BAREKENG: Journal of Mathematics and Its Application
December 2022 Volume 16 Issue 4 Page 1167–1174
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol16iss4pp1167-1174

NONPARAMETRIC REGRESSION MODEL ESTIMATION WITH THE FOURIER SERIES THE FOURIER SERIES APPROACH AND ITS APPLICATION TO THE ACCUMULATIVE COVID-19 DATA IN INDONESIA

Muhammad Danil Pasarella[1*], Sifriyani[2], Fidia Deny Tisna Amijaya[3]

[1,2] *Statistical Study Program, Faculty of Mathematics and Natural Sciences, Mulawarman University*
[3] *Mathematics Study Program, Faculty of Mathematics and Natural Sciences, Mulawarman University*
Jl. Barong Tongkok No.04, Samarinda, 75123, Indonesia

Corresponding author's e-mail: [1] danielpasarella92@gmail.com*

Abstract

The nonparametric regression model is applied to regression curves for which the regression curve is unknown. Fourier series estimation is an approach in nonparametric regression, which has high flexibility and is able to adjust to the local nature of data effectively. The purpose of the research is to obtain an estimate of the nonparametric regression model with the Fourier series approach with optimal oscillation values and the model suitability of the positive case of Covid-19 in Indonesia. Research on modeling positive cases of Covid-19 in Indonesia using nonparametric regression with the best Fourier series approach is found in the third oscillation by having a minimum GCV of 78969281 with the best model criteria R^2 = 97.86%. The influencing factors are the percentage of active smokers, the number of health workers, the number of health service facilities, population density and the percentage of the poor population.

Keywords: *Covid-19, Fourier Series, Oscillation, Nonparametric Regression.*

Article info:

Submitted: 23rd April 2022 *Accepted: 10th October 2022*

How to cite this article:
M. D. Pasarella, Sifriyani and F. D. T. Amijaya, "NONPARAMETRIK REGRESSION MODEL ESTIMATION WITH THE FOURIER SERIES THE FOURIER SERIES APPROACH AND ITS APPLICATION TO THE ACCUMULATIVE COVID-19 DATA IN INDONESIA", *BAREKENG: J. Math. & App.*, vol. 16, iss. 4, pp. 1167-1174, Dec., 2022.

1. INTRODUCTIONS

At the end of 2019, precisely in December, the world was shocked by a suspected case of pneumonia of unknown origin. The case originated in Wuhan, China. China identified pneumonia as a new type of coronavirus on Jan. 7, 2020. The Wuhan Municipal Health Commission issued a statement "Urgent notice for the treatment of pneumonia of unknown cause" [1].

In Indonesia, the Covid-19 pandemic was detected in early March, precisely on March 2, 2020. It is called Case 1 and Case 2 because, on March 2, 2020, two people have detected positive Covid-19 test results. Of the two cases, more and more positive cases of Covid-19 are recorded every day. [2]. Since the number of additional cases per day cannot be estimated, it is impossible to predict the number of customers such as services, facilities, and medical staff.

Some ways to reduce and stop the chain of transmission of this pandemic are to stay at home, not gather in large numbers, diligently wash your hands before and after activities, and always use masks. In Indonesia, the number of Covid-19 cases continues to increase and it is not yet known when the Covid-19 pandemic outbreak will end.

One of the statistical methods used for modeling is the regression method. The regression method is a statistical method to find out the relationship between response variables and predictor variables. Regression approaches can be done with three approaches, namely parametric, semiparametric, and nonparametric approaches.[3].

According to Eubank (1988), nonparametric regression is a very flexible regression in modeling data patterns so that the subjectivity of researchers can be minimized. Nonparametric regression can find the form of regression curve patterns that are not yet known the shape. This capability is supported by the presence of parameters in each type of nonparametric regression method that estimates of regression curves more flexible. [4].

Unlike the parametric regression approach, in nonparametric regression, the shape of the regression curve is assumed to be unknown. Nonparametric regression curves are only assumed to be smooth in the sense that they are contained within a given function space. The data is expected to look for its form of estimation, without being influenced by the subjective factors of the research designer. Thus, nonparametric regression approaches have high flexibility. Nonparametric regression approaches have been widely developed including Kernel, Spline, K-Nearest Neighbor, Fourier series Estimator, Histogram, MARS, Orthogonal Series, Wavelets, and Neural Network [5].

