BAREKENG: Jurnal Ilmu Matematika dan Terapan
March 2022 Volume 16 Issue 1 Page 091–098
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol16iss1pp091-098

TOTAL EDGE AND VERTEX IRREGULAR STRENGTH OF TWITTER NETWORK

Edy Saputra[1*], Nur Hilal A. Syahrir[2]

[1] *Information Systems Study Program, FMIPA, Hasanuddin University*
[2] *Actuarial Science Study Program, FMIPA, Hasanuddin University*
Jln. Perintis Kemerdekaan Km. 10 90245 Makassar, Indonesia

e-mail: [1] edy_saputra@sci.unhas.ac.id*

Abstract. *Twitter data can be converted into a graph where users can represent the vertices. Then the edges can be represented as relationships between users. This research focused on determining the total edge irregularity strength (tes) and the total vertices irregularity strength (tvs) of the Twitter network. The value could be determined by finding the greatest lower bound and the smallest upper bound. The lower bound was determined by using the properties, characteristics of the Twitter network graph along with the supporting theorems from previous studies, while the upper bound is determined through the construction of the total irregular labeling function on the Twitter network. The results in this study are the tes(TW)=18 and tvs(TW)=16.*

Keywords: *the total edge irregularity strength, twitter network, the total vertex irregularity strength*.

Article info:

Submitted: 2nd October 2021 *Accepted: 3rd February 2022*

How to cite this article:
E. Saputra and N. H. A. Syahrir, "TOTAL EDGE AND VERTEX IRREGULAR STRENGTH OF TWITTER NETWORK", *BAREKENG: J.Il. Mat. & Ter*, vol. 16, iss.1, pp. 091-098, Mar, 2022.

1. INTRODUCTION

Twitter is a social network in which users send and receive messages. One of the advantages of Twitter is that one can process and analyze social networks with an Application Programming Interface (API) which is designed to be powerful so that it can be easily accessed, in contrast to similar social media. Graph G is a set of pairs (V, E) where V is called a vertex that cannot be empty and E is an edge.

Twitter data can be converted into a graph where users can represent the vertices. Then the edges can be represented as relationships between users. By converting Twitter data into graphs, it is easy to determine vertices that have a significant role in the community/group. The vertices and edges relationship that influence each other is how much popularity the user has in the group in which the user as the vertices, then the relationship between the user follows as the edge.

This study focused on irregular labeling where Chartrand et al. [1] introduced irregular labeling. Bača et al [2] found an irregular total labeling of vertices and edges based on total labeling. Labeling t-total is irregular on the edges of Graph *G*, if for each edge of G, for example *m=ab* and *n=cd* where *m≠n* applies $\big(wt(m) = \lambda(a) + \lambda(m) + \lambda(b)\big) \neq \big(wt(n) = \lambda(c) + \lambda(n) + \lambda(d)\big)$. The notation $tes(G)$ is the total edge irregularity strength of Graph, which is the smallest positive integer (Z^+) *t* in the labels on the edges and vertices of Graph. T-labeling is Irregular vertices on Graph *G*, if for every vertex G *a,c* where *a≠c* applies $(wt(a) = \alpha(a) + \sum_m \alpha(m)) \neq (wt(c) = \alpha(c) + \sum_n \alpha(n))$. The notation $\text{tvs}(G)$ is the total value of the vertex irregularity of Graph *G*, which is the smallest positive integer (Z^+) *t* in the labels on the edges and vertices of Graph *G*.

The total value of edge and vertex irregularities in all graphs for now has not been obtained. To support this research, several previous studies have discussed the total value of edge and vertex irregularities in various graphs. For example, in the Gallian survey [3], there are researchers who examine the total value of edge and vertex irregularities in various graphs. Then, Bača et al in the research [2], [4], [5] have found the value of the upper and lower bounds on any graph *G*, namely $\left\lceil \frac{|E|+2}{3} \right\rceil \leq tes(G) \leq |E|$, and the values of irregular and modular irregularity strength of fan graphs, and wheel graphs. Slamin [6] finds irregular labeling distances on the graph. D. Indriati [7] found the value of the irregularity of graph *web* $W(n,m)$.

