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MIXED-INTEGER QUADRATIC PROGRAMMING (MIQP) MODEL FOR THE OPTIMIZATION OF INTEGRATED FARMING SYSTEMS

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ABSTRACT

This study develops a Mixed-Integer Quadratic Programming model to optimize resource allocation and profitability in integrated farming systems. The model links crop, livestock, and aquaculture components in a closed-loop framework that promotes internal resource use and reduces dependence on external inputs. Land productivity, labor, feed, fertilizer, and capital constraints are incorporated into a deterministic annual optimization model. A scenario-based case study for North Sumatra, Indonesia, is conducted using five commodities and three main resource categories. Sensitivity analysis across 243 scenarios varies price, productivity, input cost, feed cost, and capital parameters. The results show that the model identifies optimal land and labor allocations, improves permanent labor efficiency, and strengthens internal resource circulation. Profitability is most sensitive to commodity prices and productivity, while higher input and feed costs reduce system performance. Practically, the model can assist farmers, cooperatives, and local policymakers in evaluating allocation strategies and improving input efficiency. However, the study is limited by its deterministic structure, reliance on secondary data contextualized to North Sumatra, and single-year planning horizon, and it does not yet incorporate stochastic uncertainty or broader ecological and social objectives.

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1. INTRODUCTION

Optimizing integrated farming systems (IFS) remains a major challenge in the sustainable management of agricultural resources under increasing pressure from food-demand growth, climate variability, and limited land and water availability [1], [2]. Declining land productivity in intensively cultivated areas, fluctuations in livestock and aquaculture performance due to input constraints, and the broader need to improve resource-use efficiency have all strengthened the demand for analytical decision-support tools in agriculture [3], [4]. Because IFS combines crop, livestock, and aquaculture activities within a single operational setting, decision making must account for multiple technical and economic interdependencies at the same time, including land allocation, labor distribution, input requirements, and internal resource circulation [5], [6]. In this context, mathematical optimization offers a structured way to evaluate feasible production configurations and identify economically efficient allocation patterns under realistic operational constraints [7], [8].

The literature on IFS has expanded substantially, but it remains fragmented in commodity coverage, system boundaries, and modeling emphasis. Integrated farming is inherently multidisciplinary because it simultaneously involves agronomic, economic, environmental, and managerial dimensions [6]. The main challenge is not only to optimize each component separately, but also to represent how crops, livestock, and aquaculture interact through land use, feed requirements, manure flows, labor allocation, and financial limitations. This complexity is further amplified by differences in farmer preferences, production goals, and local institutional settings [9], [10]. Earlier studies have applied optimization and system-based approaches to farmland planning, agricultural production systems, and resource allocation under various constraints [11], [12]. Other works have emphasized the importance of embedding agricultural modeling within broader sustainability and stakeholder contexts, especially when multiple farm components and social objectives must be considered simultaneously [13], [14]. These studies confirm the relevance of quantitative modeling, but they also indicate that more operationally integrated formulations are still needed for farm-level decision support.

From an optimization perspective, Mixed-Integer Quadratic Programming (MIQP) is relevant when a problem combines discrete operational decisions with quadratic relationships in part of the formulation [7], [8]. In the present study, the use of MIQP is not intended to impose unnecessary nonlinearity on the profit function. Instead, the quadratic structure is introduced through sector-specific labor-allocation constraints, whereas the profit objective remains linear. This distinction is important. In integrated farming, the expansion of managed area across several interconnected activities may increase supervision, coordination, and workload complexity in a nonlinear manner, so that labor requirements are not always adequately approximated by a purely proportional linear relationship [15], [16], [17]. Accordingly, the quadratic terms in this article are used to represent diminishing marginal labor effectiveness as the managed system becomes larger and more complex, while discrete decisions such as production-unit counts and worker assignments are represented through integer variables. This formulation allows the model to remain operationally interpretable while better reflecting managerial scaling effects than a purely linear specification.

At the same time, the role of uncertainty in this article must be stated carefully. Although the broader IFS literature often discusses climate shocks, price volatility, and adaptive resilience, the model developed here is deterministic and annual in structure [18], [19]. It does not employ stochastic programming, chance constraints, or probabilistic transitions. Instead, uncertainty-related issues are addressed exogenously through scenario-based sensitivity analysis on key economic and technical parameters, including selling prices, productivity, input costs, feed costs, and capital availability [20], [21]. Thus, terms such as robustness and adaptability are used here in a restricted comparative sense, namely to describe how the optimal allocation pattern changes across alternative parameter settings, rather than to claim that the model explicitly solves a stochastic optimization problem. This clarification is necessary to align the conceptual framing of the article with the mathematical structure actually implemented.

