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DYNAMICS OF THE RUMOR SPREADING MODEL OF INDONESIA TWITTER CASE

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Abstract. *The study of the spreading of a rumor is significantly important to obtain scientific information and better strategies in reducing its negative impact. Twitter has become a medium for spreading rumors or hoaxes spatially and chronologically because it has a unique community structure. This study demonstrates the model of spreading rumors by considering credibility, correlation, and mass classification based on personality is discussed. The behavior of a model solution around equilibrium points is investigated with the Jacobian matrices. The stability also corresponds to a threshold number indicating the rumor fades away or continues to spread in the population. The analytical results are confirmed by actual data from Twitter in Indonesia with #SahkanRUUPKS. The simulation results show that the free rumor equilibrium point is stable and the threshold number is less than 1. Our study shows that the number of spreaders does not increase and the #SahkanRUUPKS rumor will vanish.*

Keywords: *Equilibrium Points, Rumor Spreading Model, #SahkanRUUPKS, Threshold Number, Twitter.*

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1. INTRODUCTION

A mathematical model is a simple representation of a phenomenon or event in the real world which is presented in a mathematical concept. In simple terms a mathematical model can be defined as a construction, in the form of graphs, symbols, simulations, or experiments designed to study a particular phenomenon in the real world [1]. Mathematical models resulting from the collection of experimental data, processing experimental data, systematic observations were developed into analysis, interpretation and behavior of the observed system [2].

Indonesia is the fifth largest country in the world after the United States, United Kingdom, Brazil, Canada, and Australia in the use of Twitter with 2.34 percent of the tweet samples. Twitter has become a medium for spreading gossip, rumors or hoaxes spatially and chronologically because it has a unique community structure [3]. The most popular news in Indonesia based on Twitter getdaytrends in 2021 is #SahkanRUUPKS [4]. The news has become popular because RUU on the Elimination of Sexual Violence or RUU PKS is still not a priority in the national legislation program. Meanwhile, in Komnas Perempuan's year-end records published in the first quarter of 2020, the number of sexual violence during 2019 reached 432.471 cases and Komnas Perempuan has also proven that cases of sexual violence increased by 792 percent over the last 12 years [5].

Rumors are unconfirmed truths and have a negative impact on society, although interventions can be made to minimize the negative impacts. Therefore, understanding the spread of rumors is very important to get better scientific information and strategies to reduce their negative impact [6]. The spread of rumors and the spread of infectious diseases are analogous phenomena. The analogy can be seen in the distribution of the population of the two models. Daley and Kendal first introduced the rumor model known as the DK model. The DK model was based on the SIR (Susceptible, Infected, Removed) epidemic model. In the DK model, the population is divided into three groups, namely the group of people who have not heard rumors (analogous to the group of individuals who are susceptible to disease), the group of people who spread rumors (analogous to the group of individuals who are infected with the disease), and the group of people who stop spreading rumors (analogous to groups of individuals who died, isolated and recovered from illness) [7]**.**

To see the level of spread of disease in a population, the basic reproduction number is used, which is denoted by R_0 [8]. The threshold parameter in the rumor spread model has the same definition as the basic reproduction number in the infectious disease spread model. In the rumor spread model, if $R_0 < 1$ then the number of spreaders of rumors does not increase so that rumors will fade or disappear, otherwise if $R_0 > 1$ then rumors will infect more people and rumors can continue to spread [9].

A research on the spread of rumors was carried out by Huo et al. (2017) who compared the model of spreading rumors with models of spreading infectious diseases by adding a group of people who were hesitant to spread rumors [10]. Then Xia et al (2017) conducted a study on the rumor spread model by considering the group of people who doubted the rumors and adding the ambiguity of the rumor content as a model parameter [11]. Chen and Wang (2020) conducted a study on dispersion models in homogeneous and heterogeneous networks [9].

This study considers the credibility of rumors, the correlation between rumors and lives, and the classification of groups based on personality [9]. Besides, the stability of the model for the rumor-free equilibrium point and the rumor-spreading endemic equilibrium point is analyzed**.** The model is implemented on the actual rumor data taken from Twitter, namely the most popular news in 2021 with #SahkanRUUPKS.