On interconnected networks such as the *Beneš* network and the *Butterfly* network, the test scores have been found by Rajasingh.I [8]. Nurdin [9]–[12] determines the *tes*, *tvs*,and *ts* of the second level Butterfly network 4, 5 and 6 as well as finds the lower bound on any graph $tes(G) \geq max\left\{\left\lceil \frac{\rho+n_\rho}{\rho+1} \right\rceil, \left\lceil \frac{\rho+n_\rho+n_{\rho+1}}{\rho+2} \right\rceil, \ldots \left\lceil \frac{\rho+\sum_{i=\rho}^{\Delta} n_i}{\Delta+1} \right\rceil\right\}$, and finds Vertex Irregular Labeling and Vertex Irregular Total Labeling on Caterpillar graphs with value $s(T_{n,m}) = n$ and $tvs(T_{n,m}) = \left\lceil \frac{n+1}{2} \right\rceil$, then Saputra. E [13] find tvs and *tes* from *Beneš* network and *Butterfly* network level 5 i.e. *tes*(BF(5))=108 and $tvs\big(BF(5)\big) = 39$. Then, Imran et al [14], have found the *tvs* value of the generalization of prism graph $tvs(P_n^m) = \left\lceil \frac{mn+3}{5} \right\rceil$. Tilukay, M [15] found a total irregular strength on a Complete *Bipartite* graph. Nandini, G. Kirithiga, et al [16] discussed the COVID-19 pandemic tree model using a complete k-ary tree network topology, Cayley tree network, Christmas trees, and the corona product of Christmas tree.

The theoretical use of graph labels in its application to social media network systems such as Twitter has not been found. Thus, this study took one of the Twitter networks on the @diskominfomks and @bsn_makassar accounts. The characteristics and properties were studied and determined. Then, the irregularity strength of the edges and vertices of the Twitter Network that has been formed.

2. RESEARCH METHOD

This study used Twitter data obtained from accounts that retweet, mention, or quote from source accounts @diskominfomks and @bsn_makassar on July 7, 2021, then Gephi software modeled/constructed the graph structure. The data obtained were 47 users as vertices and 52 retweet, mention, or quote data as edges. The data were formed into a Twitter Graph Network structure according to the number of vertices and edges shown in Figure 1.

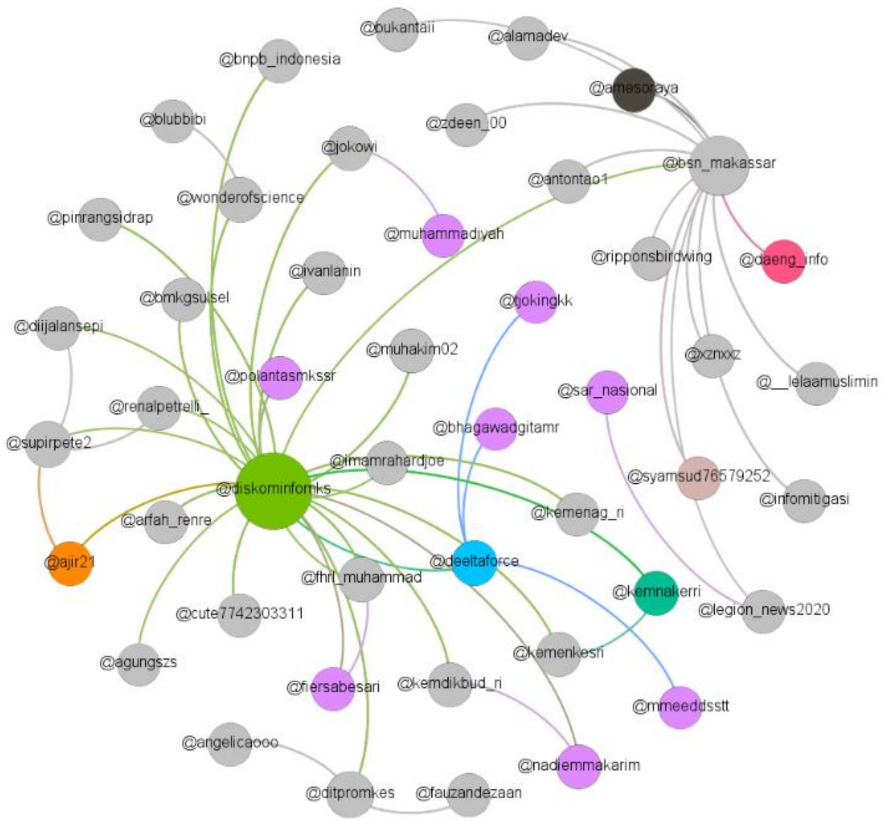


Figure 1. TW Twitter Network

2.1 The Total Edge Irregular Strength

The analysis procedure for the total edge irregular strength in this study was as follows:

1. Determine the largest lower bound of the total edge irregular strength of the Twitter network edges by analyzing the properties, characteristics of the Twitter network graph, the number of edges, and supporting theorems.
2. Determine the smallest upper bound of the total value of the Twitter edge of the network.
   a. Define the set of vertices and edges then construct the labels of the edges and vertices: $\lambda: V \cup E \rightarrow \{1, 2, 3, \cdots, t\}$, so that *t* is the smallest positive integer,
   b. Define the function labeling- *t* total edge irregular λ,
   c. Shows *t* is the largest label used,
   d. Calculates the strength and shows that the edge strength functions are different.

$$m = ab \text{ and } n = cd, \text{ where } ab, cd \in E$$
$$m \neq n$$
$$wt(m) = \lambda(a) + \lambda(m) + \lambda(b) \text{ and } wt(n) = \lambda(c) + \lambda(n) + \lambda(d)$$

2.2 Vertex Irregular Total Value

The procedure for analyzing the Total Vertex Irregularity Value in this study was as follows:

1. Determine the largest lower bound of the total value of the Twitter network vertex irregularity by analyzing the properties, characteristics of the Twitter network graph, the number of edges, and the supporting theorems.
2. Determine the smallest upper bound of the total value of the Twitter network vertex irregularity.
   a. Define the set of vertices and edges then construct the labels of the edges and vertices: $\alpha: V \cup E \rightarrow \{1, 2, 3, \cdots, t\}$, so that *t* is the smallest positive integer,

b. Define a labeling-t function irregular total vertex α,
c. Indicates l is the largest label used.
d. Calculates the strength and shows that the vertex strength functions are different.

$$\left(wt(a) = \alpha(a) + \sum_{m \in E} \alpha(m)\right) \neq \left(wt(c) = \alpha(c) + \sum_{n \in E} \alpha(n)\right)$$

3. RESULT AND DISCUSSION

Definition 1. *Supposing $G = (V, E)$ is a graph. Function $\lambda \rightarrow \{1, 2, 3, \cdots, t\}$ is the labeling-t total of the irregular total edges of the graph , if for each edge $m = ab$ and $n = cd$ where $m \neq n$ applies,*

$$wt(m) \neq wt(n) \tag{1}$$

$$wt(m) = \lambda(a) + \lambda(m) + \lambda(b) \; dan \; wt(n) = \lambda(c) + \lambda f(n) + \lambda(d) \tag{2}$$

*Notation $\boldsymbol{tes(G)}$ is the total value of edge irregularity in graph **G**. It is the smallest positive integer ($\boldsymbol{Z^+}$) **t**, then graph **G** has a labeling- **t** total irregular edges.*

Definition 2. *Supposing $G = (V, E)$ is a graph. Function $\alpha \rightarrow \{1, 2, 3, \cdots, l\}$ is the labeling-t total irregular vertices graph G, if for every vertex a, c where $a \neq c$ applies*

$$\left(wt(a) = \alpha(a) + \sum_{m} \alpha(m)\right) \neq \left(wt(c) = \alpha(c) + \sum_{n} \alpha(n)\right) \tag{3}$$

*Notation $\boldsymbol{tvs(G)}$ is the total value of vertex irregularity in graph **G.** It is the smallest positive integer **($\boldsymbol{Z^+}$)** **t**, then graph **G** has a labeling- **t** total vertex irregular.*

3.1 Labelling Total Edge Irregular

Theorem 1. *For example, TW is a Twitter Network, then the value of tes(TW)=18.*

Prove.

1. The largest lower bound value is obtained by showing that the *tes*(TW)≥18. Proving the statement *tes*(TW)≥18 used the theorem [2]

$$tes(G) \geq \left\lceil \frac{|E| + 2}{3} \right\rceil$$

From the Twitter data, the number of edges of the Twitter Network graph |E|=52, the test scores are obtained as follows:

$$tes(TW) \geq \left\lceil \frac{52 + 2}{3} \right\rceil = 18$$

So, $tes(TW) \geq 18$.

