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# OPTIMAL CONTROL ON CHOLERA DISEASE SPREADING MODEL WITH THREE VARIABLES CONTROL VARIATION

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***Abstract.*** *Cholera is an infection of the small intestine by some strains of the bacterium Vibrio Cholerae. This disease is a deadly disease that necessitates efficient prevention and control measures. In this research, the optimal control of the cholera spread model with variations of three control variables is discussed. There are four controls to minimize the spread of diseases such as sanitation, treatment consisting of quarantine, increased education, and chlorination. The dynamic system is formed with three controls variation. Then it is compared and analyzed for the most effective result. The optimal control solution is derived using the Pontryagin Minimum Principle and solved using the Runge-Kutta method.*

***Keywords:*** *optimal control, Pontryagin minimum principle, Runge Kutta.*

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## 1. INTRODUCTION

Cholera remains a global public health problem, especially related to the lack of access to clean water and proper sanitation [1]. Cholera is a diarrheal disease caused by the infection in the intestine caused by *Vibrio cholerae* bacteria [2]. Cholera does not infect people by chance; more precisely, cholera affects people with physical burdens, lack of immunity, poor health systems, and malnutrition [3]. *Vibrio cholerae* has been discovered in the environment, food and beverages, aquaculture, and clinical settings, according to investigations. Contaminated food and drink are linked to hygiene and sanitation, and people are frequently infected after consuming contaminated food or drink [13]. The means of spreading cholera are through unhygienic food or drinks contaminated with *V. cholerae* bacteria. Severe diarrhea, vomiting, and leg cramps are the most common cholera symptoms, and if not treated immediately can lead to severe illness, such as collapse, and death [4].

Extraordinary events of cholera that have been reported in Indonesia were recorded from April to August 2008 in Paniai Regency and Nabire Regency, Papua Province. This incident claimed 105 lives [5]. Proper environmental health services, such as increased access to clean water, adequate sewage and drinking water treatment, and cholera treatment, are required for effective prevention and control strategies.

Many studies have been carried out on the spread of cholera, namely mathematical spread of cholera and analysis of cholera stability epidemic model through basic reproduction numbers [7]. In 2016, Lemos-Paiao et al explain the best cholera control treatment is by treating affected people [8]. In 2019, Subchan et al explored the cholera spread model by offering therapy and intervention on sanitation, education, and quarantine [9]. Furthermore, in 2019 another research was carried out by Hasanah regarding optimal control of preventing the spread of cholera by controlling treatment and chlorination [10]. Research on the spread of cholera using the SIR model was also carried out by Hidayati, et al (2021) with vaccination control [11]. Another study looked at the stability of cholera as a result of bacterial growth and mobility as measured by basic reproduction numbers [12].

Munaqib et. al. (2021) has also conducted research on controlling the spread of disease. In his research, he took cases of the spread of the Covid-19 disease with the lockdown and quarantine treatment. As a result, the spread of the virus can be suppressed if intensive quarantine treatment is carried out and the lockdown area is prolonged [14]. The treatment considering vaccination and disinfection as an action to control the spread of cholera has also been carried out by Sun, et al (2017), with vaccination and disinfection proven to be able to suppress the spread of cholera [15]. Meanwhile, vaccine research has been conducted mathematically by studying the endemic equilibrium point of the cholera model based on the SIR model and changing it with various control strategies based on the fundamental reproduction number [16]. In addition, Pramesti (2020) claims that the Pontryagin minimal principle approach is used to tackle the problem of control variables in the mathematical model of cholera transmission via vaccination [19].

Referring to the existing research, in this paper the optimal controls used are sanitation improvement, quarantine treatment, education improvement, and chlorination. The purpose of optimal control in this paper is to reduce the populations of infected human and bacterial and the costs of preventing cholera transmission. Then from the four controls, a dynamic system variation containing three controls was formed and the results were compared to find which one was more effective. The model in this paper reconstructs the model that has been developed by Subchan, et al, [4], [9]. The first model contains optimal control of sanitation improvements, quarantine treatment, and increased education. The second model contains quarantine treatment, increased education, and chlorination. The third model contains improved sanitation, increased education, and chlorination. The fourth model contains improved sanitation, quarantine treatment, and chlorination.

