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A MATHEMATICAL APPROACH OF DUAL CHANNEL SUPPLY CHAIN BY CONSIDERING, CUSTOMER LOYALITY, CORPORATE SOCIAL RESPONSIBILITY, AND DELIVERY TIME

Bhecti Purwaningsih [1], **Ririn Setiyowati** [2*], **Nughthoh Arfawi Khurdi** [3]

[1,2,3]*Mathematics Departments, Faculty of Mathematics and Natural Sciences, Universitas Sebelas Maret*
Jln. Ir. Sutami 36A, Kentingan, Surakarta, 57126, Indonesia

*Corresponding author's e-mail: * **ririnsetiyowati@staff.uns.ac.id***

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ABSTRACT

Rapid technological advancement has fundamentally transformed consumer purchasing behavior, leading to the widespread adoption of online shopping channels. In this context, timely product delivery has become a critical determinant of customer satisfaction and loyalty in dual-channel markets. At the same time, growing environmental concerns have increased the importance of integrating Corporate Social Responsibility into supply chain decision-making. Motivated by these developments, this study develops a dual-channel supply chain model that simultaneously incorporates customer loyalty, delivery-time sensitivity, and Corporate Social Responsibility. The novelty of the study lies in its integrated analysis of these three factors within two distinct payment schemes, namely real-time payment and prepaid systems, under both decentralized and centralized decision-making structures. Optimal decisions are derived analytically and numerically to determine the pricing and operational strategies that maximize system profits in each setting. Furthermore, sensitivity analysis is conducted to examine how customer loyalty, delivery-time sensitivity, and Corporate Social Responsibility elasticity affect the decision variables and profitability of the supply chain. The results show that increases in customer loyalty and Corporate Social Responsibility elasticity enhance overall system profit. In addition, Corporate Social Responsibility elasticity influences most decision variables, except the wholesale price, manufacturer demand, and retailer demand. These findings provide new insights into the role of payment mechanisms and sustainability-oriented strategies in improving the performance of dual-channel supply chains.

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1. INTRODUCTION

The supply chain in the industrial sector includes the process of converting raw materials into finished products and distributing them to consumers. According to Copra et al. [1], this chain is a network of various interconnected divisions that share the same goal of organizing product supply effectively. Supply chain management (SCM) is necessary to achieve this goal. SCM is a series of approaches used to effectively integrate suppliers, manufacturers, warehouses, and stores so that products can be produced and distributed perfectly [2], [3], [4]. Along with technological developments, the concept of supply chain has also evolved into dual channel supply chain (DCSC) [5], [6], [7]. The advent of digital technologies and e-commerce platforms has prompted businesses to simultaneously operate through online and offline channels. In addition to expanding the market, this change creates additional difficulties for pricing, customer service, and inventory management.

Manufacturers that utilize two channels, namely online and offline channels, DCSC can reach a wider market, thereby increasing manufacturer revenue [8], [9], [10]. According to [6], the supply chain structure becomes more complex when manufacturers implement DCSC. Major manufacturers in the world that have implemented DCSC include IBM, Nike, Samsung, Apple, Dell, and Wal-Mart [11]. In Indonesia itself, there are already many manufacturers that have implemented the DCSC concept, for example Gramedia, Zoya, and Hartono Elektronik. [5] mention that based on a report from the New York Times, 42% of manufacturers have redesigned their flow structure in order to reach a wider market. This change reflects of how crucial omnichannel tactics and digital integration are becoming to contemporary business structures. Additionally, synchronizing operations between the two channels now require the use of data analytics and real-time monitoring. In order to be competitive, enterprises must create supply chain systems that are more responsive and flexible.

In utilizing DCSC, there is horizontal competition between producers and retailers [12]. According to [5], consumer purchasing behavior will change under the DCSC environment. Many consumers still purchase products through offline channels, but there are also many who have started to switch to online channels. [13] explain that every product distributed through online channels requires delivery time. In a supply chain that includes both online and offline channels, how long it takes to deliver goods is something retailers can change on purpose to affect how customers act [14]-[17]. In the online market, delivery time thus becomes a crucial element impacting customer loyalty and happiness. Additionally, merchants can use delivery time optimization as a strategic tool to better manage inventory and balance demand across channels.

Faced with increasingly fierce business competition, manufacturers cannot compete optimally with other manufacturers if they rely solely on DCSC. Manufacturers' social responsibility, or corporate social responsibility (CSR), is one of the innovations that manufacturers can implement to build long-term relationships in business [18]-[21]. [22] mention that Alibaba Group, Jingdong (JD), and Carrefour have implemented CSR in the agricultural sector. CSR is a form of producer concern for the environment and society. CSR also improves the reputation of producers, which leads to increased customer satisfaction. The implementation of CSR binds producers and retailers in a cooperation contract to minimize losses for both parties [21], [23].

The availability of payment methods in DCSC affects the level of retailer service, CSR level, and objective function in corporate decision-making[22]. The choice of payment methods also helps retailers distribute products without worrying about payment times and high product prices. Generally, this choice of payment methods binds retailers in contracts with manufacturers. Contracts are made so that neither party is disadvantaged, allowing cooperation between the two to run smoothly. In addition, when implementing this choice of payment methods, retailers also receive discounts on the total payment to manufacturers.

Motivated by the studies of [5] and [13] on the DCSC models involving channel preference, consumer loyalty, and delivery time, as well as the research of [22] on CSR and the differences between two payment methods. Existing studies have not sufficiently examined these factors within a single integrated DCSC framework, even though they jointly shape consumer behavior, channel competition, and firm profitability in practice. In particular, limited attention has been given to how consumer loyalty, delivery-time sensitivity, Corporate Social Responsibility, and alternative payment methods interact in determining optimal decisions under dual-channel structures. To address this gap, this study constructs a DCSC model that incorporates all of these elements and investigates the resulting profit-maximizing decisions. The main novelty of this study is the development of a more comprehensive analytical framework that captures both market-oriented and sustainability-oriented dimensions of supply chain decision-making under heterogeneous payment systems.

By doing so, the proposed model offers a more realistic representation of contemporary corporate operations and provides strategic guidance for manufacturers and retailers in formulating effective pricing and operational policies.

2. RESEARCH METHODS

This research began with the construction of a mathematical model of DCSC, taking into account customer loyalty, delivery time, and CSR. Next, we established the existence of an optimal solution by solving a system of multivariable equations. The obtained model and solution are applied to a case with parameter values taken from several journals. Finally, parameter sensitivity analysis is performed to determine the parameters that play a major role in maximizing profits.

2.1 Assumptions and Notations

In this section, two DCSC models are constructed, namely the DCSC model with a timely payment system and the DCSC model with a prepaid system. The timely payment system requires retailers to pay manufacturers for products at the same time as they pick up the products. Meanwhile, in the prepaid system, retailers pay a deposit to manufacturers before picking up the products. Manufacturers use this deposit for investment. The difference between these payment systems is denoted as "O" and "P" which represent the on-time payment system and the prepaid system, respectively. Decision-making in both models is based on centralization and decentralization scenarios. In the centralization scenario, decision-making is carried out centrally. However, in the decentralization scenario, each supply chain actor makes decisions separately. The symbols "C" and "D" represent the centralized and decentralized scenarios, respectively. The DCSC distribution flow in this study is shown in Fig. 1.

