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THE APPLICATION OF MARKOV CHAIN MODEL TO CALCULATE PREMIUM AND RESERVE OF ENDOWMENT INSURANCE

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Abstract. *The calculation of premiums and reserves are two essential parts of insurance. The calculation of premiums and reserves in life insurance involves using mortality tables. This research constructed a mortality table for 20-year endowment insurance using the Markov chain model. Two reasons make the policy inactive, namely death or withdrawal. The initial age used in this research is 30 years. Meanwhile, the maximum age to join this life insurance is 40 years. The mortality table that has been obtained is used to calculate premiums and reserves. Furthermore, from the research done, it was found that the age of entry to become a member of endowment insurance affects the number of premiums that must be paid. Meanwhile, the number of reserves required will increase with the increase of customers and the period of calculation of reserves.*

Keywords: *endowment insurance, markov chain, mortality table, premium, reserve*

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1. INTRODUCTION

Life insurance is insurance with the object of coverage in the form of a person, and the insured is a person's life. The risk faced in life insurance is the risk of death. According to [1], there are three primary life insurance policies: term, whole life, and endowment life insurance. Endowment life insurance can be classified into *n*-year term endowment life insurance and *n*-year pure endowment life insurance [2]. An *n*-year term endowment life insurance provides coverage to policyholders for n years. The death benefit is provided if the policyholder dies within the *n*-year contract period. Meanwhile, pure *n*-year endowment life insurance will benefit when the policyholder is still alive or reaches an *n*-year contract period.

In this study, a 20-year endowment life insurance model was developed. Endowment life insurance combines *n*-year term life insurance and *n*-year pure endowment insurance. An endowment life insurance, the death benefit can be given immediately after death or at the end of the year of death if the policyholder dies before *n*-years. On the other hand, if the policyholder achieves an *n*-year contract, the policyholder will get an *n*-year pure endowment benefit. However, in this study, it is assumed that if the policyholder dies before *n*-year, the death benefit is given at the end of the year of death.

Several things cause a customer's policy to become inactive in life insurance. This study assumes that two things make the policy inactive, namely withdrawal or death. Changes in circumstances experienced by insurance policyholders can be modeled with a Markov chain. As research conducted by [3], [4], [5], [6], [7], and [8].

The Markov chain model assumes that the probability of changing from one state to another only depends on the current state and is independent of past states [9]. In this study, the Markov chain model is used to find the probability value of the transition of states from the policyholder. Furthermore, the transition probability value is used to construct a mortality table. According to [10], actuaries use mortality tables to build premium and reserve structures for life insurance, annuities, and pensions products. Therefore, the purpose of this study was (a) to create a Markov chain model for endowment life insurance with three conditions, namely active, withdrawal, and death, (b) to create a mortality table using the Markov chain model, (c) calculate endowment life insurance premiums, and (d) calculate the financial reserve for endowment life insurance.

2. RESEARCH METHOD

This research used actuarial basic function theory and Markov chain principle. Some of the concepts and their properties are given as follows.

2.1 Life Probability

If x represents the age of an endowment life insurance customer and T be a random variable representing the lifetime of a person. According to [11], a person's life chances function can be expressed as follows:

$$\begin{aligned} {}_kp_x &= P(T > x + k | T > x) \\ &= P(T(x) > k) \\ &= 1 - P(T(x) \leq k) \\ &= 1 - {}_kq_x \end{aligned}$$

The value ${}_kp_x$ represents the probability that a person at age x will live to age $x+k$. Meanwhile, ${}_kq_x$ represents the probability that someone aged x will die before reaching age $x + k$. Furthermore, according to [11], the probability of a person's life ${}_kp_x$ can be obtained by the following formula

$${}_kp_x = \frac{l_{x+k}}{l_x} \tag{1}$$

where l_x represents the number of individuals aged x in an endowment life insurance portfolio.

2.2 Annuity

An annuity can be defined as fixed payments or receipts within a certain period. There are two terms in annuities: cash value or present value and final value. The cash value would be the value of all payments if the payments were paid at once. At the same time, the final value is the sum of all payments and interest if all payments and interest are paid once in the future.

