

PRESERVICE MATHEMATICS TEACHERS' REASONING IN SOLVING CRITICAL THINKING PROBLEM

Herizal¹, Marhami^{2*}, Mutia Fonna³, Rohantizani⁴

^{1,2,3,4}Department of Mathematics Education, Faculty of Teacher Training dan Education,
Malikussaleh University

Jalan Cot Teungku Nie, Reuleut, Kec. Muara Batu, Aceh Utara, 24355, Indonesia

Corresponding author e-mail: ^{2*}marhami@unimal.ac.id

Abstract. *Mathematical reasoning is one of the abilities that must have by students after learning mathematics. To improve students' reasoning, teachers also have to good in reasoning. Due to the importance of reasoning for teachers, a qualitative descriptive research was conducted to analyse how the reasoning of preservice mathematics teachers in solving critical thinking problem. The data were collected through instrument test that consists of one problem about perimeter of a rectangle from 40 preservice mathematics teachers of first year in one of public universities in North Aceh, Aceh, Indonesia. The problem measured the skill of manipulating mathematical process; constructing mathematical proof, and drawing conclusion. Their works were analysed for each indicator of reasoning. The result showed that more than 75% preservice mathematics teachers were wrong in solving the problem. They were not good in constructing mathematical proof to draw a conclusion. It concludes that preservice mathematics teachers' ability in mathematical reasoning was not satisfied. The ability still needs to be improved through learning activities and they must be accustomed to the problems or situations that involving reasoning.*

Keywords: *critical thinking, geometry, mathematical reasoning*

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1. INTRODUCTION

In Indonesia's Curriculum, there are several mathematical abilities that must have by students in all level of education. In detail, there are five cognitive skills that must be achieved by the students, they are (1) understanding concepts and applying mathematical procedure in daily life; (2) generalizing based on pattern, fact, phenomenon, or data; (3) doing mathematical operation to simplify and analyse; (4) reasoning in mathematical problems included both conjecturing and verifying; (5) solving the problems and communicating the idea through symbol, table, diagram, or the other media [1]. According to Pollatsek et al., there are six basic recommendations for majors, programs and courses in Mathematics. One of them is every course in mathematics should involve an activity that can help the students to develop their analytical, critical reasoning, problem solving and communication skills [2]. Based on that recommendation, it is obvious that reasoning ability need to be considered in learning mathematics especially in higher education. National Council of Teachers of Mathematics, NCTM and Kilpatric et al. also included reasoning as mathematical proficiency achieved by students [3], [4].

Mathematical reasoning is the process of reaching logical conclusions based on facts and relevant sources [5] [6]. Mathematical reasoning is also defined as reasoning about and with object mathematics [7]. Reasoning can be also interpreted as a thinking process to draw conclusions or make a correct statement that has been known to be true [8]. It can be concluded that mathematical reasoning is the process of taking conclusion of numbers of fact-based ideas through logical and critical thinking in solving mathematical problems. Expert in mathematical reasoning means that students have several abilities such as analyzing, generalizing, synthesing, justifying/proving, and solving non-routine problems. All of the abilities are the components of mathematical reasoning [9]. It can be inferred that mathematical reasoning is a high order ability in mathematics because of its complexity.

Mastering the reasoning ability for educational students is necessary to avoid the students to end up with error answers in solving reasoning mathematics problems [10]. It indicates that mathematical reasoning is very crucial in mathematics learning. Students are able to do reasoning if they are able to use reasoning skills in patterns and traits, manipulate mathematics in generalizing or explain mathematical ideas and statements. The higher the level of students' reasoning, the faster the learning process will be in achieving learning indicators [11]. In line with this, the research showed that mathematical reasoning ability has a positive influence on mathematics learning achievement [12].

Furthermore there are four factors of the importance of reasoning in daily life, such as (1) reasoning is needed in solving mathematical problem, (2) reasoning is needed in learning mathematics in school, (3) reasoning can be implemented in any other knowledges or sciences, and (4) reasoning is needed in daily life [13]. Reasoning is one of the crucial abilities in mathematics as a supporting feature in mathematics learning [14][15]. Therefore, mathematical reasoning is one of the important abilities for the students in any level of the education, especially in high school level. Mathematical reasoning ability is needed when to solve mathematics problems especially in critical thinking problems. Critical thinking is a high level thinking skill that must be possessed by students [16]. It points out that critical thinking problems is also high level problems. Mathematics teachers also preservice mathematics teachers should dominate math problems with this type of problems. Their mathematical reasoning abilities should able to direct or train their students to have reasoning abilities as well. So that they are able to solve critical thinking problems with their reasoning ability.

