARCH METHODS

This study is a theoretical investigation based on the development of inventory theory and prior research on the application of game theory in inventory systems. The analysis is further enhanced by incorporating a numerical computation approach through the use of a Genetic Algorithm (GA), which is employed to obtain near-optimal solutions in cases where analytical derivation is difficult or intractable. The hybrid approach allows game-theoretic equilibrium concepts to be integrated with heuristic search, enabling a more practical exploration of optimal strategies in inventory systems. The mathematical models presented in this study are formulated under a set of predefined assumptions. The focus is placed on a two-level inventory model involving two main parties: the Vendor (or Manufacturer) and the Buyer (or Retailer). The Vendor/Manufacturer is responsible for producing the product, while the Buyer/Retailer acts as the ordering party who purchases the product for resale or distribution to end customers.

In line with the general approach in mathematical modeling, the formulation of the two-tier inventory model in this study is based on a number of simplifying assumptions to facilitate the analysis process. The proposed model represents the inventory system as a two-tier hierarchical structure consisting of vendors and buyers. The analysis focuses on a single product type within the system. The system's characteristics are stochastic, with uncertainty modeled through variability in product lead times. Furthermore, the model considers shortage costs, defined as the penalty for stockouts. To analyze optimality, three game-theoretic equilibrium concepts are applied: Stackelberg Equilibrium, Nash Equilibrium, and Pareto Optimality. The Nash equilibrium is considered primarily as a benchmark to provide a comparative perspective against the Stackelberg and Pareto-based solutions. While the Nash framework is not explored in the same analytical depth, it helps illustrate the differences in coordination efficiency and cost outcomes under non-cooperative decision-making. The conceptual framework of this study is implicitly represented through the mathematical formulation and the sequential integration of game-theoretic equilibrium analysis with the genetic algorithm procedure. This study is theoretical in nature, where the genetic algorithm is employed as a computational tool for numerical illustration rather than for extensive validation or sensitivity analysis

3. RESULTS AND DISCUSSION

3.1. Mathematical Model Formulation

The multi-echelon inventory model has become a standard approach used in the analysis of supply chain systems. Compared to the single-echelon model, the multi-echelon inventory framework provides a more realistic representation of real-world conditions. Based on the development of research in this field, the two-echelon inventory model is the most widely adopted structure. The two-echelon system consists of two main parties in the inventory system, with the most common and well-known example being the Vendor and the Buyer, or in other terms, the Manufacturer and the Retailer. The Vendor (or Manufacturer) functions as the producer of finished goods within the supply chain, which may take the form of factories or other types of production entities. On the other hand, the Buyer (or Retailer) is not the ultimate consumer of the product, but rather an intermediary that purchases the goods for the purpose of resale, either directly to end consumers or to other parties further downstream. In recent developments, inventory models have also been extended to three-echelon systems, known as the Supplier–Manufacturer–Retailer model. In this structure, the Supplier plays the role of providing raw materials needed for the production process. For the purpose of this article, the focus is limited to the two-echelon inventory model. In addition, to address the increasing complexity of the optimization problem in such models, this study incorporates a computational perspective through Genetic Algorithms (GA). The GA framework is expected to assist in identifying near-optimal solutions for decision variables under multi-echelon interactions, where analytical solutions may become intractable due to probabilistic demand, shortage costs, and strategic behavior of the involved parties.

3.2 Determination of Assumptions in the Mathematical Inventory Model

Assumptions in a mathematical model function as boundaries that define the scope of the problem under consideration. Every mathematical model is always constructed based on a set of predetermined assumptions. These assumptions are chosen by the researcher according to the critical aspects of the problem being studied and modeled. Each researcher may adopt different perspectives in selecting and formulating assumptions for a model, and this diversity of viewpoints contributes to the advancement of knowledge in the field. In fact, a similar basic model may lead to different formulations and analyses depending on the assumptions adopted. It should be emphasized that a mathematical model has inherent limitations in representing the real-world situation of the problem under study. The process of identifying variables and parameters in any problem can never fully capture all aspects, influences, and contributions of the actual system. Consequently, a mathematical model cannot account for factors that are not explicitly stated in its initial assumptions. The more assumptions are incorporated into the identification stage for building the model, the more complex the resulting mathematical model becomes. On the one hand, a greater number of assumptions allows the model to more comprehensively represent the problem. On the other hand, the complexity of the model may also create challenges for researchers in terms of mathematical computation, finding tractable solutions, and ensuring an appropriate representation of the system. In recent literature, researchers have increasingly emphasized not merely the accumulation of assumptions, but rather the exploration of new assumptions, improvements, or extensions of existing models. This orientation ensures that research in inventory modeling continues to develop dynamically and sustainably. In this article, illustrative examples are also provided to demonstrate the construction of total cost functions and their analysis under the framework of game theory. The basic assumptions applied in this study are as follows:

1. A single type of product is considered in the inventory system.
2. Demand is known with certainty (deterministic), constant, and continuous.
3. Lead time for replenishment is known and constant.
4. A proportion of defective items (denoted by γ) exists within the batch of Q units shipped from the Vendor to the Buyer. Additionally, due to shipping disruptions, some originally non-defective items may degrade into imperfect quality with a certain probability (θ). To ensure sufficient production capacity, it is assumed that $(1-\gamma)(1-\theta)P > D$, where P is the Vendor's production rate and D is the Buyer's annual demand.
5. The Vendor bears the full cost of defective units.
6. Shortage is allowed, but these shortages are completely backordered and immediately replenished by the Vendor.

Notation:

Q	: Quantity ordered by the Buyer from the Vendor per shipment;
n	: Number of shipments in one production cycle;
D	: Annual demand;
P	: Vendor's production rate;
F	: Transportation cost per shipment;
γ	: Probability of defective items in production;
θ	: Probability of quality degradation during transportation;
h_B	: Holding cost per unit for the Buyer;
h_v	: Holding cost per unit for the Vendor;
S_v	: Setup cost for the Vendor;
S_B	: Setup cost for the Buyer;
w_v	: Warranty or replacement cost borne by the vendor for defective items;
s_c	: Sorting cost at the Buyer's facility;
B	: Maximum backorder quantity allowed;
b	: Reorder cost due to shortage;
$(.)^{opt}$	: subscript denoting optimal values of decision variables.