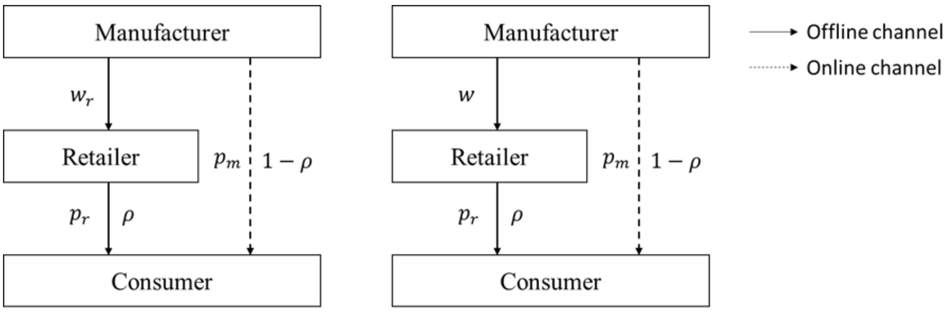

Figure 1. DCSC Distribution Flow for Model O (Left) and Model P (Right)

The following assumptions are required in constructing the profit function of the DCSC model.

1. The quantity of production is equal to the total demand for the product.
2. The wholesale price w is higher than the production cost c_p, but lower than the selling price of the product in online and offline channels.
3. Changes in demand by each supply chain actor are influenced by changes in the selling price of the product, which is parameterized by the demand elasticity β.
4. Cross-channel price competition is not considered.
5. Consumer loyalty s to both channels is indicated by the rating given by consumers.
6. Delivery time l only exists in the online channel, service and delivery costs are ignored.
7. Market demand is influenced by the CSR level of producers $(0 < y_m < 1)$ and retailers $(0 < y_r < 1)$ with demand elasticity CSR b greater than zero.
8. The CSR input costs for the online and offline channels are $\frac{1}{2}c_{cm}y_m^2$ and $\frac{1}{2}c_{cr}y_r^2$ where c_{cm} and c_{cr} are the manufacturer and retailer CSR cost coefficient greater than zero.
9. The value of b must be smaller than the values of β_m, β_r, c_{cm}, and c_{cr}, which represent the price elasticity of online and offline channel products, as well as the CSR cost coefficients of online and offline channels, respectively.
10. Product shortages in both channels are not considered.
11. All parameters are positive.

Meanwhile, the following notations are used in constructing the DCSC model.

w	: wholesale price per unit
p_m	: product selling price per unit in online channels
p_r	: product selling price per unit in offline channels
c_p	: production cost per unit
σ	: basic demand market
ρ	: product demand ratio
β_m, β_r	: elasticity of demand for online channels, offline channels
μ_m, μ_r	: consumer loyalty sensitivity for online channels, offline channels
γ_m	: delivery time sensitivity for online channels
b	: CSR elasticity
θ	: return on investment for prepaid systems
k	: down payment proportion in the prepaid system
r	: discount rate in the prepaid system
s_m, s_r	: level of consumer loyalty to online channels, offline channels
l_m	: delivery time for online channel
y_m, y_r	: CSR level for online channels, offline channels
D_m, D_r	: demand function of manufacturer, retailer
Π_m, Π_r	: profit function of manufacturer, retailer.

2.2 Demand Function

The number of online channel requests D_m is influenced by the potential market demand of $\sigma(1-\rho)$, where σ and $(1-\rho)$ are the basic market demand and the proportion of demand on online channels, respectively. D_m is inversely proportional to the selling price of product p_m with a demand elasticity of β_m. The higher the price elasticity, the greater the responsiveness of product demand to price changes. An increase in price leads to a substantial decline in consumer demand, whereas a decrease in price encourages higher product purchases. However, demand for online channels is directly proportional to consumer loyalty to online channels s_m with consumer loyalty sensitivity μ_m. When consumers have high loyalty to online channels, as indicated by giving ratings to these channels, the demand for products increases in line with the increase in consumer loyalty. When consumers have high loyalty to the online channel, as indicated by their ratings of this channel, the demand for the product increases in line with the increase in consumer loyalty. Conversely, if the online channel receives a low rating, product demand will decrease. Online channel demand is inversely proportional to product delivery time l_m, which has a sensitivity of γ_m. The longer it takes for producers to deliver products, the fewer products consumers will purchase. Therefore, delivery time management by producers is necessary so that products are received by consumers immediately. Online channel demand is also influenced by the level of CSR in online channels with CSR elasticity b. This level of CSR results in an increase in the manufacturer's reputation among consumers. This leads to increased consumer loyalty to online channels. The higher the level of CSR, the greater the demand for products in online channels. Thus, the amount of online channel demand can be written as

$$D_m = \sigma(1-\rho) - \beta_m p_m + \mu_m s_m - \gamma_m l_m + b y_m. \tag{1}$$

The basic market demand σ determines the amount of product demand in the offline channel D_r with a product demand proportion ρ. Product demand in the offline channel is inversely proportional to changes in the selling price of the product in the offline channel p_r by β_r. The high or low selling price of this product is an important factor that affects the amount of product demand in the offline channel. The higher the price of the product in the offline channel, the lower the amount of product demand. This is because consumers have a high tendency to switch to buying products on online channels after comparing product prices between channels. In addition, D_r is also influenced by the level of consumer loyalty to offline channels s_r with a sensitivity of μ_r. Consumer loyalty to offline channels is a form of consumer loyalty to purchase products on offline channels after receiving services provided by retailers. The better the retailer's service to consumers, the higher the level of consumer loyalty. This means that product demand in offline channels is directly proportional to consumer loyalty. Product demand in offline channels is also directly proportional to the CSR level y_r, which has an elasticity of b. The higher the CSR level, the higher the product demand. The amount of offline channel demand can be written as follows

$$D_r = \sigma\rho - \beta_r p_r + \mu_r s_r + b y_r. \tag{2}$$

2.3 Profit Function

Based on the product distribution flow in Fig. 1 and the assumptions in Subsection 2.1, the profit functions of manufacturer and retailer for model O with the on-time payment system are written as

$$\Pi_m^O(w, p_m, y_m) = p_m D_m + wD_r - c_p D_m - c_p D_r - \frac{1}{2} c_{cm} y_m^2, \tag{3}$$

$$\Pi_r^O(p_r, y_r) = p_r D_r - wD_r - \frac{1}{2} c_{cr} y_r^2. \tag{4}$$

The total revenue dan cost of manufacturer are $p_m D_m + wD_r$ and $c_p D_m + c_p D_r + \frac{1}{2} c_{cm} y_m^2$, respectively. Meanwhile, the total revenue and cost for retailers are $p_r D_r$ and $wD_r + \frac{1}{2} c_{cr} y_r^2$, respectively.

