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IMPACT OF FEAR BEHAVIOR ON PREY POPULATION GROWTH PREY-PREDATOR INTERACTION

Rian Ade Pratama*

Department of Mathematics Education, Faculty of Teacher Training and Education, Musamus University
Kamizaun Mopah Lama St., Merauke, 99611, Indonesia

*Corresponding author's e-mail: *pratama@unmus.ac.id*

Abstract. *Experiments on the living environment of vertebrate ecosystems, it has been shown that predators have a massive influence on the demographic growth rate of prey. The proposed fear effect is a mathematical model that affects the reproductive growth rate of prey with the Holling Type I interaction model. Mathematical analysis of the prey-predator model shows that a strong anti-predator response can provide stability for prey-predator interactions. The parameter area taken will be shown for the extinction of the prey population, the balance of population survival, and the balance between the prey birth rate and the predator death rate. Numerical simulations were given to investigate the biological parameters of the population (birth rate, natural mortality of prey, and predators). Another numerical illustration that is seen is the behavior of prey which is less sensitive in considering the risk of predators with the growth rate of prey.*

Keywords: *fear, Holling type I, predator-prey.*

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1. INTRODUCTION

Predators interplay is an important topic in ecological science both theoretically and practically by evolutionary scientists. Mathematical models provide the key to better understand complex events in the ecological world. Mathematical model of the population using the well-known logistic growth first considers a linear birth rate. The Lotka-Volterra population growth model introduces logistical types of population growth for prey and various types of response functions that represent a realistic interplay between prey and predators [1].

The development of models for population growth, the effects of fear have also been widely introduced on population births. The effect of fear plays an important role in the ecological world [2]. One of the interesting behaviors in predator-prey interactions such as in the presence of predator populations, prey populations can change their behavior. Such behavioral changes often occur to significantly alter the predator's direct predation style. Most of the population mathematical modeling in prey-predator interaction functions only considers direct predation, whereas much can happen to prey-predator behavior facing each other [3]. Therefore, considering the behavioral effect of fear on prey is more realistic for future research.

The predation process creates a fearful effect on the prey population that can affect behavior and psychology when compared to direct predation [4][5]. The effects that arise indirectly cause anti-predator behavior (changes in foraging, changes in habitat, prey alertness, and various other physiological changes). This anti-predator behavior plays an important role in regulating prey demographics. The fear effect of prey populations can lead to long-term harm to prey species [6]. Prey species that experience the effects of fear will naturally forage less because of alertness, growth rates also decline to the most severe survival mechanisms such as starvation often occurs in prey species.

Events of the fear effect occur mostly in interactions between vertebrates or bird species. Such as wolf species (*Canis lupus*) and deer (*Cervus elaphus*) observed by researchers. There was a decline in deer population growth due to attacks by groups of predatory species in the Yellowstone ecosystem. Other prey behavior is also assumed in this study as fear of prey [7][8]. Like acute fear of prey to predators, it can move prey to move places or ecosystems in the long term and return when the ecosystem is considered safe by prey. Like the behavior of birds that are afraid of the sound of predators, with the physiological state of fear in birds, they fly away or migrate since the first danger appears [9][10]. Such events can reduce the reproduction rate of birds in the long term, although in the short term it is beneficial because it saves adult prey species.

Research on a stochastic prey-predator mathematical model with the effect of fear on prey on predator feeding [11][12]. The fear effect also provides a study of the prey-predator mathematical model with structural stages with adaptive avoidance for predator populations by including the predator population fear effect for prey populations [13]. Structural stages are considered because of the final division of the predator population into small and adult predators. The analysis in the mathematical model goes deep into the constant adaptive avoidance of prey [14][15][16].

Population dynamics are also not separated from a prey-predation interaction system. There are many forms of the mathematical formulation of the function of predation from the simple to the more complex [17]. Illustration of the process of predation occurs depending on the characteristics of the prey [18]. Monotonous and non-monotonous response functions are widely adopted as a form of prey-predatory predation function [19]. In addition, there are several types of response functions that are highly dependent on prey density, there are also studies that introduce response functions depending on prey and predator density, among which are Cowley-Martin response functions, Beddington-DeAngelis response function, Monod-Haldane response function.