After defining the basic assumptions of the mathematical model, the next step is to formulate the total cost functions for each party in the inventory system, namely the Buyer and the Vendor. These cost functions represent the economic performance of each echelon and serve as the foundation for further optimization analysis. In several studies, inventory level diagrams are also provided to illustrate the dynamics of stock requirements for each party.

3.3 Buyer's Total Cost per Production Cycle

The Buyer's total cost structure in a two-echelon inventory system consists of multiple interacting components. Specifically, these include ordering cost**,** transportation cost**,** holding cost**,** and backordering cost, all of which must be taken into account to represent the Buyer's operational reality. The total cost function for the Buyer can be expressed as:

$$TC_b(Q,B) = S_b + F + s_cQ + h_b\left\{\frac{1}{2}\frac{(Q-Q\gamma\theta-B)^2}{D} + \frac{Q^2\gamma\theta(1-\gamma)(1-\theta)}{D}\right\} + b\frac{1}{2}\frac{B^2}{D}\,. \tag{1}$$

In this formulation:

1. S_b represents the ordering cost incurred by the Buyer whenever an order is placed.
2. F denotes the transportation cost required for shipping products from the Vendor.
3. s_cQ reflects the screening or inspection cost proportional to the lot size Q.
4. h_b captures the holding cost associated with maintaining inventory levels at the Buyer's facility.
5. b corresponds to the backordering cost, representing the penalty or additional expense when customer demand cannot be immediately fulfilled.

This expression in Eq. (1) represents the expected total cost of the system, which consists of ordering, holding, shortage, and quality-related cost components incurred by the decision-makers. Importantly, the formulation integrates product quality uncertainty through the parameters γ/gamma (the probability of receiving imperfect items due to production defects) and θ/theta (the probability of deterioration during transportation). The adjustment term $(1-\gamma)(1-\theta)$ serves as an effective quality factor, ensuring that the Buyer's cost structure reflects the true economic impact of imperfect quality. To obtain a tractable expression, we assume that the probability of imperfect quality follows a binomial random distribution. By applying the renewal–reward theorem, the expected total cost (ETC) for the Buyer can be expressed as:

$$\begin{aligned} ETC_b(Q,B) = \frac{E[TC_b(Q,B)]}{E[T]} &= \frac{E[TC_b(Q,B)]D}{Q(1-\gamma)(1-\theta)} \\ &= \frac{D(S_b+F)}{Q(1-\gamma)(1-\theta)} + \frac{s_cD}{(1-\gamma)(1-\theta)} + \frac{h_b(Q-Q\gamma\theta-B)^2}{2Q(1-\gamma)(1-\theta)} \\ &\quad + h_bQ\gamma\theta + \frac{bB^2}{2Q(1-\gamma)(1-\theta)} \end{aligned} \tag{2}$$

This expected total cost formulation highlights several managerial insights. First, it demonstrates the trade-off between ordering frequency and holding costs: larger order quantities (QQ) reduce ordering costs but increase holding costs. Second, the incorporation of imperfect items introduces an additional penalty that reflects the need for quality inspection and possible rework or disposal. Third, the backordering term $\frac{bB^2}{2Q(1-\gamma)(1-\theta)}$ emphasizes the importance of balancing inventory policies to mitigate customer dissatisfaction and potential loss of goodwill due to shortages. Overall, this cost model provides a more realistic and comprehensive framework compared to classical inventory models that assume perfect quality and zero shortage. It captures not only the operational costs but also the risk factors embedded in the Buyer's decision-making process. Consequently, this formulation can serve as the foundation for further optimization using game-theoretic concepts and computational approaches such as genetic algorithms, enabling robust decision support in uncertain supply chain environments.

3.4 Vendor's Total Cost per Production Cycle

In contrast to the Buyer, the Vendor's total cost structure is inherently more complex, as it depends not only on the production decisions but also on the interaction between the Vendor's and Buyer's inventory levels. The average inventory level per production cycle for the Vendor can be obtained by calculating the accumulated Vendor inventory level minus the accumulated Buyer inventory level. This difference reflects the fact that while the Vendor produces in batches, a portion of the stock is immediately transferred to the Buyer, thereby reducing the effective inventory held by the Vendor. The holding cost for the Vendor per cycle is therefore expressed as:

$$\begin{aligned} &h_v\left(\frac{nQ}{2}\left\{\left[(n-1)T+\frac{Q}{P}\right]+\left[(n-1)T+\frac{Q}{P}-\frac{nQ}{P}\right]\right\}-[1+2+\cdots+(n-1)]QT\right) \\ &\quad = h_v\left(\left[nQ\left(\frac{Q}{P}+(n-1)T\right)-\frac{nQ(nQ/P)}{2}\right]-T[Q+2Q+\cdots+(n-1)Q]\right) \end{aligned}$$

$$= h_v\left(\frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-\theta)}{2D}\right). \tag{3}$$

The expression in Eq. 3 captures the Vendor's expected total cost by considering setup, holding, and quality-related costs, which are influenced by both the production quantity and the Buyer's ordering decision. This formulation shows that the Vendor's holding cost is directly influenced by the production rate P, the number of shipments n, and the quality factor $(1-\gamma)(1-\theta)$, which reflects the proportion of defect-free items after both production and transportation. When we incorporate the Vendor's setup cost S_v, the warranty or replacement cost for defective items ($w_v nQ\gamma$), and the holding cost derived above, the Vendor's total cost per cycle is given as:

$$TC_v(n,Q) = S_v + w_v nQ\gamma + h_v\left\{\frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-\theta)}{2D}\right\}. \tag{4}$$