For the prepaid system in model P, the profit function of manufacturer and retailer are written as

$$\Pi_m^O(w, p_m, y_m) = p_m D_m + wD_r + wD_r k\theta - c_p D_m - wD_r r - \frac{1}{2} c_{cm} y_m^2, \tag{5}$$

$$\Pi_r^O(p_r, y_r) = p_r D_r - wD_r + wD_r r - \frac{1}{2} c_{cr} y_r^2. \tag{6}$$

The total revenue of manufacturer in model P is $p_m D_m + wD_r + wD_r k\theta$ and the total cost is $c_p D_m + wD_r r + \frac{1}{2} c_{cm} y_m^2$. Retailers must pay manufacturer wD_r. In addition, retailers must also pay a product down payment with a proportion k of the total wD_r. Manufacturer use the down payment to invest with a rate of return of θ. Thus, manufacturer earn $wD_r k\theta$ income from investments using product down payments by retailers. When implementing a prepaid system, manufacturers give retailers a discount of r from the total wD_r paid by retailers. CSR input costs are $\frac{1}{2} c_{cm} y_m^2$ for manufacturer and $\frac{1}{2} c_{cr} y_r^2$ for retailers.

3. RESULTS AND DISCUSSION

This section determines the optimal solution for product selling prices and CSR levels in online and offline channels. The optimal solution aims to maximize profits for both producers and retailers. This study consists of four optimal solutions, namely optimal solutions with decentralized and centralized scenarios for on-time payment systems and prepaid systems.

3.1 The On-Time Payment System

The DCSC model for manufacturers and retailers for the on-time payment system is written as

$$\text{maximize } \Pi_m^{DO}(w, p_m, y_m) = p_m D_m + wD_r - c_p D_m - c_p D_r - \frac{1}{2} c_{cm} y_m^2, \tag{7}$$

$$\Pi_r^{DO}(p_r, y_r) = p_r D_r - wD_r - \frac{1}{2} c_{cr} y_r^2, \tag{8}$$

for decentralized scenario and

$$\text{maximize } \Pi_c^{CO}(p_m, y_m, p_r, y_r) = (p_m - c_p) D_m + (p_r - c_p) D_r - \frac{1}{2} c_{cm} y_m^2 - \frac{1}{2} c_{cr} y_r^2, \tag{9}$$

for centralized scenario with $w = \lambda p_m$. In a decentralized scenario, decisions are made separately by each supply chain actor, with the manufacturer acting as the leader. The optimal solution is obtained with the retailer determining the offline channel product selling price p_r and CSR level y_r. After that, the manufacturer determines the wholesale price w, the online channel product selling price p_m, and the CSR level y_m. Unlike the centralization scenario, decision making is carried out centrally with the aim of achieving maximum profit for both supply chain actors.

The profit margin of the on-time payment system is only affected by the selling price of the product and the CSR rate. The optimal solution of the DCSC model with on-time payment system is written as follows.

Theorem 1. *If $0 < b^2 < 2\min\{\beta_m c_{cm}, \beta_r c_{cr}\}$, then the optimal solution of the centralized DCSC model in Eq. (9) is*

$$p_m^{CO*} = \frac{b^2 c_p - c_{cm}\left(\alpha + c_p\beta_m - l_m\gamma_m - \alpha\eta + s_m\mu_m\right)}{b^2 - 2\,c_{cm}\beta_m}, \tag{10}$$

$$p_r^{CO*} = \frac{b^2 c_p - c_{cr}\left(c_p\beta_r + \alpha\eta + s_r\mu_r\right)}{b^2 - 2\,c_{cr}\beta_r}, \tag{11}$$

$$y_m^{CO*} = \frac{b\left(c_p\beta_m + l_m\gamma_m + \alpha(\eta - 1) - s_m\mu_m\right)}{b^2 - 2\,c_{cm}\beta_m}, \tag{12}$$

$$y_r^{CO*} = \frac{b\left(c_p\beta_r - \alpha\eta - s_r\mu_r\right)}{b^2 - 2\,c_{cr}\beta_r}, \tag{13}$$

$$D_m^{CO*} = \frac{c_{cm}\beta_m\left(c_p\beta_m + l_m\gamma_m + \alpha(\eta - 1) - s_m\mu_m\right)}{b^2 - 2\,c_{cm}\beta_m}, \tag{14}$$

$$D_r^{CO*} = \frac{c_{cr}\beta_r\left(c_p\beta_r - \alpha\eta - s_r\mu_r\right)}{b^2 - 2\,c_{cr}\beta_r}, \tag{15}$$

$$\Pi_m^{CO*} = \left(w^{CO*} - c_p\right)D_r^{CO*} + \left(p_m^{CO*} - c_p\right)D_m^{CO*} - \frac{1}{2}c_{cm}(y_m^{CO*})^2, \tag{16}$$

$$\Pi_r^{CO*} = (p_r^{CO*} - w^{CO*})D_r^{CO*} - \frac{1}{2}c - cr\,(y_r^{CO*})^2, \tag{17}$$

$$\Pi_c^{CO*} = \Pi_m^{CO*} + \Pi_r^{CO*}. \tag{18}$$

Proof. It needs to be proven that model in Eq. (9) has a unique solution or that the function is strictly concave. The Hessian matrix of model in Eq. (9) is

$$H(\Pi_c^{CO}) = \begin{pmatrix} -2\beta_m & 0 & b & 0 \\ 0 & -2\beta_r & 0 & b \\ b & 0 & -c_m & 0 \\ 0 & b & 0 & -c_{cr} \end{pmatrix}.$$

Based on the Hessian matrix $H(\Pi_c^{CO})$, we get the principal minor determinant $H_{11} = -2\beta_m > 0$, $H_{22} = 4\beta_m\beta_r > 0$, $H_{33} = 2b^2\beta_r - 4c_{cm}\beta_m\beta_r < 0$ if $b^2 < 2c_{cm}\beta_m$, and $H_{44} = (b^2 - 2c_{cm}\beta_m)(b^2 - 2c_{cr}\beta_r) > 0$ if $b^2 < 2\,c_{cr}\beta_r$. Since $0 < b^2 < 2\min\{\beta_m c_{cm}, \beta_r c_{cr}\}$, then the Hessian matrix $H(\Pi_c^{CO})$ is negative definite. Thus, the function Π_c^{CO} is strictly concave and has a unique solution. Next, we determine the first partial derivatives of Eq. (9) with respect to p_m, p_r, y_m, and y_r, which are set equal to zero. Thus, we obtain a system of equations with four variables, namely

$$\frac{\partial \Pi_c^{CO}}{\partial p_m} = by_m - p_m\beta_m - \left(p_m - c_p\right)\beta_m - l_m\gamma_m + \alpha(1 - \eta) + s_m\mu_m = 0,$$

$$\frac{\partial \Pi_c^{CO}}{\partial p_r} = by_r - p_r\beta_r - (p_r - w)\beta_r - \left(-c_p + w\right)\beta_r + \alpha\eta + s_r\mu_r = 0,$$

$$\frac{\partial \Pi_c^{CO}}{\partial y_m} = b\left(p_m - c_p\right) - c_{cm}y_m = 0,$$

$$\frac{\partial \Pi_c^{CO}}{\partial y_r} = b(p_r - w) + b\left(w - c_p\right) - c_{cr}y_r = 0.$$