According to Bilodeau (1992), the Fourier series is one of the alternative estimators that are widely studied and developed by researchers. Nonparametric regression of the Fourier Series is well used to explain curves that indicate sine and cosine waves. Data that is suitable for analysis using the Fourier series is when the data investigated is not known patterns and there is a tendency for repetitive data patterns [6]. One of the advantages of the nonparametric regression approach by using the Fourier series is that it can overcome data that has trigonometric distribution, in this case, the sinuses and cosines. Data patterns that correspond to the Fourier series approach are repetitive, i.e. repetition of dependent variable values for different independent variables.

So based on some of the things that have been mentioned earlier, the author is interested in conducting scientific research with the title "Estimation of Nonparametric Regression Models with Fourier Series Approach and Its Application to Covid-19 Accumulative Data in Indonesia".

2. RESEARCH METHODS

This study used a nonparametric regression method with the Fourier series approach to the accumulative data sample of positive cases of Covid-19 in Indonesia in 2020 along with several factors that are suspected to have an effect. Purposive sampling techniques are used for data retrieval with considerations relating to the availability of the latest data.

2.1 Regression Analysis

Regression analysis is a data analysis used to determine the relationship between a response variable (y) and a predictor variable (x) [7]. Regression analysis also has the ability to describe the scattering of points around curves. In addition to drawing the radiating around the curve, regression analysis can also be used for prediction. [8].

According to Budiantara (2009), regression analysis in estimating regression curves there are three approaches, namely parametric regression approach, nonparametric regression, and semiparametric regression. In the parametric regression approach, there is a very strong and rigid assumption that the shape of the regression curve is known for example linear, quadratic, cubic, polynomial p degree, exponent, and others. [9].

Nonparametric regression is a regression that is very flexible in modeling data patterns so that the subjectivity of researchers can be minimized. Nonparametric regression is used to determine the pattern of the relationship between the response variable (y) and the predictor variable (x) of which the curve shape is unknown. The nonparametric regression model [10] [11] is as follows:

$$y_i = f(x_i) + \varepsilon_i, i = 1,2, \dots, n \tag{1}$$

The regression function f is assumed to be smooth thus guaranteeing the flexibility to estimate its regression function. ε_i is an error that distributes normally with mean 0 and variance σ^2 [8].

In contrast to the parametric regression approach, in nonparametric regression, the shape of the regression curve is assumed to be unknown. Nonparametric regression curves are only assumed to be smooth in the sense that they are contained within a given function space. The data is expected to look for its own form of estimation, without being influenced by the subjective factors of the research designer. Thus, nonparametric regression approaches have high flexibility [9].

2.2 Fourier Series Nonparametric Regression

There are several parameter assessment approach models in nonparametric regression used to estimate curve f_j, namely: Kernel, Spline, Wavelet, Fourier series, and local polynomial. If you look at in detail the estimation procedure in nonparametric regression, it is clearly seen that the approach is obtained by approaching the regression curve f_j with certain functions and further optimization is carried out to obtain estimates [5].

According to Tripena & Budiantara (2007) and Bilodeau (1992), univariable Fourier Series Estimators in nonparametric regression is generally used when the data investigated patterns are unknown and there is a tendency to repeat patterns. Given univariable nonparametric regression model $y_i = f(x_i) + \varepsilon_i, i = 1,2, \dots, n$. Regression curve shape $f(x)$ assumed to be unknown and contained within a continuous function space $C(0, \pi)$ random ε_i assumed to be an independent normal distribution with mean 0 and variance σ^2. Because $f(x)$ Continuous at intervals $(0, \pi)$ then it can be approached by the Fourier series function $F(x)$, with:

$$F(x) = bx + \frac{1}{2}a_0 \sum_{k=1}^{K} a_k \cos kx \tag{2}$$

where $b, a_0, a_k, k = 1,2, \dots, K$ is model parameters [9].