## 2. RESEARCH METHODS

### 2.1 Mathematical Model

The mathematical model used in this paper is based on Subchan, et al, [4]. The model is built by human population and bacterial population with SEIIQRB type (Susceptible, Educated, Infected Asymptomatic, Infected Symptomatic, Quarantined, Recovered, Bacteria) and does not consider age and

gender. The controls are sanitation improvement ($u_1$), quarantine treatment ($u_2$), education improvement ($u_3$), and chlorination ($u_4$). The interpretation of the mathematical model into the compartment diagram can be seen in Figure 1. Here are the four mathematical models based on the compartment diagram that has been given:

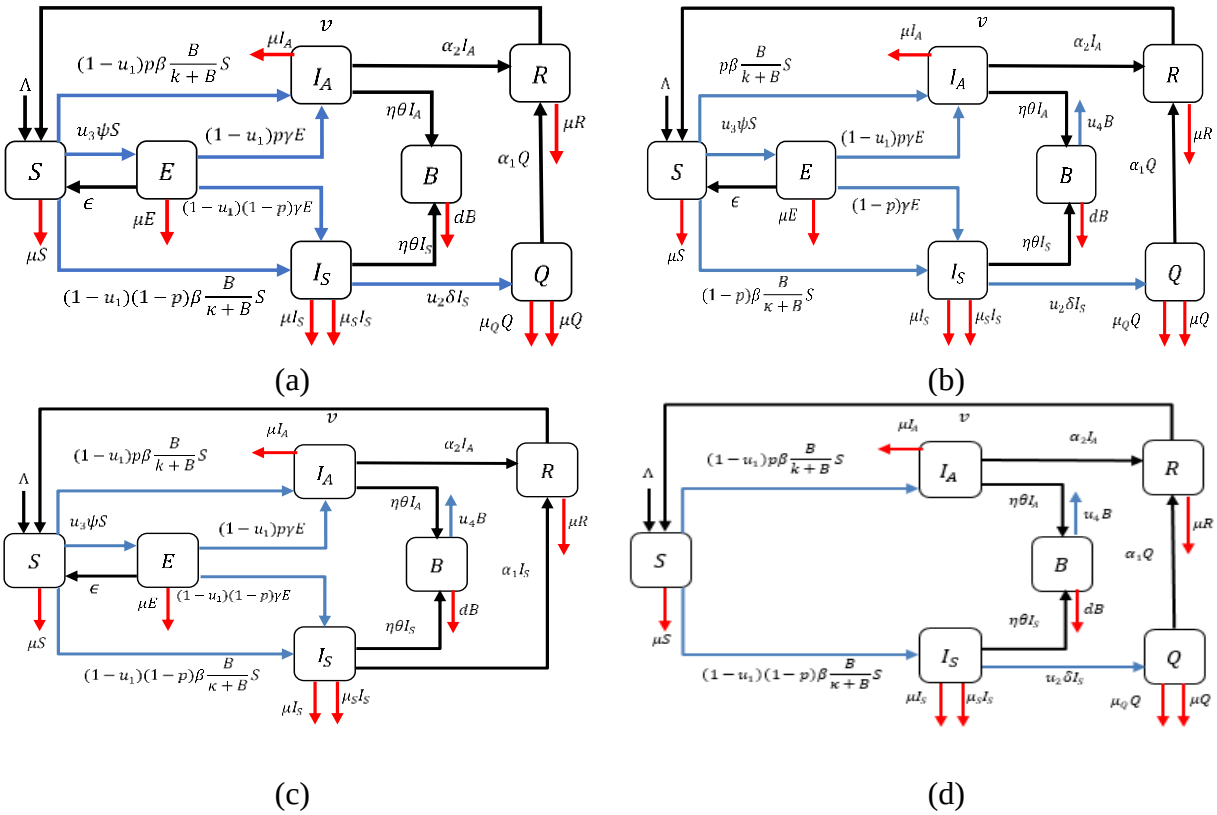


**Figure 1. Compartment scheme**

a. The first model contains optimal control of sanitation improvements ($u_1$), quarantine treatment ($u_2$), and increased education ($u_3$).