Similar to the Buyer's case, it is necessary to compute the expected total cost (ETC) for the Vendor by normalizing the total cost with respect to the expected cycle length. The expected total cost is expressed as:

$$\begin{aligned} ETC_v(n,Q\ ) &= \frac{E[TC_b(n,Q)]}{E[nT]} = \frac{E[TC_b(n,Q)]D}{nQ(1-\gamma)(1-\theta)} \\ &= \frac{D}{(1-\gamma)(1-\theta)}\left(\frac{S_v}{nQ} + w_v\gamma + h_vQ\left(\frac{1}{P} - \frac{n}{2P} + \frac{n-1}{2D}\right)\right). \end{aligned} \tag{5}$$

The equation in Eq. 5 demonstrates three essential aspects of the Vendor's decision-making: Setup and production costs scale inversely with lot size Q, incentivizing larger batch production, warranty or replacement costs grow proportionally with defect rates γ, emphasizing the importance of quality control in the Vendor's operations, and holding costs depend not only on the production rate but also on the coordination of shipment frequency n with Buyer demand. From a managerial perspective, this formulation reflects the trade-offs between production efficiency, product quality, and shipment frequency. While higher production rates reduce the Vendor's holding costs, they may also increase the exposure to defects and replacement obligations. Moreover, increasing the number of shipments reduces the Buyer's holding burden but raises the Vendor's setup costs. This dual dependency on both internal (production, defect management) and external (shipment coordination with the Buyer) factors makes the Vendor's optimization problem particularly challenging. As a result, advanced solution techniques such as game-theoretic equilibrium analysis and genetic algorithm-based optimization are necessary to determine stable and efficient decision policies that align Vendor and Buyer interests within the two-echelon supply chain.

3.5 Determination of the Optimum Solution for Decision Variables (EOQ)

The optimal value of the decision variable in the inventory system is commonly referred to as the Economic Order Quantity (EOQ). Fundamentally, the technique for determining EOQ relies on classical optimization methods, particularly calculus-based optimization using the first-order partial derivative criteria. The central difference lies in the formulation of the objective function, which serves as the basis for applying this optimization criterion. In the integration-based approach, the objective function is typically defined as the combined total cost of both the Buyer and the Vendor. By minimizing this joint cost function, the optimal ordering policy that balances the trade-off between ordering costs, holding costs, and other operational expenses can be obtained. However, in scenarios where optimal decisions must also incorporate the strategic preferences of each party, concepts from game theory may be employed. Under this framework, the objective function can be constructed not only from the perspective of cost minimization but also in terms of equilibrium outcomes, where neither the Buyer nor the Vendor has an incentive to deviate unilaterally from the chosen strategy. Furthermore, with the increasing complexity of modern supply chains, analytical methods may sometimes be insufficient to derive closed-form solutions for EOQ, especially when uncertainties, nonlinear constraints, or green inventory considerations are included. In such cases, metaheuristic algorithms, such as Genetic Algorithms (GA), can serve as alternative approaches for approximating the optimal EOQ. GA allows for flexible exploration of the solution space, reducing the risk of being trapped in local minima while providing near-optimal solutions efficiently. Thus, the determination of EOQ can be approached either analytically through calculus-based optimization or computationally through heuristic and metaheuristic techniques, depending on the structure and complexity of the inventory model under consideration.

3.6 Use of the Stackelberg Equilibrium in Determining the Optimum Solution

In a supply chain system where one party acts monopolistically as a leader while the other acts as a follower, cooperation between parties is not assumed. The leader dominates and effectively determines the system-wide optimum (EOQ), which the follower must accept. This situation can be modeled using the Stackelberg equilibrium. Under the follower-first Stackelberg scheme adopted here, the follower (Buyer) determines its optimal decision first, given identified decision variables; the leader (Vendor) then observes the follower's optimal choice and reacts optimally. In many supply chain formulations, the Vendor is assumed to be the leader while the Buyer is the follower. The mathematical steps of the Stackelberg procedure applied to our model are:

1. Determine the individual EOQ (and related decision variables) of the Buyer as the follower based on the Buyer's total expected cost function.
2. Substitute the Buyer's optimal response into the Vendor's total expected cost.
3. Determine the Vendor's optimal decision(s) given the substituted Buyer response.
4. The result of step 3 constitutes the Stackelberg system solution.

Below, we apply these steps analytically, using the Buyer's expected total cost:

$$ETC_b(Q,B) = \frac{D(S_b+F)}{Q(1-\gamma)(1-\theta)} + \frac{s_c D}{(1-\gamma)} + \frac{h_b(Q-Q\gamma\theta-B)^2}{2Q(1-\gamma)(1-\theta)} + h_b Q\gamma\theta + \frac{bB^2}{2Q(1-\gamma)(1-\theta)}, \quad (6)$$

where we denote $\alpha = \gamma\theta$ and $\Delta = (1-\gamma)(1-\theta)$ for brevity.

1. First-order condition with respect to BB (Buyer's backorder level)

Take the partial derivative of ETC_b with respect to B and set it equal to zero:

$$\frac{\partial ETC_b}{\partial B} = -\frac{h_b(Q-Q\alpha-B)}{Q\Delta} + \frac{bB}{Q\Delta} = 0. \quad (7)$$

Solving for B yields a closed-form best-response of the Buyer:

$$B^*(Q) = \frac{h_b}{b+h_b}Q(1-\alpha), \quad (8)$$

where $\alpha = \gamma\theta$. This expression indicates that the optimal backorder level is proportional to the order quantity Q with factor $\frac{h_b}{b+h_b}(1-\alpha)$.

2. Reduced Buyer cost function (substitute $B^*(Q)$)

Substituting $B^*(Q)$ into $ETC_b(Q,B)$ simplifies the Buyer's expected total cost into a scalar function of QQ only. After algebraic simplification, the reduced cost becomes

$$\widehat{ETC_b}(Q) = \frac{D(S_b+F)}{Q\Delta} + \frac{s_c D}{\Delta} + h_b Q\alpha + \frac{bh_b(1-\alpha)^2}{2(b+h_b)\Delta} = 0. \quad (9)$$

The first two terms show the classical order-frequency trade-off (ordering vs. holding), the third term reflects the direct holding penalty due to imperfect-quality fraction α/alpha, and the fourth term collects the contributions from the squared inventory terms after the optimal backorder substitution.