2. RESEARCH METHODS

The steps taken in this research are:

i. description of the model of spreading rumors by considering the credibility of rumors, correlation and classification of groups based on personality,

ii. stability analysis of the model around the rumor-free equilibrium point and the rumor-spreading endemic equilibrium point using the Jacobian matrix,

iii. determination of threshold parameters,

iv. implemention of the model on the actual data from Twitter obtained from Python using tweepy and API access (Application Programming Interface).

3. RESULTS AND DISCUSSION

3.1. Rumor Spread Model

The rumor spread model is formed with the following assumptions [9]

a. The population is divided into five groups

1. **Steady ignorant** (C^{P})**.** This class consists people who do not know the rumor; if they hear the rumor, they prefer to contemplate it and seek confirmation before making decisions.
2. **Radical Ignorant** (C^{H}). This class consists people who do not know the rumor, if they hear the rumor, they are most likely to believe it without contemplating it or seeking information.
3. **Exposed** (C^{E}). This class consists people who know the rumor but hesitate to believe it and do not spread it.
4. **Spreader** (C^{S}). This class consists people who spread the rumor
5. **Stifler** (C^{R})**.** The class consists people who know the rumor but never spread it or stop spreading it.

b. From the C^H group to the C^S group with probability $, \alpha, and\ \beta$, the three parameters are continuous random variables bounded by $0 \leq \gamma < 1$, $0 < \alpha \leq 1$ and $0 \leq \beta \leq 1$. More credible the rumor is higher the value of γ, which means that everyone who knows the rumor will believe it. In real life it is impossible for everyone to believe the rumor. Therefore, the parameter γ is limited to a value less than 1. Then higher the value of α, the more relevant the rumor is to life.

c. From the C^{P} group to the C^{S} and C^{E} groups. The C^{H} group is more likely to spread rumors than the C^{P} group. Given a parameter μ, where $0 < \mu \leq 1$, the probability that people in the C^{P} group switch to the C^{S} group is $\gamma\alpha\beta\mu$. The people in the C^{E} group are not spreaders because they hesitate to spread rumors even though they have heard the rumors, Therefore μ doesn't work on the transfer of people in the C^{P} group to the C^{E} group. People in group C^{P} are calmer than people in group C^{H}, so the probability of moving from group C^{P} to group C^{E} is $\gamma(1-\gamma)\beta$, where $\gamma = 1$ means that the rumors are completely credible or trustworthy, and $\gamma = 0$ means that the rumors are completely unreliable, so in either case no individual will switch to the C^{E} group. When the γ is in the middle value (γ=0.5), at that time the rumors are very doubtful and the number of individuals who will switch to the C^{E} group reaches a maximum. Then, if the credibility is higher or the credibility is lower, for example as when $\gamma = 0{,}7$ and $\gamma = 0{,}3$ the same level of individual doubt is produced. Therefore, it is assumed that more credible and less credible rumors have the same effect on the probability that individuals switch from the C^{P} group to the C^{E} group. A certain relationship between the credibility of rumors and the proportion of people who hesitate to spread rumors must exist. So it is assumed that the proportion of doubters in the C^{E} group is positively correlated with $\gamma(1-\gamma)$.

d. From the C^{E} group to the C^S and C^R groups. Individuals in the C^{E} group switch to the C^Sor C^R groups depending on the group to which the individual associated with the C^{E} group belongs. Individuals in the C^{E} group decide whether they will spread rumors when they are influenced by others. Therefore, the probability when an individual in group C^{E} makes contact with an individual in group C^S is θ and the probability when an individual in group C^{E} makes contact with an individual in group C^R is ϕ.

e. From group C^S to C^R, when an individual in group C^S makes contact with an individual in group C^S, C^{E}, or C^R then the individual in group C^S witches to class C^R with probability η_1, because the spreader will be aware of the rumor it was nothing new when meeting other people who knew about the rumors. In addition, individuals in the C^S group will switch to the C^R group at the rate of η_2 because people forget the rumors in the process of spreading.

The flow chart of the rumor spread model with the above assumptions is presented in Figure 1.

