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WEIGHTED ADDITIVE MODEL AND CHANCE CONSTRAINED TECHNIQUE FOR SOLVING NONSYMMETRICAL STOCHASTIC FUZZY MULTIOBJECTIVE LINEAR PROGRAM

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Abstract. *The problems of linear programming develop gradually, and its complexity is constantly growing. Various problems can be viewed as multi-objective fuzzy linear programming, multi-objective stochastic linear programming, or both. This research was focused on examining Stochastic Fuzzy Multi-Objective Linear Programming (SFMOLP), with each of the objective functions having a different level of importance to decision-makers, better known as the nonsymmetrical model. The objective function of the linear program contains fuzzy parameters. In contrast, the constraint function contains fuzzy parameters and random variables. This study aimed to develop an algorithm to transform the SFMOLP into a Program of linear Deterministic Single-Objective Linear Programming (DSOLP) to solve it using the simplex method. In transforming SFMOLP to DSOLP, several approaches have been used. They are; the weighted additive model, analytic hierarchy process, and chance-constrained technique. An example of numerical computations has been provided at the end of the discussion to illustrate how the algorithm works. The resulted model and algorithm are expected to help companies in the decision-making process.*

Keywords: *weighted additive, SFMOLP -DSOLP, analytic hierarchy process, chance constrained*

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1. INTRODUCTION

Problems faced by companies such as supplier selection and transportation can be viewed as multi-objective linear programming problems because they include both quantitative and qualitative factors. Linear programming is a way to obtain optimal results from a mathematical model composed of linear relationships. [1] defines multi-objective linear programming, and [2] provides definitions and methods for solving multi-objective linear programming problems known as deterministic models.

Decision-making does not have clear and complete information about decision criteria and boundaries. In this problem, fuzzy set theory is one of the best tools to solve the ambiguity of the information. Then in [3], it was stated that Zimmermann had used the model initiated by Belman and Zadeh to solve the Fuzzy Multi-Objective Linear Program (FMOLP) problem. In this model, the objective and the constraint function are considered to have similar levels of importance for the decision-maker. However, in reality, each objective and constraint function has a different importance for decision-makers. Therefore, the symmetrical model is not appropriate to solve the problem of decision-makers with multiple objectives. FMOLP, with each objective function having a different level of importance, is called a nonsymmetric FMOLP. Weight is needed to represent the level of importance of each objective function to complete the linear program so that several weighting models are developed.

[4] developed a FMOLP with a fuzzy objective function and a fuzzy constraint. The objective and constraint functions contain parameters in the form of fuzzy triangular numbers. To solve this, it is necessary to transform from a fuzzy multi-objective to a deterministic single-objective. [5] solved the nonsymmetric FMOLP by using a max-min weighting model. In contrast, for transforming fuzzy constraints into deterministic constraints, the standard deviation concept of the two fuzzy parameters in the form of fuzzy triangular numbers was used. This method has a fairly lengthy process. [6] uses the concept of interval arithmetic to transform fuzzy constraints into deterministic constraints that are easier to understand. This concept was developed from [7].

[3] has developed and completed the FMOLP developed by [5] by using an additive weighting model and the concept of interval arithmetic. In this model, the objective and constraint functions are both fuzzy. Determination of the weights in this model used the Analytic Hierarchy Process (AHP) approach. Procedures regarding AHP are discussed by [8] and [9].

In 1994, [10] developed a Stochastic Fuzzy Multi-Objective Linear Program (SFMOLP) and solved it using the Chance Contrainted technique. The Chance Constrained and SFMOLP techniques are described in detail in [11]. Therefore, in this study, the SFMOLP model will be developed and solved with an additive weighting model and the Chance Constrained technique.

2. RESEARCH METHODS

This research is an exploratory study by reviewing several journals, books, and other sources on the internet. The model under study is non-symmetrical, namely a model with each objective function having a different level of importance for decision-makers. The model developed is also limited to the constraint function, which contains random variables. The existing non-symmetrical models can be developed for cases where random variables are also found in the objective function for further research. In completing SFMOLP, it is necessary to transform to DSOLP form. The transformation process is carried out using the additive weighting model described in [3], the chance-constrained technique presented by [10], and the infimum – supremum fuzzy concept of fuzzy triangular numbers presented by [12] and developed by [13].

3. RESULTS AND DISCUSSION

3.1. Stochastic Fuzzy Multi-Objective Linear Program Formulation (SFMOLP)

There are several assumptions to formulate the SFMOLP Model as follows:

1. Decision-making has a fuzzy objective for each objective function, with *p* fuzzy min objective functions and *q-p* max fuzzy objective functions.
2. Decision-making faces as many as *h* fuzzy constraints and as many as *m-h* probabilistic constraints.
3. The fuzzy constraints can be in the form of fuzzy max, fuzzy min, fuzzy equal, or a combination of the three. The variables have fuzzy coefficients in the form of fuzzy triangular numbers.
4. The membership function for fuzzy purposes and fuzzy constraints is linear.
5. Each fuzzy goal has a different level of importance for decision making.