3. First-order condition with respect to Q (Buyer's EOQ)

Differentiate $\widehat{ETC_b}(Q)$with respect to Q and set to zero:

$$\frac{\widehat{ETC_b}(Q)}{dQ} = h_b\alpha + \frac{bh_b(1-\alpha)^2}{2(b+h_b)\Delta}. \quad (10)$$

Rearranging gives a closed-form expression for the buyer's optimal order quantity Q_B^*:

$$\frac{D(S_b+F)}{{Q_B^*}^2\Delta} = h_b\alpha + \frac{bh_b(1-\alpha)^2}{2(b+h_b)\Delta}. \quad (11)$$

Therefore

$$Q_B^* = \sqrt{\frac{D(S_b + F)}{\Delta\left(h_b\alpha + \frac{bh_b(1-\alpha)^2}{2(b+h_b)\Delta}\right)}}\ , \tag{12}$$

and, using the closed-form $B^*(Q)$, the optimal backorder at the buyer is

$$B^* = \frac{h_b}{b + h_b} Q_B^*(1-\alpha). \tag{13}$$

These formulas give an explicit analytic characterization of the follower's (Buyer's) Stackelberg response in terms of model parameters. Note that $\alpha = \gamma\theta$ and $\Delta = (1-\gamma)(1-\theta)$.

4. Second-order condition and feasibility

To ensure that the stationary point is a minimum, check the second derivative:

$$\frac{d^2\widehat{ETC_b}(Q)}{dQ^2} = \frac{2D(S_b + F)}{Q^3\Delta} > 0, Q > 0\ , \tag{14}$$

so the critical point computed above corresponds to a local (and global, given convexity of the dominant term) minimum for positive QQ. Also, verify feasibility constraints (e.g., $Q > 0, 0 \le B \le Q$, production-capacity constraint $(1-\gamma)(1-\theta)P > D$, and others) when applying the formula numerically.

5. Leader reaction (Vendor)

After obtaining the follower strategy (Q_B^*, B^*), substitute Q_B^* (and B^* if needed) into the Vendor's expected total cost $ETC_v(n, Q)$. The Vendor, as leader, then optimizes its decision variables (e.g., shipment frequency n, or its own batch size if modeled separately) by minimizing the Vendor's expected cost given the Buyer's response $\min_{n\in\mathcal{N}} ETC_v(n, Q_B^*)$**.** This produces the leader's optimal decision(s) (e.g., n^*), completing the Stackelberg outcome.

The closed-form $B^*(Q)$ reveals that optimal backordering increases with the buyer's holding cost h_b and with order quantity Q, but decreases with the backorder penalty b. The quality factor $(1-\alpha)$ scales down the effective backorder because defective/deteriorated items reduce usable inventory. The EOQ formula $B^*(Q)$ preserves the classical structure $\sqrt{\frac{A}{B}}$ (ordering-related numerator divided by holding-like denominator), but the effective holding coefficient is increased by terms that capture product quality risk and the combined effect of inspection/backorder trade-offs. In short, imperfect quality and backordering behavior raise the effective marginal holding cost and therefore reduce the EOQ relative to a perfect-quality model with identical base parameters. After computing (Q_B^*, B^*), the Vendor optimizes its own policy; the resulting system solution is the Stackelberg equilibrium (follower-first).

3.7 Use of the Pareto Optimal Scheme in Determining the Optimal Solution

The optimal result derived from the integration scheme is not necessarily Pareto–Efficient. Therefore, to achieve a result that satisfies Pareto efficiency, a cooperative game approach must be applied in determining the optimal solution. The objective function under this scheme is expressed as a weighted function in the form of a convex combination of the Vendor's and Buyer's total costs. The requirement that must be fulfilled is that both parties agree on the optimal result, which should satisfy a Nash Equilibrium condition:

$$Z = \lambda_p(ETC_b) + (1-\lambda_p)ETC_v, \quad 0 < \lambda < 1$$

$$Z = \lambda_p\left[\frac{D(S_b + F)}{Q(1-\gamma)(1-\theta)} + \frac{s_c D}{(1-\gamma)(1-\theta)} + \frac{h_b(Q - Q\gamma\theta - B)^2}{2Q(1-\gamma)(1-\theta)} + h_b Q\gamma\theta + \frac{bB^2}{2Q(1-\gamma)(1-\theta)}\right]$$
$$+(1-\lambda_p)\left[\frac{D}{(1-\gamma)(1-\theta)}\left(\frac{S_v}{nQ} + w_v\gamma + h_v Q\left(\frac{1}{P} - \frac{n}{2P} + \frac{n-1}{2D}\right)\right)\right]. \tag{15}$$

The next step to determine the optimal value of Z is to apply the Karush–Kuhn–Tucker (KKT) conditions for constrained optimization. Due to the complexity of the model, the optimal solution for the decision variable (i.e., EOQ) cannot be expressed in explicit closed form. Instead, the result is often obtained in an implicit relation among the decision variables. Such outcomes may not provide sufficiently practical information for end-users. Therefore, numerical simulation employing algorithmic search procedures has become the most viable alternative to approximate the optimal decision variables in inventory systems. The application of the Pareto scheme using the convex combination as shown in Eq. (15) ensures that the resulting solution lies within the feasible region of the joint cost space of Buyer and Vendor. The parameter λp plays a crucial role in adjusting the bargaining power between the two parties: a value of λp closer to 1 emphasizes the Buyer's cost minimization, while a value closer to 0 favors the Vendor. In practice, the selection of λp is determined through negotiation or predefined contractual arrangements. To find the optimal Q^*, one can differentiate Z with respect to Q, leading to the condition:

$$\frac{\partial Z}{\partial Q} = \lambda_p \frac{\partial ETC_b}{\partial Q} + \left(1 - \lambda_p\right) \frac{\partial ETC_v}{\partial Q} = 0. \tag{16}$$

These conditions characterize the Stackelberg equilibrium by defining the optimal decisions of the leader and the follower, where each party minimizes its respective expected total cost given the strategic response of the other. This first-order condition shows that the optimal solution balances the marginal contributions of both Buyer and Vendor to the joint weighted cost. However, due to the nonlinearity and interaction terms, such as $Q\gamma\theta$ and B^2/Q, explicit closed-form solutions are rarely tractable, which explains the reliance on numerical procedures such as genetic algorithms, simulated annealing, or gradient-based solvers to approximate the solution.