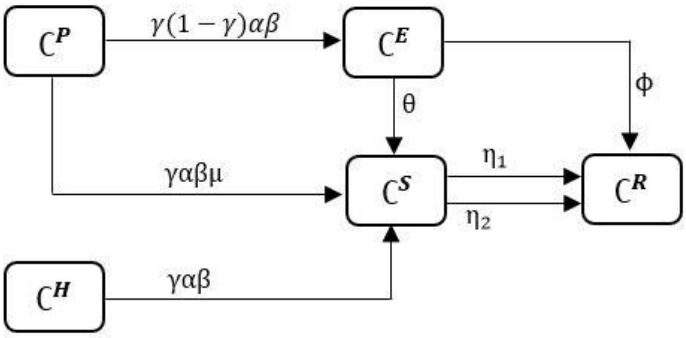

Figure 1. Flowchart of the Rumor Spread Model

Let P, H, E, S and R be the individual densities in the groups C^P, C^H, C^E, C^S and C^R respectively at time t. This individual density satisfies the following normal conditions

$$P + H + E + S + R = 1.$$

The rumor spreading model based on the above assumptions is expressed in the following system of differential equations

$$\begin{aligned}
\frac{dP}{dt} &= -PS(A+B) \\
\frac{dH}{dt} &= -HSC \\
\frac{dE}{dt} &= PSB - k_1SE - k_2RE \\
\frac{dS}{dt} &= PSA + HSC + k_1SE - k_3S(R+S+E) - S\eta_2 \\
\frac{dR}{dt} &= k_3S(R+S+E) + S\eta_2 + k_2RE
\end{aligned} \tag{1}$$

where, $A = k\gamma\alpha\beta\mu$, $B = k\alpha\beta\gamma(1-\gamma)$, $C = k\gamma\alpha\beta$, $k_1 = k\theta$, $k_2 = k$ [2]ϕ, and $k_3 = k\eta_1$. The model parameters are described in Table 1 below.

Table 1. Parameters of the Rumor Spread Model

Parameters	Information	Condition
$\boldsymbol{\alpha}$	The correlation coefficient between rumor and peoples's lives	$0 < \alpha \leq 1$
$\boldsymbol{\gamma}$	The credibility of a rumor	$0 \leq \gamma < 1$
$\boldsymbol{\beta}$	The probability that an ignorant individual hears a rumor via contact with individual in group spreader	$0 \leq \beta \leq 1$
$\boldsymbol{\mu}$	The spreading desire ratio	$0 < \mu \leq 1$
$\boldsymbol{\theta}$	The propbability that individuals in group C^E switch to group C^S	
$\boldsymbol{\phi}$	The probability that individuals in group C^E switch to group C^R	
$\boldsymbol{\eta_1}$	The probability that individuals in group C^S switch to group C^R	
$\boldsymbol{\eta_2}$	Forgetting rate	
$\boldsymbol{k}$	The average degree or average number of contacts of each individual in the network	

to determine the value of the average degree (k) in social networks is the same as determining the value of the average degree (k) in graph theory, consider the following theorem.

Theorem 1. [13] *In a graph, the number of vertices is equal to twice the number of edges. As a result, the number of vertices of an odd degree produces an even number of sides.*

3.2. Stability Analysis

3.2.1. Equilibrium Points

There are two equilibrium points of model (1).

a. The rumor free equilibrium point, which is a condition where the group of people who know and spread rumors is zero, so that rumors do not spread in the population. It is expressed as

$$E_0 = \left(P^0, H^0, E^0, S^0, R^0\right) = \left(x; n; 0; 0; (1-x-n)\right),$$

where x is the density of individuals in the Steady Ignorant group and n is the density of individuals in the Radical Ignorant group.

b. The rumor endemic equilibrium points, i.e. the condition where rumors spread within the population, are expressed as

$$E_* = P_1, H_1, E_1, S_1, R_1 = \left(0; 0; -\frac{\theta\eta_1 w}{\phi(\theta-\eta_1)} + \frac{\eta_1 w}{(\theta-\eta_1)} + \frac{\eta_2}{k(\theta-\eta_1)}; w; -\frac{\theta w}{\phi}\right),$$

where

$$w = \frac{-b}{a},$$

$$a = -\frac{k\eta_1\theta}{\phi} + k\eta_1 - \frac{k{\eta_1}^2\theta}{\phi(\theta-\eta_1)} + \frac{k{\eta_1}^2}{\theta-\eta_1} + \frac{k\theta^2\eta_1}{\phi(\theta-\eta_1)} - \frac{k\theta\eta_1}{\theta-\eta_1},$$

$$b = \frac{\eta_1\eta_2}{\theta-\eta_1} + \eta_2 - \frac{\theta\eta_2}{\theta-\eta_1}.$$