Based on these 5 assumptions, SFMOLP is a problem

Determine $\mathbf{x} = [x_1 \ x_2 \ \cdots x_n]^T$ that
Minimize $\widetilde{z_k} = \sum_{i=1}^{n} c_{ki} x_i \lesssim z_k^0, k = 1, 2, \dots, p$
Maximize $\widetilde{z_l} = \sum_{i=1}^{n} c_{li} x_i \gtrsim z_l^0, l = p+1, p+2, \dots, q$

with constraint:

$$\widetilde{g_r}(x) = \sum_{i=1}^{n} \widetilde{a_{ri}}\, x_i \lesssim \widetilde{b_r}, \qquad r = 1, 2, \dots, h,$$
$$Prob\left[g_p(x) = \textstyle\sum_{i=1}^{n} a_{pi}\, x_i \le b_p\right] \ge 1 - \alpha_i, \;\; p = h+1, h+2, \dots, m, \tag{1}$$
$$x_i \ge 0, i = 1, 2, \dots, n.$$
$$\alpha \in (0, 1)$$

with c_{ki}, c_{li} and a_p worth the *crisp* while b_p is a normally distributed random variable from a probabilistic constraint p, $\widetilde{a_{ri}} = \left(a_{ri}^{(1)}, a_{ri}^{(2)}, a_{ri}^{(3)}\right)$ and $\widetilde{b_{ri}} = \left(b_{ri}^{(1)}, b_{ri}^{(2)}, b_{ri}^{(3)}\right)$ is the fuzzy parameter of the fuzzy constraint r. z_k^0 and z_l^0 is the level of aspiration that the decision maker wants to achieve and the tilde sign ~denotes a fuzzy environment.

The membership function for each fuzzy objective function and fuzzy constraint is a linear function presented in equations (2), (3), (4) and (5).

$$\mu_{Z_k}(x) = \begin{cases} 1 & ; \quad Z_k(x) \le Z_k^- \\ f_{Z_k} = \frac{Z_k^+ - Z_k(x)}{Z_k^+ - Z_k^-} & ; \quad Z_k^- \le Z_k(x) \le Z_k^+, k = 1,2,\cdots,p \\ 0 & ; \quad Z_k(x) \ge Z_k^- \end{cases} \tag{2}$$

$$\mu_{Z_l}(x) = \begin{cases} 1 & ; \quad Z_l(x) \ge Z_l^+ \\ f_{Z_l} = \frac{Z_l(x) - Z_l^-}{Z_l^+ - Z_l^-} & ; \quad Z_l^- \le Z_l(x) \le Z_l^+, \; l = p+1, p+2, \cdots, q \\ 0 & ; \quad Z_l(x) \le Z_l^- \end{cases} \tag{3}$$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; \quad x < a^{(1)}, x > a^{(3)} \\ \frac{x - a^{(1)}}{a^{(2)} - a^{(1)}} & ; \quad a^{(1)} \le x \le a^{(2)} \\ \frac{a^{(3)} - x}{a^{(3)} - a^{(2)}} & ; \quad a^{(2)} \le x \le a^{(3)} \\ 1 & ; \quad x = a^{(2)} \end{cases} \tag{4}$$

$$\mu_{\tilde{B}}(x) = \begin{cases} 0 & ; \quad x < b^{(1)}, x > b^{(3)} \\ \frac{x - b^{(1)}}{b^{(2)} - b^{(1)}} & ; \quad b^{(1)} \le x \le b^{(2)} \\ \frac{b^{(3)} - x}{b^{(3)} - b^{(2)}} & ; \quad b^{(2)} \le x \le b^{(3)} \\ 1 & ; \quad x = b^{(2)} \end{cases} \tag{5}$$

The graphs of membership functions (2), (3), (4) and (5) are presented in Figure 1.

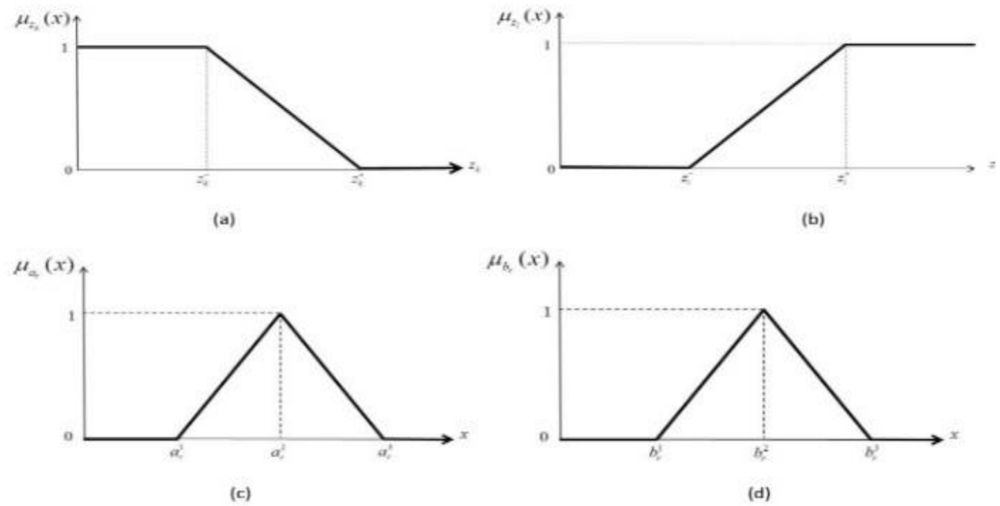

Figure 1. Membership Function of (a). $Z_l(x)$, (b). $Z_k(x)$, (c). $a_r(x)$ and (d). $b_r(x)$

The simplex method cannot solve the linear program (1). Therefore it is necessary to transform it into the DSOLP model. The transformation in question is divided into three stages, with the first stage being to transform multi-objective into single-objective, the second stage is to transform the fuzzy constraint function into a deterministic constraint function, and the third stage is to transform probabilistic constraints into deterministic constraints.