3.8 Integration of Genetic Algorithm with Game Theory in Solving the Inventory Model

Analytical approaches that rely on calculus-based optimization or the Karush–Kuhn–Tucker (KKT) conditions often face significant challenges when the objective function becomes nonlinear, complex, and highly interdependent. This difficulty becomes even more apparent in inventory systems that involve asymmetric decision-making between the Vendor and the Buyer, where the problem is modeled through a game-theoretic perspective. Whether in a non-cooperative setting such as the Stackelberg equilibrium, or in a cooperative framework such as the Pareto-efficient solution, purely analytical methods may not provide explicit and tractable results. In such cases, metaheuristic methods, particularly the Genetic Algorithm (GA), offer a powerful alternative for obtaining approximate optimal solutions.

The Genetic Algorithm, which draws inspiration from the principles of natural evolution, works by generating populations of candidate solutions that evolve across multiple generations. Each candidate solution, or chromosome, represents possible values of the decision variables in the inventory model, such as the order quantity, number of shipments, or backordering level. Through processes of selection, crossover, and mutation, successive populations are formed, and the overall fitness of the solutions improves iteratively until convergence is achieved. By integrating GA with game theory, the complex equilibrium problems in inventory models can be explored systematically in a way that maintains both computational feasibility and decision relevance.

The integration begins with the definition of a suitable fitness function. In the case of a Stackelberg framework, the Buyer, acting as the follower, determines its optimal order quantity based on its cost function, and this result is then substituted into the Vendor's cost function. The Vendor, as the leader, then applies GA to search for the global optimum of its cost function. In this way, the leader's final decision reflects not only its own interest but also incorporates the strategic response of the follower. On the other hand, in a Pareto framework, the GA operates directly on a convex weighted function that combines the Buyer's and the Vendor's expected total costs. By varying the weight parameter, GA is able to generate a range of Pareto-efficient outcomes that capture the trade-offs between the two parties.

The robustness of GA lies in its ability to handle functions that are otherwise intractable by conventional optimization methods. Instead of requiring explicit differentiation or closed-form algebraic solutions, GA relies on iterative evaluations of candidate solutions, guided by evolutionary operators. This allows it to explore a wide solution space, increasing the likelihood of reaching near-global optima, while also avoiding the pitfalls of local minima. Moreover, when embedded in a game-theoretic framework, GA does not merely minimize cost functions in isolation but also respects the equilibrium structure of the interaction, whether it be the hierarchical dominance of Stackelberg or the fairness principle inherent in Pareto solutions.

Ultimately, the integration of GA with game theory provides a robust and flexible computational framework for solving multi-echelon inventory models. It complements the limitations of purely analytical techniques and ensures that the resulting solutions are both computationally practical and strategically meaningful. This approach not only enhances the applicability of inventory theory in complex supply chain settings but also opens avenues for exploring equilibrium dynamics that would otherwise remain inaccessible through traditional methods. The GA procedure follows its canonical steps: initialization, fitness evaluation, selection of parents, crossover, mutation, and population updating. This iterative process gradually refines the solution space, converging towards a near-optimal equilibrium solution. Elitism may also be employed to preserve the best solutions across generations. The pseudocode below outlines the computational scheme:

Algorithm 1. GA General Procedure for Inventory Problem

Input: Model parameters $(S_b, S_v, h_b, h_v, F, w_v, \gamma, \theta, P, D, n, \lambda_p)$
Output: Approximate optimal decision variables (Q^*, B^*, n^*)

1: Initialize population of chromosomes, each encoding decision variables (Q, B, n).

2: For each chromosome in the population:
  Evaluate fitness:
    If the Stackelberg framework:
      Compute Buyer's EOQ as follower → substitute into Vendor's cost function.
      Fitness = Vendor's expected total cost (ETC_v).
    If Pareto framework:
      Compute combined cost:
        $Z = \lambda_p * ETC_b + (1 - \lambda_p) * ETC_v$.
      Fitness = Z.

3: Repeat until termination condition is satisfied (max generations or convergence):
  Select parent chromosomes based on fitness (selection operator).
  Apply the crossover operator to generate offspring.
  Apply the mutation operator to introduce variability.
  Evaluate the fitness of offspring using the same procedure.
  Update population with offspring (elitism may be applied).

4: Return the best chromosome(s) as an approximate equilibrium solution.

3.9 Numerical Computation and Examples

The numerical example presented in this section is intended for illustrative purposes only, to demonstrate the applicability of the proposed model and solution approach rather than to provide an exhaustive. In order to demonstrate the applicability of the proposed model, this chapter provides numerical computations and illustrative examples. The main purpose is to validate the theoretical framework developed in the previous sections by applying it to a simulated two-echelon inventory system consisting of a Vendor (manufacturer) and a Buyer (retailer). The computations focus on determining the Economic Order Quantity (EOQ) for both parties under different game-theoretic scenarios and examining the effect of incorporating a Genetic Algorithm (GA) for numerical optimization. For numerical illustration, the following baseline parameters are assumed:

Table 1. Baseline Parameters

Parameter	Description	Value	Unit
D	Annual demand	10,000	units/year
P	Production rate	30,000	units/year
S_b	Ordering cost for Buyer	150	$/order
S_v	Setup cost for Vendor	200	$/batch
F	Transportation cost per shipment	100	$/shipment
h_b	Holding cost for Buyer	2	$/unit/year
h_v	Holding cost for Vendor	1.5	$/unit/year
w_v	Warranty cost per defective item	5	$/unit
b	Backordering cost per unit	4	$/unit/year
s_c	Sorting cost per defective unit	1	$/unit
γ	Defective production rate	0.02	(2%)

Parameter	Description	Value	Unit
θ	Deterioration probability in shipment	0.01	(1%)

The genetic algorithm parameters were selected based on preliminary trials and commonly accepted practices in inventory optimization problems to ensure stable convergence and reasonable computational effort.