Several recent studies have examined resource allocation, integrated production trade-offs, and sustainability-oriented agricultural planning from different methodological angles. Land-use and resource-efficiency issues have been studied in relation to productivity, environmental performance, and spatial planning [10], [15], [18]. Other works have explored integrated crop-livestock systems, circular resource use, and optimization-based assessments of agricultural interactions [14], [16], [19]. At the same time, studies on irrigation scheduling, spatially informed agricultural management, and shock-sensitive farm planning have shown the value of mathematical programming for handling constrained agricultural decisions under changing conditions [8], [21], [22]. However, these studies typically emphasize only part of the integrated system, focus on different sectoral settings, or adopt modeling objectives that differ from farm-level annual profit optimization in a closed-loop IFS context. In addition, local observations continue to show that agricultural integration is shaped by practical constraints such as labor availability, access to technology, input costs, environmental pressure, and institutional fragmentation [23]-[26]. These factors support the need for a mathematically explicit yet operationally grounded model that can represent cross-component allocation decisions within a unified framework.

Based on this gap, the present study develops a deterministic annual MIQP model for optimizing resource allocation in an IFS that integrates crop production, livestock, and aquaculture within a closed-loop setting. The model incorporates continuous decisions on land allocation, integer decisions on production units and permanent labor, linear representations of revenue, costs, and internal input balances, and convex quadratic labor constraints that reflect diminishing marginal labor effectiveness. The proposed model is designed to contribute in four ways. First, it provides an optimization framework for allocating land, labor, and production inputs across interdependent farm components. Second, it explicitly incorporates internal feed and manure linkages so that the efficiency of resource circulation can be evaluated together with profit performance. Third, it offers a mathematically consistent MIQP formulation in which the quadratic structure is tied to labor requirements rather than to an artificially nonlinear profit objective. Fourth, it applies the formulation to a case study calibrated with secondary data from North Sumatra, Indonesia, thereby providing a contextually grounded illustration of how the model behaves under alternative price, productivity, cost, and capital scenarios.

The remainder of this article is organized as follows. Section 2 presents the research methods, including the mathematical formulation of the deterministic annual MIQP model, the definition of variables and parameters, and the grouping of land, labor, budget, and internal resource-balance constraints. Section 3 reports the numerical results for the North Sumatra case study and discusses the implications of the optimal allocation pattern and the scenario-based sensitivity analysis. Finally, Section 4 concludes the article by summarizing the main findings, clarifying the current limitations of the model, and outlining directions for further development, including stochastic, environmental, and multi-objective extensions.

2. RESEARCH METHODS

This study adopts a quantitative approach grounded in mathematical modeling within an explanatory-computational design framework. The methodology presented is designed to achieve the research objective outlined at the end of the introduction: the development of an operational model for IFS based on MIQP, aimed at optimizing resource allocation and maximizing farmer profits. The model development is guided by core principles derived from the literature review and best practices in integrated farming system modeling: (1) explicit integration of crop, livestock, and aquaculture components; (2) resource-use efficiency through optimal allocation of land, labor, and production inputs; and (3) a balanced trade-off between profit maximization, operational cost reduction, sustained productivity, and ecosystem equilibrium. Figure 1 presents the conceptual structure of the optimization model for IFS, consisting of three main components: constraints, decision variables, and the objective function. Constraints include limitations on land, labor, and budget. Decision variables cover land allocation, types of production activities, and labor usage. The objective function aims to maximize the annual net profit of the entire system. These components form the mathematical foundation for efficient operational decision-making in IFS.

For clarity, let x collect the continuous decision variables, including land allocation and additional input-purchase variables, and let y collect the integer decision variables, including the numbers of livestock sheds, fish ponds, and permanent workers. The final model can be written compactly as [27]:

$$\max Z = c^T x + d^T y$$

subject to:

$$Ax + By \leq b,$$

$$x^T Q_s x + a_s^T x \leq b_s, \quad s \in S_L,$$

$$x \geq 0, y \in \mathbb{Z}_+.$$

Here, S_L denotes the set of labor-related sectors, and each Q_s is diagonal with nonnegative coefficients. This representation emphasizes that the model is mixed-integer and quadratic only through the labor-allocation structure.

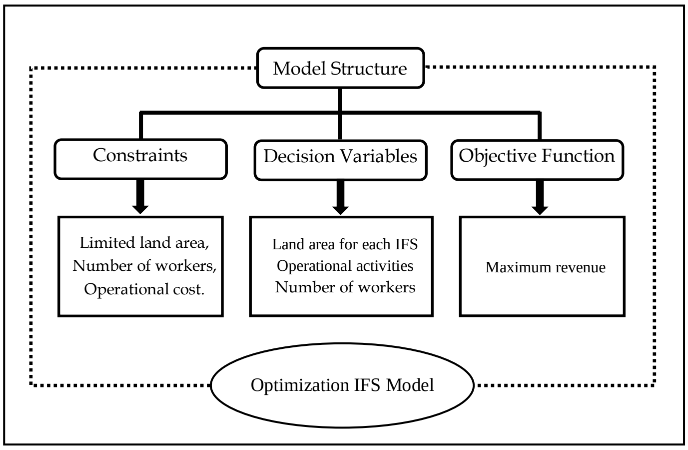


Figure 1. Conceptual Framework of the Optimization Model for IFS

This general framework directly underpins the optimization model developed in this article. To improve clarity and consistency, Table 1 consolidates the notation used throughout the formulation. The proposed model is a convex mixed-integer quadratic optimization model with a linear annual net-profit objective and separable quadratic labor-allocation constraints. Accordingly, the quadratic component of the model does not arise from the revenue term but from the labor requirement functions, which are used to represent increasing workload intensity as the operated area expands in each subsystem.