2. The next is showing *tes*(TW)≤18. It is shown by creating labeling-18 construction total irregular *TW* edge in Figure 2. In the Twitter data, the @diskominfomks and @bsn_makassar accounts are the center. They are surrounded by other accounts, which indicates that the two users often interact. In the world of social media, by retweeting, mentioning, or quoting, the two accounts are labeled with the smallest and largest vertex. Each edge is given a different weight starting from 3,4,5,6, 7, ..., 54. Therefore, the value of *tes*(TW)≤18 is obtained. □

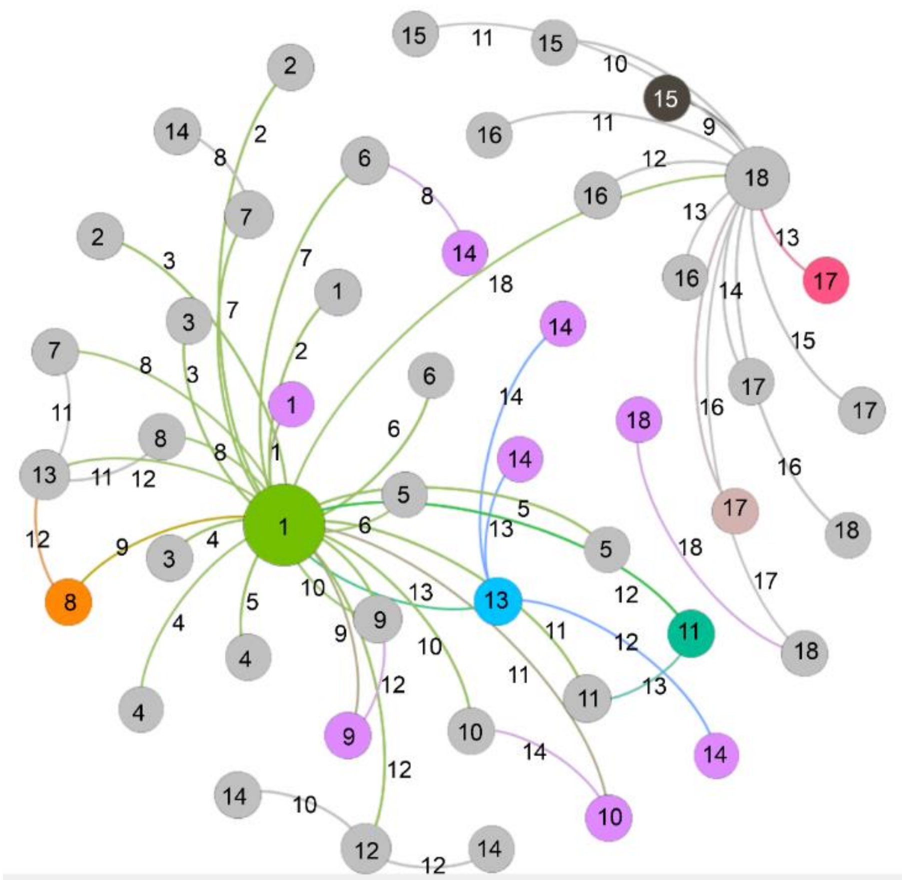


Figure 2. $tes(TW) = 18$

3.2 Labelling Total Vertex Irregular

Theorem 2. *For example, TW is a Twitter Network, then the value of tvs(TW)=16.*

Prove.

1. The largest lower bound value is obtained by showing that tvs(TW)≥16. Proving the statement tvs(TW)≥16 uses the theorem [9].

$$tvs(G) \geq max\left\{\left\lceil\frac{\rho + n_\rho}{\rho + 1}\right\rceil, \left\lceil\frac{\rho + n_\rho + n_{\rho+1}}{\rho + 2}\right\rceil, \ldots \left\lceil\frac{\rho + \sum_{i=\rho}^{\Delta} n_i}{\Delta + 1}\right\rceil\right\}$$

Where:

ρ = smallest degree

n_ρ = the number of vertex that have degrees

Δ = greatest degree

In Figure 1 on the Twitter Network graph, the smallest degree of the TW graph is 1 and the largest degree is 26. Then, there are 6 types of vertex degrees which are presented in Table 1 as follows:

Figure 1. The number of vertices that have degrees on the Twitter Network graph

Variable	Total	Description
$\boldsymbol{\rho}$	1	Least degree
$\boldsymbol{n_1}$	30	Number of degree vertex 1
$\boldsymbol{n_2}$	12	Number of degree vertex 2
$\boldsymbol{n_3}$	1	Number of degree vertex 3
$\boldsymbol{n_4}$	2	Number of degree vertex 4

Variable	Total	Description
n_{13}	1	Number of degree vertex 13
n_{26}	1	Number of degree vertex 26