3.2. Multi-Objective Transformation to Single-Objective

The following is a definition of the maximum nonsymmetric fuzzy decision according to [3].

Definition 1. *The fuzzy decision for the fuzzy objective function q and the crisp constraint h with the fuzzy objective function having different levels of importance is defined by the membership function*

$$\mu_D(\boldsymbol{x}) = \sum_{j=1}^{q} w_j \mu_{z_j}(x) \text{ dengan } \sum_{j=1}^{q} w_j = 1, w_j \geq 0 \tag{6}$$

where w_j is a coefficient or weight that states the level of importance of the fuzzy goal. This fuzzy decision is called a nonsymmetric fuzzy decision.

Definition 2. *The maximum decision of a non-geometric fuzzy decision is defined by*

$$\mu_D(x^*) = \max_{x \in X_d} \mu_D(\boldsymbol{x}) = \max_{x \in X_d} \left\{ \sum_{j=1}^{q} w_j \mu_{z_j}(x) \right\} \tag{7}$$

Based on Definition 1 and Definition 2, the linear program 1 can be restated as a problem:

Determine $\mathbf{x} = [x_1^* \ x_2^* \dots x_n^*]^T$ that
Maximize $\sum_{j=1}^{q} w_j \mu_{z_j}(x)$
with obstacle

$$\begin{aligned} &\widetilde{g_r}(x) = \sum_{i=1}^{n} \widetilde{a_{ri}}\, x_i \lessapprox \widetilde{b_r}, \qquad r = 1, 2, \dots, h, \\ &Prob\left[g_p(x) = \sum_{i=1}^{n} a_{pi}\, x_i \leq b_p\right] \geq 1 - \alpha_i, \ p = h+1, h+2, \dots, m, \\ &x_i \geq 0, i = 1, 2, \dots, n. \\ &\alpha_i \in (0, 1), i = 1,2, \dots, n \\ &\sum_{j=1}^{q} w_j = 1, \ w_j \geq 0 \end{aligned} \tag{8}$$

By using auxiliary variables λ, where $\lambda = min\left\{\mu_{z_1}, \mu_{z_2} \cdots \mu_{z_q}\right\}$, then the linear program (8) can be presented as

Determine $\mathbf{x} = [x_1^* \ x_2^* \dots x_n^*]^T$ that
Maximize $\sum_{j=1}^{q} w_j \lambda_j$
with obstacle

$$\begin{aligned} &\lambda_j \leq f\mu_{zj}(x), j = 1, 2, \dots, q \\ &\widetilde{g_r}(x) = \sum_{i=1}^{n} \widetilde{a_{ri}}\, x_i \lessapprox \widetilde{b_r}, \ r = 1, 2, \dots, h, \end{aligned} \tag{9}$$

$$Prob\left[g_p(x) = \sum_{i=1}^{n} a_{pi}\, x_i \le b_p\right] \ge 1 - \alpha_i, \qquad p = h+1, h+2, \dots, m,$$
$$\lambda_j \in [0,1], j = 1, 2, \dots, q$$
$$x_i \ge 0, i = 1, 2, \dots, n$$
$$\sum_{j=1}^{q} w_j = 1, \ \ w_j \ge 0$$

Obstacle $0 \le \lambda \le 1$ given because λ is the minimum value of the membership degree for the fuzzy objective function and fuzzy constraints, where the membership degree is a member of the set of real numbers whose values are between 0 to 1.

3.3. Transformation of Fuzzy Control Function into Deterministic Constraint Function

The concepts of infimum and supremum of fuzzy sets transform the fuzzy constraint function into a deterministic constraint function, which has been described in [3]. In [3], it has been stated that there are two types of triangular fuzzy numbers, namely triangular fuzzy numbers defined in [14] and triangular fuzzy numbers defined in [12]. [15] his research has used triangular fuzzy numbers defined in [12], so that in this study used triangular fuzzy numbers defined in [14].

Based on [15] and [3], the following theorem is presented:

Theorem 1. [7] *It is given $\widetilde{M}$ and $\widetilde{N}$ two fuzzy numbers, $\widetilde{M} \ \vee \ \widetilde{N} = \widetilde{N}$ if and only for each $\alpha \in [0,1]$ apply*

$$inf\{x: \mu_{\widetilde{N}}(x) \ge \alpha\} \ge inf\{x: \mu_{\widetilde{M}}(x) \ge \alpha\}$$
$$sup\{x: \mu_{\widetilde{N}}(x) \ge \alpha\} \ge sup\{x: \mu_{\widetilde{M}}(x) \ge \alpha\}$$

If $\widetilde{M} = \left(m^{(1)}, m^{(2)}, m^{(3)}\right)$ and $\widetilde{N} = \left(n^{(1)}, n^{(2)}, n^{(3)}\right)$ two fuzzy numbers, then we get

$$\widetilde{M} \le \widetilde{N} \Leftrightarrow \begin{cases} m^{(2)} \le n^{(2)} \\ m^{(2)} - \beta \le n^{(2)} - \delta \\ m^{(2)} + \gamma \le n^{(2)} + \tau \end{cases}$$

with $\beta = m^{(2)} - m^{(1)}$, $\gamma = m^{(3)} - m^{(2)}$, $\delta = n^{(2)} - n^{(1)}$ dan $\tau = n^{(3)} - n^{(2)}$.