Equation (2) can also be written in the following form of Equation (3) if the nonparametric regression function is univariable $f(x_i)$ approached by the Fourier series function $F(x)$:

$$f(x_i) = bx_i + \frac{1}{2}a_0 \sum_{k=1}^{K} a_k \cos kx_i \tag{3}$$

so that from Equation (3) obtained:

$$y_i = bx_i + \frac{1}{2}a_0 \sum_{k=1}^{K} a_k \cos kx_i + \varepsilon_i; \varepsilon_i \sim N(\mathbf{0}, \sigma^2\mathbf{I}) \quad (4)$$

Equation (4) can be expressed in the following matrix form

$$\boldsymbol{Y} = \boldsymbol{X}[\boldsymbol{K}]\boldsymbol{\beta} + \boldsymbol{\varepsilon}; \boldsymbol{\varepsilon} \sim N(\mathbf{0}, \sigma^2\mathbf{I}) \quad (5)$$

By using the OLS (Ordinary Least Square) method, the error is minimized, so that Equation (6) is obtained.

$$\mathbb{Q}(\boldsymbol{\beta}) = (\boldsymbol{Y} - \boldsymbol{X}(\boldsymbol{K})\boldsymbol{\beta})'(\boldsymbol{Y} - \boldsymbol{X}(\boldsymbol{K})\boldsymbol{\beta}) \quad (6)$$

then obtained the form of the equation multiplication result from Equation (6) [8]:

$$\mathbb{Q}(\boldsymbol{\beta}) = \boldsymbol{Y}'\boldsymbol{Y} - \mathbf{2}\boldsymbol{\beta}'\boldsymbol{X}'(\boldsymbol{K})\boldsymbol{Y} + \boldsymbol{\beta}'\boldsymbol{X}'(\boldsymbol{K})\boldsymbol{X}(\boldsymbol{K}) \quad (7)$$

On the OLS estimation method, $\hat{\beta}$ was obtained by minimizing $\mathbb{Q}(\beta)$ by partially lowering $\mathbb{Q}(\beta)$ against vectors β, as well as a little elaboration and considering the matrix $\boldsymbol{X}(K)$ nonsingular (matrix with full rank) then obtained estimator β as follows:

$$\hat{\boldsymbol{\beta}}(\boldsymbol{K}) = \left(\boldsymbol{X}'(\boldsymbol{K})\boldsymbol{X}(\boldsymbol{K})\right)^{-\mathbf{1}}\boldsymbol{X}'(\boldsymbol{K})\boldsymbol{Y} \quad (8)$$

with estimator for regression curve f_j given by:

$$\hat{f}_j(x_{ji}) = \hat{b}_j(x_{ji}) + \frac{1}{2}\hat{a}_{0j} + \sum_{k=1}^{K} \hat{a}_{kj} \cos kx_{ji} \quad (9)$$

so estimates for regression curves $\mu(x_{1i}, x_{2i}, \dots, x_{pi})$ given by [17]:

$$\mu(x_{1i}, x_{2i}, \dots, x_{pi}) = \sum_{j=1}^{p} \hat{f}_j(x_{ji}) \quad (10)$$

Meanwhile, for the good of the nonparametric regression model with the Fourier series approach is measured based on the selection of optimal oscillation parameters that provide the smallest Mean Square Error (MSE) value. Optimal oscillation parameters are selected based on generalized cross validation (GCV) formulation.

Generalized Cross Validation (GCV) is one of the criteria for determining the optimal K value. [12]. The optimal K value is the oscillation parameter value of the model as a representation of bandwidth. The determination of optimal K will result in a high R^2 value.

2.3 Bandwidth Selection and Best Model

To obtain the optimal K can be seen from the minimum GCV scores. GCV values can be formulated as in the following Equation (11) [13] [14]:

$$\boldsymbol{GCV}(\boldsymbol{K}) = \frac{MSE(K)}{\left(n^{-1}trace(\mathbf{I} - \mathbf{A}(\mathrm{K}))\right)^2} \quad (11)$$

with:

$$\boldsymbol{MSE}(\boldsymbol{K}) = n^{-1}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (12)$$

where K is the oscillation parameter, $\boldsymbol{I}$ is an identity-sized matrix $n \times n$, and $\mathbf{A}(\mathrm{K})$ is a hat matrix containing the oscillation parameter K.