Furthermore, the present value of actuarial liability is defined as the present value of the expected payments made by the insurance company. The present value of actuarial liability in endowment life insurance is denoted $A_{x:n]}$. According to [11], the present value of the actuarial liability of endowment life insurance is obtained by the following formula:

$$A_{x:n]} = \sum_{k=0}^{n-1} v^{k+1} \ {}_kp_x \, q_{x+k} + v^n \ {}_np_x \tag{2}$$

where $v = \frac{1}{1+i}$ is the discount factor of interest rate (i) when the policy is issued until the death benefit is paid.

Then, when viewed from the time of payment, annuities can be divided into late annuity-immediate and early annuity-due. The final annuity-immediate payment is made at the end of each period. In contrast, the initial annuity-due payment is made at the beginning of each period. Annuity from endowment life insurance in this study is paid at the beginning of each year, denoted by $\ddot{a}_{x:n]}$. According to [11], the annuity of endowment life insurance is obtained by the following formula:

$$\ddot{a}_{x:n]} = \sum_{k=0}^{n-1} v^k \ {}_kp_x \tag{3}$$

2.3 Markov Chain

Stochastic process $\{X_t\}$ is a collection of random variables with t representing the index of time. Random variables $X_0, X_1, \dots, X_t, X_{t+1}$ follow a stochastic process. A stochastic process is said to be a Markov chain if for all conditions $i_0, i_1, \dots, i, j$ fulfill the following equation

$$P(X_{t+1} = j | X_t = i, X_{t-1} = i_{t-1}, \dots, X_0 = i_0) = P(X_{t+1} = j | X_t = i) = P_{ij} \tag{4}$$

where P_{ij} represents the probability that the transition from a process in state i at time t and will be in event j at time $t+1$. In other words, the occurrence of X_{t+1} depends only on the occurrence X_t and is independent of the events $X_0, X_1, \dots, X_{t-1}$.

2.4 Markov Chain Model in Endowment life insurance

In this study, suppose a 20-year endowment life insurance product, where participants who can join are participants between the ages of 30-40 years. The participant's policy status is declared inactive for two reasons, namely withdrawal or death. The amount of benefits provided is assumed to be 100 million. Meanwhile, the benefits are given to participants who reach the contract or those who die during the 20-year contract. The diagram and transition matrix of these conditions can be described as follows:

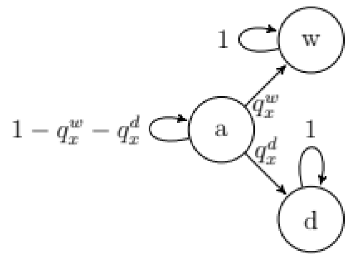

Figure 1. Transition Diagram of the Markov Chain Model for Three Conditions Endowment Life Insurance

where q_x^w states the probability that a person aged x years will be out of insurance before reaching the age of $x+1$ years because of withdrawal. Meanwhile q_x^d states the probability that a person aged x years will get out of insurance before reaching the age of $x+1$ years because he/she dies

Furthermore, according to [9] the transition probability matrix of the Markov chain model of endowment life insurance with three states is

$$\boldsymbol{P} = \begin{pmatrix} p_{aa} & p_{aw} & p_{ad} \\ p_{wa} & p_{ww} & p_{wd} \\ p_{da} & p_{dw} & p_{dd} \end{pmatrix}$$
$$= \begin{pmatrix} p_x^\tau & q_x^w & q_x^d \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Where conditions "a" active, "w" withdrawal, and "d" death.

2.5 Data Source

The initial data used in this study was sourced from the 2011 Indonesian Mortality Table (TMI) which was taken from the Financial Services Authority (OJK) annual report on insurance in 2016 [12]. The data from TMI used include the initial population at the age of $x = 30$ (l_x) which is 989770 and the number of individuals who died at the age of $x = 30$ (D_x) which is 1356. Meanwhile, the number of individuals who withdraw (W_x), for $x = 30$ assumed for 100 people.