Proof. ($\Rightarrow$) It is known $\widetilde{M} \le \widetilde{N}$, according to [15], then apply $\widetilde{M} \le_L \widetilde{N}$ or $inf\{x: \mu_{\widetilde{N}}(x) \ge \alpha\} \ge \inf\{x: \mu_{\widetilde{M}}(x) \ge \alpha\}$ for each $\alpha \in [0,1]$ and $\widetilde{M} \le_R \widetilde{N}$ or $sup\{x: \mu_{\widetilde{N}}(x) \ge \alpha\} \ge sup\{x: \mu_{\widetilde{M}}(x) \ge \alpha\}$ for each $\alpha \in [0,1]$. It will be proven

1. $m^{(2)} \le n^{(2)}$
2. $m^{(2)} - \beta \le n^{(2)} - \delta$
3. $m^{(2)} + \gamma \le n^{(2)} + \tau$

with $\beta = m^{(2)} - m^{(1)}$, $\gamma = m^{(3)} - m^{(2)}$, $\delta = n^{(2)} - n^{(1)}$ dan $\tau = n^{(3)} - n^{(2)}$.

According to [3], if $\widetilde{M} = \left(m^{(1)}, m^{(2)}, m^{(3)}\right)$, $\widetilde{N} = \left(n^{(1)}, n^{(2)}, n^{(3)}\right)$ then for each $\alpha \in [0,1]$ apply:

$$\widetilde{M}_\alpha = \left[\left(m^{(2)} - m^{(1)}\right)(\alpha - 1) + m^{(2)}, m^{(2)} - \left(m^{(3)} - m^{(2)}\right)(\alpha - 1)\right]$$

and

$$\widetilde{N}_\alpha = \left[\left(n^{(2)} - n^{(1)}\right)(\alpha - 1) + n^{(2)}, n^{(2)} - \left(n^{(3)} - n^{(2)}\right)(\alpha - 1)\right].$$

for $\alpha = 1$ it is obtained $\widetilde{M}_1 = \{m^{(2)}\}$ and $\widetilde{N}_1 = \{n^{(2)}\}$ so $inf\widetilde{M}_1 \le inf\widetilde{N}_1 \Leftrightarrow m^{(2)} \le n^{(2)}$ and $sup\,\widetilde{M}_1 \le sup\,\widetilde{N}_1 \Leftrightarrow m^{(2)} \le n^{(2)}$.

for $\alpha = 0$ obtained

$\widetilde{M}_0 = \left[-\left(m^{(2)} - m^{(1)}\right) + m^{(2)}, m^{(2)} + \left(m^{(3)} - m^{(2)}\right)\right]$ and

$\widetilde{N}_0 = \left[-\left(n^{(2)} - n^{(1)}\right) + n^{(2)}, n^{(2)} + \left(n^{(3)} - n^{(2)}\right)\right]$.

For example $= m^{(2)} - m^{(1)}$, $\gamma = m^{(3)} - m^{(2)}$, $\delta = n^{(2)} - n^{(1)}$, $\tau = n^{(3)} - n^{(2)}$, so it is obtained

$$\begin{aligned} & inf\widetilde{M}_0 \le inf\widetilde{N}_0 \\ \Leftrightarrow\ & inf\left[-\beta + m^{(2)}, m^{(2)} + \gamma\right] \le inf\left[-\delta + n^{(2)}, n^{(2)} + \tau\right] \\ \Leftrightarrow\ & m^{(2)} - \beta \le n^{(2)} - \delta \end{aligned}$$

and

$$\begin{aligned} & sup\,\widetilde{M}_0 \le sup\,\widetilde{N}_0 \\ \Leftrightarrow\ & sup\left[-\beta + m^{(2)}, m^{(2)} + \gamma\right] \le sup\left[-\delta + n^{(2)}, n^{(2)} + \tau\right] \\ \Leftrightarrow\ & m^{(2)} + \gamma \le n^{(2)} + \tau \end{aligned}$$

(⇐) known

1. $m^{(2)} \leq n^{(2)}$
2. $m^{(2)} - \beta \leq n^{(2)} - \delta$
3. $m^{(2)} + \gamma \leq n^{(2)} + \tau$

Will be proven for each $\alpha \in [0,1]$ apply

$$inf\{x: \mu_{\widetilde{N}}(x) \geq \alpha\} \geq inf\{x: \mu_{\widetilde{M}}(x) \geq \alpha\}$$
$$sup\{x: \mu_{\widetilde{N}}(x) \geq \alpha\} \geq sup\{x: \mu_{\widetilde{M}}(x) \geq \alpha\}$$

For $0 \leq \alpha \leq 1$ obtained

$$\begin{aligned} inf\{x: \mu_{\widetilde{N}}(x) \geq \alpha\} &= inf \widetilde{N}_\alpha \\ &= inf\left[\delta(\alpha-1) + n^{(2)}, n^{(2)} - \tau(\alpha-1)\right] \\ &= \delta(\alpha-1) + n^{(2)} \\ &= \left(n^{(2)} - n^{(1)}\right)(\alpha-1) + n^{(2)} \end{aligned}$$

and

$$\begin{aligned} inf\{x: \mu_{\widetilde{M}}(x) \geq \alpha\} &= inf \widetilde{M}_\alpha \\ &= inf\left[\beta(\alpha-1) + m^{(2)}, m^{(2)} - \gamma(\alpha-1)\right] \\ &= \beta(\alpha-1) + m^{(2)} \\ &= \left(m^{(2)} - m^{(1)}\right)(\alpha-1) + m^{(2)} \end{aligned}$$