These values are chosen to reflect a realistic medium-scale vendor-buyer system with moderate defect and deterioration risks. The analytical procedure yields the following results $Q_b^* = 1224$ units and $Q_v^* = 1633$. Hence, the compatible solution requires aligning the Buyer's and Vendor's shipment policy through $n = \frac{Q_v}{Q_b} \approx 1.33$. Practically, $n = 1$ or $n = 2$ can be selected. The Buyer's annual total cost (ordering + holding) $TC_b = 2449$ \$/ year. The Vendor's annual total cost (setup + holding) is $TC_v = 2450$ \$/year. Then, both Buyer and Vendor have almost equal costs under the classical EOQ solution. Due to the nonlinearities introduced by the defect rate (γ), deterioration (θ), and sorting/warranty costs, analytical closed- forms are no longer exact. Therefore, we apply a genetic algorithm (GA) to approximate the optimal solution. First, we define decision variables Q (order size), B (backordering level), n (shipment multiple). The fitness function is the negative of the total function (to minimize cost). GA parameters: population = 50, crossover probability = 0.8, mutation rate = 0.05, generations = 200. After running the GA, the following results are obtained:

Table 2. GA-Based EOQ Results with Defect and Deterioration

Party	EOQ (Q^*)	Backorder (B^*)	Shipments (n)	Total Cost (\$/year)
Buyer	1198 units	150 units	–	2530
Vendor	1605 units	–	1	2510

The results highlight several important aspects of applying Genetic Algorithms (GA) in the inventory model with game-theoretic settings. First, in terms of accuracy, the GA demonstrates convergence close to the analytical Economic Order Quantity (EOQ) in the simplified scenario, which validates the reliability of the proposed heuristic method. This indicates that GA is not only capable of approximating known solutions but also provides confidence in its application to more complex situations. Second, the flexibility of GA becomes evident when additional cost components, such as defect and deterioration, are introduced. In such cases, deriving closed-form analytical solutions through conventional calculus methods becomes intractable. Nevertheless, GA successfully identifies near-optimal solutions, proving its robustness and adaptability to handle more realistic and nonlinear extensions of the inventory problem. In game-theoretic analysis, the Stackelberg and Pareto solutions are inherently analytical and point-based; therefore, the outcomes are presented in numerical and analytical form rather than graphical equilibrium plots.

Finally, the integration of game-theoretic perspectives adds further insight into the decision-making dynamics within the supply chain system. Under the Stackelberg equilibrium, the Vendor, as the leader, exerts greater influence, shaping the shipment policy and forcing the Buyer to bear relatively higher holding costs. Conversely, in a Pareto-optimized framework, the allocation of order quantities and costs tends to be more balanced, reducing the disparity between the two parties. This comparison underscores the practical implications of selecting different strategic schemes when aiming to achieve efficiency and fairness in supply chain coordination.

The numerical computation conducted in this study provides insights into the practical applicability of the proposed two-echelon inventory model. Using hypothetical parameter values for demand, production rate, holding costs, and defect probabilities, both analytical and numerical solutions were derived. The analytical approach yielded closed-form or semi-closed-form expressions for the optimal decision variables, whereas the Genetic Algorithm (GA) provided approximate solutions through iterative search. The results show that the EOQ obtained from the GA approach converges closely to the analytical benchmark, indicating the reliability of the heuristic method in handling complex, nonlinear structures. Specifically, the buyer's total cost function and the vendor's total cost function both exhibit convexity, which ensures the existence of an optimal point. The GA solutions demonstrate consistency with this theoretical expectation by locating decision values that minimize the overall system cost.

Graphical illustrations further highlight the behavior of the cost functions. As the order quantity increases, the total cost initially decreases due to the reduction in ordering frequency, but beyond a certain threshold, the cost rises again because of escalating holding expenses. This U-shaped curve validates the classic inventory theory while also showing the impact of defect probabilities and shortage costs on the system dynamics. Moreover, the numerical experiments confirm the relevance of integrating game-theoretic perspectives. Under the Stackelberg equilibrium, the vendor as leader tends to set order sizes that benefit its own cost structure, while the buyer adapts accordingly. In contrast, the Pareto-optimal scheme balances the cost between parties, resulting in a more equitable allocation of benefits. The GA effectively captures these strategic outcomes by optimizing under different equilibrium scenarios. Overall, the numerical evidence reinforces the theoretical framework and demonstrates that a hybrid approach combining analytical methods with GA-based computation provides a powerful decision-support tool for two-echelon inventory systems.