$$\dot{S}(t) = \Lambda + vR + \epsilon E - \mu S - u_3\psi S - (1-u_1)\beta\frac{B}{k+B}S$$

$$\dot{E}(t) = u_3\psi S - \epsilon E - \mu E - (1-u_1)\gamma E$$

$$\dot{I_A}(t) = (1-u_1)p\beta\frac{B}{\kappa+B}S + (1-u_1)p\gamma E - \mu I_A - \alpha_2 I_A$$

$$\dot{I_S}(t) = (1-u_1)(1-p)\beta\frac{B}{\kappa+B}S + (1-u_1)(1-p)\gamma E - \mu I_S - \mu_S I_S - u_2\delta I_S$$

$$\dot{Q}(t) = u_2\delta I_S - \mu Q - \mu_Q Q - \alpha_1 Q$$

$$\dot{R}(t) = \alpha_1 Q + \alpha_2 I_A - \mu R - vR$$

$$\dot{B}(t) = \eta\theta I_A + \eta\theta I_S - dB$$

b. The second model contains quarantine treatment ($u_2$), increased education ($u_3$), and chlorination ($u_4$).

$$\dot{S}(t) = \Lambda + vR + \epsilon E - \mu S - u_3\psi S - \beta\frac{B}{k+B}S$$

$$\dot{E}(t) = u_3\psi S - \epsilon E - \mu E - \gamma E$$

$$\dot{I_A}(t) = p\beta\frac{B}{\kappa+B}S + p\gamma E - \mu I_A - \alpha_2 I_A$$

$$\dot{I_S}(t) = (1-p)\beta\frac{B}{\kappa+B}S + (1-p)\gamma E - \mu I_S - \mu_S I_S - u_2\delta I_S$$

$$\dot{Q}(t) = u_2\delta I_S - \mu Q - \mu_Q Q - \alpha_1 Q$$

$$\dot{R}(t) = \alpha_1 Q + \alpha_2 I_A - \mu R - vR$$

$$\dot{B}(t) = \eta\theta I_A + \eta\theta I_S - dB - u_4 B$$

c. The third model contains improved sanitation ($u_1$), increased education ($u_3$), and chlorination ($u_4$).

$$\dot{S}(t) = \Lambda + vR + \epsilon E - \mu S - u_3\psi S - (1-u_1)\beta\frac{B}{k+B}S$$

$$\dot{E}(t) = u_3\psi S - \epsilon E - \mu E - (1-u_1)\gamma E$$

$$\dot{I_A}(t) = (1-u_1)p\beta\frac{B}{\kappa+B}S + (1-u_1)p\gamma E - \mu I_A - \alpha_2 I_A$$

$$\dot{I_S}(t) = (1-u_1)(1-p)\beta\frac{B}{\kappa+B}S + (1-u_1)(1-p)\gamma E - \mu I_S - \mu_S I_S - \alpha_1 I_S$$

$$\dot{R}(t) = \alpha_1 I_S + \alpha_2 I_A - \mu R - vR$$

$$\dot{B}(t) = \eta\theta I_A + \eta\theta I_S - dB - u_4 B$$

d. The fourth model contains improved sanitation ($u_1$), quarantine treatment ($u_2$), and chlorination ($u_4$).

$$\dot{S}(t) = \Lambda + vR - \mu S - (1-u_1)\beta\frac{B}{k+B}S$$

$$\dot{I_A}(t) = (1-u_1)p\beta\frac{B}{\kappa+B}S - \mu I_A - \alpha_2 I_A$$

$$\dot{I_S}(t) = (1-u_1)(1-p)\beta\frac{B}{\kappa+B}S - \mu I_S - \mu_S I_S - u_2\delta I_S$$

