

## THE SOLUTION OF MATHEMATICAL MODEL OF OTOBUS TICKET SALES WITH REFERRAL MARKETING STRATEGY

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**Abstract.** One of the ways to involve customers in marketing strategies is known as the referral strategy. This strategy has been applied in various fields for marketing, one of which is in the field of transportation. This study aims to determine the solution to the mathematical model of bus ticket sales using a referral strategy. The data in this study is bus passenger data throughout 2020 which was obtained from one of the Otobus companies in Jakarta. Mathematical model that is compiled using the analogy of the model of the spread of disease. The results of this study are a mathematical model of bus ticket sales using a referral strategy consisting of 4 compartments. The model solution is determined by iterating over the system of differential equations that has been formed. Based on the solution obtained, the simulation results show that the referral strategy in bus ticket sales is able to increase bus passengers up to 39.92%.

**Keywords:** mathematical model, referral strategy, system of difference equation, marketing.

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## 1. INTRODUCTION

The development of information technology has had an impact in all fields, ranging from education, transportation, communication, marketing and others. The field of transportation, especially public transportation, has undergone many changes. These changes are not only in the fleet facilities but also in the ticket sales management system. Initially, transportation ticket sales were carried out directly at stations, terminals, and airports. Along with the times, ticket sales can then be done by agents who partner with transportation companies. Nowadays even people who want to buy tickets can directly order online without having to go to ticket selling agents. Technology has an impact on the management and distribution of tickets to be more effective which will be more profitable for the company [1].

As one of the business fields, transportation companies need to think about strategies to carry out promotions. One of the promotional strategies that can be done is to involve customers. Rusmawan [2] states that customers have the potential to participate in promoting a product if they are satisfied with the product they buy. In the online marketing system, it can also be seen that consumers are always given space by the company to leave comments on the products and services of the company. This marketing communication model is known as the Word of Mouth [3]. Companies can provide encouragement so that their customers are more interested in promoting the company's products. One of the stimuli that can be given is by providing incentives to customers if they succeed in bringing in new customers to buy the company's products. This system is referred to as the referral system. In the referral system, companies and customers can work together to earn profits. The results of the CMO survey in 2021 stated that the promotion strategy of a company mostly involved its customers, which was 28% [4].

The use of the referral system in helping the promotion of ticket sales at transportation companies has at least two opposite impacts. On the one hand, this system can increase the potential reach of customers because customers have a motive to get incentives from the referral system. On the other hand, companies really need to think carefully about the incentives given to customers so that the company still gets optimal profits [5], [6]. So far, not many entrepreneurs, especially Otobus entrepreneurs, have a good system in providing an overview of the referral system, including the impact on company finances. Based on interviews with the owners of the Sumber Alam Otobus (PO) Company, so far the company only tends to estimate based on confidence in making decisions. The basics of decision making in marketing management are still not based on representative mathematical calculations.

The ticket marketing system using the referral method will have an impact on information about a company. This information will be able to spread quickly in line with customer motivation to get incentives. Such behavior can also be observed in the process of spreading an infectious disease. A disease can become a pandemic if the rate of transmission is high in a population. The models of disease spread that are often used are the compartment model [7]–[9]. This is also similar to the referral model, where information can go viral if the customer is successful in bringing new customers to the company. This can give the assumption that the ticket sales system with this referral method has similarities with the spread of infectious disease models.

Based on the above issues, POs need to have a guideline in calculating and predicting marketing policies taken through this referral system. Therefore, the researcher proposes research activities to model and simulate ticket sales with a referral system. A model based on customer dynamics towards ticket sales with a referral system.

## 2. RESEARCH METHODS

This research is an applied research with the object of an Otobus company. This research consists of 4 stages, namely as follows.

a. Data collection

The data used in this study is the passenger data of the Sumber Alam PO Bus throughout 2020. Bus passenger data is seen from the total ticket sales per day for all destinations.

b. Model Construction

Mathematical model for ticket marketing strategy with referral method using the model that has been built by Putra [10].

c. Determine the numerical solution

The model solution was determined numerically using the help of Microsoft excel.

d. Model simulation

Model simulation is done by determining the value of model parameters based on ticket sales data. The results of this simulation are expected to be used for consideration of ticket sales strategies.