The optimal solution is obtained by solving the above system of equations, and we get as written in Eqs. (10) and (13). The optimal demand function is obtained by substituting Eqs. (10) and (13) into Eqs. (1) and (2), yielding Eqs. (14) and (15). While the optimal profit function is obtained as written in Eqs. (16) and (18). ■

Theorem 2. *If $0 < b^2 < 2\min\{\beta_m c_{cm}, \beta_r c_{cr}\}$, then the optimal solution of the decentralized DCSC model in Eqs. (7) and (8) is*

$$p_m^{DO*} = \frac{b^2 c_p - c_{cm}\left(\alpha + c_p\beta_m - l_m\gamma_m - \alpha\eta + s_m\mu_m\right)}{b^2 - 2\,c_{cm}\beta_m}, \tag{19}$$

$$p_r^{DO*} = \frac{b^2(c_p\beta_r + \alpha\eta + s_r\mu_r) - c_{cr}\left(c_p\beta_r + 3\alpha\eta + 3s_r\mu_r\right)}{2\beta_r(b^2 - 2\,c_{cr}\beta_r)}, \tag{20}$$

$$y_m^{DO*}=\frac{b\left(c_p\beta_m+l_m\gamma_m+\alpha(\eta-1)-s_m\mu_m\right)}{b^2-2\,c_{cm}\beta_m},\tag{21}$$

$$y_r^{DO*}=\frac{b\left(-\alpha\eta-s_r\mu_r+\frac{1}{2}(c_p\beta_r+\alpha\eta+s_r\mu_r)\right)}{b^2-2\,c_{cr}\beta_r},\tag{22}$$

$$w^{DO*}=\frac{c_p\beta_r+\alpha\eta+s_r\mu_r}{2\beta_r}\tag{23}$$

$$D_m^{DO*}=\frac{c_{cm}\beta_m\left(c_p\beta_m+l_m\gamma_m+\alpha(\eta-1)-s_m\mu_m\right)}{b^2-2\,c_{cm}\beta_m},\tag{24}$$

$$D_r^{DO*}=\frac{c_{cr}\beta_r\left(c_p\beta_r-\alpha\eta-s_r\mu_r\right)}{2(b^2-2\,c_{cr}\beta_r)},\tag{25}$$

$$\Pi_m^{DO*}=\left(w^{CO*}-c_p\right)D_r^{CO*}+\left(p_m^{CO*}-c_p\right)D_m^{CO*}-\frac{1}{2}c_{cm}(y_m^{CO*})^2,\tag{26}$$

$$\Pi_r^{DO*}=(p_r^{CO*}-w^{CO*})D_r^{CO*}-\frac{1}{2}c-c_{cr}\,(y_r^{CO*})^2,\tag{27}$$

Proof. Let's start with the function $\Pi_r^{DO}(p_r,y_r)$ in Eq. (8). The Hessian matrix of the function $\Pi_r^{DO}(p_r,y_r)$ is

$$H(\Pi_r^{DO})=\begin{pmatrix}-2\beta_r & b\\ b & -c_{cr}\end{pmatrix}.$$

Based on the Hessian matrix $H(\Pi_r^{DO})$, we obtain the principal minor determinant $H_{11}=-2\beta_r<0$ and $H_{22}=-b^2+2c_{cr}\beta_r>0$ because $b^2-2c_{cr}\beta_r<0$. In other words, the Hessian matrix $H(\Pi_r^{DO})$ is negative definite, so the function Π_r^{DO} is strictly concave. The solutions p_r and y_r are obtained by solving the system of equations obtained by determining the first partial derivative Π_r^{DO} with respect to p_r and y_r, then setting it equal to zero, namely

$$p_r^{DO}=\frac{b^2w-c_{cr}(w\beta_r+\alpha\eta+s_r\mu_r)}{b^2-2\,c_{cr}\beta_r},\qquad y_r^{DO}=\frac{b(w\beta_r-\alpha\eta-s_r\mu_r)}{b^2-2\,c_{cr}\beta_r}.\tag{28}$$

Next, we substitute p_r^{DO} and y_r^{DO} in Eq. (28) into D_r in Eq. (7). The Hessian matrix of Π_m^{DO} is

$$H(\Pi_m^{DO})=\begin{pmatrix}\frac{2c_{cr}\beta_r^2}{b^2-2c_{cr}\beta_r} & 0 & 0\\ 0 & -2\beta_m & b\\ 0 & b & -c_m\end{pmatrix}$$

with the principal minor determinant $H_{11}=\frac{2c_{cr}\beta_r^2}{b^2-2c_{cr}\beta_r}<0$, $H_{22}=\frac{4c_{cr}\beta_m\beta_r^2}{b^2-2c_{cr}\beta_r}>0$, and $H_{33}=-\frac{2c_{cr}\beta_r^2(b^2-2c_{cm}\beta_m)}{b^2-2c_{cr}\beta_r}<0$ because $0<b^2<2\min\{\beta_m c_{cm},\beta_r c_{cr}\}$. Since the hessian matrix $H(\Pi_m^{DO})$ is negative definite, then the function Π_m^{DO} is strictly concave. Similarly, the first derivative of the Pi function with respect to w, p_m, and y_m is set to zero, yielding the solution p_m^{DO*}, y_m^{DO*}, and w^{DO*} in Eqs. (19), (21), and (23). Next, we substitute w^{DO*}into Eq. (28) and we get the optimal solution p_r^{DO*} and y_r^{DO*} in Eqs. (20) and (22). By substituting w^{DO*}, p_m^{DO*}, y_m^{DO*}, p_r^{DO*}, and y_r^{DO*} into Eqs. (1), (2), (7), and (8), we obtain the optimal solution for demand function in Eqs. (24) and (25), and profit function for manufacturer and retailer in Eqs. (26) and (27). ■

3.2 The Prepaid System

The profits of manufacturers and retailers in the prepaid system are influenced not only by the selling price of the product, the wholesale price, and the level of CSR, but also by the size of the discount. Manufacturers also receive additional profits from the rate of return on investment. Based on the profit function of manufacturers and retailers in Eqs. (5) and (6), the DCSC models for prepaid systems are

Maximize
$$\Pi_m^{DP}(w,p_m,y_m)=p_mD_m+wD_r+wD_rk\theta-c_pD_m-wD_rr-\frac{1}{2}c_{cm}y_m^2,\tag{29}$$

$$\Pi_r^{DP}(p_r,y_r)=p_rD_r-wD_r+wD_rr-\frac{1}{2}c_{cr}y_r^2.\tag{30}$$

for decentralized scenario and

$$\text{Maximize } \Pi_c^{CP}(p_m, y_m, p_r, y_r) = (p_m - c_p)D_m + (p_r - c_p)D_r + wD_r\, k\theta - \tfrac{1}{2}c_{cm}y_m^2 - \tfrac{1}{2}c_{cr}y_r^2, \tag{31}$$

for centralized scenario with $w = \lambda p_m$.

The optimal solution for the DCSC model with a prepaid system for both centralized and decentralized scenarios is presented in the following theorem.