$$\dot{Q}(t) = u_2\delta I_S - \mu Q - \mu_Q Q - \alpha_1 Q$$

$$\dot{R}(t) = \alpha_1 Q + \alpha_2 I_A - \mu R - vR$$

$$\dot{B}(t) = \eta\theta I_A + \eta\theta I_S - dB - u_4 B$$

**Table 1. Variable and parameter**

| Variable and Parameter | Description | Variable and Parameter | Description |
|---|---|---|---|
| $S(t)$ | Susceptible population | $\beta$ | Rate of consumption of cholera bacteria |
| $E(t)$ | Educated population | $k$ | Constant of the bacterial population |
| $I_A(t)$ | Infected population with mild symptoms | $\beta\frac{B}{k+B}$ | Rate of movement of susceptible populations into infected populations |
| $I_S(t)$ | Infected population with severe symptoms | $\gamma$ | Infection rate of educated population |
| $Q(t)$ | Quarantined population | $p$ | Proportion of infected individuals with mild symptoms |
| $R(t)$ | Population cured | $1-p$ | Proportion of infected individuals with severe symptoms |
| $B(t)$ | Bacterial population | $\alpha_1$ | Cure rate of infected people with mild symptoms |
| $\Lambda$ | Increase in vulnerable population due to natural birth | $\alpha_2$ | Cure rate of infected people with severe symptoms |
| $v$ | Rate of loss of immunity of individuals recovering so that they are vulnerable again | $\mu_S$ | Death rate due to cholera infection with severe symptoms |
| $\epsilon$ | Rate of educated population stopping taking preventive measures | $\mu_Q$ | Death rate due to cholera when quarantined |
| $\mu$ | Rate of natural death | $\eta$ | Bacterial growth rate |
| $\psi$ | Rate of increase in individual education | $\delta$ | Individual quarantine rate |
| $d$ | Bacterial death rate | $\theta I$ | Infected population disposal rate |

### 2.2 Optimal Control Problem

Based on the dynamic system model of cholera transmission, it can be expressed mathematically through the following equation,

$$\dot{x}(t) = f(x(t), u(t), t)$$

with the state variable as follows,

$$x(t) = \big(S(t), E(t), I_A(t), I_S(t), Q(t), R(t), B(t)\big)^T; u(t) = (u_1(t), u_2(t), u_3(t), u_4(t))$$

with the initial state $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$.

Furthermore, utilizing Pontryagin's Minimum Principle, this part provides the solution to the optimal control problem [17]. The types of optimal control problems are fixed-final time and free-final time [6]. The purpose of the problem is to find an optimal control that aims to reduce the infected human and bacterial populations, and the preventing cholera transmission. Mathematically, the objective function (performance index) can be defined as follows:

$$min\, J = \frac{1}{2}\int_{t_0}^{t_f} \left[C_1 {I_S}^2(t) + C_2 {I_A}^2(t) + C_3 B^2(t) + C_4 {u_1}^2(t) + C_5 {u_2}^2(t) + C_6 {u_3}^2(t) + C_7 {u_4}^2(t)\right] dt$$

where $t_0$ is the initial time and $t_f$ is the final time, and $C_i$ is the day's weight parameter or price coefficient assigned to each control, where $C_i > 0$ for each $i = 1,2,3,4,5,6,7$.

## 3. RESULT AND DISCUSSION

The following are the steps to use Pontryagin's Minimum Principle to solve the optimal control problem by giving an example of the third model and for the other models the same steps are carried out. The first step to solving is to form the Hamiltonian function as follows:

$$\begin{aligned}\mathcal{H} = \frac{1}{2}&\left(C_1 {I_S}^2(t) + C_2 {I_A}^2(t) + C_3 B^2(t) + C_4 {u_1}^2(t) + C_6 {u_3}^2(t) + C_7 {u_4}^2(t)\right)\\ &+ \lambda_S\left(\Lambda + vR + \epsilon E - \mu S - u_3\psi S - (1-u_1)\beta\frac{B}{k+B}S\right)\\ &+ \lambda_E(u_3\psi S - \epsilon E - \mu E - (1-u_1)\gamma E)\\ &+ \lambda_{I_A}\left((1-u_1)p\beta\frac{B}{\kappa+B}S + (1-u_1)p\gamma E - \mu I_A - \alpha_2 I_A\right)\\ &+ \lambda_{I_S}\left((1-u_1)(1-p)\beta\frac{B}{\kappa+B}S + (1-u_1)(1-p)\gamma E - \mu I_S - \mu_S I_S\right.\\ &\left.- \alpha_1 I_S\right) + \lambda_R(\alpha_1 I_S + \alpha_2 I_A - \mu R - vR)\\ &+ \lambda_B(\eta\theta I_A + \eta\theta I_S - dB - u_4 B)\end{aligned} \quad (1)$$