3.2.2. Jacobian Matrix

The Jacobian matrix of the system of equations (1) is expressed as

$$J = \begin{bmatrix} -S(A+B) & 0 & 0 & -P(A+B) & 0 \\ 0 & -SC & 0 & -HC & 0 \\ SB & 0 & -k_1S-k_2R & PB-k_1E & -k_2E \\ SA & SC & k_1S-k_3S & PA+HC+k_1E-k_3(R+2S+E)-\eta_2 & -k_3S \\ 0 & 0 & k_3S+k_2R & k_3(R+2S+E)+\eta_2 & k_3S+k_2E \end{bmatrix}.$$

The Jacobian matrix for the rumor-free equilibrium point (E_0),

$$J(E_0) = \begin{bmatrix} 0 & 0 & 0 & -x(A+B) & 0 \\ 0 & 0 & 0 & -nC & 0 \\ 0 & 0 & -k_2(1-x-n) & PB-k_1E & 0 \\ 0 & 0 & 0 & xA+nC-k_3(1-x-n)-\eta_2 & 0 \\ 0 & 0 & k_2(1-x-n) & k_3(1-x-n)+\eta_2 & 0 \end{bmatrix}. \tag{2}$$

The characteristic equation of the Jacobian matrix (2) is

$$\lambda^3(k_2(1-x-n)+\lambda)(-xA-nC+k_3(1-x-n)+\eta_2+\lambda) = 0. \tag{3}$$

Furthermore, from (3) the eigenvalues are obtained, namely

$\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = -k_2(1-x-n)$ and $\lambda_5 = xA+nC-k_3(1-x-n)-\eta_2$.

because $\lambda_1 = \lambda_2 = \lambda_3 = 0$ and $\lambda_4 < 0$, then the stability of the model around the rumor-free equilibrium point depends on λ_5. If $\lambda_5 < 0$, then

$$xA + nC - k_3(1 - x - n) - \eta_2 < 0$$
$$xA + nC < k_3(1 - x - n) + \eta_2$$
$$\frac{xA+nC}{k_3(1-x-n)+\eta_2} < \frac{k_3(1-x-n)+\eta_2}{k_3(1-x-n)+\eta_2}$$
$$\frac{xA+nC}{k_3(1-x-n)+\eta_2} < 1 \tag{4}$$

Jacobian matrix for endemic equilibrium points (E_*),

$$J(E_*) = \begin{bmatrix} -S_1(A+B) & 0 & 0 & 0 & 0 \\ 0 & -S_1C & 0 & 0 & 0 \\ S_1B & 0 & -k_1S_1 - k_2R_1 & -k_1E_1 & -k_2E_1 \\ S_1A & S_1C & k_1S_1 - k_3S_1 & k_1E_1 - k_3(R_1 + 2S_1 + E_1) - \eta_2 & -k_3S_1 \\ 0 & 0 & k_3S_1 + k_2R_1 & k_3(R_1 + 2S_1 + E_1) + \eta_2 & k_3S_1 + k_2E_1 \end{bmatrix}. \tag{5}$$

The characteristic equation of the Jacobi matrix (5)

$$(\lambda - m_{11})(\lambda - m_{22})(\lambda^3 - a_2\lambda^2 - a_1\lambda - a_0) = 0, \tag{6}$$

where,

$$m_{11} = -S_1(A + B)$$
$$m_{22} = -S_1C$$
$$m_{31} = S_1B$$
$$m_{33} = -k_1S_1 - k_2R_1$$
$$m_{34} = -k_1E_1$$
$$m_{35} = -k_2E_1$$
$$m_{41} = S_1A$$
$$m_{42} = S_1C$$
$$m_{43} = k_1S_1 - k_3S_1$$
$$m_{44} = k_1E_1 - k_3(R_1 + 2S_1 + E_1) - \eta_2$$
$$m_{45} = -k_3S_1$$
$$m_{53} = k_3S_1 + k_2R_1$$
$$m_{54} = k_3(R_1 + 2S_1 + E_1) + \eta_2$$
$$m_{55} = k_3S_1 + k_2E_1$$

and

$$a_0 = m_{33}m_{44}m_{55} - m_{33}m_{45}m_{54} + m_{43}m_{54}m_{35} - m_{43}m_{34}m_{55} + m_{53}m_{34}m_{45} - m_{53}m_{35}m_{44}$$
$$a_1 = m_{53}m_{55} - m_{33}m_{44} - m_{33}m_{55} - m_{44}m_{55} + m_{45}m_{54} + m_{43}m_{34}$$
$$a_2 = m_{33} + m_{44} + m_{55}$$