The results of observations on the mathematical model of the bird population and the fear behavior of the wolf population, as well as the behavior of the Holling type I predation function, became the basis for the development of a prey-predation mathematical model in this study [20]. The formulation of the initial research is to provide realistic basic assumptions to find a mathematical model [21]. The prey and predator populations were analyzed by differential equations without eliminating the dimensions of each representative variable. Furthermore, the equilibrium point is shown and its stability is tested. Local stability testing is used to show population growth over a long time.

2. RESEARCH METHODS

This research model is a literature study that is supported by research with the latest and reputable journals. This section describes the concepts of basic assumptions and basic research methods. The mathematical model of the prey-prey population is represented by two variables. Variables $x(t)$ and $y(t)$ become the population density of prey and predators at any time $t > 0$. Logistic growth was adopted for population growth in prey [22]. The prey birth rate r, natural death rate and intraspecies interactions are components of prey logistics growth. The following mathematical model is recommended for differential prey growth:

$$\frac{dx}{dt} = x(r - \delta_1 - \gamma x), \tag{1}$$

where δ_1 and γ, are natural prey mortality rates and mortality rates due to interactions between prey, respectively. Intraspecies interactions between prey often occur in the struggle for place or the existence of survival.

The formulation of a mathematical model on prey will be developed in the form of a function with a fear effect based on the assumptions that have been given [23]. The function with the fear effect will be given on the growth of the prey species by multiplying the form $f(\alpha,\beta,y)$, so that the form model (1) becomes

$$\frac{dx}{dt} = x(rf(\alpha,\beta,y) - \delta_1 - \gamma x), \tag{2}$$

where α and β, are levels prey fear or anti-predatory behavior and minimum fear of prey. So the overall form of the mathematical model is as follows:

$$\begin{aligned} \frac{dx}{dt} &= x(rf(\alpha,\beta,y) - \delta_1 - \gamma x) - g(x)y, \\ \frac{dy}{dt} &= y(g(x) - \delta_2 - y), \end{aligned} \tag{3}$$

where $g(x)$ is a function of predation interaction from predator to prey, which means that the prey is consumed by the predator per unit time unit. While δ_2 represents the natural mortality rate of the population of predators, ρ is a conversion coefficient for prey to predators.

The response function $g(x)$ defined as the predation function which adopts the Holling type I. This predation function is linear in shape, the amount of predation by prey is equal to the number of preys. The greater the number of preys, the greater the predation of the predator. The characteristics of predators who adopt this function tend to be passive in their prey [24]. The form of the Holling type I predation function is as follows:

$$g(x) = ax, \tag{4}$$

where a, is the rate of interaction between populations. The fear effect function $f(a,\beta,y)$ described mathematically as follows:

$$f(\alpha,\beta,y) = \beta + \frac{\alpha(1-\beta)}{\alpha + y}, \tag{5}$$

which β meet $\beta \in [0,1]$. The fear effect function is assumed that $f(0,\beta,y) = \beta$, $f(\alpha,\beta,0) = 1$, $\lim_{y\to\infty} f(\alpha,\beta,y) = \beta$, and $\lim_{\alpha\to\infty} f(\alpha,\beta,y) = 1$. The function $f(0,\beta,y) = \beta$, shows that the prey population is always less than the minimum fear power β. The form of the function $f(\alpha,\beta,0) = 1$, shows that when there is no predator population, the fear function has no effect on the growth of the prey population. Fear function $f(\alpha,\beta,y) = \beta$, shows that if the predator population increases, the prey population experiences minimum fear pressure from the predator species. Fear function on $f(\alpha,\beta,y) = 1$, indicating that after the fear level is saturated at a certain point in the prey population, the fear function has no effect due to the physiological impact when the prey is accustomed to the predator threat. This kind of event often occurs in prey-prey interactions.