where $\lambda_i$ for $i = 1,2,3,4,5,6,7$ is a costate vector or a state-dependent Lagrange multiplier.

The second step is to minimize $\mathcal{H}$ in the first step for each variable u shown as follows:

$$\begin{aligned}u_1^* &= \frac{1}{C_4}\left(\beta\frac{B}{k+B}S\left(\lambda_{I_A}p + \lambda_{I_S}(1-p) - \lambda_S\right) + \gamma E\left(\lambda_{I_A}p + \lambda_{I_S}(1-p) - \lambda_E\right)\right)\\ u_3^* &= \frac{\psi S(\lambda_S - \lambda_E)}{C_6}\\ u_4^* &= \frac{\lambda_B B}{C_7}\end{aligned} \quad (2)$$

The optimal control $u^*$ is acquired from $\frac{\partial\mathcal{H}}{\partial u}$ and has the following characteristics

$$u_1^* = min\left(u_{1_{min}}, max\left(\hat{u}_1^*, u_{1_{max}}\right)\right)$$
$$u_2^* = min\left(u_{2_{min}}, max\left(\hat{u}_2^*, u_{2_{max}}\right)\right)$$

$$u_3^* = min\left(u_{3_{min}}, max\left(\hat{u}_3^*, u_{3_{max}}\right)\right)$$
$$u_4^* = min\left(u_{4_{min}}, max\left(\hat{u}_4^*, u_{4_{max}}\right)\right)$$

The third step is to substitute the optimal control equation (2) into the Hamiltonian equation (1) to obtain the optimal Hamiltonian $\mathcal{H}^*$. Then $\mathcal{H}^*$ is used to find the fourth step, namely the state equation as follows:

$x^*(t) = \left(\frac{\partial \mathcal{H}}{\partial \lambda}\right)$
$\dot{S}^* = \Lambda + vR + \epsilon E - \mu S - u_3^* \psi S - (1 - u_1^*)\beta \frac{B}{k+B} S$
$\dot{E}^* = u_3^* \psi S - \epsilon E - \mu E - (1 - u_1^*)\gamma E$
$\dot{I_A}^* = (1 - u_1^*)p\beta \frac{B}{\kappa+B} S + (1 - u_1^*)p\gamma E - \mu I_A - \alpha_2 I_A$
$\dot{I_S}^* = (1 - u_1^*)(1 - p)\beta \frac{B}{\kappa+B} S + (1 - u_1^*)(1 - p)\gamma E - \mu I_S - \mu_S I_S - u_2^* \delta I_S$
$\dot{R}^* = \alpha_1 I_S + \alpha_2 I_A - \mu R - vR$
$\dot{B}^* = \eta\theta I_A + \eta\theta I_S - dB - u_4^* B$