Improving students' mathematical reasoning ability depends on several factors. Conceptual understanding is one of the factors. Lack of understanding a concept contribute to students' ability in solving mathematics problems [17], [18]. Beside good in conceptual understanding, another factor affecting the students' ability is teacher. In several literatures, it showed that teacher is one of the important factors for students in teaching and learning process [19]. In the case of mathematical reasoning, the teachers have to good in mathematical reasoning before teaching or integrating in the mathematics learning process. The ability is not instant. It takes much time to accustomed to the mathematical reasoning ability. The process is start when the teachers was still in preservice teacher's education. Due to the importance of mathematical reasoning, it is interesting for further analyzing about the reasoning ability of preservice mathematics teachers in solving problems since they will improve students' reasoning ability in the future. Therefore, this study was conducted to analyze the mathematical reasoning ability of preservice mathematics teachers in solving critical thinking problem.

2. RESEARCH METHODS

In this study, researchers wanted to analyze how the reasoning of preservice mathematics teachers in solving critical thinking problem. Therefore, the researcher used qualitative method for conducting descriptive research. The use of qualitative method because qualitative research is designed to provide an in-depth description of a specific program, practice, or setting [20].

The participants were 40 preservice mathematics teachers of first year in one of public universities in North Aceh, Aceh, Indonesia. They have learned about basic plane geometry. To know how their reasoning in solving mathematical critical thinking problem, an instrument consist of one problem was given for them. The problem was about geometry focused on perimeter of rectangle (see Figure 1). This research measured three from six indicators of reasoning, such as: mathematics manipulation; constructing mathematical proof, and drawing conclusion [21]. The use of the critical thinking problem shown in Figure 1 because to answer the question, preservice mathematics teachers need several skills related to reasoning indicator explained above. Before using in the research, the instrument was validated by two lecturers of the department of mathematics education. Data were analyzed using Miles and Huberman step, i.e. data reduction, data display, and drawing conclusion [22].

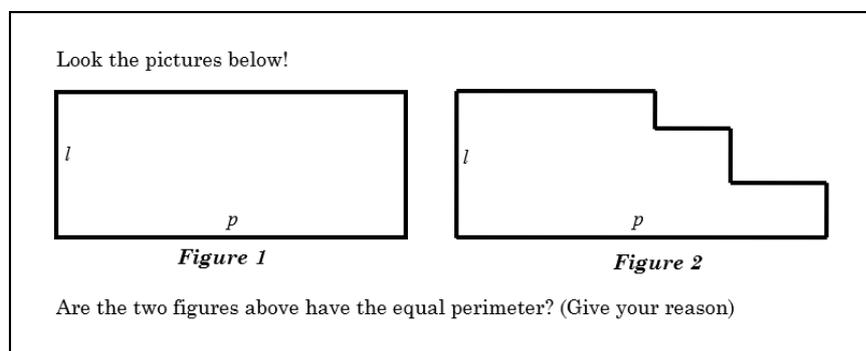


Figure 1. Mathematical Critical Thinking Problem

3. RESULT DAN DISCUSSION

The ability of preservice mathematics teachers' reasoning in solving critical thinking problem was obtained from a test. To answers the problem, they must be good in looking the similarity of the sides of both figures. In this step, their critical thinking was needed to construct that the length and width of the rectangle was equal. In solving the problem, there are three main skills that have to expert by them except critical thinking ability. First, ability in doing mathematics manipulation. The ability is used when determined the perimeter of each figures. Second, ability in constructing mathematical proof. To explain the correct answer, the preservice teachers have to prove that the perimeter of the both figures is equal to. The last, drawing conclusion. This part related to the second skill.

After analyzing preservice mathematics teachers' answer sheets, there were six types of their answers. From six types, there were two types that wrote right answers (total 25% of participants). The answers showed in Figure 2 and Figure 3.

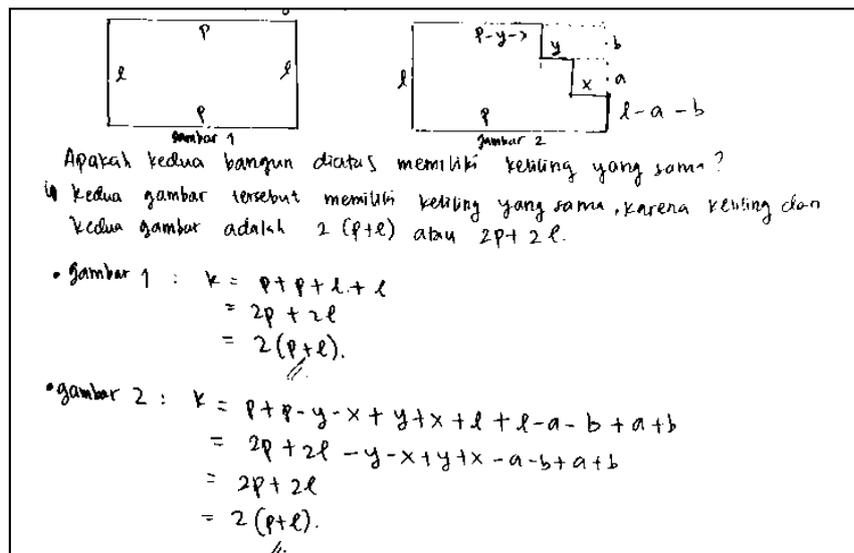


Figure 2. Type 1 (Correct Answer)