To find out the quality of a model, we can use the coefficient of determination (R^2). The R^2 value indicates the model's ability to explain data variability. The higher the R^2 value, the better the quality of a model. The value of R^2 is obtained by the formula:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2} \tag{13}$$

with y_i as a response variable, $\hat{y}_i$ as the alleged result of y_i, and $\bar{y}_i$ as the average result value of y_i [15].

2.4 Research Variables

The research variable consists of the response variable (y) and the predictor variable (x). The response variable for this study is the Number of Positive Cases of Covid-19. The predictor variable of this study is the Number of Active Smokers (x_1) Number of Health Workers (x_2), Number of Health Care Facilities (x_3), Population Density (x_4), and Percentage of the Poor (x_5). Variable notation, data types, variable operational definitions, and variable units can be seen in Table 1.

Table 1. Research Variables

Notation	Operational Definition	Data Source
y	People who tested positive for the Covid-19 virus with evidenced by the results of the Reverse Transcriptase-Polymerase Chain Reaction Laboratory examination	Covid19.go.id
x_1	Percentage of people who intentionally smoke rolls or rolls of tobacco that are usually wrapped in paper	bps.go.id
x_2	The number of health workers who carry out their duties as health workers who provide services during the Covid-19 pandemic	Kemenkes RI
x_3	a tool and/or place used to organize health care efforts, whether promotive, preventive, curative or rehabilitative conducted by the Government, local government, and/or the community.	Kemenkes RI
x_4	Number of inhabitants per unit area	bps.go.id
x_5	Percentage of people living below the poverty line	bps.go.id

2.5 Stages of Analysis

The stages of data analysis are as follows.

1. Perform descriptive statistical analysis of research data in the form of descriptive statistics on variable data using Microsoft Office Excel 365 and Software R 4.1.1;
2. Model response variables using fourier series nonparametric regression with varying amounts of oscillations;
3. Calculates the MSE value of each Fourier series nonparametric regression model;
4. Calculates the GCV value of each Fourier series nonparametric regression model;
5. Determine the optimal number of oscillations based on minimum GCV values;
6. Perform nonparametric regression modeling of the fourier series using the optimal number of oscillations;
7. Calculates the coefficient of determination value of the Fourier series nonparametric regression model that has the optimal oscillation value;
8. Comparing the actual data value with the value of the prediction data;
9. Draw conclusions and provide explanations of the results of the conclusions.

3. RESULTS AND DISCUSSION

In research with nonparametric regression methods of the Fourier series on accumulative data on positive cases of Covid-19 in Indonesia in 2020. The research data used consists of response variables, namely the number of positive cases of Covid-19. The predictor variables of this study are the percentage of the active smoker population, the number of health workers, the number of healthcare facilities, population density, and the percentage of the poor population. Then the descriptive statistical analysis is carried out in order to see the description of the data spread. After descriptive statistical analysis, modeling was carried out using nonparametric regression with the Fourier series approach using 3 oscillations so that 3 nonparametric regression models were obtained with the Fourier series approach.

3.1 Descriptive Statistical Analysis

The descriptive statistical analysis aims to describe the state of the data such as the average and standard deviation of the data. The results of the analysis that has been done can be seen in Table 2.

Table 2. The Descriptive Statistical Analysis

Variable	Minimum	Average	Maximum	Standard Deviation
Y	1.941	21.858,76	183.735	36.267,87
x_1	18,58	28,01	34,07	3,06
x_2	475	3.637,97	17.032	4.696,25
x_3	115	965,21	5.625	1.526,39
x_4	9,3	739,27	15.906,52	2.708,86
x_5	4,45	10,81	26,8	5,41

Based on Table 2 obtained the following results:

1. The average value for positive cases of Covid-19 in each province in Indonesia is 21,858.76 cases with a standard deviation of 36,267.87 which means that the data spreads from the data center;
2. The average value for active smokers in each province in Indonesia is 28.01% with a standard deviation of 3.06 which means that the data is gathered around the data center;
3. The average value for health workers in each province in Indonesia is 3637.97 or 3638 people with a standard deviation of 4696.25 which means that the data spreads from the data center;
4. The average value for health care facilities in each province in Indonesia is 965.21unit with a standard deviation of 1526.39 which means that the data is spread from the data center;
5. The average value for population density in each province in Indonesia is 739.27 people/km2 with a standard deviation of 2708.86 which means that the data spreads from the data center;
6. The average value for the poor in each province in Indonesia is 10.81% with a standard deviation of 5.41 which means that the data is gathered around the data center.