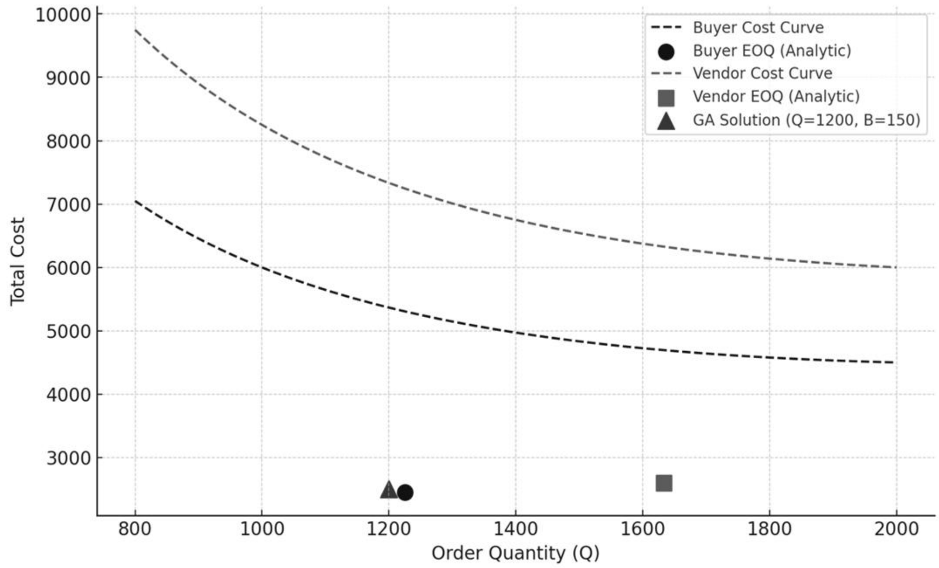

Figure 1. Comparison of EOQ Solutions: Analytic vs Genetic Algorithm

The graph above illustrates the comparison between the analytical solution and the solution obtained using a Genetic Algorithm (GA). The blue curve represents the Buyer’s total cost, with the analytical optimum marked by a blue circle. The green curve represents the Vendor’s total cost, with the analytical optimum indicated by a green square. The red triangle depicts the GA solution, which yields an EOQ of approximately 1200 with a backorder level of 150 and a total cost close to 2500. From the graph, it can be observed that the GA solution lies reasonably close to the analytical optimum, indicating that the heuristic approach can identify competitive solutions even in more complex models where closed-form analytical solutions are difficult to obtain.

Managerial insights

The results of this study provide practical insights for decision makers in two-echelon supply chains, particularly vendors and buyers facing imperfect quality items and shortage risks. The Stackelberg-based analysis highlights how leader–follower decision structures influence optimal ordering and production quantities, enabling dominant players to anticipate and respond to the reactions of their counterparts. Meanwhile, the Pareto-based solutions offer guidance for coordinated decision-making when joint cost efficiency is prioritized. The integration with a genetic algorithm further allows managers to obtain near-optimal decisions efficiently in complex settings where analytical solutions are difficult to derive, making the proposed framework applicable to real-world inventory planning problems.

4. CONCLUSION

This study developed a two-echelon inventory model that integrates vendor–buyer interactions with game-theoretic and heuristic optimization concepts. The integration of game theory and heuristic optimization provides an effective framework for modeling strategic interactions between supply chain actors, enabling both cooperative and non-cooperative scenarios to be analyzed systematically. The application of Stackelberg, Nash, and Pareto-Optimal equilibria allows for the determination of optimal order quantities and cost allocations under probabilistic conditions, reflecting realistic decision-making behaviors. Genetic Algorithm (GA) optimization successfully approximated the analytical solutions, demonstrating high accuracy in handling nonlinear and interdependent decision variables. The hybrid approach combining analytical and heuristic methods enhances computational efficiency and robustness in solving complex inventory problems where traditional methods become intractable. This study contributes by integrating mathematical modeling, game theory, and evolutionary computation within a unified inventory modeling framework. Future research may extend this approach to multi-echelon systems, incorporate stochastic demand and lead time uncertainty, or explore other metaheuristics such as Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO). Furthermore, integrating environmental considerations could support the development of sustainable and green supply chain models. It should be noted that this study is limited to a two-echelon deterministic inventory structure and focuses on theoretical modeling with numerical illustrations, without extensive validation or sensitivity.

Author Contributions

Rubono Setiawan: Conceptualization, Writing-Original Draft, Writing-Review, Data Curation, Formal Analysis, Investigation, Methodology, Resources, Project Administration, Software Computation, Validation. M. Aditya Tri Ariyanto: Visualization and Editing. All authors discussed the results and contributed to the final manuscript.

Funding Statement

This research was funded by RKAT Universitas Sebelas Maret Tahun Anggaran 2025, through a research grant: Skema Penelitian Fundamental (PF-UNS) (Grant No: 369/UN27.22/PT.01.03/2025).

Acknowledgment

The author would like to express gratitude to the Institute for Research and Community Service (LPPM) of Universitas Sebelas Maret.

Declarations

The authors declare no competing interests.

Declaration of Generative AI and AI-assisted Technologies

Generative AI tools (e.g., ChatGPT) were used solely for language refinement, including grammar, spelling, and clarity. The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. All final text was reviewed and approved by the authors.

REFERENCES

[1] S. K. Goyal, "AN INTEGRATED INVENTORY MODEL FOR A SINGLE SUPPLIER-SINGLE CUSTOMER PROBLEM," *The International Journal of Production Research*, vol. 15, no. 1, pp. 107–111, 1977. doi: https://doi.org/10.1080/00207547708943107.

[2] M. Esmaeili, M.-B. Aryanezhad, and P. Zeephongsekul, "A GAME THEORY APPROACH IN SELLER–BUYER SUPPLY CHAIN," *European Journal of Operations Research*, vol. 195, no. 2, pp. 442–448, 2009. doi: https://doi.org/10.1016/j.ejor.2008.02.026.

[3] M. Elyasi, F. Khoshalhan, and M. Khanmirzaee, "MODIFIED ECONOMIC ORDER QUANTITY (EOQ) MODEL FOR ITEMS WITH IMPERFECT QUALITY: GAME-THEORETICAL APPROACHES," *International Journal of Industrial Engineering Computations*, vol. 5, no. 2, p. 211, 2014. doi: https://doi.org/10.5267/j.ijiec.2014.1.003

[4] R. Setiawan, "A GAME THEORY APPROACH IN VENDOR-BUYER PROBABILISTIC INVENTORY SYSTEM WITH IMPERFECT QUALITY, INSPECTION ERROR, MINIMUM SERVICE LEVEL CONSTRAINT AND PARTIAL BACKORDERING," *Far East Journal of Mathematical Sciences*, vol. 100, no. 10, p. 1695, 2016. doi: https://doi.org/10.17654/MS100101695.

[5] R. Setiawan, "NON-COOPERATIVE INVENTORY GAMES FOR DEFECTIVE ITEMS AND QUANTITY DISCOUNTS USING STRATEGIC COMPLEMENTARITY," *Croatian Operational Research Review*, vol. 16, no. 1, pp. 53–64, 2025. doi: https://doi.org/10.17535/crorr.2025.0005.