Theorem 3. *If $0 < b^2 < 2\beta_r c_{cr}$ and $(-1 + r - k\theta) < 0$, then the optimal solution of the decentralized DCSC model in Eqs. (29) and (30) is*

$$w^{DP*} = \frac{c_p(\beta_r - r\beta_r) - (-1 - r + k\theta)(\alpha\eta + s_r\mu_r)}{2\beta_r(-1 + r)(-1 - r + k\theta)}, \tag{32}$$

$$p_m^{DP*} = \frac{b^2 c_p - c_{cm}(\alpha + c_p\beta_m - l_m\gamma_m - \alpha\eta + s_m\mu_m)}{b^2 - 2\, c_{cm}\beta_m}, \tag{33}$$

$$p_r^{DP*} = \frac{c_{cr}\beta_r\left(c_p\beta_r(1 - r) - 3(-1 - r + k\theta)(\alpha\eta + s_r\mu_r)\right)}{2\beta_r(b^2 - 2\, c_{cr}\beta_r)(-1 - r + k\theta)} + \frac{b^2\left(c_p(r - 1)\beta_r + (-1 - r + k\theta)(\alpha\eta + s_r\mu_r)\right)}{2\beta_r(b^2 - 2\, c_{cr}\beta_r)(-1 - r + k\theta)}, \tag{34}$$

$$y_m^{DP*} = \frac{b(c_p\beta_m + l_m\gamma_m + \alpha(\eta - 1) - s_m\mu_m)}{b^2 - 2\, c_{cm}\beta_m}, \tag{35}$$

$$y_r^{DP*} = \frac{b\left(c_p\beta_r(r - 1) - (-1 - r + k\theta)(\alpha\eta + s_r\mu_r)\right)}{2(b^2 - 2\, c_{cr}\beta_r)(-1 - r + k\theta)}, \tag{36}$$

$$D_m^{DP*} = \frac{c_{cm}\beta_m(c_p\beta_m + l_m\gamma_m + \alpha(-1 + \eta) - s_m\mu_m)}{b^2 - 2c_{cm}\beta_m}, \tag{37}$$

$$D_r^{DP*} = \frac{c_{cr}\beta_r\left(c_p(r - 1)\beta_r(-1 - r + k\theta)(\alpha\eta + s_r\mu_r)\right)}{2(b^2 - 2\, c_{cr}\beta_r)(-1 - r + k\theta)}, \tag{38}$$

$$\Pi_m^{DP*} = (p_m^{DP*} - c_p)D_m^{DP*} + (w^{DP*} - c_p)D_r^{DP*} + w^{DP*}D_r^{DP*}(k\theta - r) - \frac{1}{2}c_{cm}(y_m^{DP*})^2, \tag{39}$$

$$\Pi_r^{DP*} = (p_r^{DP*} - w^{DP*})D_r^{DP*} + rw^{DP*}\, D_r^{DP*} - \frac{1}{2}c_{cr}(y_r^{DP*})^2. \tag{40}$$

Proof. In a decentralized scenario, manufacturers act as leaders and retailers act as followers in determining pricing policy. In the same way as the proof in Theorem 2, we start from the function Π_r^{DP} in Eq. (30). The Hessian matrix of the function Π_r^{DP} is equal to $H(\Pi_r^{DO})$ with principal minor determinant $H_{11} = -2\beta_r < 0$ and $H_{22} = -b^2 + 2c_{cr}\beta_r > 0$ because $b^2 - 2c_{cr}\beta_r < 0$. So, the function Π_r^{DP} is strictly concave and has a unique solution, namely

$$p_r^{DP} = \frac{b^2 w(1 - r) + c_r(w\beta_r(r - 1) - \alpha\eta - s_r\mu_r)}{b^2 - 2c_{cr}\beta_r}, \tag{41}$$

$$y_r^{DP} = -\frac{b(w\beta_r(r - 1) + \alpha\eta + s_r\mu_r)}{b^2 - 2c_{cr}\beta_r}. \tag{42}$$

Next, p_r^{DP} and y_r^{DP} in Eqs. (41) and (42) are substituted into the Eq. (29) and the Hessian matrix of function Π_m^{DP} is obtained as

$$H(\Pi_m^{DP}) = \begin{pmatrix} \dfrac{2\, c_{cr}(r - 1)\beta_r^2(-1 + r - k\theta)}{b^2 - 2c_{cr}\beta_r} & 0 & 0 \\ 0 & -2\beta_m & b \\ 0 & b & -c_{cm} \end{pmatrix}.$$

The principal minor determinant of matrix $H(\Pi_m^{DP})$ are

$$H_{11} = \frac{c_{cr}(-1 + r)\beta_r^2(-1 + r - k\theta)}{b^2 - 2c_{cr}\beta_r} < 0,$$

$$H_{22} = -\left(\frac{4c_{cr}(-1+r)\beta_m\beta_r^2(-1+r-k\theta)}{b^2-2c_{cr}\beta_r}\right) > 0,$$

and

$$H_{33} = -\frac{2c_{cr}(-1+r)(b^2-2c_m\beta_m)\beta_r^2(-1+r-k\theta)}{b^2-2c_{cr}\beta_r} < 0,$$

because $0 < b^2 < 2\beta_r c_{cr}$ *and* $(-1+r-k\theta) < 0$. Thus, the function Π_m^{DP} is strictly concave and has an optimal solution as written in Eqs. (32), (33), and (35). Next, we substitute Eqs. (32), (33), and (35) into Eqs. (41) and (42) so we obtain the optimal solution in Eqs. (34) and (36). By substituting Eqs. (31)-(36) into the demand function in Eqs. (1) and (2) and the profit function in Eqs. (29) and (30), the optimal solution is obtained as in Eqs. (37) and (40). ■

Theorem 4. *If* $0 < b^2 < 2\min\{\beta_m c_{cm}, \beta_r c_{cr}\}$*, then the optimal solution of the centralized DCSC model in Eq. (31) is*

$$p_m^{CP*} = \frac{b^2c_p - c_{cm}\left(\alpha + c_p\beta_m - l_m\gamma_m - \alpha\eta + s_m\mu_m\right)}{b^2-2c_{cm}\beta_m}, \tag{43}$$

$$p_r^{CP*} = \frac{1}{b^2-2c_{cr}\beta_r}\left(b^2\left(c_p - \frac{k\theta\lambda\left(b^2c_p - c_{cm}\left(\alpha + c_p\beta_m - l_m\gamma_m - \alpha\eta + s_m\mu_m\right)\right)}{b^2-2c_{cm}\beta_m}\right) - c_{cr}\left(c_p\beta_r + \alpha\eta - \frac{k\theta\lambda\beta_r\left(b^2c_p - c_{cm}\left(\alpha + c_p\beta_m - l_m\gamma_m - \alpha\eta + s_m\mu_m\right)\right)}{b^2-2c_{cm}\beta_m} + s_r\mu_r\right)\right), \tag{44}$$

$$y_m^{CP*} = \frac{b\left(c_p\beta_m + l_m\gamma_m + \alpha(-1+\eta) - s_m\mu_m\right)}{b^2-2c_{cm}\beta_m}, \tag{45}$$