Table 1. Notation of the Proposed Optimization Model

Symbol	Definition	Unit / Domain
I	Set of crop commodities	index set
J	Set of livestock commodities	index set
K	Set of aquaculture commodities	index set
i, j, k	Index for crop, livestock, and aquaculture commodities	$\forall i \in I, \forall j \in J, \forall k \in K$
Z	Annual net profit	IDR
CPJ	Total revenue	IDR
CPM	Total cost	IDR
L	Total available land	ha
LX_i	Total land allocated to crop (i)	ha
LJX_i	Crop land of commodity (i) allocated to market sales	ha
LPX_i	Crop land of commodity (i) allocated to internal feed use	ha
LY_j	Land allocated to livestock activity (j)	ha
LZ_k	Land allocated to aquaculture activity (k)	ha
FPY_j	Number of livestock housing units/cages for livestock (j)	integer
FPZ_k	Number of fish ponds for aquaculture (k)	integer
PX_i	Number of permanent workers assigned to crop (i)	integer
PY_j	Number of permanent workers assigned to livestock (j)	integer
PZ_k	Number of permanent workers assigned to aquaculture (k)	integer
a_i, b_j, c_k	Productivity coefficients for crop, livestock, and aquaculture	kg/ha or kg/unit
CJX_i, CJY_j, CJZ_k	Selling prices	IDR/kg
CBX_i, CBY_j, CBZ_k	Seed/breeding-stock costs	IDR
CPX_i, CPY_j, CPZ_k	External fertilizer/feed purchase costs	IDR/kg
P_1, P_2, P_3, P_4	Seed/stock, fertilizer, feed, and labor cost components	IDR
A_i	Additional fertilizer/manure purchase for crop (i)	kg

Symbol	Definition	Unit / Domain
B_j	Additional livestock feed purchase for livestock (j)	kg
C_k	Additional feed purchase for aquaculture (k)	kg
γ	Land conversion coefficient for one livestock unit or pond	ha/unit
r_j	Manure supplied by livestock activity j	kg/unit
m_i	Manure requirement for crop i	kg/unit
δ_{ij}	Feed-conversion coefficient from crop i to livestock j	kg feed/kg crop output
d_j	Feed requirement for livestock activity j	kg/ha
e_k	Feed requirement for aquaculture activity k	kg/ha
QX_i, QY_j, QZ_k	Labor workload coefficients	day/ha^2
q	Effective working days per worker per year	day/year
CTX_i, CTY_j, CTZ_k	Labor wages	IDR/person
β	Maximum available annual budget/capital	IDR
$\mathbb{Z}_+$	Set of nonnegative integers	domain

The model considers an annual planning horizon and three component sets, namely crops $i \in I$ where $I = \{\text{Corn}, \text{Grass}\}$, livestock $j \in J$ where $J = \{\text{Chicken}, \text{Goat}\}$, and aquaculture $k \in K$ where $K = \{\text{Fish}\}$ as illustrated in the schematic diagram in Figure 2. The objective is to maximize annual net profit, defined as total revenue minus total production cost: $\max Z = CPJ - CPM$.

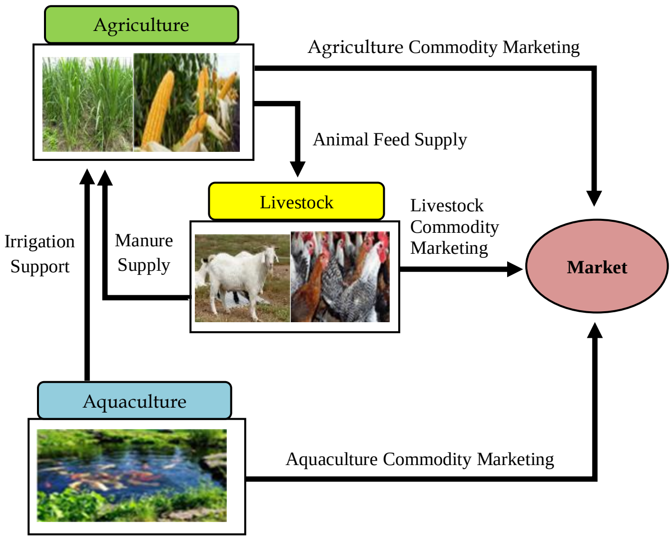


Figure 2. Scheme of the Interrelation between Agriculture, Livestock, and Aquaculture in the Example of IFS