From point 1 it is known $m^{(2)} \leq n^{(2)} \Leftrightarrow m^{(2)} - n^{(2)} \leq 0$.
Besides that$0 \leq \alpha \leq 1 \Leftrightarrow -1 \leq \alpha - 1 \leq 0$, so that it is obtained

$$(\alpha-1)\left(m^{(2)} - n^{(2)}\right) \leq n^{(2)} - m^{(2)} \tag{10}$$

From point 2 it is known $m^{(2)} - \beta \leq n^{(2)} - \delta \Leftrightarrow m^{(2)} - n^{(2)} \leq \beta - \delta$, so that it is obtained

$$(\alpha-1)(\beta-\delta) \leq (\alpha-1)\left(m^{(2)} - n^{(2)}\right) \tag{11}$$

from (6) and (7), it is obtained:

$$\begin{aligned} & (\alpha-1)(\beta-\delta) \leq (\alpha-1)\left(m^{(2)} - n^{(2)}\right) \leq n^{(2)} - m^{(2)} \\ \Leftrightarrow\ & (\alpha-1)(\beta-\delta) \leq n^{(2)} - m^{(2)} \\ \Leftrightarrow\ & (\alpha-1)\beta - (\alpha-1)\delta \leq n^{(2)} - m^{(2)} \\ \Leftrightarrow\ & m^{(2)} + (\alpha-1)\beta \leq n^{(2)} + (\alpha-1)\delta \end{aligned}$$

Because $\beta = m^{(2)} - m^{(1)}$ dan $\delta = n^{(2)} - n^{(1)}$ then obtained

$$m^{(2)} + (\alpha-1)\left(m^{(2)} - m^{(1)}\right) \leq n^{(2)} + (\alpha-1)\left(n^{(2)} - n^{(1)}\right)$$

in other words for fuzzy numbers of the form $\widetilde{M} = \left(m^{(1)}, m^{(2)}, m^{(3)}\right)$ dan $\widetilde{N} = \left(n^{(1)}, n^{(2)}, n^{(3)}\right)$ apply

$$inf\{x: \mu_{\widetilde{N}}(x) \geq \alpha\} \geq inf\{x: \mu_{\widetilde{M}}(x) \geq \alpha\}$$

In the same way will be obtained:

$$sup\{x: \mu_{\widetilde{N}}(x) \geq \alpha\} \geq sup\{x: \mu_{\widetilde{M}}(x) \geq \alpha\}$$

which completes the proof of the theorem.■

According to Theorem 1, the linear program (5) is changed to:

Determine $\mathbf{x} = [x_1^* \ x_2^* \dots x_n^*]^T$ yang
Maximize $\sum_{j=1}^{q} w_j \lambda_j (x)$
with obstacle

$$\begin{aligned} & \lambda_j \leq f_{\mu_{z_j}(x),} j = 1, 2, \cdots q \\ & g_r(\boldsymbol{x}) = \sum_{i=1}^{n} a_{ri}^{(2)} x_i \leq b_r^{(2)}, \quad r = 1, 2, \dots, h, \\ & g_r(\boldsymbol{x}) = \sum_{i=1}^{n} (a_{ri}^{(2)} - \beta) x_i \leq (b_r^{(2)} - \delta), \quad r = 1, 2, \dots, h, \\ & g_r(\boldsymbol{x}) = \sum_{i=1}^{n} (a_{ri}^{(2)} + \gamma) x_i \leq (b_r^{(2)} + \tau), \quad r = 1, 2, \dots, h, \\ & Prob\left[g_p(x) = \sum_{i=1}^{n} a_{pi} x_i \leq b_p\right] \geq 1 - \alpha_i, \qquad p = h+1, h+2, \dots, m, \\ & x_i \geq 0, i = 1, 2, \dots, n \\ & \textstyle\sum_{j=1}^{q} w_j = 1, \ \ w_j \geq 0 \end{aligned} \tag{12}$$

$$\lambda_j \in [0,1], j = 1,2 \dots, q,$$
$$\alpha_i \in (0,1), i = 1,2, \dots, n$$

3.4. Transformation of Probabilistic Constraint Functions into Deterministic Constraint Functions

To transform the probabilistic constraint function into a deterministic constraint function, the Chance Constrained technique proposed by [10] is used..

Based on [10], let (b_i) and Var(b_i) show the mean and variance of the random variable with a normal distribution b_i. The probabilistic constraint on the linear program (12) can be restated as follows:

$$Prob\left[\frac{b_p - E(b_p)}{\sqrt{Var(b_p)}} \geq \frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_p)}{\sqrt{Var(b_p)}}\right] \geq s_i, i = 1,2, \dots, m \quad (13)$$

with $s_i = 1 - \alpha_i$ dan $\frac{b_p - E(b_p)}{\sqrt{Var(b_p)}}$ is a standard normal random variable. The inequality (13) can be stated again as follows

$$1 - Prob\left[\frac{b_p - E(b_p)}{\sqrt{Var(b_p)}} \leq \frac{\sum_{j=1}^{n} a_{ij} - E(b_p)}{\sqrt{Var(b_p)}}\right] \geq s_i \quad (14)$$

or

$$Prob\left[\frac{b_p - E(b_p)}{\sqrt{Var(b_p)}} \leq \frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_p)}{\sqrt{Var(b_p)}}\right] \leq 1 - s_i$$