### 3. RESULTS AND DISCUSSION

The marketing strategy with the referral method aims to get more customers by taking advantage of loyal customers from the company. This strategy works by viral information so that other people who know will have the opportunity to become customers. These characteristics are similar to models of the spread of a disease. The mathematical model construction for this referral strategy has been built by [11]–[14] using the classical compartment model approach, namely SIR. The model has been built this referral strategy model with 4 compartments, namely *Target Market* (T), *Exposed* (E), *Active Sharing* (A), and *Dormant* (D). The flow chart of this model can be seen in the following figure.

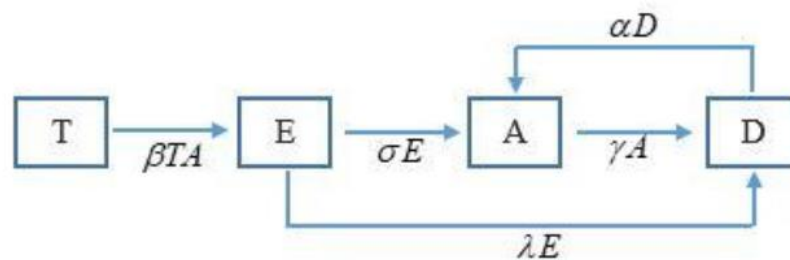


Figure 1. Flowchart of the Model

Based on Figure 1 above, the movement of the target market population to the exposed group is determined by the level of interaction of individuals in group T with referred customers  $\beta$ . Then each individual has time to think about whether to become a customer or not. The rate of incubation of the information so that the individual decides to become a referred customer is determined by  $\sigma$ . The individual who decides not to become a customer is determined by  $\lambda$ . A person may stop being a customer due to several factors. This situation is given by the parameter  $\gamma$ . In this model, it also provides conditions where someone who no longer wants to take part in being a customer can become a referred customer. This can be caused by incomplete information received previously or seeing friends who have successfully received rewards from this strategy. The rate of this occurrence is given by the parameter  $\alpha$ .

Based on the flow chart above, the system of differential equations is obtained as the following mathematical model.

$$\begin{cases} \frac{dT}{dt} = -\beta TA \\ \frac{dE}{dt} = \beta TA - (\sigma + \lambda) E \\ \frac{dA}{dt} = \sigma E - \gamma A + \alpha D \\ \frac{dD}{dt} = \gamma A + \lambda E - \alpha D \end{cases} \quad (1)$$

The solution of equation (1) is determined numerically using the help of Microsoft Excel. The system of differential equations (1) is first transformed into a system of differential equations [15]. The system of differential equations of the differential equation in system (1) is obtained as follows.

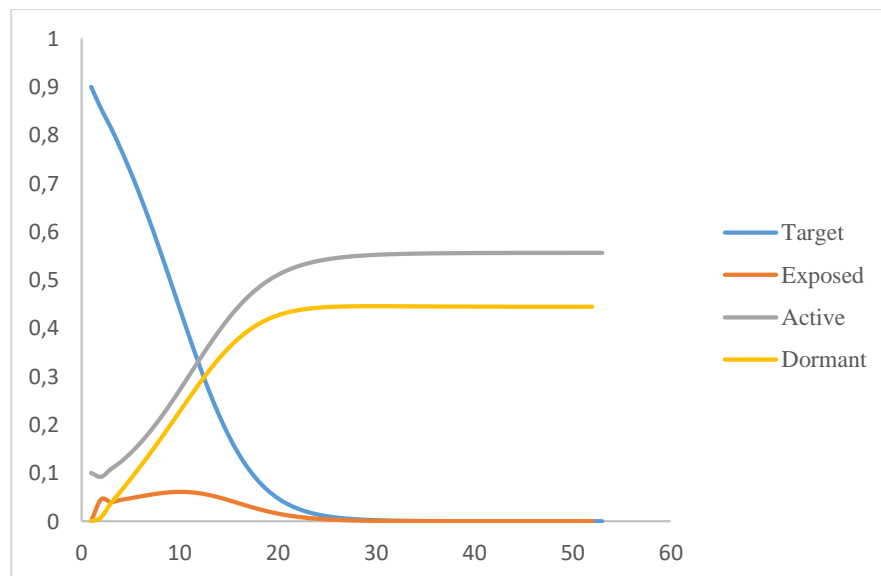
$$\begin{cases} \Delta T = (-\beta TA) \Delta t \\ \Delta E = (\beta TA - \sigma E - \lambda E) \Delta t \\ \Delta A = (\sigma E - \gamma A + \alpha D) \Delta t \\ \Delta D = (\gamma A + \lambda E - \alpha D) \Delta t \end{cases} \quad (2)$$

The following are the parameter values used in the simulation model.