The mathematical model of the theoretical reconstruction is formed as follows:

$$\frac{dx}{dt} = x\left(r\left(\rho + \frac{\alpha(1-\rho)}{\alpha + y}\right) - \delta_1 - ax - \beta y\right), \tag{6}$$

$$\frac{dy}{dt} = y(\tau\beta x - \delta_2 - by),$$

where for $x(0) = x_0 > 0$ and $y(0) = y_0 > 0$ the initial condition of the prey-predator population model. Description of model parameters (6) is presented in the following table of variables, parameters and dimensions:

Table 1. Description of model variables (6)

Symbol	Description	Dimensions
x	Population Prey	$[N]$
y	Population Predator	$[N]$
r	Birth rate of prey population	$[T]^{-1}$
α	Fear level	-
ρ	Minimum power of fear	-
δ_1	Death rate natural prey	$[T]^{-1}$
δ_2	Predatory natural death	$[T]^{-1}$
a	Rate iintraspecies prey competitiveness	$[T]^{-1}[N]^{-1}$
β	Predation-prey interaction rate Prey-predator	$[N]^{-1}[T]^{-1}$
τ	Cconversion	$[N]^{-1}[T]^{-1}$
b	Rate In-predator intraspecies competitiveness rate	$[T]^{-1}[N]^{-1}$

3. RESULTS AND DISCUSSION

3.1. Equilibrium Analysis

The equilibrium point that emerges from model (6) is the point that will be investigated and tested to see the survival of the species. Equilibrium point investigations mathematically using the differential equations $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$. The results show the point of equilibrium on the model (6) is $E_0 = (0,0)$, $E_1 = \left(\frac{r-\delta_1}{a}, 0\right)$, $E_2 = \left(0, -\frac{\delta_2}{b}\right)$, $E_3 = (x^*, -y^*)$ and $E_4 = (x^*, y^*)$. The equilibrium point of the model (6), it is clear that there are five points. The most rational point selection is the positive equilibrium value, $E_4 = (x^*, y^*)$. The mathematical form of the equilibrium is $E_4 = (x^*, y^*)$:

$$x^* = \frac{\sqrt{A} + (-\alpha b + 2\delta_2)\tau\beta^2 + b\tau\beta(r\rho - \delta_1) - ab^2 + ab\delta_2}{2\beta\tau(\tau\beta^2 + ab)}$$
$$y^* = \frac{\sqrt{A} - ab^2 + b(-\alpha\tau\beta^2 + \tau\beta(r\rho - \delta_1) - a\delta_2)}{2b(\tau\beta^2 + ab)} \qquad (7)$$

where,

$$A = \alpha^2 b^2 \left((\tau\beta^2 + ab)^2 - 2\left(((p-2)r + \delta_1)\beta\tau + a\delta_2\right)(\tau\beta^2 + ab)\alpha + (-\tau\beta(r\rho - \delta_1) + a\delta_2)^2\right).$$

There is only one equilibrium point which is analyzed for stability in model (6), which is a positive point. The Jacobian matrix for model (6) with equilibrium point E_4 is as follows:

$$J(E_4) = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix}$$

where,

$$j_{11} = r\left(\rho + \frac{\alpha(1-\rho)}{\alpha + y}\right) - \delta_1 - 2ax - \beta y,$$
$$j_{12} = \frac{\alpha r x(1-\rho)}{(\alpha + y)^2} - \beta x,$$
$$j_{21} = y\tau\beta,$$
$$j_{22} = \beta\tau x - 2by - \delta_2.$$

The characteristic equation associated with the Jacobian matrix $J(E_4)$ is,

$$f(\lambda) = N_1\lambda^2 + N_2\lambda + N_2, \qquad (8)$$

Criterion for testing the equilibrium point towards its stability is using the Hurwitz criterion. The equilibrium points E_4 asymptotically stablesatisfies the conditions $N_1 > 0, N_2 > 0, N_3 > 0$ and $N_1 N_2 > N_3$.