3.2 Determining the Optimal K Oscillation Value

In nonparametric regression of the Fourier series, it largely depends on the oscillation parameter (K). The oscillation parameter (K) is the sum of the oscillations of the cosine waves in the model. The larger K value will result in the model getting more complex and the oscillation of the estimation curve will be tighter and follow the actual data pattern so that the bias is smaller and the variant is getting bigger. In this study, the authors limited the use of oscillation parameter values only by using oscillation parameter values (K) which are as much as 1, 2, and 3.

To get the best Fourier series estimator in nonparametric regression with this Fourier series approach, it is necessary to select the optimal oscillation parameter (K). The selection of optimal oscillation parameters (K) is carried out using the GCV method whose formula can be seen in Equation 11. The optimal K value selected is the K value that produces the minimum GCV value. The analysis results for the value K are given in Table 3 below:

Table. 3 Determining the Optimal Oscillation Parameter Value (K) Based on the GCV Method

Oscillation Parameters (K)	GCV Value	R^2 Value
1	381.583.809	0,9741
2	153.877.550	0,9765
3	78.969.281	0,9786

The results obtained from Table 3 will be used to find the best model based on the Minimum GCV score and high determination coefficient value.

3.3 Best Model Selection Based on Minimum GCV

Based on Table 3, this $K = 3$ value returns a GCV value of $78{,}969{,}281$. The nonparametric regression of the Fourier series with $K = 3$ results in an R^2 value of 97.86%. That is, the diversity of response values has been able to be explained by predictor variables of 97.86%. Based on the model on the nonparametric regression equation with the Fourier series approach with 5 predictor variables with 3 oscillations based on Equation (10), the estimated form of the Fourier series nonparametric regression model for the accumulative number model of positive cases of Covid-19 in Indonesia is as follows:

$$\hat{y}_i = 18973{,}80 - 711{,}97x_{1i} + 2128{,}36\cos x_{1i} - 634{,}74\cos 2x_{1i} - 715{,}96\cos 3x_{1i} + 2{,}28x_{2i} + 332{,}76\cos x_{2i} + 1335{,}92\cos 2x_{2i} + 180{,}29\cos 3x_{2i} + 8{,}59x_{3i} + 610{,}48\cos x_{3i} + 3372{,}37\cos 2x_{3i} + 274{,}91\cos 3x_{3i} + 6{,}80x_{4i} + 381{,}26\cos x_{4i} - 1249{,}82\cos 2x_{4i} - 116{,}68\cos 3x_{4i} + 23{,}93x_{5i} + 2183{,}92\cos x_{5i} + 324{,}83\cos 2x_{5i} - 3991{,}92\cos 3x_{5i}$$

Based on Table 3 it can be seen that the optimal K value is 3. This is because the value $K = 3$ produces a GCV value that is smaller than the values $K = 1$ and $K = 2$. So that the equation model with $K = 3$ is obtained as the best model.

The number of K values can actually be more than 3, but here the author limits only to 3 oscillations because it will have an impact on the number of parameters that will be estimated and the resulting model is not parsimony (simple).

Nonparametric regression models with the Fourier series approach can be used to predict a value of a response variable based on the predictor variable. From the model of the equation can be presented a visual comparison between the value y with $\hat{y}$ of oscillation $K = 3$ as follows:

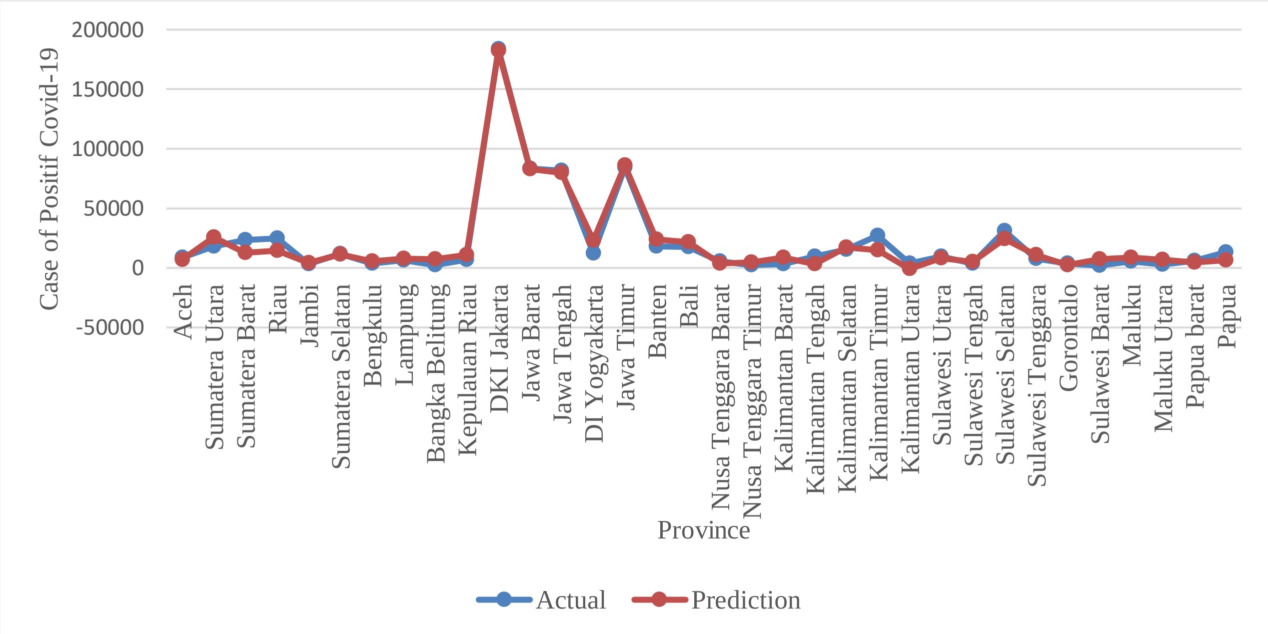

Figure 1. Comparison of actual data with predictive data

Based on Figure 1, it is known that the results of predictions using nonparametric regression models with a Fourier series approach tend to follow actual data patterns. The use of nonparametric regression models with a Fourier series approach is adjusted to the variables contained in the model both response and predictor. So the addition or reduction of 1 point on each predictor variable will greatly affect the increase or decrease in the number of positive cases of Covid-19 throughout Indonesia. So that from the model built this can be used as a consideration to make decisions in the future.

4. CONCLUSION

Based on the results and discussions of the research that has been conducted, the best Fourier series nonparametric regression model is obtained based on Covid-19 data in Indonesia as follows.

$$\hat{y}_i = 18.973{,}80 - 711{,}97x_{1i} + 2.128{,}36\cos x_{1i} - 634{,}74\cos 2x_{1i} - 715{,}96\cos 3x_{1i} + 2{,}28x_{2i} + 332{,}76\cos x_{2i} + 1.335{,}92\cos 2x_{2i} + 180{,}29\cos 3x_{2i} + 8{,}59x_{3i} + 610{,}48\cos x_{3i} + 3.372{,}37\cos 2x_{3i} +$$

$274{,}91 \cos 3x_{3i} + 6{,}80x_{4i} + 381{,}26 \cos x_{4i} - 1.249{,}82 \cos 2x_{4i} - 116{,}68 \cos 3x_{4i} + 23{,}93x_{5i} + 2183{,}92 \cos x_{5i} + 324{,}83 \cos 2x_{5i} - 3.991{,}92 \cos 3x_{5i}$

`In addition, based on modeling that has been done using nonparametric regression with the Fourier series approach in Covid-19 cases in Indonesia, it can be concluded that of the 3 oscillations carried out, the optimal oscillation obtained is with 3 oscillations with a minimum GCV value of 78,969,281 and has a determination coefficient value of 0.9786 which means that the diversity of response values has been able to be explained by predictor variables of 97.86%. In addition, from research it was obtained that the actual value is not much different from the predicted value so that the model has been considered in accordance with the actual circumstances.

REFERENCES

[1] F. R. Yamali and R. N. Putri, "Dampak Covid-19 terhadap Ekonomi Indonesia," *Ekonomis: Journal of Economics and Buisiness,* pp. Vol. 4 No. 2 Pages 384-388, 2020.