For compactness, the final model can be written as:

$$\max Z \tag{1}$$

subject to:

objective-linking and accounting equations,

$$Z = CPJ - CPM, \tag{2}$$

$$CPJ = \sum_{i \in I} (CJX_i \cdot a_i \cdot LJX_i) + \sum_{j \in J} (CJY_j \cdot b_j \cdot LY_j) + \sum_{k \in K} (CJZ_k \cdot c_k \cdot LZ_k), \tag{3}$$

$$CPM = P_1 + P_2 + P_3 + P_4, \tag{4}$$

$$P_1 = \sum_{i \in I} (CBX_i \cdot LX_i) + \sum_{j \in J} (CBY_j \cdot FPY_j) + \sum_{k \in K} (CBZ_k \cdot FPZ_k), \tag{5}$$

$$P_2 = \sum_{i \in I} (CPX_i \cdot A_i)\,, \tag{6}$$

$$P_3 = \sum_{j \in J} (CPY_j \cdot B_j) + \sum_{k \in K} (CPZ_k \cdot C_k)\,, \tag{7}$$

$$P_4 = \sum_{i \in I} (PX_i \cdot CTX_i) + \sum_{j \in J} (PY_j \cdot CTY_j) + \sum_{k \in K} (PZ_k \cdot CTZ_k)\,, \tag{8}$$

land-allocation constraints,

$$\sum_{i \in I} LX_i + \sum_{j \in J} LY_j + \sum_{k \in K} LZ_k \le L, \tag{9}$$

crop-partition constraints,

$$LX_i = LJX_i + LPX_i\,, \quad \forall i \in I, \tag{10}$$

facility-conversion constraints,

$$LY_j = \gamma \cdot FPY_j, \qquad \forall j \in J, \tag{11}$$

$$LZ_k = \gamma \cdot FPZ_k, \qquad \forall k \in K, \tag{12}$$

internal input-balance and procurement constraints,

$$\sum_{j \in J} r_j \cdot FPY_j + A_i \ge m_i \cdot LX_i, \qquad \forall i \in I, \tag{13}$$

$$\sum_{i \in I} \delta_{ij} \cdot a_i \cdot LPX_i + B_j \ge d_j \cdot LY_j, \quad \forall j \in J, \tag{14}$$

$$C_k \ge e_k \cdot LZ_k, \qquad \forall k \in K, \tag{15}$$

quadratic labor constraints,

$$QX_i \cdot {LX_i}^2 \le q \cdot PX_i, \qquad \forall i \in I, \tag{16}$$

$$QY_j \cdot {LY_j}^2 \le q \cdot PY_j, \qquad \forall j \in J, \tag{17}$$

$$QZ_k \cdot {LZ_k}^2 \le q \cdot PZ_k, \qquad \forall k \in K, \tag{18}$$

annual budget constraint,

$$CPM \le \beta, \tag{19}$$

domain and integrality constraints,

$$LX_i, LJX_i, LPX_i, LY_j, LZ_k, A_i, B_j, C_k \ge 0, \quad \forall i \in I, \forall j \in J, \forall k \in K, \tag{20}$$

$$FPY_j, FPZ_k, PX_i, PY_j, PZ_k \in \mathbb{Z}_+, \quad \forall i \in I, \forall j \in J, \forall k \in K. \tag{21}$$

Eqs. (2)-(8) define the accounting structure of the optimization model. Eq. (2) links the objective variable to annual net profit, while Eq. (3) defines total revenue from crop, livestock, and aquaculture sales. Eqs. (4)-(8) decompose total production cost into seed and breeding-stock cost, fertilizer cost, feed cost, and labor cost. Eq. (9) is the land-availability constraint, ensuring that the total operated land does not exceed the available land endowment. Eq. (10) is the crop-partition constraint, which divides crop land into land allocated to market sales and land allocated to internal feed use. Eqs. (11)-(12) are facility-conversion constraints that connect livestock and aquaculture land allocation to the required numbers of livestock sheds and fish ponds through the coefficient γ.

Eqs. (13)-(15) represent internal input-balance and procurement constraints. These equations ensure that manure and feed requirements are satisfied either through internal resource circulation within the integrated farming system or through additional external purchases represented by the auxiliary variables A_i, B_j, C_k. Eqs. (16)-(18) are the separable quadratic labor-allocation constraints. These equations constitute the explicit quadratic component of the model and indicate that larger operated areas require disproportionately greater labor support. Eq. (19) imposes the annual budget limitation. Finally, Eqs. (20) and (21) define the non-negativity, real-valued, and integrality restrictions on the decision variables.

The convexity of the final formulation follows from the labor functions, which are specified as separable quadratic expressions with nonnegative quadratic coefficients. Hence, the Hessian of each sectoral labor term is positive semidefinite. Since the remaining constraints are linear, the continuous relaxation of the final formulation is convex. This property distinguishes the present model from nonconvex formulations that arise from bilinear products among decision variables. The final formulation is a convex mixed-integer quadratic program with a linear profit objective, linear resource-balance constraints, and separable quadratic labor-allocation constraints, thereby providing an integrated yet mathematically consistent optimization framework for the proposed farming system.