$$y_r^{CP*} = \frac{b\left(c_p\beta_r - \alpha\eta - \frac{k\theta\lambda\beta_r\left(b^2c_p - c_{cm}\left(\alpha + c_p\beta_m - l_m\gamma_m - \alpha\eta + s_m\mu_m\right)\right)}{b^2-2c_{cm}\beta_m} - s_r\mu_r\right)}{b^2-2c_{cr}\beta_r}, \tag{46}$$

$$D_m^{CP*} = \frac{c_{cm}\beta_m\left(c_p\beta_m + l_m\gamma_m + \alpha(-1+\eta) - s_m\mu_m\right)}{b^2-2c_{cm}\beta_m}, \tag{47}$$

$$D_r^{CP*} = \frac{c_{cr}\beta_r(A+B+C)}{(b^2-2c_{cm}\beta_m)(b^2-2c_{cr}\beta_r)} \tag{48}$$

$$\Pi_m^{CP*} = \left(w^{CP*} - c_p\right)D_r^{CP*} + \left(p_m^{CP*} - c_p\right)D_m^{CP*} + w^{CP*}D_r^{CP*}(k\theta - r) - \frac{1}{2}c_{cm}(y_m^{CP*})^2, \tag{49}$$

$$\Pi_r^{CP*} = (p_r^{CP*} - w^{CP*})D_r^{CP*} + r\,w^{CP*}D_r^{CP*} - \frac{1}{2}c_{cr}(y_r^{CP*})^2, \tag{50}$$

$$\Pi_c^{CP*} = \Pi_m^{CP*} + \Pi_r^{CP*}. \tag{51}$$

where $A = -b^2\left(\alpha\eta + c_p\beta_r(-1+k\theta\lambda) + s_r\mu_r\right)$, $B = c_{cm}\left(2\alpha\beta_m\eta - k\alpha\beta_r(-1+\eta)\theta\lambda + c_p\beta_m\beta_r(-2+k\theta\,\lambda)\right)$, *and* $C = k\beta_r\theta\lambda(-l_m\gamma_m + s_m\mu_m) + 2s_r\beta_m\mu_r$.

Proof. In the same way as the proof in Theorem 1, we first prove that the function Π_c^{CP} in Eq. (31) is strictly concave and has a unique solution. The Hessian matrix of the function Π_c^{CP} is

$$H(\Pi_c^{CP}) = \begin{pmatrix} -2\beta_m & 0 & b & 0 \\ 0 & -2\beta_r & 0 & b \\ b & 0 & -c_m & 0 \\ 0 & b & 0 & -c_{cr} \end{pmatrix}.$$

Since $0 < b^2 < 2\min\{\beta_m c_{cm}, \beta_r c_{cr}\}$, we obtain the principal minor determinant of the Hessian matrix $H(\Pi_c^{CP})$ are $H_{11} = -2\beta_m < 0$, $H_{22} = 4\beta_m\beta_r > 0$, $H_{33} = 2\beta_r(b^2 - 2c_{cm}\beta_m) < 0$, and $H_{44} = (b^2 - 2c_{cm}\beta_m)(b^2 - 2c_{cr}\beta_r)$. Thus, it is clear that function Π_c^{CP} is strictly concave and has a unique solution. Next, we determine the first partial derivatives of Eq. (31) with respect to p_m, p_r, y_m, and y_r, which are set equal to zero. Thus, we obtain a system of equations with four variables, namely

$$\frac{\partial \Pi_c^{CP}}{\partial p_m} = by_m - p_m\beta_m - (p_m - c_p)\beta_m - l_m\gamma_m + \alpha(1-\eta) + s_m\mu_m = 0,$$
$$\frac{\partial \Pi_c^{CP}}{\partial p_r} = by_r - p_r\beta_r - (p_r - w)\beta_r - (-c_p + w)\beta_r - rw\beta_r + \alpha\eta - w\beta_r(-r + k\theta) + s_r\mu_r = 0,$$
$$\frac{\partial \Pi_c^{CP}}{\partial y_m} = b(p_m - c_p) - c_{cm}y_m = 0,$$
$$\frac{\partial \Pi_c^{CP}}{\partial y_r} = b(p_r - w) + b(w - c_p) + brw - c_{cr}y_r + bw(-r + k\theta) = 0.$$

The optimal solution is obtained by solving the above system of equations, and we get as written in the Eqs. (43)-(46). The optimal demand function is obtained by substituting Eqs. (43)-(46) into Eqs. (1) and (2), yielding Eqs. (47) and (48). While the optimal profit function is obtained as written in Eqs. (49)-(51). ■

3.3 Numerical Simulation and Sensitivity Analysis

In this section, the application is carried out through simulation on the DCSC model, taking into account consumer loyalty, delivery time, and CSR that has been obtained. Numerical simulation aims to determine the selling price of products, wholesale prices, CSR levels, and optimal profits for each supply chain actor. The parameter values used in numerical simulation are taken from several sources as presented in Table 1.

Table 1. The Parameter Values Used in Numerical Simulation

Parameter	Value	Source	Parameter	Value	Source
c_p	\$10	[24]	μ_r	0.7/unit	[25]
l_m	5 days	[26]	η	0.4	[13]
λ	0.46	[26]	α	100 unit	[22]
β_m	0.8 unit^2/\$/year	[19]	b	0.5/unit	[22]
β_r	0.75 unit^2/\$/year	[19]	c_{cm}	\$20	[22]
γ_m	0.7 unit/day	[19]	c_{cr}	\$25	[22]
s_m	4.2	[27]	θ	0.01/month	[22]
s_r	4.5	[28]	k	0.55	.[29]
μ_m	0.3/unit	[25]	r	0.05	[30]

Next, the parameter values in Table 1 were substituted into the optimal solution of the DCSC model obtained in Theorems 1 - 4. The optimal solution for the DCSC model, taking into account customer loyalty, delivery time, and CSR, is presented in Table 2. The notation "O" indicates a timely payment system and "P" indicates a prepaid system.

Table 2. The Optimal Solution of DCSC Model in Decentralized and Centralized Scenario

Decision Variables	Decentralized Scenario		Centralized Scenario	
	O	P	O	P
w	\$33.77	\$35.51	\$19.02	\$19.02
p_m	\$41.35	\$41.35	\$41.35	\$41.35
p_r	\$45.73	\$45.72	\$33.93	\$33.93
y_m	0.78	0.78	0.78	0.78
y_r	0.24	0.24	0.48	0.48
D_m	25	25	25	25
D_r	9	9	18	18
Π_m	\$993.10	\$994.86	\$941.70	\$926.83
Π_r	\$106.62	\$106.88	\$264.65	\$281.39
Π_c	\$1099.72	\$1101.74	\$1206.34	\$1208.22

Based on Table 2, it appears that the product selling price per unit in online channels p_m, CSR level for online channels y_m, and demand function of manufacturer D_m obtained the same values of $41.35, 0.78, and 25 for the on-time payment system and the prepaid system in both the decentralized and centralized scenarios. The selling price of products on online channels in a decentralized scenario is lower than the selling price of products on offline channels. Conversely, in a centralized scenario, the selling price of products on online channels is higher than the selling price of products on offline channels. The optimal CSR level on online channels is always higher than on offline channels in both scenarios. Similarly, the optimal product demand on online channels is always higher than on offline channels. This results in the optimal profit on online channels also always being higher than offline channels. The profit of online channels in a decentralized scenario is also higher than in a centralized scenario. However, conversely, the profit of offline channels is higher when applying a centralized scenario than a decentralized one.