Data source: processed data

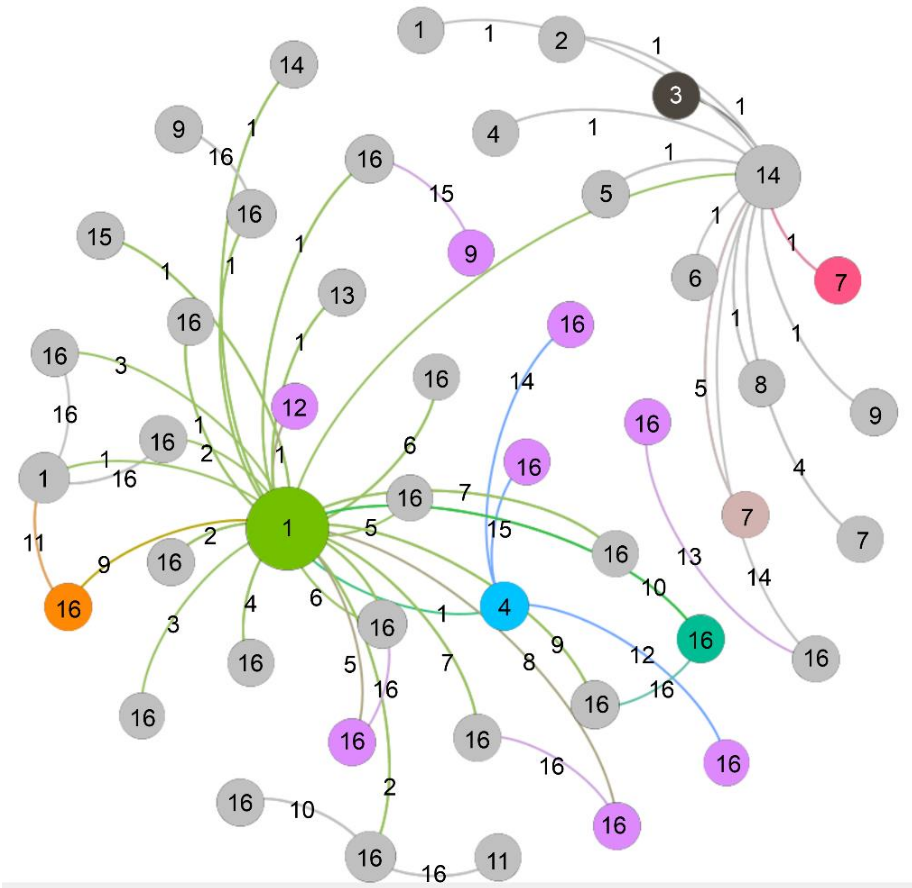
Therefore, the lower bound value is obtained as follows:

$$tvs(TW) \geq maks\left\{\left\lceil\frac{31}{2}\right\rceil, \left\lceil\frac{43}{3}\right\rceil, \left\lceil\frac{44}{4}\right\rceil, \left\lceil\frac{46}{5}\right\rceil, \left\lceil\frac{47}{14}\right\rceil, \left\lceil\frac{48}{27}\right\rceil\right\}$$

$$tvs(TW) \geq maks\{16,15,11,10,4,2\}$$

So, we get the value of $tvs(TW) \geq 16$.

2. The next is showing tvs(TW)≤16. It is shown by creating a labeling-16 construction total irregular vertex TW in Figure 3. The @diskominfomks and @bsn_makassar accounts are the center or the vertex with the greatest degree among the other vertices. Then, it is given a vertex strength starting from the vertex with the smallest degree, namely degree 1 to degree 2, degree 3, degree 4, degree 13, and the vertex with the largest degree of 26. Therefore, the strength of the vertices on the Twitter Network graph is different. So, the value $tvs(TW) \leq 18$. □



Gambar 3. $tvs(TW) = 16$

4. CONCLUSIONS

By converting Twitter data into a graph where vertices can be represented as users, edges can be represented as relationships between users. The @diskominfomks and @bsn_makassar accounts have the data of retweet, mention, or quote as the edges that connect other accounts and have a different number of edges strength and vertices strength. Based on the research results carried out, the total value of the irregularity of the edges on the Twitter network graph *tes*(TW) = 18 and the total value of the irregularity of the vertices on the Twitter network graph tvs(TW) = 16. It is hoped that future research can find the value of *tvs* and *tes* in general on the Twitter network because the structure of the graph is irregular due to the very complex and unstructured Twitter data.