From equation (6), we get$\lambda_1 = m_{11}$ and $\lambda_2 = m_{22}$. The endemic equilibrium point is stable if $m_{11} \le 0$ and $m_{22} \le 0$. Next, consider the characteristic polynomial in equation (6) below,

$$\lambda^3 - a_2\lambda^2 - a_1\lambda - a_0 = 0. \tag{7}$$

The stability around the rumor endemic equilibrium point depends on the elements of the first column in the Routh criteria table.

Table 2. Routh Criteria

$\boldsymbol{\lambda^3}$	1	$\boldsymbol{-a_1}$
$\boldsymbol{\lambda^2}$	$-a_2$	$-a_0$
$\boldsymbol{\lambda^1}$	b_1	0
$\boldsymbol{\lambda^0}$	c_1	0

where,

$$b_1 = -\frac{\begin{vmatrix} 1 & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}}{a_{n-1}} = -\frac{\begin{vmatrix} 1 & -a_1 \\ -a_2 & -a_0 \end{vmatrix}}{-a_2} = -\frac{(-a_0 - a_1 a_2)}{-a_2} = \frac{a_0 + a_1 a_2}{-a_2}$$

$$c_1 = -\frac{\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix}}{b_1} = -\frac{\begin{vmatrix} -a_2 & -a_0 \\ \frac{(a_0+a_1a_2)}{-a_2} & 0 \end{vmatrix}}{\frac{(a_0+a_1a_2)}{-a_2}} = -\frac{0 - \left(-a_0 \frac{(a_0+a_1a_2)}{-a_2}\right)}{\frac{(a_0+a_1a_2)}{-a_2}} = -a_0$$

Theorem 2. [14] *All roots of the polynomial equation (7) have negative real parts if and only if the elements of the first column in Table 2 are non-zero and have the same sign.*

According to Theorem 1, the endemic equilibrium point is stable if $\frac{a_0+a_1a_2}{-a_2} > 0$, $a_0 < 0$ and $a_2 < 0$.

3.2.3. Basic Reproduction Number

Basic reproduction number is used to see the level of spread of disease in a population [15]. Basic reproduction number were determined using the Next Generation Matrix focused on groups of exposed and infected individuals [12] . The exposed and infected compartments are in the Exposed and Spreader groups as follows.

$$\begin{aligned} \frac{dE}{dt} &= PSB - k_1SE - k_2RE, \\ \frac{dS}{dt} &= PSA + HSC + k_1SE - k_3S(R+S+E) - S\eta_2. \end{aligned} \tag{8}$$

Let T be the matrix with entries describing the emergence of new infections and Σ be the matrix with entries describing the movement of individuals between groups, so we get T and Σ from the system (8), *i.e.*

$$T = \begin{bmatrix} 0 & PB \\ 0 & PA + HC \end{bmatrix}$$
$$\Sigma = \begin{bmatrix} -k_1S - k_2R & -k_1E \\ k_1S - k_3S & k_1E - k_3(R + 2S + E) + \eta_2 \end{bmatrix}$$

Next, substitute the rumor-free equilibrium point E_0 to get

$$T = \begin{bmatrix} 0 & xB \\ 0 & xA + nC \end{bmatrix}$$
$$\Sigma = \begin{bmatrix} -k_2(1 - x - n) & 0 \\ 0 & -k_3(1 - x - n) + \eta_2 \end{bmatrix}$$

and

$$-T\Sigma^{-1} = \begin{bmatrix} 0 & \frac{xB}{k_3(1-x-n)+\eta_2} \\ 0 & \frac{xA+nC}{k_3(1-x-n)+\eta_2} \end{bmatrix} \tag{9}$$

The largest eigenvalue of the matrix (9) is a threshold parameter, denoted by R_0

$$\begin{aligned} R_0 &= \frac{xA+nC}{k_3(1-x-n)+\eta_2} \\ &= \frac{k(\mu x+n)\gamma\alpha\beta}{k(1-x-n)\eta_1+\eta_2} \end{aligned} \tag{10}$$

The inequality (4) can be rewritten in the form $R_0 < 1$, so that the rumor-free equilibrium point is stable if $R_0 < 1$, which means that rumors do not spread more widely. On the other hand, if $R_0 > 1$, then the rumor-free equilibrium point is unstable which means that rumors continue to spread in the population.