[6] R. Setiawan and I. Endrayanto, "ANALYSIS OF THE N-PERSON NONCOOPERATIVE SUPERMODULAR MULTIOBJECTIVE GAMES," *Operations Research Letters*, vol. 51, no. 3, pp. 278–284, 2023. doi: https://doi.org/10.1016/j.orl.2023.03.007.

[7] R. Setiawan and I. Endrayanto, "ANALYSIS OF A SINGLE MANUFACTURER MULTI-RETAILER INVENTORY COMPETITION USING SUPERMODULAR MULTI-OBJECTIVE GAMES," *Mathematical Modelling of Engineering Problems*, vol. 9, no. 3, 2022. doi: https://doi.org/10.18280/mmep.090301.

[8] G. Rota-Graziosi, "THE SUPERMODULARITY OF THE TAX COMPETITION GAME," *J Math Econ*, vol. 83, pp. 25–35, 2019. doi: https://doi.org/10.1016/j.jmateco.2019.04.003.

[9] G. Koshevoy, T. Suzuki, and D. Talman, "SUPERMODULAR NTU-GAMES," *Operations Research Letters*, vol. 44, no. 4, pp. 446–450, 2016. doi: https://doi.org/10.1016/j.orl.2016.04.007.

[10] S. Yin, T. Nishi, and G. Zhang, "A GAME THEORETIC MODEL FOR COORDINATION OF SINGLE MANUFACTURER AND MULTIPLE SUPPLIERS WITH QUALITY VARIATIONS UNDER UNCERTAIN DEMANDS," *International Journal of Systems Science: Operations & Logistics*, vol. 3, no. 2, pp. 79–91, 2016. doi: https://doi.org/10.1080/23302674.2015.1050079.

[11] K. Halat and A. Hafezalkotob, "MODELING CARBON REGULATION POLICIES IN INVENTORY DECISIONS OF A MULTI-STAGE GREEN SUPPLY CHAIN: A GAME THEORY APPROACH," *Computers & Industrial Engineering*, vol. 128, pp. 807–830, 2019. doi: https://doi.org/10.1016/j.cie.2019.01.009

[12] P. Ghasemi, F. Goodarzian, J. Muñuzuri, and A. Abraham, "A COOPERATIVE GAME THEORY APPROACH FOR LOCATION-ROUTING-INVENTORY DECISIONS IN HUMANITARIAN RELIEF CHAIN INCORPORATING STOCHASTIC PLANNING," *Applied Mathematical Modelling*, vol. 104, pp. 750–781, 2022. doi: https://doi.org/10.1016/j.apm.2021.12.023

[13] X. Xu, R. Yue, B. Yang, and Z. Li, "INTEGRATING GAME THEORY AND DATA-DRIVEN OPTIMIZATION MODELS FOR ONLINE RETAILERS WITH REUSABLE PACKAGING ADOPTION," *Journal of Retailing and Consumer Services*, vol. 84, p. 104222, 2025. doi: https://doi.org/10.1016/j.jretconser.2025.104222

[14] Z. Wei, "CONSTRUCTION OF SUPPLY CHAIN FINANCE GAME MODEL BASED ON BLOCKCHAIN TECHNOLOGY AND NASH EQUILIBRIUM ANALYSIS," *Procedia Computer Science*, vol. 262, pp. 901–908, 2025. doi: https://doi.org/10.1016/j.procs.2025.05.124

[15] S. Ren, C. Liang, Y. Liu, J. Wang, and C. Wang, "EVOLUTIONARY GAME-BASED WAREHOUSING RESOURCES SHARING STRATEGY FOR LOGISTICS INDUSTRY UNDER PRODUCT-SERVICE SYSTEM PARADIGM," *Advanced Engineering Informatics*, vol. 67, p. 103560, 2025. doi: https://doi.org/10.1016/j.aei.2025.103560

[16] J. Yang and S. T. Ng, "PROSPECTS FOR DIGITAL TWIN TECHNOLOGY IN THE BUILDING MODULAR CONSTRUCTION AND OPERATION PHASES: A GAME THEORY-BASED ANALYSIS," *Journal of Cleaner Production*, vol. 470, p. 143344, 2024. doi: https://doi.org/10.1016/j.jclepro.2024.143344

[17] S. Mahato, F. Mahato, and G. C. Mahata, "GAME THEORETICAL ANALYSIS IN SUSTAINABLE SUPPLY CHAIN FOR PERISHABLE PRODUCTS INCORPORATING TRANSPORTATION STRATEGIES AND PARTIAL BACKLOGGING UNDER CARBON EMISSIONS," *Journal of Cleaner Production*, vol. 519, p. 145890, 2025. doi: https://doi.org/10.1016/j.jclepro.2025.145890.

[18] L. Silbermayr, "A REVIEW OF NON-COOPERATIVE NEWSVENDOR GAMES WITH HORIZONTAL INVENTORY INTERACTIONS," *Omega*, vol. 92, p. 102148, 2020. doi: https://doi.org/10.1016/j.omega.2019.102148

[19] R. Setiawan, "A PROBABILISTIC INVENTORY PROBLEM FOR IMPERFECT QUALITY WITH PARTIAL SHORTAGE BACKORDERING, CARBON EMISSION, AND ITS COMPUTATION USING GENETIC ALGORITHM IN PYTHON.," *Mathematical Modelling of Engineering Problems*, vol. 11, no. 7, 2024. doi: https://doi.org/10.18280/mmep.110708

[20] R. Setiawan, "GREEN PROBABILISTIC INVENTORY MODEL WITH SHORTAGE BACKORDERING, CARBON EMISSION COST, AND IMPERFECT QUALITY ITEMS: A NEWTON-RAPHSON METHOD APPROACH USING PYTHON," *Journal of the Indonesian Mathematical Society*, pp. 77–88, 2024. doi: https://doi.org/10.22342/jims.30.1.1677.77-88

[21] J. Blank and K. Deb, "PYMOO: MULTI-OBJECTIVE OPTIMIZATION IN PYTHON," *IEEE access*, vol. 8, pp. 89497–89509, 2020. doi: https://doi.org/10.1109/ACCESS.2020.2990567