3. RESULTS AND DISCUSSION

3.1 Results

The numerical simulations were conducted to demonstrate the methodology developed and evaluate the behavior of the optimized MIQP IFS model. The primary objective of this simulation was to obtain optimal solutions in the form of resource allocations that maximize total profit while adhering to all technical and economic constraints defined in the model. All simulations were implemented using Python 3.12 and the Pyomo library, executed on a device equipped with an Intel Core i7 processor and 16 GB of RAM. The Python code used for the simulation in this study is available from the corresponding author on request. The simulation data (including productivity, input coefficients, prices, and resource constraints) are summarized in Table 2 and were derived from the literature and official national data, adjusted to reflect the context of North Sumatra for the 2023–2024 period [2].

Table 2. Simulation Data

Parameter	Unit	Corn	Grass	Chicken	Goat	Fish
CJX_i, CJY_j, CJZ_k	IDR/kg	6,000	100	25,000	120,000	20,000
a_i, b_j, c_k	Kg	100,000	200,000	30,000	4,000	20,000
CBX_i, CBY_j, CBZ_k	IDR	50,000,000	10,000,000	100,000	150,000	200,000
CPX_i, CPY_j, CPZ_k	IDR/kg	10,000	10,000	4,000	20,000	6,000
f_i, g_j, h_k	kg	10,000	10	200	500	80
$m_{j,k}$	kg	-	-	10	10	8
QX_i, QY_j, QZ_k	day	4000	500	2000	2000	1500
CTX_i, CTY_j, CTZ_k	IDR	2,000,000	2,000,000	2,000,000	2,000,000	2,000,000

In this simulation, the total land area available for allocation was set at $L = 1$ hectare, representing a typical smallholder farm scenario. The coefficient $\gamma = 0.002$ was adopted to reflect the conversion rate of livestock and aquaculture units to their corresponding land area requirements within the IFS framework. The permanent labor wage was uniformly set at IDR 2,000,000 per month for all operational activities, based on prevailing local wage standards. For modeling purposes, it was assumed that the maximum effective working period for a permanent worker is $q = 250$ days per year, ensuring all labor allocations remain realistic within annual operational limits. All labor-related costs were incorporated comprehensively in both the objective function and the labor constraints, thereby capturing the full economic burden of human resource utilization. Furthermore, the simulation imposed a maximum annual budget constraint of IDR 200,000,000, encompassing all expenditures related to production inputs and labor within the IFS. This constraint ensures that the model operates within realistic financial limits, reflecting typical capital availability for small- and medium-scale integrated farms in the region. Collectively, these parameter settings provide a robust and practical basis for evaluating the efficiency and sustainability of resource allocation strategies within the integrated farming system modeled in this study.

Table 3 presents the main optimization results obtained from the MIQP model for the integrated farming system. The system achieves a net profit of IDR 154,158,108, with total revenue amounting to IDR

348,825,375 and total costs of IDR 194,667,267. This outcome demonstrates the financial viability and efficiency of the optimal resource allocation strategy, where the difference between revenue and cost reflects the model's capability to maximize economic returns within the given constraints.

Table 3. Summary of Optimal Results of The MIQP Model for IFS

Component	Value (IDR)	Description
Profit (Z)	154,158,108	Total net profit
Revenue (CPJ)	348,825,375	Total revenue
Cost (CPM)	194,667,267	Total cost

Table 4 details the optimal allocation of land among the different commodities. The results indicate a balanced distribution, with maize and grass occupying the largest portions allocated for both sale and feed purposes. All available land (1 ha) is fully utilized, with no unused area remaining. The allocation to animal husbandry and aquaculture is also optimized, reflected in the number of cages for poultry and goats, and ponds for catfish, ensuring that all production units are accommodated within the land constraint. As shown in Table 5, the labor allocation is distributed among the main sectors (crop production, livestock, and fisheries), requiring a total of five workers. This distribution reflects the system's integrated nature and the need for specialized labor in each sector to ensure efficient and effective operations throughout the farming cycle.

Table 4. Land Allocation for Each Commodity in the IFS

Commodity	Total Area (ha)	Sale Area (ha)	Feed Area (ha)	Note
Maize	0.220	0.165	0.055	–
Grass	0.318	0.071	0.248	–
Chicken	0.132	–	–	66 cages
Goat	0.220	–	–	110 cages
Catfish	0.110	–	–	55 ponds
Total Used	1.000	–	–	Total land used: 1.0 ha
Land Left	0.000	–	–	–

Table 6 summarizes the total output of each commodity per production cycle. Grass exhibits the highest productivity (63,664 kg), followed by maize (21,968 kg), which together contribute substantially to the system's feed and sales streams. The livestock and aquaculture components yield 3,960 kg of chicken, 880 kg of goat, and 2,200 kg of catfish per period, respectively. These results highlight the integrated system's ability to support diversified and synergistic production outputs. Table 7 reports the quantities of purchased fertilizer and feed required for each commodity. Notably, the model results in zero external purchases of fertilizer for maize and grass, as well as feed for chicken and goat, indicating that internal resource recycling is maximized. The only external input required is catfish feed, amounting to 4,400 kg, which demonstrates the system's high degree of self-sufficiency except in aquaculture.