3.3. Implementation of the Rumor Spreading Model on Actual Data

The actual data from Twitter for the #SahkanRUUPKS rumor obtained with Python using tweepy and API access (Application Programming Interface) is presented in Table 3.

Table 3. Rumor Actual Data #SahkanRUUPKS Based On Twitter

Twitter Data	
Tweet sumber ($\boldsymbol{T}$)	10
Reply tweet ($\boldsymbol{R}$)	1.603
Retweet ($\boldsymbol{RT}$)	31.147
Vulnerable Twitter users ($\boldsymbol{F}$)	42.052

Furthermore, the parameter values of the rumor spread model (1) are obtained from the actual Twitter data which can be seen in Table 4.

Table 4. Parameter Value

Parameter	Value	Parameter	Value
$\boldsymbol{\alpha}$	0.78	$\boldsymbol{\gamma}$	0.74
$\boldsymbol{\beta}$	0.57	$\boldsymbol{\mu}$	0.038
$\boldsymbol{\theta}$	0.049	$\boldsymbol{\phi}$	0.15
$\boldsymbol{\eta_1}$	0.74	$\boldsymbol{\eta_2}$	0.167
$\boldsymbol{k}$	2		

Substituting the above parameter values into the model (1), we get

$$\begin{aligned}
\frac{dP}{dt} &= -0.026PS - 0.17PS \\
\frac{dH}{dt} &= -0.68HS \\
\frac{dE}{dt} &= 0.17PS - 0.098SE - 0.30RE \\
\frac{dS}{dt} &= 0.026PS + 0.68HS + 0.098E - 1.5S(R+S+E) - 0.17S \\
\frac{dR}{dt} &= 1.5S(R+S+E) + 0.17S + 0.30RE
\end{aligned} \tag{11}$$

The rumor-free equilibrium point of model (10) is $E_0 = (0.283; 0.242;\ 0;\ 0;\ 0.475)$. The stability of the system of equations (10) around the rumor-free equilibrium point is determined by the eigenvalues of the Jacobian matrix. The eigenvalues of the Jacobian matrix $J(E_0)$ are $\lambda_1 = \lambda_2 = \lambda_3 = 0$, $\lambda_4 = -0.14$ dan $\lambda_5 = -0.72$, so it can be concluded that the rumor-free equilibrium point is stable. Next, basic reproduction number is

$$\begin{aligned}
R_0 &= \frac{k(\mu x + n)\gamma\alpha\beta}{k(1 - x - n)\eta_1 + \eta_2} \\
&= 0.19
\end{aligned}$$

the value of $R_0 < 1$ means that the number of spreaders does not increase and news about #SahkanRUUPKS will disappear and be forgotten or #SahkanRUUPKS will not spread in the population.

The equilibrium point of endemic rumor spread is $E_* = (0; 0; 0.31; -0.27; 0.087)$. Note that the density of individuals in the C^Sgroup has a negative sign it contradicts the assumption that the C^S group is a group of people who spread rumors the negative value of the individual density in the C^S group means that there are no groups of people who spread rumors. Furthermore, the stability of the system of equations (1) around the equilibrium point of endemic rumor spread is determined by the eigenvalues of the Jacobian matrix. The eigenvalues of the Jacobian matrix $J(E_*)$ are $\lambda_1 = 0.05292$ and $\lambda_2 = 0.1836$, so that the endemic equilibrium point for spreading rumors is unstable. The graph of the solution of the system of equations (10) can be seen in Figure 2 below.