3.2. Numerical Simulation

The simulation model (6) will be shown with parameters that have been taken from assumptions and references. It also shows several variables and parameters of fear that greatly affect the condition of the population's mathematical model. The visualization of the model (6) will show the parameter values that affect the dynamics of the model system. The analysis of the existence of an equilibrium point is also shown with a numerical approach. Most of the parameter values are obtained from relevant research and support the model (6).

The parameters taken in the simulation are $r = 1.5, \alpha = 0.003, \rho = 0.5, \delta_1 = 0.015, \delta_2 = 0.05, a = 0.01, \beta = 0.05, \tau = 0.0741$ and $b = 0.04$. There are five equilibrium points that emerge from model (6), each of which is $E_0 = (0,0)$, $E_1 = (148.5,0)$, $E_2 = (0,-1.25)$, $E_3 = (13.42247708, -0.006743060477)$ and $E_4 = (54.54703201,3.80241884)$. It is clear that the positive equilibrium point that allows for analysis is $E_4 = (54.54703201, 3.80241884)$. The next test is to look at the characteristic equations associated with the Jacobian matrix. The characteristic equation that emerges from the numerical simulation is $f(\lambda) = \lambda^2 + 0.69756707735\lambda + 0.121506488$. From the characteristic equation, it was obtained that the criteria Routh-Hurwitzand were met the eigenvalues that met the criteria were $\lambda_1 = -0.360761330962206$ and $\lambda_2 = -0.336805742537794$. All eigenvalues exists and is negative, then the equilibrium point E_4 locally asymptotic stable. The prey and predator populations of model (6) show sustained growth over a long period of time.

For the initial population around the equilibrium point E_4, we take $x(0) = 5$ and $y(0) = 1$. The movement of the population curve in time t, will be shown as a form of visual simulation of prey and predator populations.