ACKNOWLEDGEMENT

The author expresses his deepest gratitude to Hasanuddin University for assisting the author in the Internal Grant for Beginner Lecturer Research Scheme (PDPU) for 2021. With number 915/UN.22/PT.01.03/2021.

REFERENCES

[1] G. Chartrand and P. Zhang, *Introduction to Graph Theory*. Boston: Mc Graw-Hill Press, 2004.
[2] M. Bača, S. Jendrol', M. Miller, and J. Ryan, "On irregular total labellings," *Discrete Math.*, vol. 307, no. 11–12, pp. 1378–1388, 2007, doi: 10.1016/j.disc.2005.11.075.
[3] J. A. Gallian, "A dynamic survey of graph labeling," *Electron. J. Comb.*, vol. 1, no. DynamicSurveys, 2018.
[4] M. Bača, Z. Kimáková, M. Lascsáková, and A. Semaničová-Feňovčíková, "The Irregularity and Modular Irregularity Strength of Fan Graphs," *Symmetry (Basel).*, vol. 13, no. 4, 2021, doi: 10.3390/sym13040605.
[5] M. Bača, M. Imran, and A. Semaničová-Feňovčíková, "Irregularity and Modular Irregularity Strength of Wheels," *Mathematics*, vol. 9, no. 21, 2021, doi: 10.3390/math9212710.
[6] Slamin, "On distance irregular labelling of graphs," *Far East J. Math. Sci.*, vol. 102, no. 5, pp. 919–932, 2017, doi: 10.17654/MS102050919.
[7] D. Indriati, Widodo, I. E. Wijayanti, K. A. Sugeng, and M. Bača, "On Total Edge Irregularity Strength of Generalized Web Graphs and Related Graphs," *Math. Comput. Sci.*, vol. 9, no. 2, pp. 161–167, 2015, doi: 10.1007/s11786-015-0221-5.
[8] I. Rajasingh, B. Rajan, and S. Teresa Arockiamary, "Total Edge Irregularity Strength of Butterfly Networks," *Int. J. Comput. Appl.*, vol. 49, no. 3, pp. 21–22, 2012.
[9] Nurdin, "Total irregular labeling of butterfly network on level two," *AIP Conf. Proc.*, vol. 1867, no. August, 2017, doi: 10.1063/1.4994470.
[10] Nurdin, E. T. Baskoro, A. N. M. Salman, and N. N. Gaos, "On the total vertex irregularity strength of trees," *Discrete Math.*, vol. 310, no. 21, pp. 3043–3048, 2010, doi: 10.1016/j.disc.2010.06.041.
[11]. Nurdin, M. Zakir, and Firman, "Vertex-irregular labeling and vertex-irregular total labeling on caterpillar graph," *Int. J. Appl. Math. Stat.*, vol. 40, pp. 99–105, Jan. 2013.
[12] N. Hinding, H. K. Kim, N. Sunusi, and R. Mise, "On Total Vertex Irregularity Strength of Hexagonal Cluster Graphs," *Int. J. Math. Math. Sci.*, vol. 2021, 2021, doi: 10.1155/2021/2743858.
[13] E. Saputra, N. Hinding, and S. Amir, "Total Irregular Labelling Of Butterfly and Beneš Network 5-Dimension," *J. Mat. Stat. dan Komputasi*, vol. 17, no. 1, pp. 61–70, 2020, doi: 10.20956/jmsk.v17i1.10909.
[14] M. Imran, A. Ahmad, M. K. Siddiqui, and T. Mehmood, "Total vertex irregularity strength of generalized prism graphs," *J. Discret. Math. Sci. Cryptogr.*, 2021, doi: 10.1080/09720529.2020.1848103.
[15] M. I. Tilukay, P. D. M. Taihuttu, A. N. M. Salman, F. Y. Rumlawang, and Z. A. Leleury, "Complete bipartite graph is a totally irregular total graph," *Electron. J. Graph Theory Appl.*, vol. 9, no. 2, p. 387, 2021, doi: 10.5614/ejgta.2021.9.2.11.
[16] G. K. Nandini, R. S. Rajan, A. A. Shantrinal, T. M. Rajalaxmi, I. Rajasingh, and K. Balasubramanian, "Topological and thermodynamic entropy measures for covid-19 pandemic through graph theory," *Symmetry (Basel).*, vol. 12, no. 12, pp. 1–29, 2020, doi: 10.3390/sym12121992.