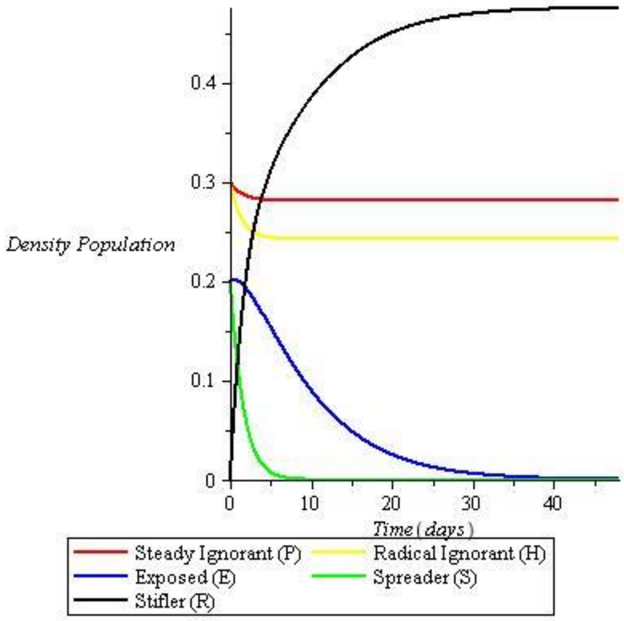

Figure 2. Graph of the Rumor Spreading Model Solution

Based on the solution graph in Figure 2, the model (10) is stable towards the rumor-free equilibrium point, $E_0 = (0.283; 0.242;\ 0;\ 0;\ 0.475)$. Furthermore, the interpretation of the rumor-free equilibrium points to the actual data obtained is as follows:

a. The density of individuals in the Steady Ignorant group decreased due to the movement of individuals from the Steady Ignorant group to the Stifler group. Individual density in the Steady Ignorant group decreased to 0.285 at the 2nd time, then stabilized at 0.283 at the 4th time. So it can be concluded that 28.3 percent of individuals in the Steady Ignorant group decided not to spread #SahkanRUUPKS after pondering and seeking confirmation about the news after 4 hours of the news being spread on Twitter.

b. The density of individuals in the Radical Ignorant group decreased due to the movement of individuals from the Radical Ignorant group to the Stifler group. Individual density in the Radical Ignorant group decreased at 0.245 at the 5th time, then stabilized at 0.242 at the 8th time. So it can be concluded that 24.2 percent of individuals in the Radical Ignorant group decided not to spread #SahkanRUUPKS after 8 hours the news was spread on Twitter.

c. The density of individuals in the Exposed group decreases and remains constant to a rumor-free point, note that the Exposed point drops to 0.0068 at the 29th time, then stabilizes towards 0 at the 42nd time meaning that the people in the Exposed group do not increase and #SahkanRUUPKS was forgotten after 42 hours the news spread on Twitter.

d. The density of individuals in the Spreaders group decreases and remains constant to a rumor-free equilibrium point. Note that the Spreader point drops to 0.00544 at the 29th time, then it stabilizes towards 0 at the 6th time meaning that the people in the Spreaders group do not increase and #SahkanRUUPKS was forgotten after 6 hours the news spread on Twitter.

e. The density of individuals in the Stifler group continued to rise towards the point of 0.473 at the 38th time, then it stabilized at the point of 0.475 at the 42nd time so that it can be concluded that 47.5 percent of individuals in the Stifler group did not spread #SahkaRUUPKS after 42 hours the news about #SahkanRUUPKS spread on twitter.

4. CONCLUSIONS

Based on the results of the discussion, it can be concluded that the rumor-free equilibrium point in the system of equations (1) is $E_0 = \left(x, n, 0,0, (1 - x - n)\right)$, with the threshold parameter is $\frac{k(\mu x+n)\gamma\alpha\beta}{k(1-x-n)\eta_1+\eta_2}$. Furthermore, the endemic equilibrium point for spreading rumors from the system of equations (1) is $E_* = \left(0,0, -\frac{\theta\eta_1 w}{\phi(\theta-\eta_1)} + \frac{\eta_1 w}{(\theta-\eta_1)} + \frac{\eta_2}{k(\theta-\eta_1)}, w, -\frac{\theta w}{\phi}\right)$.

The implementation of the rumor spreading model with actual data #SahkanRUUPKS via Twitter in Indonesia resulted in a rumor-free equilibrium point $E_0 = (0.283;\ 0.242;\ 0;\ 0;\ 0.475)$ and an endemic equilibrium point for spreading rumors $E_* = (0;\ 0;\ 0.31; -0.27;\ 0.087)$. The rumor-free equilibrium point is stable with the basic reproduction number $R_0 < 1$, meaning that the number of spreaders of #SahkanRUUPKS does not increase and that #SahkanRUUPKS disappears or is forgotten.

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