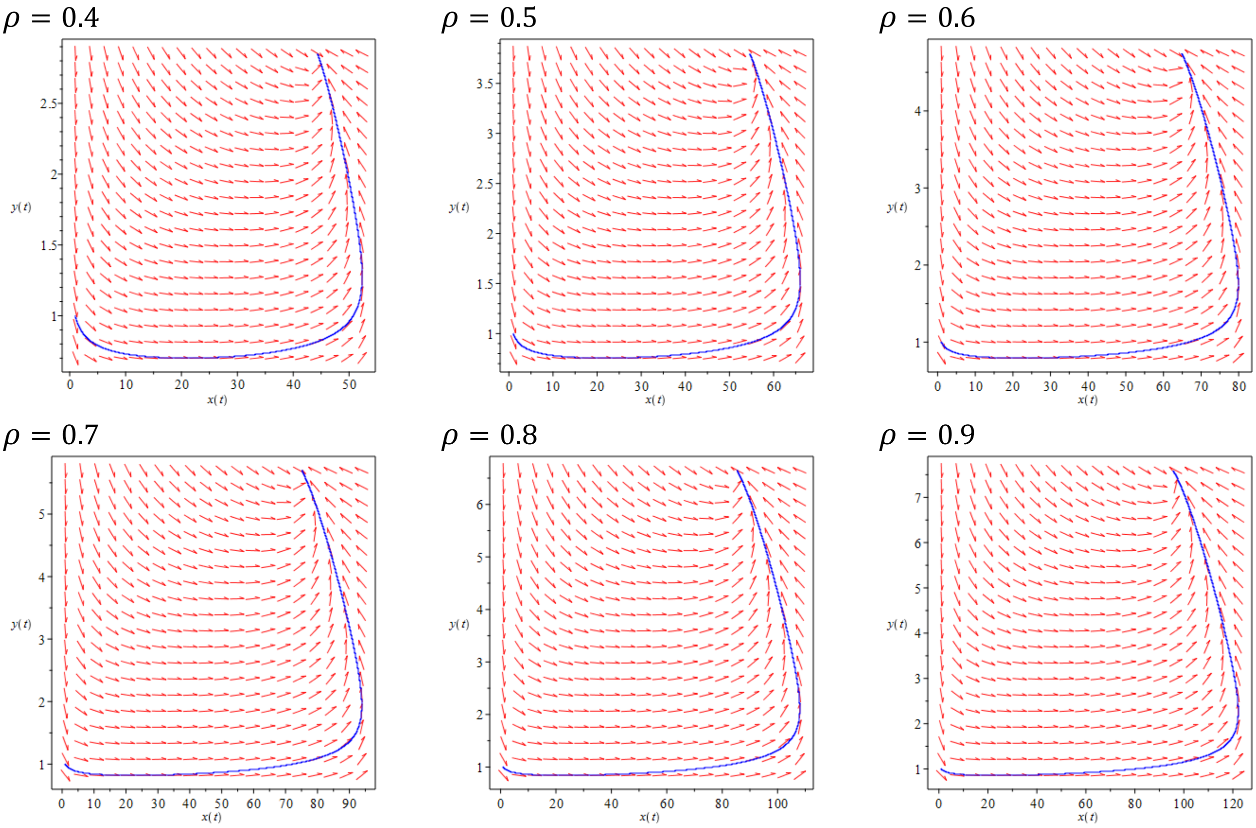

Figure 1. Population movement curve with fear effect coefficient

In Figure 1, it is visually shown the movement of prey and predator populations that move stably. The fear effect parameter has a significant impact on the movement of each population. The comparison of prey and predator populations is strongly influenced by the parameters taken on the fear effect. The greater the fear effect experienced by the prey, the greater the comparison of the two populations. The population movement in the curve can be seen at a larger value $\rho > 0.9$, but if the condition $\rho > 0.9$ applied to model (6), instability will be obtained at the equilibrium point. This condition also occurs in the condition for $\rho < 0.4$.