Table 5. Labor Allocation by Sector in the IFS

Sector	Number of Workers
Crop	2
Livestock	2
Fisheries	1
Total	5

Table 6. Commodity Productivity per Production Cycle

Commodity	Output (kg)
Maize	21,968
Grass	63,664
Chicken	3,960
Goat	880
Catfish	2,200

Table 7. Purchased Fertilizer and Feed Requirements per Commodity

Commodity	Amount (kg)
Purchased Fertilizer for Corn	0
Purchased Fertilizer for Grass	0
Purchased Feed for Chicken	0
Purchased Feed for Goat	0
Purchased Feed for Catfish	4,400

Table 8 displays the balance between manure demand and supply within the system. While total demand for manure reaches 2,200 kg, the available supply is 1,760 kg, resulting in a deficit of 440 kg. This finding underscores the importance of optimizing manure management and may justify the need for supplementary sources or further system adjustments to close the nutrient loop. Table 9 provides a breakdown of revenue contributions from each commodity. The goat sector generates the highest revenue (IDR 105,600,000), followed closely by chicken (IDR 99,000,000) and maize (IDR 98,809,009), while catfish and grass make up the remainder. This distribution emphasizes the significance of livestock and maize as the primary income sources in the integrated system.

Table 8. Manure Supply and Demand Balance

Component	Amount (kg)
Demand	2,200
Supply	1,760
Deficit	440

Table 10 outlines the main cost components incurred in the system. Labor constitutes the largest expense (IDR 120,000,000), followed by seeds and feed, with no costs associated with fertilizer, due to effective internal cycling. The total cost profile highlights the major financial commitments and points to labor and seed procurement as key cost drivers.

Table 9. Revenue Breakdown per Commodity

Commodity	Revenue (IDR)
Corn	98,809,009
Grass	1,416,366
Chicken	99,000,000
Goat	105,600,000
Catfish	44,000,000
Total Revenue	348,825,375

Table 10. Cost Breakdown by Input Category

Component	Cost (IDR)
P1 (Seed)	48,267,267
P2 (Fertilizer)	0
P3 (Feed)	26,400,000
P4 (Labor)	120,000,000
Total Cost	194,667,267

The baseline solution can be interpreted more explicitly from its revenue and cost composition. Relative to total revenue, the profit margin is 44.19%, indicating that the integrated configuration is economically viable but still cost-sensitive. The land constraint is binding in the base case because the full 1.0 ha is allocated, leaving no idle land. Revenue is concentrated in goats (30.27%), chicken (28.38%), and corn (28.33%), which together account for 86.98% of total revenue. On the cost side, labor is the dominant expenditure component (61.64% of total cost), followed by seeds and breeding stock (24.79%) and feed

(13.56%). This composition helps explain why profitability reacts strongly to productivity and output-price changes, while remaining vulnerable to higher labor and feed requirements. In addition, the manure balance shows a deficit of 440 kg, equivalent to 20% of total manure demand, indicating that internal nutrient circulation is substantial but not fully closed in the baseline solution.

To evaluate the robustness and responsiveness of the MIQP model under varying economic conditions, a comprehensive sensitivity analysis was conducted involving 243 simulation scenarios. This was achieved by systematically varying five key economic parameters: selling price (0.8×, 1.0×, 1.2×), productivity (0.9×, 1.0×, 1.1×), input costs (0.9×, 1.0×, 1.1×), feed costs (0.9×, 1.0×, 1.1×), and capital availability (IDR 100 million, 150 million, and 200 million). Each combination of these parameter levels was simulated independently while holding other parameters constant to isolate the effect of each variable. For each scenario, the model computed optimal land use, labor allocation, and profit outcomes, enabling a structured assessment of parameter sensitivity. This factorial design approach enabled a structured comparison of first-order parameter effects across alternative economic conditions and revealed a diminishing marginal profit effect with respect to capital expansion.

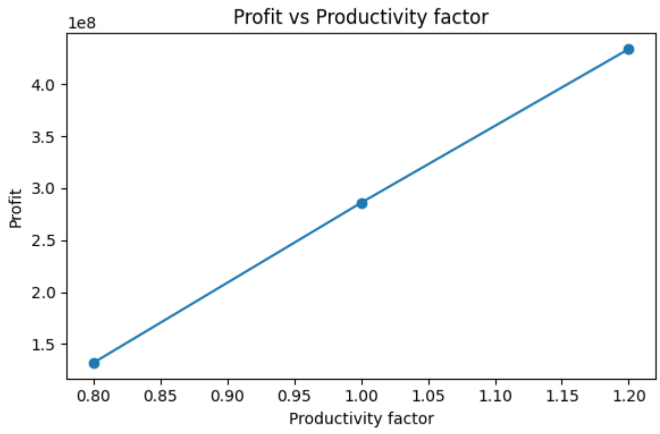
Across the simulated scenarios, the direction of the sensitivity pattern is clear: higher prices and productivity improve profit, whereas higher input and feed costs reduce it. The capital factor also increases profit, but the associated gains become less pronounced at higher levels, indicating diminishing marginal returns to additional capital under the current system structure. These patterns suggest that the model is more responsive to productivity and market-side improvements than to simple expansion of available capital.