**Table 2. Variable and parameter values**

| Parameter | Parameter Value | Parameter | Parameter Value | Parameter | Parameter Value |
|---|---|---|---|---|---|
| $S(0)$ | 5750 [1] | $C_6$ | 0.5 (assumption) | $p$ | 0.78 [1] |
| $E(0)$ | 0 [1] | $C_7$ | 0.5 (assumption) | $\alpha_2$ | 0.2 [1] |
| $I_A(0)$ | 1000 [1] | $u_1$ | 0.001 – 0.4 [5] | $\delta$ | 0.15 [5] |
| $I_S(0)$ | 700 [1] | $u_2$ | 0 – 1 [1] | $\mu_S$ | 0.00127 [1] |
| $Q(0)$ | 0 [1] | $u_3$ | 0 – 1 [1] | $\mu_Q$ | 0.0001 [5] |
| $R(0)$ | 0 [1] | $u_4$ | 0 – 1 [1] | μ | $2.2493 \times 10^{-5}$ [5] |
| $B(0)$ | 275000 [1] | Λ | $\frac{24.4N(0)}{365000}$ [5] | $v$ | $\frac{0.4}{365}$ [5] |
| $C_1$ | 1 (assumption) | β | 0.08 [5] | $\eta$ | 50 [1] |
| $C_2$ | 0.1 (assumption) | k | $10^6$ [1] | $\epsilon$ | 0.003 [1] |
| $C_3$ | 0.5 (assumption) | $d$ | $\frac{1}{30}$ [5] | $\gamma$ | 0.0005 [1] |
| $C_4$ | 0.5 (assumption) | $t_f$ | 100 days [5] | | |
| $C_5$ | 1 (assumption) | $\psi$ | 0.008 [1] | | |

The optimal Hamiltonian $\mathcal{H}^*$ is also used to find the fifth step, which is the costate equation as follows:

$\lambda^*(t) = -\left(\frac{\partial \mathcal{H}}{\partial x}\right)$
$\lambda_S^* = -\left(-\mu\lambda_S - u_3^*\psi\lambda_S - (1 - u_1^*)\beta \frac{B}{k+B}\lambda_S + u_3^*\psi\lambda_E + (1 - u_1^*)p\beta \frac{B}{\kappa+B}\lambda_{I_A} + (1 - u_1^*)(1 - p)\beta \frac{B}{\kappa+B}\lambda_{I_S}\right)$
$\lambda_E^* = -\left(\epsilon\lambda_S - \epsilon\lambda_E - \mu\lambda_E - (1 - u_1^*)\gamma\lambda_E + (1 - u_1^*)p\gamma\lambda_{I_A} + (1 - u_1^*)(1 - p)\gamma\lambda_{I_S}\right)$
$\lambda_{I_A}^* = -\left(C_2 I_A - \mu\lambda_{I_A} - \alpha_2\lambda_{I_A} + \alpha_2\lambda_R + \eta\theta\lambda_B\right)$
$\lambda_{I_S}^* = -\left(C_1 I_S - \mu\lambda_{I_S} - \mu_S\lambda_{I_S} - \alpha_1\lambda_{I_S} + \alpha_1\lambda_R + \eta\theta\lambda_B\right)$
$\lambda_R^* = -(v\lambda_S - \mu\lambda_R - v\lambda_R)$
$\lambda_B^* = -\left(C_3 B - (1 - u_1^*)\beta \frac{Sk}{(k+B)^2}\lambda_S + (1 - u_1^*)p\beta \frac{Sk}{(k+B)^2}\lambda_{I_A} + (1 - u_1^*)(1 - p)\beta \frac{Sk}{(k+B)^2}\lambda_{I_S} - d\lambda_B - u_4^*\lambda_B\right).$

The optimal state and the optimal costate have been acquired by considering the boundary conditions $x(0) = x_0$ and $\lambda_{t_f} = 0$. The optimal state and costate solution were obtained using Forward-Back Sweep Runge-Kutta order 4. Table 2 lists the parameter values utilized in the simulation [18].