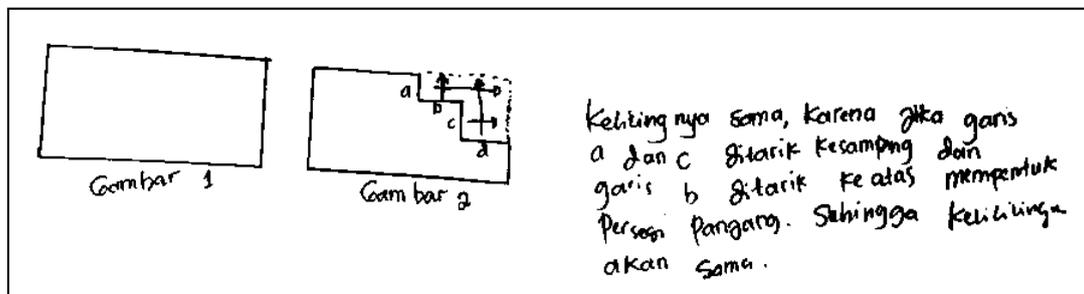


Figure 3. Type 2 (Correct Answer)

Figure 2 and Figure 3 were examples of preservice mathematics teachers with right answer. In figure 2, they were constructing mathematical proof systematically through mathematics manipulation process. The figure showed that they started by determining the missing component of the second figure. Then, they calculated the perimeter of the first and second figure to draw a conclusion. This activity means that preservice teacher who answered as shown in the figure 2 have good ability in mathematical reasoning. This conclusion based on Brodie stated that proof was a form of justification and argumentation then mathematical proving ability could be said as a part of mathematical reasoning [7]. So, if the someone is good in mathematical proving then they also good in mathematical reasoning.

Meanwhile in Figure 3, preservice mathematics teachers proved that the two figures in the problem have the equal perimeter by informal proof. In this case, they proved by words with illustration. They described how to make sure that figure 1 and figure 2 can be seen as a same rectangle with the congruent both length and the width. This type of proof is also said as semantic reasoning i.e. constructing a proof by justify it through informal arguments [23].

Besides, there were 75% of participants that wrote the wrong answer. For example, in Figure 3 it can be seen that preservice mathematics teachers supposed the first line in second picture with 'm-x-y' then the second line with 'x-y' and the last with 'y'. This incorrect representation showed that their mathematical manipulation is lack. The error caused that the perimeter of the both figures was different. For the 1st figure they got $2m+2n$ as perimeter and $2m-(x+y)+2n-(p+q)$ as the perimeter of the 2nd figure. The result is different even they used the right and the same formula.

6. Apakah kedua bangun ini memiliki keliling yang sama?

Gambar 1

Gambar 2

Rumus:
 $K = 2(P + L)$
 $= 2P + 2L$
 $= P + P + L + L$

* Gambar 1.
 $K = m + m + n + n$
 $= 2m + 2n$

* Gambar 2
 $K = m + m - x - y + n + n - p - q$
 $= 2m - x - y + 2n - p - q$
 $= 2m - (x + y) + 2n - (p + q)$

Maka, keliling kedua bangun tersebut
 Tidak Sama.

Figure 4. Type 1 (Incorrect Answer)

Tidak, karena Gambar 1 ~~adalah~~ adalah Persegi Panjang dengan bentuk yang sempurna, namun Gambar 2 itu awalnya merupakan Persegi Panjang, tapi karena telah dipotong bentuknya pun berubah sehingga kelingnya pun pasti berbeda nilainya.

Figure 5. Type 2 (Incorrect Answer@)

Another case was in Figure 5. It showed that the preservice mathematics teachers draw a conclusion based on illustration such as thought that different shapes also have different perimeter before compiling proof or using manipulative in mathematics. The students that answered like shown in Figure 4 did not have good mathematical critical thinking ability. They did not see detail of the component of each figure.

The findings in Figure 5 also Figure 4 can be interpreted that the achievement of preservice mathematics teachers' reasoning ability has not been optimally achieved. The problem occurred because students were not used to solve critical mathematical problems by making new solutions, but they were used to the usual completion steps in their daily problem-solving [24].

4. CONCLUSION

Mathematical reasoning is an important ability in mathematics. The ability is related to another sub skill which are mathematical proving and justification. Improving students' mathematical reasoning ability is one of the goals in the mathematics curriculum of Republic of Indonesia. Preservice mathematics teachers as teacher candidates must have good ability in mathematical reasoning since it is an asset to improve students' ability in the future. But, the study showed that most of them are difficult in part of constructing correct mathematical proof and drawing conclusion as a part of mathematical reasoning. It implies that the ability of preservice mathematics teachers in mathematical reasoning is still need to be improved. The further research is needed specifically in developing strategies or learning models to improve the ability of mathematical reasoning.

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