If K_{s_i} is the value of a standard normal random variable $\emptyset(K_{s_i}) = 1 - s_i$, then the constraint (14) can be shown as follows:

$$\emptyset\left[\frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{Var(b_i)}}\right] \leq \emptyset(K_{s_i}), \quad (15)$$

This inequality will be satisfied only if

$$\frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_p)}{\sqrt{Var(b_p)}} \leq K_{s_i} \quad (16)$$

or

$$\sum_{j=1}^{n} a_{ij}x_j - E(b_p) \leq K_{s_i}\sqrt{Var(b_p)} \quad (17)$$

Based on the equation (13) – (17), then linear program (12) can be written as problem

Determine $\mathbf{x} = [x_1^* \; x_2^* \dots x_n^*]^T$ yang

Maximize $\sum_{j=1}^{q} w_j \lambda_j (x)$

with obstacle

$$\begin{aligned}
&\lambda_j \leq f_{\mu_{z_j}(x)}, j = 1, 2, \cdots q \\
&g_r(\boldsymbol{x}) = \sum_{i=1}^{n} a_{ri}^{(2)} x_i \leq b_r^{(2)}, \quad r = 1, 2, \dots, h, \\
&g_r(\boldsymbol{x}) = \sum_{i=1}^{n} (a_{ri}^{(2)} - \beta) x_i \leq (b_r^{(2)} - \delta), \quad r = 1, 2, \dots, h, \\
&g_r(\boldsymbol{x}) = \sum_{i=1}^{n} (a_{ri}^{(2)} + \gamma) x_i \leq (b_r^{(2)} + \tau), \quad r = 1, 2, \dots, h, \\
&\sum_{j=1}^{n} a_{ij}x_j \leq E(b_i) + K_{s_i}\sqrt{Var(b_i)}, \\
&x_i \geq 0, \quad i = 1, \dots, n \\
&\sum_{j=1}^{q} w_j = 1, \quad w_j \geq 0 \\
&\lambda_j \in [0,1], j = 1, 2 \dots, q, \\
&\alpha_i \in (0,1), i = 1,2, \dots, n
\end{aligned} \quad (18)$$

Linear Program (18) is a Deterministic Single Objective Linear Program (DSOLP) so it can be solved using the simplex method. The solution obtained by solving the linear program (18) is also the solution of the linear program (1). This is guaranteed by the following two theorems.

Theorem 2. *If* $\boldsymbol{x}^* \in X$ *is the single optimal solution of DSOLP (18) for a w=* $(w_1, w_2, \ldots, w_q) > 0$, *so* $\boldsymbol{x}^*$ *a is the Pareto optimal solution of* SFMOLP *(1).*

Theorem 3. *If* $\boldsymbol{x}^* \in X$ *is the Pareto optimal solution of* SFMOLP *(1), so* $\boldsymbol{x}^*$ *is the optimal solution of DSOLP (18) for a w=* $(w_1, w_2, \ldots, w_q) > 0$.

3.5. Model Completion Algorithm

The algorithm for solving the nonsymmetric SFMOLP model is presented in Figure 2.