2.6 Mortality Table

From the TMI data, it can be obtained that q_x^d or the probability that an individual will die is between x and $x+1$. The value of q_x^d can be calculated using the following formula [13]:

$$q_x^d = \frac{D_x}{l_x} \tag{5}$$

Meanwhile, from the TMI data, it can also be obtained that q_x^w is the probability that an individual's status will become inactive due to withdrawal between the ages of x to $x+1$. The value q_x^w can be calculated using the following formula:

$$q_x^w = \frac{W_x}{l_x} \tag{6}$$

Furthermore, p_x^τ which is the probability that an individual is still alive or active in a endowment life insurance product, is calculated by the following formula:

$$p_x^\tau = 1 - q_x^d - q_x^w \tag{7}$$

From these assumptions, a one-step transition probability matrix is obtained as follows:

$$\boldsymbol{P} = \begin{pmatrix} p_x^\tau & q_x^w & q_x^d \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0{,}9985 & 0{,}0001 & 0{,}0014 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Furthermore, according to [14] the Markov chain applies the property $P^n = \boldsymbol{P^n}$ namely the transition probability of n steps can be obtained by multiplying the one-step transition probability matrix by itself n times. Therefore, the transition probability for $x = 31, 32, \dots, 60$ is obtained by using the matrix multiplication property.

2.7 Analysis Steps

In this study, the analytical steps carried out in calculating premiums and reserves for three-conditions endowment life insurance are as follows:

a. Determine the term of the endowment life insurance product
b. Determine the age requirements of participants who can join the endowment life insurance product
c. Determine the amount of benefits that will be provided to participants
d. Draw a transition diagram of three-condition Markov chain model
e. Determine the transition probability matrix
f. Create a mortality table
g. Calculate premiums from endowment life insurance products
h. Calculate the financial reserves of an endowment life insurance product

3. RESULTS AND DISCUSSION

In this section, the discussion is divided into three things, namely the mortality table for endowment life insurance in three circumstances, premium calculation and the last is the calculation of reserves for endowment life insurance with three conditions.

3.1. Mortality Table of Three Conditions Endowment Life Insurance

From equations (5), (6) and (7), the mortality table for endowment life insurance is obtained with three conditions as follows:

Table 1. Mortality Table of Three Conditions Endowment Life Insurance

x	l_x	W_x	D_x	p_x^τ	q_x^w	q_x^d
30	989770	100	1356	0,9985	0,0001	0,0014
31	988314	198	2767	0,9970	0,0002	0,0028
32	985349	296	4138	0,9955	0,0003	0,0042
33	980915	392	5493	0,9940	0,0004	0,0056
34	975029	488	6825	0,9925	0,0005	0,0070
35	967717	581	8129	0,9910	0,0006	0,0084
36	959007	671	9398	0,9895	0,0007	0,0098
37	948938	759	10533	0,9881	0,0008	0,0111
38	937645	844	11721	0,9866	0,0009	0,0125
39	925081	925	12859	0,9851	0,0010	0,0139
40	911297	1002	13943	0,9836	0,0011	0,0153
41	896352	1076	14969	0,9821	0,0012	0,0167
42	880307	1144	15846	0,9807	0,0013	0,0180
43	863317	1209	16748	0,9792	0,0014	0,0194
44	845360	1268	17583	0,9777	0,0015	0,0208
45	826509	1322	18266	0,9763	0,0016	0,0221
46	806921	1372	18963	0,9748	0,0017	0,0235
47	786586	1416	19586	0,9733	0,0018	0,0249
48	765584	1455	20058	0,9719	0,0019	0,0262
49	744071	1488	20536	0,9704	0,0020	0,0276
50	722047	1516	20939	0,9689	0,0021	0,0290
51	699591	1539	21198	0,9675	0,0022	0,0303
52	676854	1557	21389	0,9661	0,0023	0,0316
53	653909	1569	21579	0,9646	0,0024	0,0330
54	630761	1577	21635	0,9632	0,0025	0,0343
55	607549	1580	21689	0,9617	0,0026	0,0357
56	584280	1578	21618	0,9603	0,0027	0,0370
57	561084	1571	21546	0,9588	0,0028	0,0384
58	537967	1560	21357	0,9574	0,0029	0,0397
59	515050	1545	21117	0,9560	0,0030	0,0410
60	492387	1526	20877	0,9545	0,0031	0,0424