Furthermore, a sensitivity analysis was conducted to determine the effect of parameter changes on the solution of the DCSC model. Changes in parameter values were made to parameters that affect the demand function, including consumer loyalty (s_m), delivery time (l_m), and CSR elasticity (b). Changes in consumer loyalty (s_m) affect product prices, demand, and the profits of supply chain actors, as shown in Table 3 for the decentralization scenario and Table 4 for the centralization scenario.

Table 3. Sensitivity of s_m in Decentralization Scenario

s_m	w^{DO*}	p_m^{DO*}	p_r^{DO*}	y_m^{DO*}	y_r^{DO*}	D_m^{DO*}	D_r^{DO*}	Π_m^{DO*}	Π_r^{DO*}	Π_c^{DO*}
4.1	33.77	41.33	45.73	0.78	0.24	25	9	992.16	106.62	1098.78
4.3	33.77	41.36	45.73	0.78	0.24	25	9	994.04	106.62	1100.66
4.5	33.77	41.40	45.73	0.79	0.24	25	9	995.93	106.62	1102.55
4.7	33.77	41.44	45.73	0.79	0.24	25	9	997.81	106.62	1104.43
4.9	33.77	41.48	45.73	0.79	0.24	25	9	999.70	106.62	1106.32
s_m	w^{DP*}	p_m^{DP*}	p_r^{DP*}	y_m^{DP*}	y_r^{DP*}	D_m^{DP*}	D_r^{DP*}	Π_m^{DP*}	Π_r^{DP*}	Π_c^{DP*}
4.1	35.51	41.33	44.90	0.78	0.26	25	9	993.92	106.88	1100.80
4.3	35.51	41.36	44.90	0.78	0.26	25	9	995.80	106.88	1102.68
4.5	35.51	41.40	44.90	0.79	0.26	25	9	997.68	106.88	1104.56
4.7	35.51	41.44	44.90	0.79	0.26	25	9	999.57	106.88	1106.45
4.9	35.51	41.48	44.90	0.79	0.26	25	9	1001.45	106.88	1108.33

Table 4. Sensitivity of s_m in Centralization Scenario

s_m	w^{DO*}	p_m^{DO*}	p_r^{DO*}	y_m^{DO*}	y_r^{DO*}	D_m^{DO*}	D_r^{DO*}	Π_m^{DO*}	Π_r^{DO*}	Π_c^{DO*}
4.1	19.01	41.33	33.93	0.78	0.48	25	18	940.60	264.80	1205.40
4.3	19.03	41.36	33.93	0.78	0.48	25	18	942.80	264.49	1207.29
4.5	19.05	41.40	33.93	0.79	0.48	25	18	944.99	264.18	1209.17
4.7	19.06	41.44	33.93	0.79	0.48	25	18	947.19	263.87	1211.05
4.9	19.08	41.48	33.93	0.79	0.48	25	18	949.39	263.86	1212.94
s_m	w^{DP*}	p_m^{DP*}	p_r^{DP*}	y_m^{DP*}	y_r^{DP*}	D_m^{DP*}	D_r^{DP*}	Π_m^{DP*}	Π_r^{DP*}	Π_c^{DP*}
4.1	35.51	41.33	33.87	0.78	0.48	25	18	925.74	281.54	1207.28
4.3	35.51	41.36	33.87	0.78	0.48	25	18	927.92	281.24	1209.17
4.5	35.51	41.40	33.87	0.79	0.48	25	18	930.10	280.95	1211.05
4.7	35.51	41.44	33.87	0.79	0.48	25	18	932.29	280.65	1212.94
4.9	35.51	41.48	33.87	0.79	0.48	25	18	934.48	280.35	1214.83

Table 3 shows that an increase in the value of s_m results in an increase in online channel profits and system profits. An increase in the value of s_m in the decentralization scenario causes the profits of each supply chain actor and system profits in model O to be lower than in model P. Table 4 shows that the online channel profit in model O is higher than in model P. However, the offline channel profit and system profit in model O are higher than in model P. An increase in the value of s_m causes the online channel profit and system profit in both models in the centralization scenario to always increase. Furthermore, the effect of delivery time sensitivity on wholesale prices, product selling prices, demand, and profit margins for decentralization and centralization scenarios are presented in Tables 5 and 6.

Table 5. Sensitivity of l_m in Decentralization Scenario

l_m	w^{DO*}	p_m^{DO*}	p_r^{DO*}	y_m^{DO*}	y_r^{DO*}	D_m^{DO*}	D_r^{DO*}	Π_m^{DO*}	Π_r^{DO*}	Π_c^{DO*}
1	33.77	43.11	45.73	0.83	0.24	25	9	1083.62	106.62	1189.96
2	33.77	42.67	45.73	0.82	0.24	25	9	1060.32	106.62	1166.94
3	33.77	42.23	45.73	0.81	0.24	25	9	1037.60	106.62	1144.22
4	33.77	41.79	45.73	0.79	0.24	25	9	1015.20	106.62	1121.82
5	33.77	41.35	45.73	0.78	0.24	25	9	993.10	106.62	1099.72
l_m	w^{DP*}	p_m^{DP*}	p_r^{DP*}	y_m^{DP*}	y_r^{DP*}	D_m^{DP*}	D_r^{DP*}	Π_m^{DP*}	Π_r^{DP*}	Π_c^{DP*}
1	35.51	43.11	44.90	0.83	0.26	25	9	1085.09	106.88	1191.97
2	35.51	42.67	44.90	0.82	0.26	25	9	1062.07	106.88	1168.95
3	35.51	42.23	44.90	0.81	0.26	25	9	1039.36	106.88	1146.24
4	35.51	41.79	44.90	0.79	0.26	25	9	1016.95	106.88	1123.83
5	35.51	41.35	44.90	0.78	0.26	25	9	994.86	106.88	1101.74

Table 6. Sensitivity of l_m in Centralization Scenario

l_m	w^{DO*}	p_m^{DO*}	p_r^{DO*}	y_m^{DO*}	y_r^{DO*}	D_m^{DO*}	D_r^{DO*}	Π_m^{DO*}	Π_r^{DO*}	Π_c^{DO*}
1	19.83	43.11	33.93	0.83	0.48	26	18	1046.49	250.09	1296.58
2	19.63	42.67	33.93	0.82	0.48	26	18	1019.83	253.73	1273.56
3	19.42	42.23	33.93	0.81	0.48	25	18	993.48	257.37	1250.84
4	19.22	41.79	33.93	0.79	0.48	25	18	967.43	261.01	1228.44
5	19.02	41.35	33.93	0.78	0.48	25	18	941.70	264.65	1206.34
l_m	w^{DP*}	p_m^{DP*}	p_r^{DP*}	y_m^{DP*}	y_r^{DP*}	D_m^{DP*}	D_r^{DP*}	Π_m^{DP*}	Π_r^{DP*}	Π_c^{DP*}
1	19.83	43.11	33.87	0.83	0.48	26	18	1031.02	267.52	1298.54
2	19.63	42.67	33.87	0.82	0.48	26	18	1004.51	270.98	1275.50
3	19.42	42.23	33.87	0.81	0.48	25	18	978.31	274.45	1252.76
4	19.22	41.79	33.87	0.79	0.48	25	18	952.42	277.92	1230.34
5	19.02	41.35	33.87	0.78	0.48	25	18	926.83	281.39	1208.22

Table 5 shows that there is no effect of changes in l_m value on retailer profits. An increase in l_m value results in a decrease in producer profits and system profits. The P model in the decentralization scenario generates greater profits for supply chain actors. Table 6 shows that an increase in l_m causes a decrease in producer profits and system profits. However, this increase in l_m results in an increase in retailer profits. As in the decentralization scenario, the P model in the centralization scenario also generates greater profits than the O model.