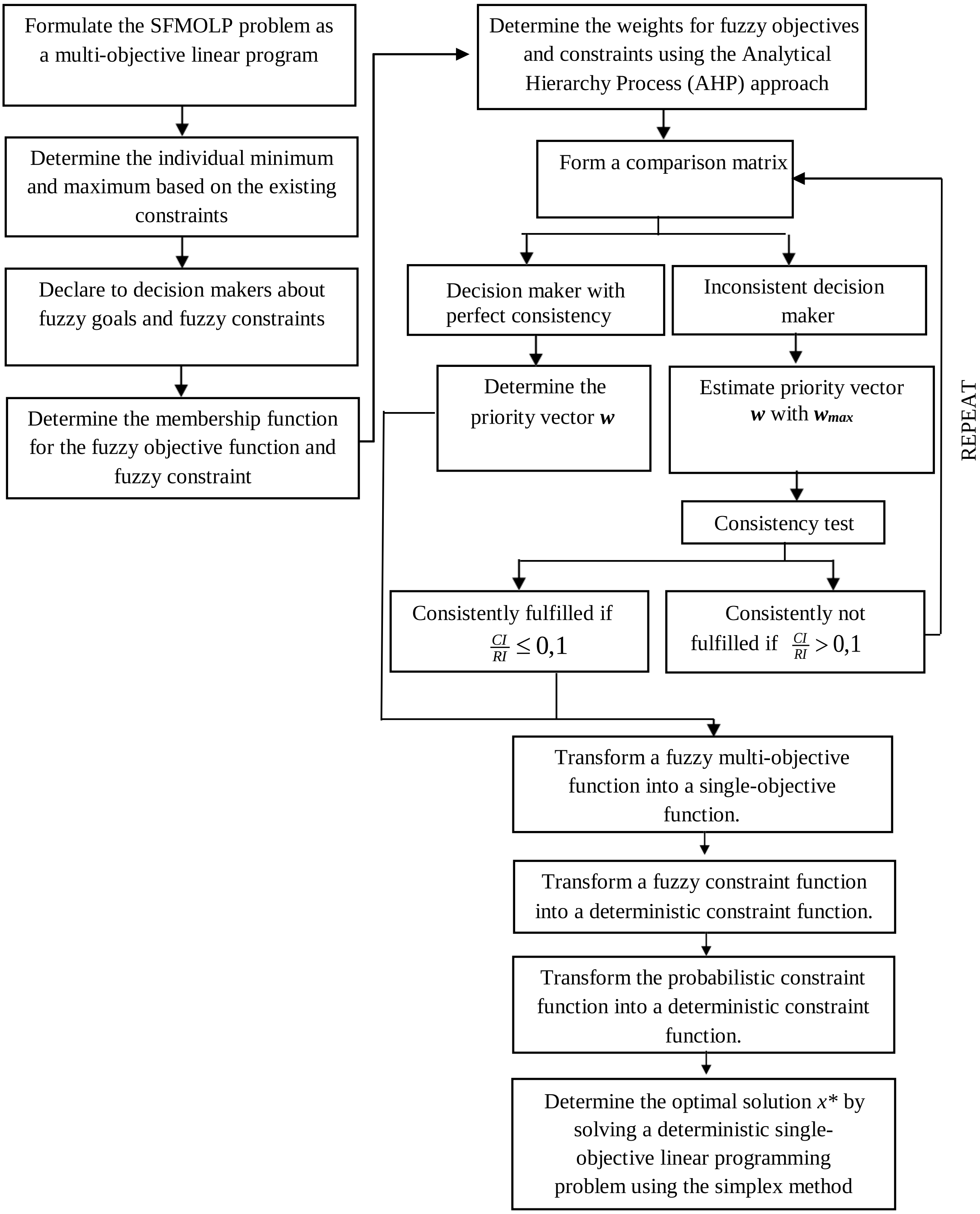

Figure 2. SFMOLP Completion Flowchart

3.6. Numerical Example

Given a stochastic fuzzy multi-objective supplier selection problem as follows:
Minimize $Z_1 = 15x_1 + 10x_2 + 12x_3 \lessapprox 10.822,5$
Maximize $Z_2 = 0.75x_1 + 0.8x_2 + 0.9x_3 \gtrapprox 837,53$
Maximize $Z_3 = 0.7x_1 + 0.85x_2 + 0.75x_3 \gtrapprox 798,33$
with constraints:

$$
\begin{aligned}
&Prob\,[x_1 + x_2 + x_3 \le p_i] \ge 0,95 \\
&(0.89, 0.90, 0.91)x_1 \lessapprox (602, 655, 710) \\
&(0.89, 0.90, 0.91)x_2 \lessapprox (486, 538, 592) \\
&(0.89, 0.90, 0.91)x_3 \lessapprox (393, 421, 497) \\
&x_i \ge 0, i = 1,2,3.
\end{aligned} \tag{19}
$$

Z_1, Z_2, and Z_3 each is price, quality and service. x_1, x_2, and x_3 symbolizes the number of products to be purchased from suppliers. 1, 2 and 3, p_i is the variation in the number of requests most often obtained by the company presented in Table 1. The first constraint is a demand constraint. The second, third and fourth constraints are capacity constraints. The fifth constraint ensures that the number of products purchased is not negative.

Table 1 Variation in the number of requests received by the company

Variation	Request (p)
1	750
2	850
3	1000
4	950
5	940
E(p)	**898**
Var(p)	**9.770**

Order variation data in Table 1 are arranged based on the number of most often obtained requests. By using the Kolmogorov-Smirnov test, the significance value (α) is 0.958> 0.05, so it can be concluded that the demand data is normally distributed. The probability of the company getting the number of requests (α), as shown in Table 1, is 95% (α=0.05). In Table 2, the level of importance of each supplier according to the decision-maker is presented.

Table 2 The Level of Importance of the Fuzzy Objective Function

	Price	Quality	Service Level
Price	1	1/4	1/3
Quality	3	1	2
Service Level	5	1/2	1

To determine the weights of the objectives in the additive model, the Analytic Hierarchy Process (AHP) approach is used. The procedure for AHP is discussed by [8] and [9] and is presented in Figure 2, so we get w_1=0,12,w_2=0,56,w_3=0,32. Then, based on the linear program (18), the linear program (19) is transformed into:

Maximize $0,12\lambda_1 + 0,56\lambda_2 + 0,32\lambda_3$
with obstacle

$$
\begin{aligned}
&\lambda_1 \le \frac{13.929,99 - (15x_1 + 10x_2 + 12x_3)}{3.107,49} \\
&\lambda_2 \le \frac{(0.75x_1 + 0.8x_2 + 0.9x_3) - 759,73}{77,79} \\
&\lambda_3 \le \frac{(0.7x_1 + 0.85x_2 + 0.75x_3) - 710,13}{88,20} \\
&x_1 + x_2 + x_3 \le 991,9 \\
&0.89x_1 \le 601.64
\end{aligned}
$$

$$0.90x_1 \leq 655.2$$
$$0.91x_1 \leq 709.8$$
$$0.89x_2 \leq 485.94$$
$$0.90x_2 \leq 538.2$$
$$0.91x_2 \leq 591.5$$
$$0.89x_3 \leq 393.38$$
$$0.90x_3 \leq 421.2$$
$$0.91x_3 \leq 496.86$$
$$x_1, x_2, x_3 \geq 0$$
$$\lambda \in [0,1].$$

By using the POM for Windows software, the optimal solution for the above model formulation is obtained as follows::

$x_1 = 0$, $x_2 = 442$, dan $x_3 = 549$

$Z_1 = 11.008$, $Z_2 = 847{,}7$, dan $Z_3 = 787{,}45$

The level of achievement of the objective function is

$$\mu_{Z_1} = 0.97, \qquad \mu_{Z_2} = 1, \qquad \mu_{Z_3} = 0.88$$

In other words, based on calculations using the additive weighting model, the decision-maker must buy from supplier 2 as many as 442 packages, from supplier 3 as many as 549 packages but not from supplier 1. This decision will minimize purchasing costs and maximize the quality and service obtained.