**Table 1. Values of Parameters**

Parameter	Value	Reference
$\beta$	0.5	Assume
$\sigma$	500	Assume
$\lambda$	0.5	Assume
$\gamma$	0.08	Assume
$\alpha$	0.1	Assume

The initial values used for the simulation are  $T_0 = 0.9$ ,  $E_0 = 0$ ,  $I_0 = 0.1$  and  $D_0 = 0$ . To determine the solution of the model, use the help of Microsoft excel to iterate over the numerical solution of equation (2). The time used is in weeks and a duration of 53 weeks. The following are the simulation results from the mathematical model that has been built.



**Figure 2. Simulation of Mathematical Model**

Based on the results in Figure 2 above, the increase in referred customers can reach 50% of the total population and 40% of the population is no longer involved in the referral program. This situation begins to become latent at the 25th week. So that companies can think of a strategy to reactivate an increase in their prospective customers. Based on the simulation results of the model above, the following will give a comparison of bus passengers with the application of the referral strategy with the initial situation (without a marketing strategy).

The model solution in the simulation of Figure 2 above is then used to predict the behavior of bus passengers for a year. Based on bus passenger data throughout 2020, the following is the prediction of the comparison of bus passengers with normal situations (without the application of the referral strategy) with the situation of applying the referral strategy.

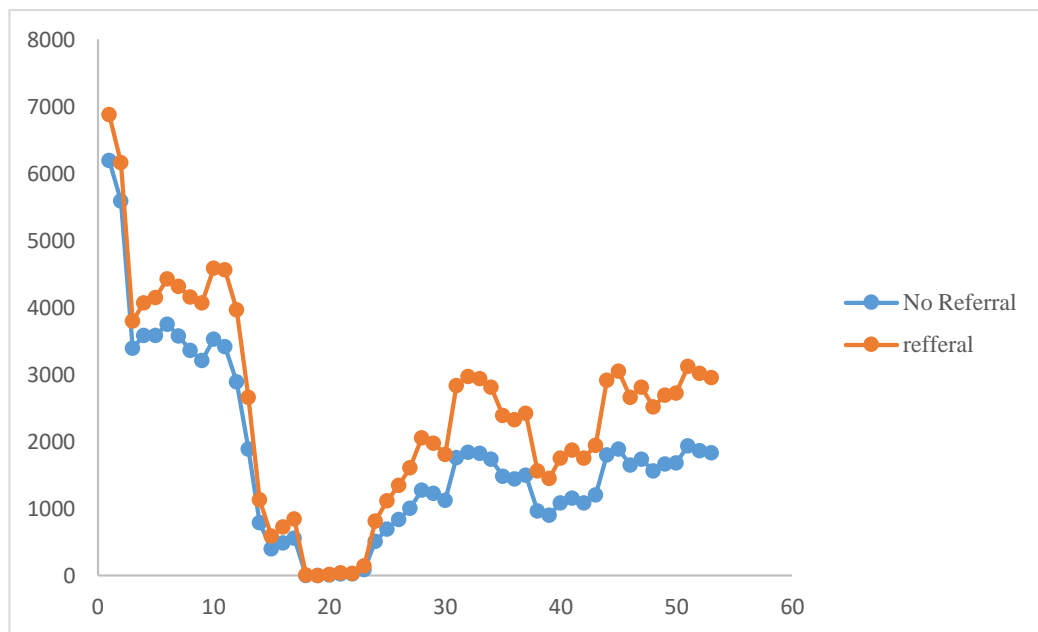


Figure 3. Comparisons Bus Customer with Referral Strategy

The referral strategy is able to increase the number of passengers by around 39.92% per year. The number of passengers per year can reach 129.567 people based on passenger data in 2020. This marketing strategy is able to spread information quite quickly because every referred customer has a motive to get a reward. Based on the results of this prediction, the company can determine the amount of reward given to each customer who successfully brings new customers to the bus company.

#### 4. CONCLUSIONS

The mathematical model on the referral strategy for bus ticket sales uses a compartment model consisting of 4 subpopulations, namely Target Market (T), Exposed (E), Active Broadcaster (A), and Dormant (D). The solution of the mathematical model is determined by iterating over a system of different equations. Based on the mathematical model that has been compiled, the researcher predicts that this referral strategy has the potential to increase bus ticket sales up to 39.92%.

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