3.2. Three Conditions Endowment Life Insurance Premium

In this study, it is assumed that the benefits provided are 100 million. This benefit is given to customers who die during the 20-year contract period or who are still alive until the contract ends (reaching a 20-year contract). Premiums are calculated using the following formula [11]:

$$Premium = \frac{100jt\ A^d_{x:n]}}{\ddot{a}_{x:n]}} \tag{8}$$

For example, for customers who enter at the age of 30 years. Premiums are obtained in the following way

$$A^d_{30:20]} = \sum_{k=0}^{19} v^{k+1}\ {}_kp^\tau_{30}\ q^d_{30+k} + v^{20}\ {}_{20}p^\tau_{30}$$
$$= 0{,}4117$$

Meanwhile,

$$\ddot{a}_{30:20]} = \sum_{k=0}^{19} v^k \ {}_kp_{30}^{\tau}$$
$$= 12{,}1594$$

Therefore, the value of $Premium = 282.156$. In the same way, the premium is calculated for those who join insurance at the age of 31, 32, …, 40. From the results of the calculations that have been carried out, the results are as follows:

Table 2. Monthly and Annual Premiums of Three Circumstances Endowment Life Insurance

Entry Age	Monthly Premium	Annual Premium
30	282.156	3.385.870
31	289.004	3.468.044
32	296.033	3.552.397
33	303.101	3.637.211
34	310.204	3.722.450
35	317.341	3.808.097
36	324.509	3.894.109
37	331.705	3.980.461
38	339.001	4.068.015
39	346.328	4.155.938
40	353.682	4.244.186

3.3. Three-Conditions Endowment Life Insurance Reserve

If ${}_kV_{x:n]}$ represent the financial reserve of an n-year endowment insurance product in the next k-years of a person aged x. Financial reserves are calculated using the following formula [15]:

$$ {}_kV_{x:n]} = 100jt\ \left(A_{x+k:n-k]}\right) - Premi\ \left(\ddot{a}_{x+k:n-k]}\right) \qquad (9)$$

The formula can be interpreted as the amount of reserves needed is equal to the amount of expenditure minus the amount of income.

Suppose the financial reserves needed for individuals who enter at the age of 30 years for a period of (k = 5) years in the future. The amount of financial reserves required is obtained in the following way:

$$A_{35:15\,]} = \sum_{k=0}^{n-1} v^{k+1}\ {}_kp_x^{\tau}\ q_{x+k}^{d} + v^n\ {}_np_x^{\tau}$$
$$= \sum_{k=0}^{14} v^{k+1}\ {}_kp_{30}^{\tau}\ q_{35}^{d} + v^{15}\ {}_{15}p_{35}^{\tau}$$
$$= 0{,}514375216$$

Meanwhile,

$$\ddot{a}_{35:15]} = \sum_{k=0}^{n-1} v^k \ {}_kp_x^{\tau}$$
$$= \sum_{k=0}^{14} v^k \ {}_kp_{35}^{\tau}$$
$$= 9{,}971499543$$

Thus, ${}_5V_{30:20} = 17.692.022$. In the same way, the required financial reserves are calculated for customers who join at the age of 31,32,…,40. The financial reserves are calculated for each year i.e., years 0,1,2,…,19. The results that have been obtained are then cumulated for each different entry age. The following shows the results for the total financial reserves where it is assumed that at each age, there is one customer.

Table 3. Total Financial Reserves of Three Conditions Endowment Life Insurance

Year -	Total Reserve
0	0
1	35.118.331
2	70.882.084
3	107.438.991
4	144.950.228
5	183.591.891
6	223.565.354
7	265.082.948
8	308.378.985
9	353.713.239
10	401.373.646
11	451.680.496
12	504.996.659
13	561.725.812
14	622.319.255
15	687.282.123
16	757.176.441
17	832.635.018
18	914.370.791
19	1.003.189.660

4. CONCLUSIONS

From the research that has been done, it can be concluded several things as follows:

1. The n-step transition probability is used to construct a three-conditions endowment life insurance mortality table obtained by multiplying the transition probability matrix by itself n times.
2. The n-step transition probability is used to calculate premiums and reserves for a three-conditions endowment life insurance product.
3. The age of entry to become a member of endowment life insurance affects the amount of premium that must be paid. The higher the age of a person joining the insurance, the amount of premium to be paid will also be higher.
4. The number of reserve funds required by an endowment life insurance company will increase along with the increase in the number of policyholders and the period of calculation of financial reserves.

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