4. CONCLUSION

In solving the non-symmetrical Stochastic Fuzzy Multi-Objective Linear Program (SFMOLP), it is necessary to transform SFMOLP into a Deterministic Single-Objective Linear Program (DSOLP). Therefore, the linear program can be solved using the simplex method. The transformation process is divided into three stages. The first stage is to transform the multi-objective into a single-objective using an additive weighting model. The second stage is to transform the fuzzy constraint function into a deterministic constraint function using the concepts of infimum and supremum fuzzy sets. The third stage is the probabilistic constraint function. Become a deterministic constraint function by using the chance-constrained technique. The algorithm that has been built can be used for various SFMOLP problems such as supplier selection, supply chain, and transportation.

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REFERENCES

[1] F. S. Hillier and G. J. Lieberman, *Introduction to Operations Research*, 11th ed. New York: McGraw-Hill Education, 2021.

[2] F. M. Zenis, M. Y. Fajar, D. Y. Ramdani, and Y. Ramdani, “Program linear multi-objective dengan fixed-weight method,” *Jurnal Matematika*, vol. 14, no. 1, pp. 1-7, May 2015.

[3] G. S. Mada, “Model pembobotan aditif termodifikasi untuk menyelesaikan masalah pemilihan supplier multi-objektif fuzzy dengan fungsi objektif fuzzy dan kendala fuzzy pada rantai supply,” ["A modified additive weighting model to solve a fuzzy multi-objective supplier selection problem with a fuzzy objective function and fuzzy constraints on the supply chain,"] Prosiding Seminar Nasional Saintek IV FST Undana, pp. 87-98, Oct. 2019.

[4] H. Nasseri, M. Morteznia, and M. Mirmohseni, “A New Method for Solving Fully Fuzzy Multi Objective Supplier Selection Problem,” *Int. J. Res. Ind. Eng*, vol. 6, no. 3, Nov. 2017.

[5] H. Cheng, W. Huang, Q. Zhou, and J. Cai, “Solving fuzzy multi-objective linear programming problems using deviation degree measures and weighted max–min method,” *Applied Mathematical Modelling*, vol. 37, no. 10, pp. 6855–6869, Jun. 2013.

[6] H. Suprajitno, “Solving Multiobjective Linear Programming Problem Using Interval Arithmetic,” *Applied Mathematical Sciences*, vol. 6, no. 80, pp. 3959-3968, 2012.

[7] H. M. Nehi and M. Alineghad, “Solving Interval and Fuzzy Multi Objective Linear Programming Problem by Necessarily Efficiency Points,” *International Mathematical Forum 3*, pp. 99-106, 2008.

[8] A. E. Munthafa and H. Mubarok, "Penerapan metode analytical hierarchy process dalam sistem pendukung keputusan penentuan mahasiswa berprestasi," ["The application of the analytical hierarchy process method in the decision support system for determining outstanding students,"] *Jurnal Siliwangi Seri Sains dan Teknologi*, vol. 3, no. 2, pp. 192–201, 2017.

[9] P. T. Prasetyaningrum and A. Sari, "Penerapan analytical hierarchy process (AHP) untuk mendukung keputusan pemilihan desrinasi tempat wisata daerah istimewa yogyakarta untuk para wisatawan mancanegara non asia," ["Implementation of the analytical hierarchy process (AHP) to support the decision to choose tourist destinations for the Yogyakarta Special Region for non-Asian foreign tourists,"] *Jurnal SIMETRIS*, vol. 10, no. 2, pp. 519-528, Nov. 2019.

[10] S. Hulsurkar, M. P. Biswal, and S. B. Sinha, "Fuzzy programming approach to multi-objective stochastic linear programming problems," *Fuzzy Sets and Systems*, vol. 88, no. 2, pp. 173–181, Jun. 1997.

[11] M. Sakawa, I. Nishizaki, and H. Katagiri, *Fuzzy Stochastic Multiobjective Programming*, New York: Springer New York, 2011.

[12] D. A. Kalaichelvi and K. Janofer, "α-cuts of triangular fuzzy numbers and α-cuts of triangular fuzzy number matrices," Int. Jr. of Mathematic Science & Applications, vol.2 no.2, pp. 635-643, May 2012.

[13] B. T. Balan, "Solving Multi-Objective Fuzzy Linear Optimization Problem Using Fuzzy Programming Technique," *IOSR Journal of Mathematics*, pp. 18-21. 2016

[14] Y. Septiani, B. Irawanto, and S. Hariyanto, "Program linier fuzzy penuh dengan algoritma multi objective linear programming menggunakan metode level sum," ["Full fuzzy linear program with multi objective linear programming algorithm using level sum method,"] *Jurnal Mahasiswa Matematika Undip*, vol. 5, no. 4, pp. 1-10, 2016.

[15] E. Lalian, G. S. Mada, F. Luan, and E. Binsasi, "Pengembangan Program Linear Multi-Objektif Fuzzy Stokastik Model Simetris dan Penyelesaiannya dengan Menggunakan Teknik Chance Constrained," ["Development of a Symmetrical Fuzzy Stochastic Multi-Objective Linear Program and its Solution using the Chance Constrained Technique,"] *Jurnal Sains Lahan Kering*, vol. 4, no. 2, pp. 22–24, Jan. 2022.