(b)

Figure 3 (a) demonstrates a strong and nearly linear positive relationship between the price factor and the resulting profit. As commodity prices increase, profit rises substantially. This trend highlights the central role of market price fluctuations in determining farm income. The result confirms that price support mechanisms or access to premium markets can directly boost profitability in integrated farming systems. Similarly, (b)

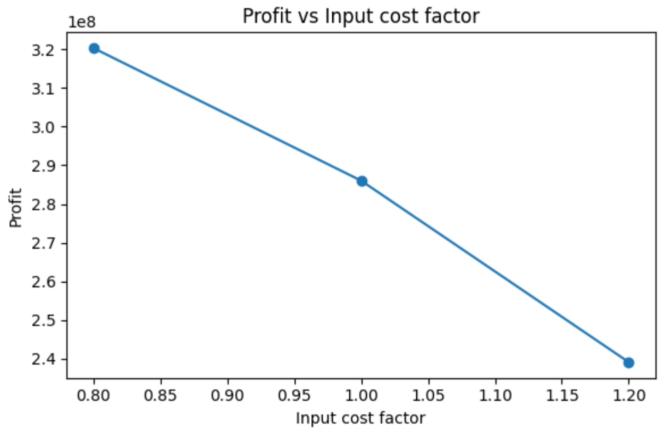
Figure 3 (b) reveals a positive and approximately linear association between productivity factor and profit. Enhancing productivity, whether through improved farming practices, technology adoption, or better resource allocation, directly contributes to higher profit margins. This finding underscores the importance of productivity enhancement interventions as a strategic priority for integrated agricultural enterprises. In contrast, Figure 4 (a) shows a clear negative relationship between input cost factor and profit. Higher input costs, such as seeds, fertilizers, and chemicals, erode profit margins. This result is consistent with theoretical expectations and suggests that cost-efficiency and input price stabilization are critical for maintaining sustainable profits in integrated farming systems. Figure 4 (b) indicates that an increase in the feed factor is associated with a reduction in profit. Since feed constitutes a substantial component of the total cost in livestock production, fluctuations in feed prices significantly affect overall profitability. These results advocate for efficient feed management and sourcing strategies to minimize feed-related expenses. Finally, the analysis of the capital factor in Figure 5 demonstrates that increased capital availability enhances profit, albeit with diminishing marginal returns at higher capital levels. While additional capital enables greater investment in productive resources and scaling up operations, the profit increase becomes less pronounced as the system approaches its optimal capacity. This highlights the importance of balanced capital allocation and strategic investment planning.

(a)

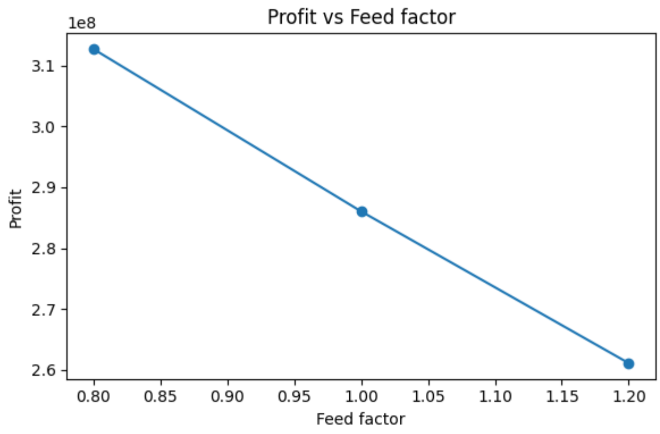


(b)

Figure 3. Sensitivity Analysis of Optimal Profit with Respect to (a) Price Factor and (b) Productivity Factor in The IFS Model



(a)



(b)

Figure 4. Sensitivity Analysis of Optimal Profit with Respect to (a) Cost Factor and (b) Feed Factor in The IFS Model

Overall, the sensitivity analysis reveals that profit optimization in integrated farming systems is most responsive to changes in commodity prices and productivity, whereas input and feed cost escalations pose significant risks to profitability. Capital infusion remains beneficial, but its impact is subject to diminishing returns. These insights provide evidence-based guidance for policymakers and farm managers to prioritize interventions on market access, productivity improvement, cost control, and optimal capital utilization.

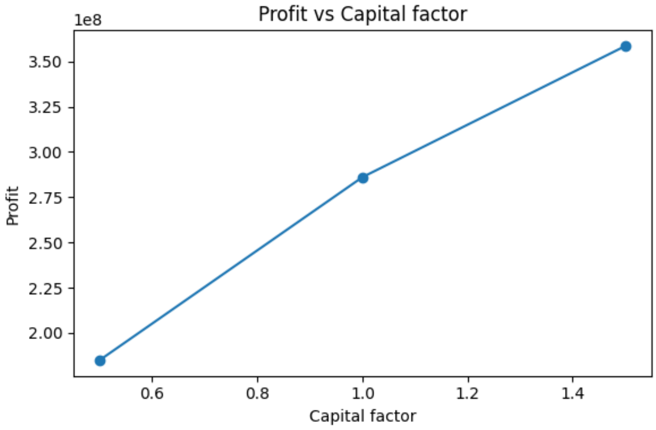


Figure 5. Sensitivity Analysis of Optimal Profit with Respect to the Capital Factor in The IFS Model