Next, changes in CSR levels affect the profits of both channels. This is because both supply chain actors implement CSR. Changes in CSR levels are caused by changes in CSR elasticity values. The effect of CSR elasticity on wholesale prices, product selling prices, demand, and profit margins for decentralization and centralization scenarios are presented in Tables 7 and 8.

Table 7. Sensitivity of b in Decentralization Scenario

b	w^{DO*}	p_m^{DO*}	p_r^{DO*}	y_m^{DO*}	y_r^{DO*}	D_m^{DO*}	D_r^{DO*}	Π_m^{DO*}	Π_r^{DO*}	Π_c^{DO*}
0.1	33.77	41.11	45.65	0.16	0.05	25	9	985.89	105.94	1091.83
0.2	33.77	41.14	45.66	0.31	0.10	25	9	986.78	106.02	1092.81
0.3	33.77	41.19	45.68	0.47	0.14	25	9	988.28	106.17	1094.45
0.4	33.77	41.26	45.70	0.63	0.19	25	9	990.38	106.36	1096.75
0.5	33.77	41.35	45.73	0.78	0.24	25	9	993.10	106.62	1099.72
b	w^{DP*}	p_m^{DP*}	p_r^{DP*}	y_m^{DP*}	y_r^{DP*}	D_m^{DP*}	D_r^{DP*}	Π_m^{DP*}	Π_r^{DP*}	Π_c^{DP*}
0.1	35.51	41.11	44.81	0.16	0.05	25	9	987.63	106.20	1093.83
0.2	35.51	41.14	44.82	0.31	0.10	25	9	988.53	106.28	1094.81
0.3	35.51	41.19	44.84	0.47	0.15	25	9	990.03	106.42	1096.45
0.4	35.51	41.26	44.86	0.63	0.20	25	9	992.13	106.62	1098.76
0.5	35.51	41.35	44.90	0.78	0.26	25	9	994.86	106.88	1101.74

Table 8. Sensitivity of b in Centralization Scenario

b	w^{DO*}	p_m^{DO*}	p_r^{DO*}	y_m^{DO*}	y_r^{DO*}	D_m^{DO*}	D_r^{DO*}	Π_m^{DO*}	Π_r^{DO*}	Π_c^{DO*}
0.1	18.91	41.11	33.78	0.16	0.10	25	18	932.88	250.09	1296.58
0.2	18.92	41.14	33.79	0.31	0.19	25	18	933.98	253.73	1273.56
0.3	18.95	41.19	33.82	0.47	0.29	25	18	935.80	257.37	1250.84
0.4	18.98	41.26	33.87	0.63	0.38	25	18	938.37	261.01	1228.44
0.5	19.02	41.35	33.93	0.78	0.48	25	18	941.70	264.65	1206.34

b	w^{DP*}	p_m^{DP*}	p_r^{DP*}	y_m^{DP*}	y_r^{DP*}	D_m^{DP*}	D_r^{DP*}	Π_m^{DP*}	Π_r^{DP*}	Π_c^{DP*}
0.1	19.83	41.11	33.72	0.16	0.10	25	18	918.19	281.43	1199.62
0.2	19.63	41.14	33.74	0.31	0.19	25	18	919.26	281.43	1200.69
0.3	19.42	41.19	33.77	0.47	0.29	25	18	921.06	281.42	1202.47
0.4	19.22	41.26	33.81	0.63	0.38	25	18	923.58	281.40	1204.98
0.5	19.02	41.35	33.87	0.78	0.48	25	18	926.83	281.39	1208.22

Table 7 shows that a significant increase in CSR results in increased profits for actors and system profits. In this decentralization scenario, model P generates greater profits than model O. Based on Table 8, it can be seen that in the centralization scenario, the higher the level of CSR, the greater the profits obtained by supply chain actors. Table 8 also shows that model O is more profitable than model P, because it generates greater profits. Based on sensitivity analysis, we found that consumer loyalty and delivery time only affect changes in product selling prices in online channels, CSR levels for online channels, and manufacturer profits in decentralized scenarios, while product selling prices in offline channels, CSR levels for offline channels, and manufacturer and retailer demand are not affected in centralized scenarios. CSR elasticity has an effect on almost all decision variables except wholesale prices, manufacturer demand, and retailer demand.

4. CONCLUSION

In this paper, we analyzed pricing decisions in a single-product dual-channel supply chain involving a manufacturer and a retailer operating through online and offline channels. The main contribution of this study lies in integrating customer loyalty, delivery time, and Corporate Social Responsibility into a unified pricing framework under two payment schemes, namely the on-time payment system and the prepaid system. The model was examined under both centralized and decentralized decision structures, with the manufacturer acting as the Stackelberg leader in the latter case. Numerical and sensitivity analyses were conducted to complement the analytical results. The findings indicate that customer loyalty and delivery time mainly influence online selling prices, the Corporate Social Responsibility level in the online channel, and the manufacturer's profit in the decentralized setting, while several decision variables in the centralized setting remain unchanged. In addition, Corporate Social Responsibility elasticity affects most decision variables, except the wholesale price and the demand levels of both channel members. Overall, the results enrich the literature on dual-channel supply chain pricing by showing how payment mechanisms and behavioral factors jointly shape optimal decisions. Future research may extend the proposed framework by incorporating service competition, green investment, product returns, demand uncertainty, and dynamic decision-making in more realistic and sustainable supply chain environments.

Author Contributions

Bhecti Purwaningsih: Conceptualization, Formal Analysis, Investigation, Methodology, Software, Visualization, Writing-Original Draft. Ririn Setiyowati: Conceptualization, Funding Acquisition, Project Administration, Resources, Supervision, Visualization, Writing-Original Draft, Writing-Review and Editing, Final Manuscript. Nughthoh Arfawi Khurdi: Supervision, Validation, Visualization, Writing-Original Draft, Writing-Review and Editing. All authors discussed the results and contributed to the final manuscript.

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Declarations

The authors declare no competing interest.

Declaration of Generative AI and AI-assisted technologies

Generative AI tools were used solely for language refinement (grammar, spelling, and clarity). The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. The authors reviewed and approved all final text.

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