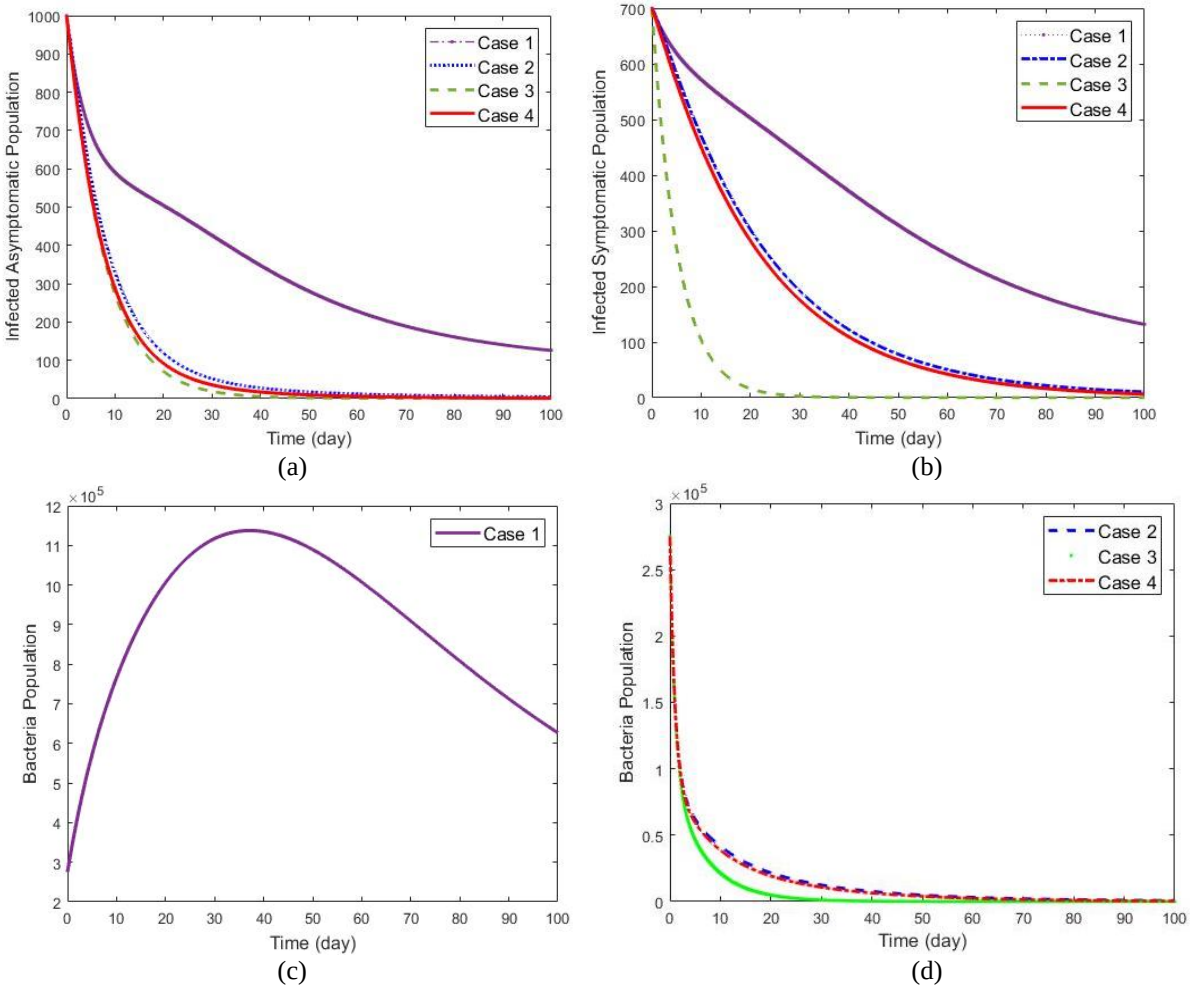


**Figure 2. (a) Infected Asymptomatic, (b) Infected Symptomatic, (c) and (d) Bacteria.**

Figure 2 shows the simulation based on the four cases. The first case shows that the decline in the infected population with mild or severe symptoms did not show a significant number, but the concentration of bacteria increased. This shows that chlorination control is needed. The decrease in the infected population with mild symptoms, severe symptoms, and the highest concentration of bacteria occurred in the third case. The final time showed that in the third model there were no infected individuals and the concentration of bacteria decreased to 46.0578 cells/ml.

## 4. CONCLUSIONS

Based on the results of numerical simulations that have been carried out, the third model with control of improved sanitation, increased education, and chlorination is able to minimize the infected individuals, both with mild symptoms and severe symptoms with the bacterial concentrations at the end to 46.0578 cells/ml. In addition, the third model can minimize the cost of preventing the spread of cholera. This shows that quarantine treatment can be avoided if the control of sanitation and chlorination improvement is carried out optimally.

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