3.2 Discussion

These results show that the proposed optimization model provides a structured representation of how an integrated farming system can allocate land, labor, and internal inputs under binding resource constraints. Beyond reporting numerical outputs, the solution pattern clarifies the economic role of integration within the farming system. In particular, the allocation results indicate that not all commodities should be interpreted only through their direct market contribution. Grass, for example, contributes relatively little to total revenue, yet it supports the livestock component through internal feed provision. Likewise, the goat component contributes not only through product sales but also through its role in supporting internal manure circulation. This pattern is consistent with recent studies showing that higher levels of crop-livestock integration improve nutrient recycling, resource-use efficiency, and the functional coordination of farm components within integrated production systems [28], [29], [30]. Thus, the performance of an integrated farming system depends not only on the market value of each commodity, but also on the functional contribution of each component to overall system coordination and internal input efficiency.

The baseline solution also reveals an important structural feature of the system. Revenue is concentrated in goats, chicken, and corn, whereas labor constitutes the dominant cost component. This means that profitability is shaped jointly by output-oriented commodities and by the cost of maintaining an operationally integrated system. From this perspective, the model does not simply identify the most profitable commodity mix in isolation; rather, it identifies a coordinated allocation pattern in which commercially dominant components coexist with supporting components that strengthen internal input flows. The economic prominence of the chicken component in the present solution is also consistent with the broader literature showing that poultry production is a highly efficient and economically important livestock sector with strong market relevance [31], [32]. The manure deficit observed in the baseline solution further reinforces this interpretation. Internal nutrient circulation is substantial, but it is not fully closed, which suggests that the system already captures a meaningful degree of integration while still facing practical biological and operational limits.

The sensitivity analysis provides further insight into how the solution responds to alternative economic conditions. Across the simulated scenarios, profit increases with higher output prices and productivity and declines when input and feed costs rise. This result suggests that the model is more responsive to improvements on the market and productivity side than to simple expansion of physical or financial scale. The capital factor also has a positive effect, but the associated gains become less pronounced at higher levels, indicating diminishing marginal returns to additional capital under the present allocation structure. In practical terms, these findings imply that improving productivity, stabilizing output prices, and controlling feed and input expenditures are likely to be more effective than relying on capital expansion alone. Accordingly, the model can support decision making not only at the farm-operation level but also in the design of policy interventions related to input efficiency, productivity support, and local agricultural planning.

At the same time, the results should be interpreted within the boundaries of the model. The present formulation is deterministic, annual, and calibrated from locally derived secondary data, so the reported solution should be understood as a structured decision-support benchmark rather than as a stochastic forecast of future farm performance. In addition, the current model does not yet incorporate environmental indicators, carbon impacts, or broader social objectives. Nevertheless, the convex MIQP formulation developed in this study provides a mathematically consistent way to integrate discrete production-unit decisions, internal resource balances, and quadratic labor requirements within a single farm-planning framework. With this structure, the model offers a useful basis for evaluating efficient integrated farming configurations and for supporting future extensions toward stochastic, environmental, or multi-objective optimization settings.

4. CONCLUSION

This study developed a deterministic convex MIQP model for integrated farming resource allocation in which the quadratic structure arises from separable labor-allocation constraints, while the profit objective remains linear. In the North Sumatra case study, the model identified a feasible allocation that fully utilized available land, generated IDR 154,158,108 in annual profit, and relied heavily on internal input circulation, although a 20% manure deficit remained in the baseline configuration. The economic interpretation of the base solution shows that profitability is driven mainly by goat, chicken, and corn production, whereas labor is the dominant cost component. Scenario analysis across 243 cases indicates that profit is most responsive to output prices and productivity, while increases in input and feed costs reduce returns, and higher capital levels exhibit diminishing marginal gains. The theoretical contribution of this work lies in formulating a convex mixed-integer framework that combines discrete production-unit decisions with quadratic labor requirements for an integrated crop-livestock-aquaculture setting. Practically, the model provides a structured decision-support tool for evaluating land, labor, and input allocation under alternative economic scenarios. However, the present formulation remains deterministic, single-period, and single-objective, and it does not yet include stochastic uncertainty, environmental indicators, or full post-optimal diagnostics. These extensions remain important directions for future research.

Author Contributions

Lasker Pangarapan Sinaga: Conceptualization, Formal Analysis, Funding Acquisition, Methodology, Project Administration, Resources, Supervision, Validation, Writing-Original Draft. Rizki Habibi: Conceptualization, Data Curation, Software, Validation, Writing-Review and Editing. Suvriadi Panggabean: Investigation, Visualization, Validation, Writing-Review and Editing. All authors have read and agreed to the published version of the manuscript.

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Declarations

The authors declare no competing interests.

Declaration of Generative AI and AI-assisted Technologies

Generative AI tools (e.g., ChatGPT) were used solely for language refinement, including grammar, spelling, and clarity. The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. All final text was reviewed and approved by the authors.

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