[2] F. D. Arianto and N. , "Prediksi Kasus COVID-19 di Indonesia Menggunakan Metode Backpropagation dan Fuzzy Tsukamoto," *Jurnal Teknologi Invormasi,* pp. Vol.4, No.1, 2020.

[3] M. F. Qudratullah, Analisis Regresi Terapan: Teori, Contoh Kasus, dan Aplikasi dengan SPSS, Yogyakarta: Penerbit ANDI, 2013.

[4] A. T. R. Dani and N. Y. Adrianingsih, "Pemodelan Regresi Nonparametrik dengan Estimator Spline Truncated dan Deret Fourier," *Jambura Journal of Mathematics,* Vols. Vol. 3, No.1, pp. 26-36, 2021.

[5] I. W. Sudiarsa, Estimator Gabungan Deret Fourier dan Spline Truncated dalam Regresi Nonparametrik Multivariabel, Surabaya: Institut Teknologi Sepuluh November, 2018.

[6] L. R. Khairunnisa, A. Prahutama and R. Santoso, "Pemodelan Regresi Semiparametrik dengan Pendekatan Deret Fourier (Studi Kasus: Pengaruh Indeks Dow Jones dan BI Rate terhadap Indeks Harga Saham Gabungan)," *Jurnal Gaussian,* pp. Volume 9, Nomor 1, hal 50-63, 2020.

[7] S. Riskiyah, Penerapan Regresi Nonparametrik Spline Truncated untuk Mengetahui Faktor-Faktor yang Mempengaruhi Hasil Tangkapan Ikan Layang di Kabupaten Pamekasan, Pamekasan: Fakultas Matematika dan Ilmu Pengetahuan Alam Universitas Islam Madura, 2018.

[8] A. Sholiha, K. dan M. F. F. Mardianto, "Estimator Deret Fourier dalam Regresi Nonparametrik dengan Pembobot untuk Perencanaan Penjualan Camilan Khas Madura," *Zeta-Math Journal,* p. Volume 4 No.1, 2018.

[9] N. P. A. M. Mariati, Pemodelan Regresi Deret Fourier dan Spline Truncated dalam Regresi Multivariabel (Aplikasi: Data Kemiskinan di Provinsi Papua), Surabaya: Institut Teknologi Sepuluh November, 2015.

[10] Sifriyani, Haryatmi, I. N. Budiantara and Gunardi, "Geographically Weighted Regression With Spline Approach," *Far East Journal of Mathematical Sciences,* vol. 101(6), pp. 1183-1196, 2017.

[11] Sifriyani, S. H. Kartiko, I. N. Budiantara and Gunardi, "Development of Nonparametric Geographically Weighted Regression Using Truncated Spline Approach," *Songklanakarin Journal of Science and Technology,* vol. 40(4), pp. 909-920, 2018.

[12] M. Mardianto, Kumpulan Karya Ilmiyah dalam Bentuk Artikel sebagai Bahan Penilaian Mata Kuliah Regresi Nonparametrik dan Literature Review., Yogyakarta: Fakultas Matematika dan Ilmu Pengetahuan Alam Universitas Gajah Mada, 2017.

[13] S. Sifriyani, I. N. Budiantara, S. H. Kartiko and Gunardi, "A New Method of Hypotesis Test for Truncated Spline Nonparametric Regression Influenced by Spatial Heterogeity and Aplication," *Abstract and Applied Analysis,* vol. 2018, p. https://doi.org/10.1155/2018/9769150, 2018.

[14] S. Sifriyani, M. Rasjid, D. Rosadi, S. Anwar, R. D. Wahyuni and S. Jalaluddin, "Spatial-Temporal Epidemiology of Covid-19 Using a Geographically and Temporally Weighted Regression Model," *Symmetry,* vol. 14(742), p. https://doi.org/10.3390/sym14040742, 2022.

[15] I. R. N. Wisiono, A. I. Nurwahidah, Y. Andriyana and N. Sunengsih, "Regresi Nonparametrik dengan Pendekatan Deret Fourier pada Data Debit Air Sungai Citarum," *Jurnal Matematika "Mantik",* p. Vol.04 No. 02, 2018.