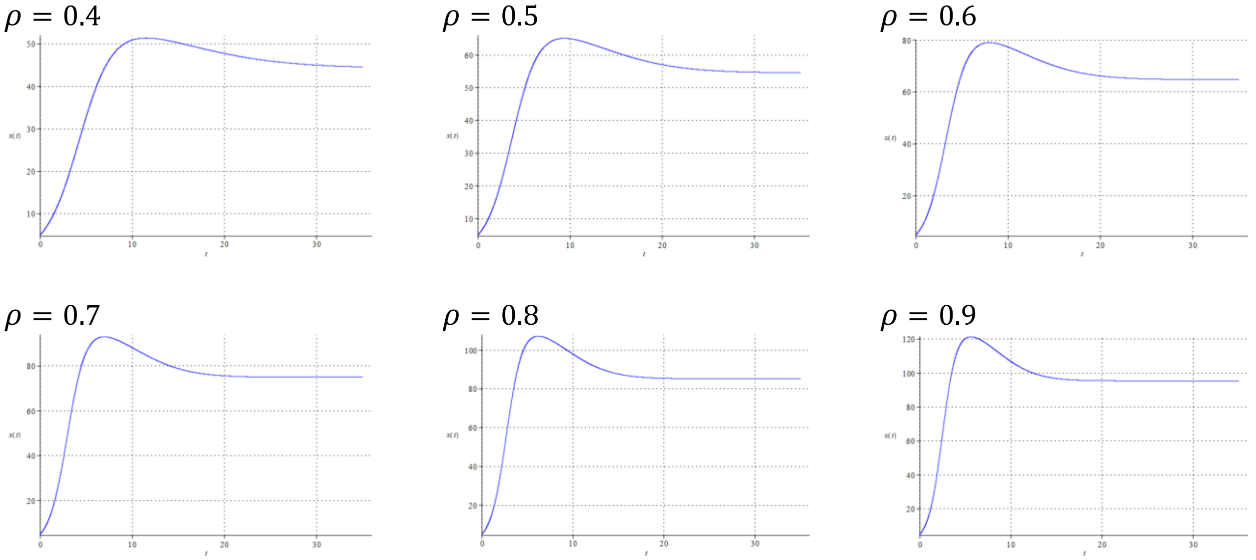

Figure 2. The prey population growth curve with fear effect coefficient

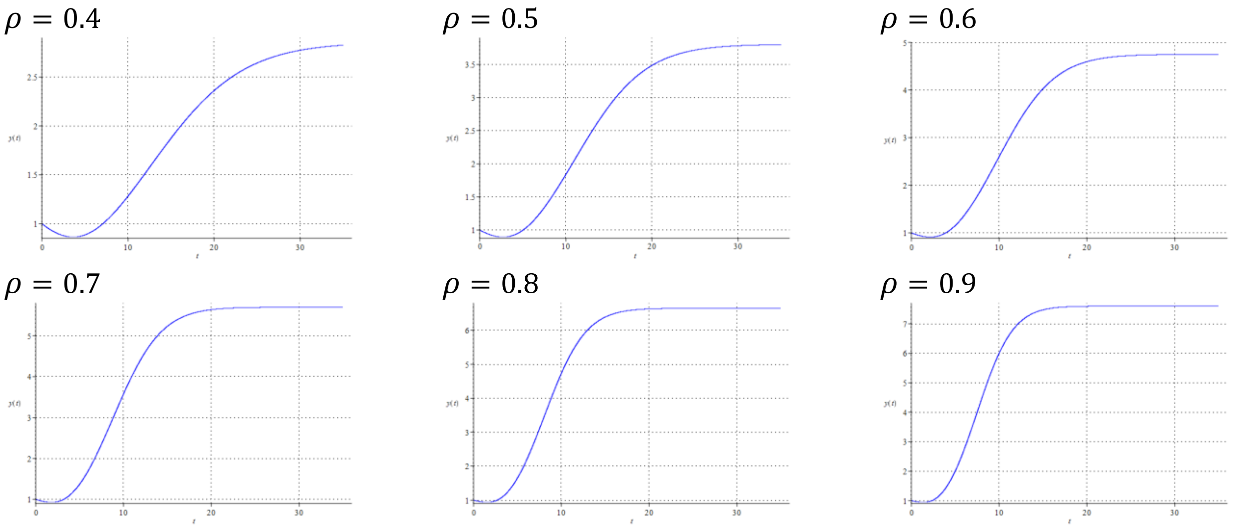

Figure 3. Predator population growth curve with fear effect coefficient

Figure 2 shows the shape of the curve that changes as a result of increasing the fear effect parameter. The assumption of this fear effect is given according to the condition that, if the species is intervened continuously, the fear power will increase which causes fear and the species becomes more afraid. These conditions show significant population growth at certain time conditions. The smaller the fear effect experienced by the prey, the slower the population growth will be and it will not be significant or lead to a stable growth. Meanwhile, Figure 3 also shows the reverse condition of the prey population. The predator population depicted by the curve in Figure 3 is the increasing rate of population growth. The smaller the fear effect experienced by the population, the slower the growth rate of the predator population. Events like this logically in a species ecosystem are very realistic. If the predator population increases its prey on the prey then what happens is that the prey becomes increasingly afraid. Meanwhile, the response to the growth of prey and predator populations continues to increase according to the increasing effect of fear on prey species.

4. CONCLUSION

Investigation was carried out on the effect of fear experienced by prey with the commonly used predation response function Holling I. Equilibrium analysis on the formed model shows four equilibrium points. The equilibrium analyzed is an equilibrium that meets the stability requirements. Stability performed on the model is local asymptotic from a single biologically significant equilibrium point. The range of parameter values that have met the stability of the model, it provides a clear picture of the two populations. The greater the fear power value, the stronger the predator population growth and vice versa. Meanwhile, the prey population that experienced the power of fear, continued to grow linearly on the power of fear given by the predator.

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